A Hybrid Heuristic Algorithm to Solve Capacitated Location-routing Problem With Fuzzy Demands

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Abstract

In this paper, the capacitated location-routing problem with fuzzy demands (CLRP-FD) is considered. The CLRP-FD is composed of two well-known problems: facility location problem and vehicle routing problem. The problem has many real-life applications of which some have been addressed in the literature such as management of hazardous wastes and food and drink distribution. In CLRP-FD, a set of customers with fuzzy demands should be supplied by a fleet of vehicles that start and end their tours at a single depot. Moreover, the vehicles and the depots have a limited capacity. To model this problem, a fuzzy chance-constrained programming is designed based on fuzzy credibility theory. To solve the CLRP-FD, a hybrid heuristic algorithm (HHA) including two main phases is proposed. In the first phase, an initial population of solutions is generated by the greedy clustering method (GCM) obtained from the literature of the problem, while in the second phase, a genetic algorithm is applied for further improvement of the solutions of first phase. While the first phase of the HHA consists of four steps, the second phase includes two main steps. To achieve the best value of the major parameter of the model, named dispatcher preference index, and to analyze its influence on the changes of the final solution, numerical experiments with different sizes on the number of customers and candidate depots are carried out. The computational results show that the HHA is efficient so that it has improved all solutions that obtained from the GCM. Finally, performance of the proposed model to the similar model exists in the literature is evaluated by several standard test problems of the CLRP.

Keywords : Capacitated location-routing problem; Fuzzy demand; Credibility theory; Stochastic simulation; Fuzzy-chance constrained programming; Genetic algorithm.

1 Introduction

$\mathbf{E}$ver increasing demand of customers to receive their desired products with less waiting time

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and competitive prices, make the logistics as the main problem in supply chain management. In recent years, efficient, reliable, and flexible decisions on location of depots and vehicle routings are of vital importance to managers [23, 57]. Many researchers indicated that if the routes are ignored while locating the depots, the costs of distribution systems might be immoderate [22, 42]. At the first attempt, Salhi and Rand [44] showed that solving the location problem without route consideration may lead to a sub-optimal solution. The location-routing problem (LRP) overcomes

this drawback through making the location and routing decisions, simultaneously. The LRP is defined as a facility location problem (FLP) that solves the vehicle routing problem (VRP), simultaneously [26, 47]. Since both problems belong to the class of NP-hard problem, the LRP is also an NP-hard problem [4, 6, 60]. The LRP is applicable for a wide variety of fields such as food and drink distribution, newspapers delivery, waste collection, bill delivery, military applications, parcel delivery and various consumer goods distribution, etc. [29, 45]. In the capacitated location-routing problem (CLRP), the problem is limited with the vehicles and the depots' capacities to supply the customers. Furthermore, the customers must only be supplied by a single vehicle which means that the vehicle meets every customer in a tour only once. A homogenous fleet of vehicles transports the products with specific capacity from depots to the customers and returns there as soon as finishing the entire tour. The objectives in the CLRP are to determine the location of depots and a set of customers to be assigned to each depot as well as the distribution routes [9, 32, 38]. Since the CLRP is an NP-hard problem, some authors have employed approximation heuristic algorithms for solving that [30, 54]. In this kind of problems, the solution time increases exponentially as with an increase in the size of the problem, while an exact algorithm is applied to solve them. For this reason, most papers in the field of CLRP have only focused on new solution methods that are often based on heuristic or meta-heuristic approaches [36]. Some reviews on solution methods of the CLRP exist in literature that can be found in [12, 13, 34, 43].

Nagy and Salhi [34] classified the heuristics into four different types as follows: sequential, clustering-based, iterative, and hierarchical. Sequential methods solve the location problem by minimizing the sum of facility to customer distance and the routing problem based on the selected depots sequentially. Clustering-based methods partition the customers into clusters and then find a depot for each cluster [46, 4]. The VRP is then solved for each cluster. Iterative methods decompose the LRP into two sub-problems. Then, sub-problems are solved iteratively by feeding information from one sub-problem to the other [53, 40, 13]. Hierarchical

methods consider the location problem as the main problem and the VRP as a subordinated problem [33, 1].

Many heuristics that hybrid two different heuristic approaches are proposed in the literature of the LRP. Since this study uses a two-phase approach to solve the problem, the similar works are summarized as follows:

Tuzun and Burke [50] proposed a two-phase tabu search (TS) approach for the LRP. One phase seeks a good facility configuration while the other one obtains a good routing for this configuration. Wu et al. [53] presented a combined TS and simulated annealing (SA) decomposition approach to solve the multi-depot location routing problem with multiple fleet types and limited number of vehicles for each vehicle type. Lin et al. [27] developed a meta-heuristic approach based on threshold accepting (TA) and SA to assist in making decisions of facility location, vehicle routing and loading decision for bill delivery services in Hong Kong. Albareda-Sambola et al. [1] proposed another two-phase TS heuristic for the LRP which incurs not capacity constraints on vehicles. Wang et al. [52] proposed a two-phase hybrid heuristic which decomposes the LRP into location–allocation problem and vehicle routing problem. In the location phase, the TS was applied to obtain the configuration of facility locations. For each selected facility location, a vehicle routing problem was solved by ant colony optimization (ACO) in the routing phase. Bouhafs et al. [7] proposed a hybrid algorithm which combined the SA and ant colony system (ACS) to solve the CLRP. A good configuration of facilities was first found by the SA, and then the ACS was applied to construct the routings based on the configuration. These two ACO-related heuristics construct the routing problem and feedback the information for the facility selection phase.

Prins and his coworkers conducted different heuristic methods to the LRP [38]. They combined greedy randomized adaptive search procedures (GRASP) and path relinking to develop a two-phase algorithm for the CLRP. In the first phase, the GRASP and a learning process were implemented to select depots. The second phase was to generate new solutions using a path relinking. Later, Prins et al. [39] presented a memetic algorithm with population management (MA) to solve the same problem. Prins et al.

[40] proposed a cooperative approach, which combines the Lagrangean relaxation and granular tabu search (GTS), to solve the CLRP. The algorithm alternates between a location sub-problem, solved by Lagrangean relaxation, and a multi-depot VRP, solved by the GTS. Duhamel et al. [13] presented a GRASP with evolutionary location search (GRASP-ELS) approach for the CLRP. Barreto et al. [4] integrated several hierarchical and non-hierarchical clustering techniques in a sequential heuristic algorithm. Marinakis and Marinaki [30] developed a hybrid algorithm, which combined the particle swarm optimization (PSO), multiple phase neighborhood search-greedy randomized adaptive search procedure (MPNS-GRASP), the expanding neighborhood search (ENS) and path relinking, to solve the LRP. Yu et al. [54] proposed a simulated annealing algorithm to solve the LRP. The LRP is generally considered as a deterministic case in the literature. A few researches have addressed fuzzy versions of the LRP [12]. Recently, fuzzy logic has been used to model many different problems. The need to use fuzzy logic in problems arises whenever there are some vague or uncertain parameters. Credibility theory has already been used in many problems with fuzzy parameters, in parallel with some meta-heuristics. In the CLRP, some papers have been done with fuzzy variables and credibility theory so far. The work of Zarandi et al. [58] was the first attempt to model the CLRP with fuzzy variables, using credibility theory. They investigated a CLRP in which the travel time between every pair of nodes was a fuzzy variable. A simulation-embedded simulated annealing (SA) procedure was proposed in order to solve the problem. They tested the proposed method using standard test problems of the CLRP, and the results showed that their method was robust and could be used in real-world applications. In the second work, Fazel Zarandi et al. [57] considered the LRP with time windows under uncertainty. It was assumed that demands of customers and travel times are fuzzy variables. In their work, a fuzzy chance-constrained programming model was designed using credibility theory and a simulation-embedded SA algorithm was presented in order to solve the problem. To initialize solutions of SA, a heuristic method based on fuzzy c-means clustering with Mahalanobis distance and a sweep method were employed. They attested that the proposed solution approach was effective and robust. In third work, Zare Mehrjerdi and Nadizadeh [59] considered the CLRP with fuzzy demands. They modeled the problem with a fuzzy chance-constrained programming based on the fuzzy credibility theory. To solve this problem, a greedy clustering method (GCM) including the stochastic simulation was proposed. In the GCM, iterative and clustering-based approaches were used to solve the problem. To obtain the best value of the dispatcher preference index of the model numerical experiments were carried out. Consequently, to show the performance of their proposed method, associated results were compared with the lower bound of the solutions. In work of Nadizadeh and Hosseini Nasab [31], the dynamic capacitated location-routing problem with fuzzy demands (DCLRP-FD) was considered. In the DCLRP-FD, facility location problem and vehicle routing problem are solved on a time horizon. Decisions concerning facility locations are permitted to be made only in the first time period of the planning horizon but, the routing decisions may be changed in each time period. It was assumed that the demands of customers were fuzzy variables. To model the DCLRP-FD, a fuzzy chance-constrained programming was designed based upon the fuzzy credibility theory. To solve this problem, a hybrid heuristic algorithm (HHA) with four phases including the stochastic simulation and a local search method were proposed. The efficiency of the HHA was demonstrated via comparing with the lower bound of solutions and by using a standard benchmark set of test problems. The numerical examples showed that the proposed algorithm was robust and could be used in real world problems. In this paper, the CLRP with fuzzy demands (CLRP-FD) is considered. Since the information about demand of each customer is often not precise enough, customer demand is assumed as a fuzzy number. As an example, based on business experience, it can be concluded that demand of a customer is "around 50 units", usually "between 20 and 60 units", etc. For this reason, there is not enough data to be used to fit a probability distribution on the demand of customers in most real applications. On the other hand, based on the expert's judgment, one can easily estimate the demand of customers. Therefore, while using probability theory is cumbersome and

costly, fuzzy logic is worthwhile in these problems. As a result, the fuzzy set theory provides a convenient alternative framework for modeling real-world systems mathematically and offers several advantages to the use of heuristic approaches [3]. More precisely, some reasons for choosing fuzzy variables instead of probabilistic function in the customers' demands are as follows:

1. The stochastic-probabilistic theory requires significant knowledge about the statistical distribution of the uncertain parameters. In contrast, fuzzy theory provides an efficient way to model imprecision even when no historical information is available.

2. The use of stochastic-probabilistic theory involves extensive computation and requires complete knowledge on the statistical distribution of the uncertain time-varying parameters.

3. Fuzzy theory enables the use of fuzzy rules in heuristic algorithms.

4. Instead of optimizing the average behaviors, as in stochastic-probabilistic theory, fuzzy theory rather aims to find solutions where all constraints are satisfied to some extent with a sufficient level of confidence.

This paper contributes to the CLRP-FD in the following directions: (a) a fuzzy-chance constrained programming (FCCP) is proposed based on credibility theory to model the problem which is slightly different from what previously investigated by researchers; (b) a hybrid heuristic algorithm (HHA) is integrated with stochastic simulation to solve the problem; (c) the sensitivity analysis on the dispatcher preference index of the model is carried out; (d) the performance of the proposed model and the efficiency of the HHA are compared with both a commercial solver and a conventional approach by some numerical examples. It is of interest to notice that, the HHA consists of two main phases and uses two well-known algorithms, ant colony system and genetic algorithm, which their efficiency have shown in solving the location-routing problems [10, 48].

The remainder of this paper is organized as follows: In Section 2, some basic concepts of the fuzzy theory are given. Section 3 introduces the CLRP-FD and presents a FCCP model using the credibility theory. Details of the HHA to solve CLRP-FD are presented in Section 4. In Section 5, some numerical experiments are given to reveal the performance of the model and proposed HHA. In the final section, the conclusion remarks of the paper are presented.

2   Fuzzy credibility theory

The concept of the fuzzy set was initiated by Zadeh [55] via the membership function. Then it has been well developed and applied in a wide variety of real problems. In order to measure a fuzzy event, the term fuzzy variable was introduced by Kaufmann [24], and then Zadeh [56] proposed the possibility measure theory of fuzzy variable. Although, possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely necessary in both theory and practice. In order to define a self-dual measure, a modified form of the possibility theory called credibility theory was founded by Liu [28] and has been recently applied by many scholars all around the world. Before proceeding with development of a fuzzy model for CLRP with credibility theory, a brief introduction to basic concepts and definitions used in this paper is presented as follows:

Let Θ be a non-empty set, and P the power set of Θ. Each element in P is called an event, and ϕ is an empty set. In order to present an axiomatic definition of possibility, it is necessary to assign a number $\text{Pos}\{A\}$ to each event A, which indicates the possibility that A will occur. To ensure that the number $\text{Pos}\{A\}$ has certain mathematical properties, the following four axioms are approved [28]:

Axiom 2.1. $\text{Pos}\{\Theta\} = 1$;
Axiom 2.2. $\text{Pos}\{\phi\} = 0$;
Axiom 2.3. For each $A_i \in p(\Theta)$, $\text{Pos}\{ \bigvee A_i\} = \text{supremum}_i \, \text{Pos}\{A_i\}$;
Axiom 2.4. If Θ_i is a non-empty set, and the set function $\text{Pos}_i\{\}$; $i = 1, 2, \ldots, n$, satisfies above three axioms, and $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, then for each $A \in p(\Theta)$, $\text{Pos}\{A\} = \text{supremum}_{(\theta_1, \theta_2, \ldots, \theta n) \in A} \text{Pos}_1 \{\theta_1\} \bigwedge \text{Pos}_2\{\theta_2\} \bigwedge \cdots \text{Pos}_n\{\theta_n\}$.

The above four axioms form the basis of credibility measure theory such that all concepts of

credibility theory can be obtained from them [28].

Definition 2.1 *Let (Θ, P(Θ), Pos) be a possibility space, and A be a set in p(Θ), then the necessity measure of A is defined by Nec{A} = 1 − Pos{A^c}, that A^c is the complement of event A.*

Definition 2.2 *Let (Θ, P(Θ), Pos) be a possibility space, and A be a set in p(Θ), then the credibility measure of A is defined by Cr{A} = $\frac{1}{2}$ (Pos{A} + Nes{A}).*

Considering definition 2.2, the credibility of a fuzzy event is de?ned as the average of its possibility and necessity. The credibility measure is self-dual. A fuzzy event may fail even though its possibility achieves 1, and holds even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fails if its credibility is 0. In fact, the law of credibility plays a role similar to what plays the role of probability in measurement theory for ordinary sets [15].

Now let consider a triangular fuzzy variable $\tilde{d}$ = (d_1, d_2, d_3) for demand of a customer, $\tilde{d}$ is described by its left boundary d_1, and its right boundary d_3. In other word, the dispatcher or manager can subjectively estimate, based on his experience or available data, the demand of a customer will not be less than d_1 and greater than d_3. The value of d_2 corresponding to a grade of membership of 1 can also be determined by a subjective estimate. If the actual demand of a customer is considered by r, the possibility, necessity and credibility are easily obtained as follows [14]:

$$\text{Pos}\{\tilde{d} \geq r\} = \begin{cases} 1, & \text{if } r \leq d_2 \\ \frac{d_3-r}{d_3-d_2}, & \text{if } d_2 \leq r \leq d_3 \\ 0, & \text{if } r \geq d_3 \end{cases} \quad (2.1)$$

$$\text{Nec}\{\tilde{d} \geq r\} = \begin{cases} 1, & \text{if } r \leq d_1 \\ \frac{d_2-r}{d_2-d_1}, & \text{if } d_1 \leq r \leq d_2 \\ 0, & \text{if } r \geq d_2 \end{cases} \quad (2.2)$$

$$\text{Cr}\{\tilde{d} \geq r\} = \begin{cases} 1, & \text{if } r \leq d_1 \\ \frac{2d_2-d_1-r}{2(d_2-d_1)}, & \text{if } d_1 \leq r \leq d_2 \\ \frac{d_3-r}{2(d_3-d_2)}, & \text{if } d_2 \leq r \leq d_3 \\ 0. & \text{if } r \geq d_3 \end{cases} \quad (2.3)$$

3 Problem description and FCCP model for the CLRP-FD

In the CLRP, demand of each customer should be supplied by a single vehicle and the total demand of customers in each route must not exceed the capacity of the vehicle. The vehicles start and end to the same depot, and total demand of the allocated customers must be less than or equal to the capacity of the depot. The objective of the problem is to minimize the total cost including opening costs of depots and routing costs.

In the CLRP-FD, in addition to the above assumptions, the demand of each customer is a triangular fuzzy number such as $\tilde{d}$ = (d_1, d_2, d_3). To model the problem with credibility theory, the fuzzy number representing demand of the jth customer is denoted by $\tilde{d}_j = (d_{1j}, d_{2j}, d_{3j})$. Let the vehicles have equal capacity denoted by Q. After serving the first k customers, the available load of the vehicle will equal $Q_k = Q - \sum_{j=1}^{k} \tilde{d}_j$ in which Q_k is also a triangular fuzzy number by using the rules of fuzzy arithmetic, and

$$Qk = \left(Q - \sum_{j=1}^{k} d_{3j}, \ Q - \sum_{j=1}^{k} d_{2j}, \ Q - \sum_{j=1}^{k} d_{1j} \right) =$$

$$(q_{1,k}, \ q_{2,k}, \ q_{3,k}).$$

The credibility that the next customer demand does not exceed the remaining load of the vehicle can be obtained as follows:

$$\text{Cr} = Cr\left\{ \tilde{d}_{k+1} \leq Q_k \right\} = \quad (3.4)$$

$$Cr\left\{ (d_{1,k+1} - q_{3,k}, \ d_{2,k+1} - q_{2,k}, \ d_{3,k+1} - q_{1,k}) \leq 0 \right\}$$

$$Cr\left\{ \tilde{d}_{k+1} \leq Q_k \right\} =$$

$$\begin{cases} 0, & \text{if } d_{1,k+1} \geq q_{3,k} \\ \frac{q_{3,k}-d_{1,k+1}}{2*(q_{3,k}-d_{1,k+1}+d_{2,k+1}-q_{2,k})}, \\ \quad d_{1,k+1} \leq q_{3,k}, \ d_{2,k+1} \geq q_{2,k} \\ \\ \frac{d_{3,k+1}-q_{1,k}-2*(d_{2,k+1}-q_{2,k})}{2*(q_{2,k}-d_{2,k+1}+d_{3,k+1}-q_{1,k})}, \\ \quad d_{2,k+1} \leq q_{2,k}, \ d_{3,k+1} \geq q_{1,k} \\ 1. & d_{3,k+1} \leq q_{1,k} \end{cases} \quad (3.5)$$

Similarly, let the capacity of candidate depots be equal and denoted by P. After allocating k customers to a depot, the inventory level of the depot

will equal $P_k = P - \sum_{j=1}^{k} \tilde{d}_j$ in which P_k is also a triangular fuzzy number by using the rules of fuzzy arithmetic, and

$$P_k = \left(P - \sum_{j=1}^{k} d_{3j}, \ P - \sum_{j=1}^{k} d_{2j}, \ P - \sum_{j=1}^{k} d_{1j} \right)$$

$$= (p_{1,k}, \ p_{2,k}, \ p_{3,k})$$

The credibility that the next allocated customer demand does not exceed the inventory level of the depot can be shown as follows:

$$\mathrm{Cr} = Cr\left\{ \tilde{d}_{k+1} \leq P_k \right\} = \qquad (3.6)$$

$$Cr\{(d_{1,k+1} - p_{3,k}, \ d_{2,k+1} - p_{2,k}, \ d_{3,k+1} - p_{1,k}) \leq 0\}$$

$$\mathrm{Cr}\left\{ \tilde{d}_{k+1} \leq P_k \right\} =$$

$$\begin{cases} 0, d_{1,k+1} \geq p_{3,k} \\ \frac{p_{3,k} - d_{1,k+1}}{2*(p_{3,k} - d_{1,k+1} + d_{2,k+1} - p_{2,k})}, \\ \quad d_{1,k+1} \leq p_{3,k}, \qquad d_{2,k+1} \geq p_{2,k} \\ \\ \frac{d_{3,k+1} - p_{1,k} - 2*(d_{2,k+1} - p_{2,k})}{2*(p_{2,k} - d_{2,k+1} + d_{3,k+1} - p_{1,k})}, \\ \quad d_{2,k+1} \leq p_{2,k}, \ d_{3,k+1} \geq p_{1,k} \\ \\ \quad 1. \quad \text{if} \ \ d_{3,k+1} \leq p_{1,k} \end{cases} \qquad (3.7)$$

There is no doubt that if the level of remaining goods in the vehicle is high and the demand at the next customer is low, then the vehicle's chance of being able to serve the next customer become greater. This means that the greater the difference between available goods and demand of the next customer, the greater preference to send the vehicle to serve the next customer. According to formulation (3.5), the preference index denoted by Cr, indicates the magnitude of the preference for sending the vehicle to the next customer after serving the current customer. It is obvious that $Cr \in [0, 1]$. When $Cr = 0$ driver is completely sure that he should return to the depot. When $Cr = 1$, the driver is absolutely certain that he can serve the next customer by the remaining goods in the vehicle. Let the dispatcher preference index denoted by Cr^*, where $Cr^* \in [0,1]$. So, according to the Cr^* and the credibility that the next customer demand does not exceed the remaining capacity of the vehicle, a decision must be made either to send the vehicle to the next customer or to return that to the depot. Thus, if relation $Cr = Cr^*$ is fulfilled, then the vehicle should be sent to the next customer; otherwise, the vehicle should be returned to the depot, and send it back again to the next customer after reloading sufficient goods. The process does not terminate until all the customers' demands are fulfilled.

Similarly, in formulation (3.7) if the inventory level of depot is high and the demand of the next customer being low, then the depot's chance of being able to serve the next customer become greater. This means that the greater the difference between the available capacity of the depot and the demand of the next customer, the greater the preference to allocate the next customer to the depot for receiving service. The preference index is denoted by Cr, and $Cr \in [0,1]$. When $Cr = 0$, then the depot manager is completely sure that he should not accept supplying the next customer. On the other hand, when $Cr = 1$, the depot manager is absolutely certain that he can serve the next customer. Let the assignment preference index for allocating customers to a depot denoted by Cr^{**}, $Cr^{**} \in [0, 1]$. So, regarding to the Cr^{**} and the credibility that the next customer's demand does not exceed the remaining capacity of the depot, a decision must be made either to allocate the customer to the current depot or to supply it by the next opened depot. Thus, if the relation $Cr = Cr^{**}$ is fulfilled, then the depot should serve the next customer; otherwise, the customer should receive service from another opened depot. This procedure does not end until all the customers are allocated.

Furthermore, the vehicle routes (or planned routes) are designed in advance by applying the proposed heuristic method. But, the actual value of demand of a customer is only known when the vehicle reaches the customer. Due to demand uncertainty of the customers, a vehicle might not be able to serve a customer once it arrives there because of insufficient capacity. When this is the case, the vehicle returns to the depot to reload itself and then returns to the customer where it had a "failure" and continues its service for the rest of the planned route. This arises an additional distance because of route failure. Hence, an additional distance is considered for the vehicle due to the "failure" happened at a customer's location along the route, while evaluating the planned route [25].

Both parameters Cr^* and Cr^{**} which are empirically determined, have an extremely great im-

pact not only on the total length of the planned routes but also on the additional distances. For example, lower values of parameter Cr^* express the dispatcher's desire to use vehicle capacity as much as possible. These values result in shorter planned distances. But lower values of parameter Cr^* increase the number of circumstances where a vehicle meets a customer but is unable to serve that, thereby the total distance travelled by the vehicle is increased due to the "failure". On the other hand, higher values of parameter Cr^* are characterized by less utilization of vehicle capacity along the planned routes and fewer additional distances to travel due to failures. So, the sensitive parameters Cr^* and Cr^{**} significantly influence the sum of planed route lengths and additional distances and their proper values should be given by the decision maker to model. In this work, some numerical experiments were performed to compute the Cr^*, through solving several instances of capacitated location-routing problem.

The following notations are used to represent the mathematical programming formulation for the CLRP-FD.

Sets and parameters:

I: Set of candidate depots indexed by i and $I = \{1, 2, ..., M\}$, that M is the number of candidate depots.

J: Set of customers indexed by j and $J = \{1, 2, ..., N\}$, that N is the number of customers.

V: Set of all points: $V = I \cup J$ and $V = \{1, 2, ..., M, M+1, M+2, ..., M+N\}$.

K: Set of vehicles indexed by k.

E: Set of arcs (i, j) connecting every pair of nodes $i, j \in V$.

$\tilde{d}_j$: Fuzzy demand of customer j.

O_i: Opening cost of the depot at candidate site i.

F_k: Fixed cost of employing the vehicle k.

P: Capacity of depots. It is assumed that all depots have equal capacity.

c_{ij}: Cost of traveling associated with arc $(i, j) \in E$.

Q: Capacity of vehicles; It is assumed that all vehicles are homogeneous.

f_k: Additional distances traveled by vehicle k.

Decision variables:

$$Z_i = \begin{cases} 1 & \text{if the depot at candidate site } i \text{ is opend} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand at customer } j \text{ is served by} \\ & \text{the depot located at candidate site } i \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ goes directly from} \\ & \text{customer } i \text{ to customer } j \\ 0 & \text{otherwise} \end{cases}$$

$U_{jk}=$ Auxiliary variables for sub-tour elimination constraints in route k. The corresponding fuzzy chance-constrained programming (FCCP) mathematical formulation of the CLRP-FD based on credibility theory is given by:

$$Minimize \quad \sum_{i \in I} O_i Z_i + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F_k X_{ijk} + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} X_{ijk} \tag{3.8}$$

$$Minimize \quad \sum_{k \in K} f_k \tag{3.9}$$

Subject to

$$Cr\left(\sum_{i \in V} \sum_{j \in J} \tilde{d}_j x_{ijk} \leq Q\right) \geq Cr^* \quad \forall \ k \in K \tag{3.10}$$

$$Cr\left(\sum_{j \in J} \tilde{d}_j Y_{ij} \leq PZ_i\right) \geq Cr^{**} \quad \forall \ i \in I \tag{3.11}$$

$$\sum_{i \in V} \sum_{k \in K} X_{ijk} = 1 \quad \forall \ j \in J \tag{3.12}$$

$$U_{lk} - U_{jk} + N x_{ljk} \leq N-1 \quad \forall \ l, j \in J; \ \forall \ k \in K \tag{3.13}$$

$$\sum_{j \in V} X_{ijk} - \sum_{j \in V} X_{jik} = 0 \quad \forall \ i \in V; \ \forall \ k \in K \tag{3.14}$$

$$\sum_{i \in I} \sum_{j \in J} X_{ijk} \leq 1 \quad \forall \ k \in K \tag{3.15}$$

$$\sum_{u \in J} X_{iuk} + \sum_{u \in V \backslash \{j\}} X_{ujk} \leq 1 + Y_{ij} \quad \forall \ i \in I; \ \forall \ j \in J; \ \forall \ k \in K \tag{3.16}$$

$$Z_i \in \{0, 1\} \quad \forall \ i \in I \tag{3.17}$$

$$Y_{ij} \in \{0, 1\} \quad \forall \ i \in I; \ \forall \ j \in J \tag{3.18}$$

$$X_{ijk} \in \{0, 1\} \quad \forall \ i \in V; \ \forall \ j \in V; \ \forall \ k \in K \tag{3.19}$$

Table 1: The relative values of two test instances.

No. of customers	No. of potential sites	Vehicle capacity	Depot capacity	Fixed cost of depots	Fixed cost of vehicles
30	5	300	900	50	10
100	7	800	10000	50	10

Table 2: The average of results with different Cr^* when the number of customers is 30.

Cr^*	Planned routes	Additional distances	Routing costs	Depot costs	Vehicle costs	Total costs	CPU Time (second)
0.1	643.8	215	858.8	150	30	**1038.8**	**790**
0.2	655	201.7	856.7	150	30	**1036.7**	**736**
0.3	686.1	180.5	866.6	150	40	**1056.6**	**620**
0.4	699.9	150.4	850.3	200	40	**1090.3**	**451**
0.5	760.8	101.3	862	200	50	**1112**	**400**
0.6	750.1	60.4	810.5	150	50	**1010.5**	**477**
0.7	810.4	13.1	823.5	150	60	**1033.5**	**370**
0.8	868.2	2.7	870.9	150	70	**1090.9**	**372**
0.9	890.6	0	890.6	150	80	**1120.6**	**342**
1	905.9	0	905.9	150	80	**1135.9**	**274**

Table 3: The average of results with different Cr^* when the number of customers is 100.

Cr^*	Planned routes	Additional distances	Routing costs	Depot costs	Vehicle costs	Total costs	CPU Time (second)
0.1	951.8	299.3	1251.1	50	50	**1351.1**	**13019**
0.2	954	320.1	1274.4	50	50	**1374.4**	**10019**
0.3	987.7	323.7	1311.4	50	60	**1421.4**	**9243**
0.4	1000.3	319.7	1320	50	60	**1430**	**7649**
0.5	1091.4	235	1326.8	50	70	**1446.8**	**6530**
0.6	1145	60.7	1206	50	80	**1336**	**5738**
0.7	1215.4	14.6	1230	50	90	**1370**	**4865**
0.8	1243.4	0.4	1243.8	50	100	**1393.8**	**4559**
0.9	1300.5	0	1300.5	50	110	**1460.5**	**4415**
1	1397.5	0	1397.5	50	110	**1557.5**	**3633**

$$U_{jk}^t \in \{N \cup 0\} \quad \forall j \in J; \ \forall k \in K; \ \forall t \in T$$
$$(3.20)$$

The objective function (3.8) represents sum of the fixed depot location costs, fixed costs of employing vehicles, and travel costs, respectively. The objective function (3.9) seeks to minimize total additional travel distance due to routes' fail-

ure. Note that, the second objective function value is computed by stochastic simulation algorithm described in section 4.1.1. Chance constraints (3.10) and (3.11) assure that all customers are visited in accordance with the vehicle capacity and are allocated within the depot capacity with credibility of Cr^* and Cr^{**},

Table 4: Comparison of results between two solving approaches for the CLRP-FD.

Cr^*	30 customers				100 customers		
	GCM	HHA	Gap (%)		GCM	HHA	Gap (%)
0.1	1044.1	1038.8	-0.5		1373.5	1351.1	-1.6
0.2	1048.6	1036.7	-1.1		1411.9	1374.4	-2.7
0.3	1074.2	1056.6	-1.6		1475.1	1421.4	-3.6
0.4	1105.7	1090.3	-1.4		1483.6	1430	-3.6
0.5	1131	1112	-1.7		1479.4	1446.8	-2.2
0.6	1023.9	1010.5	-1.3		1362.3	1336	-1.9
0.7	1048.3	1033.5	-1.4		1384.9	1370	-1.1
0.8	1107.3	1090.9	-1.5		1413.3	1393.8	-1.4
0.9	1128.4	1120.6	-0.7		1484.3	1460.5	-1.6
1	1148.3	1135.9	-1.1		1579.7	1557.5	-1.4
Ave.			**-1.2**				**-2.1**

Table 5: The summarized results of two test instances with their lower bounds.

	Quality of solutions				CPU Time (second)		
Instance	HHA solution	Lower bound	Gap (%)		HHA solution	Lower bound	Gap (%)
30 customers	1010.5	620.4	**62.9**		477	420	**13.6**
100 customers	1336	969	**37.9**		5738	2980	**25.8**

Table 6: The summarized results of LINGO 11 on lower bounds.

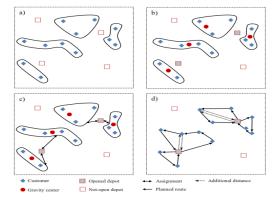|	Lower bound				CPU Time (second)		
Instance	HHA	Lingo 11	Gap (%)		HHA	Lingo 11	Gap (%)
30 customers	620.4	728	**17.3**		420	2880	**585.7**
100 customers	969	infeasible	-		4560	28800	**531.6**

respectively. Each customer should be served within one route only and the customers should have only one predecessor, which is stated by constraint (3.12). The sub-tour elimination constraints are assured in (3.13). The continuity of the routes and return to the original depot are guaranteed through constraints (3.14) and (3.15). Constraints (3.16) ensure that a customer must be assigned to a depot if there is a route connecting them. Constraints (3.17), (3.18), and (3.19) specify the binary variables used in the formulation and finally, auxiliary variables taking positive values are declared in (3.20).

4 Proposed hybrid heuristic algorithm for the CLRP-FD

A hybrid heuristic algorithm, named HHA, is presented in this section to solve the CLRP-FD. In general, HHA is composed of two major phases. In the first phase, an initial population of solutions based on greedy clustering method (GCM) is generated within four steps, which is illustrated in Fig. 1 [59]. In the first step of GCM, customers are clustered using a greedy search algorithm (Fig. 1(a)). In the second step, the gravity center of each cluster is calculated, which is used to select the depot(s) (Fig. 1(b)). The clusters are allocated to the opened depot(s) in the third

Table 7: Performance of HHA on some CLRP-FD instances obtained from standard test sets of CLRP.

Instance name	Optimal solution of the LRP instance			HHA solution of the CLRP-FD instance with $Cr^* = 0.6$			Gap(%)
	Cost	No. of depots	No. of vehicles	Cost	No. of depots	No. of vehicles	
Gaskell67-21 × 5 (Barreto, 2004)	424.9	2	4	522.9	2	8	23.1
Gaskell67-22 × 5 (Barreto, 2004)	585.1	1	3	718.8	1	4	22.8
Gaskell67-32 × 5a (Barreto, 2004)	567.2	1	4	783.4	1	6	38.1
Gaskell67-32 × 5b (Barreto, 2004)	504.3	1	3	712.4	1	5	41.3
20-5-1a (Prins & Prodhon, 2014)	54793	3	5	73289	3	14	33.8
20-5-1b (Prins & Prodhon, 2014)	39104	2	3	50394	2	5	28.9
20-5-2a (Prins & Prodhon, 2014)	48908	3	5	51198	3	5	4.7
20-5-2b (Prins & Prodhon, 2014)	37542	2	3	48129	2	5	28.2
50-5-1a (Prins & Prodhon, 2014)	90111	3	12	116617	3	13	29.4

Figure 1: Illustrative example of the first phase of the HHA

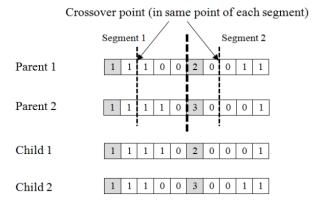**Figure 2:** Illustration of two instances with chromosome representation.

step, considering the distance between the depot and the gravity center of the clusters as well as the capacity of the depot (Fig. 1(c)). Finally, in the fourth step, ant colony system (ACS) forms an admissible tour among each cluster and relevant depot (Fig. 1(d)). In this step, the stochastic simulation is also used to determine the demand of customers. This helps to evaluate the planned routes and calculate additional distances due to route failures.

In the second phase of the HHA, a genetic algorithm (GA) is used to improve the initial population generated through the first phase. In the first step of this phase, a generation of offspring is produced by crossover and mutation operators. Then, the population of solutions is evaluated by using a proper fitness function. In the next step,
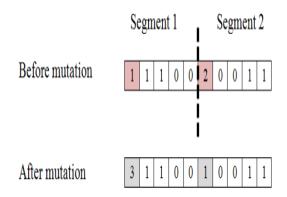
the best solutions are applied to form the next generation of offspring. This improvement procedure will be continued until the termination condition is matched. When a better solution is obtained, the new best solution will be replaced with the past best-known solution. The problem is initialized by defining a plane comprising the set of depots, M, customers, N, and their coordinate points. Details of the HHA's phases are described in following sub-sections.

4.1   Creating the initial population

As pointed out earlier, the initial population of solutions is created through the first phase of the HHA, named as GCM. To do this, all steps of this phase are repeated for *pop-size* times. Indeed,

Figure 3: Crossover operator employed in the customers' genes of each segment

Figure 4: Mutation operator employed in the section of the depots' genes.

Figure 5: Flowchart of the hybrid heuristic algorithm (HHA).

the obtained solutions are considered as an initial population of GA to be applied in the second phase. First phase includes four steps as follows:

4.1.1 Clustering the customers

The first step for creating the initial population in GCM is clustering the customers. The customers are grouped considering their distances and their fuzzy demands in addition to the capacity of the vehicles. A greedy search algorithm is used to select a set of customers. At first, to form a cluster, a customer is selected randomly from a set of non-clustered customers belongs to N. The algorithm searches for the closest customer to the last added customer of the current cluster. Note that, the greedy selection of the next customer may cause a shorter tour and then lower routing costs. The closest customer is not assigned to the cluster if its demand exceeds the remaining capacity of the vehicle, considering the dispatcher preference index value and the credibility of the customer. When a new customer is selected to be assigned to a cluster, total fuzzy demand of
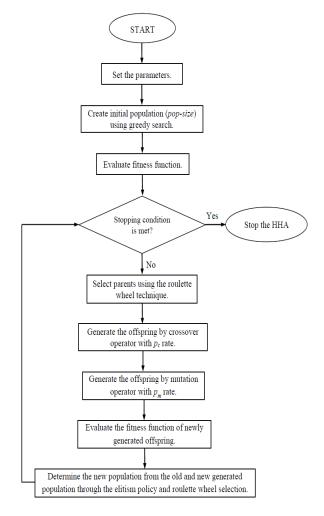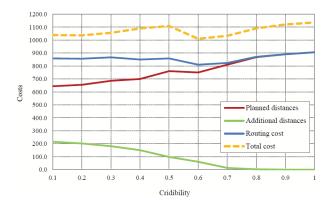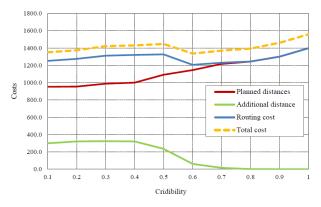
current members of the cluster is calculated and compared to the capacity of the vehicle. If the relation $Cr = Cr^*$ is fulfilled (according to the formulation (3.5)), then the new customer is allowed to assign to the current cluster. Otherwise, last selected customer is withdrawn from the cluster. The greedy search algorithm seeks for a new customer close to the last added member of the cluster among the ungrouped customers. This procedure helps to use the maximum capacity of vehicles. The algorithm forms a new cluster if there is no customer to be assigned to current cluster considering the capacity of vehicles and fuzzy demand of customers. When there is no unassigned customer, the process of clustering stops.

4.1.2 Establishing the depot(s)

This step of the GCM searches among potential sites to establish the depot(s). Firstly, the gravity center of each cluster is calculated according to

Figure 6: The changes of costs with different of Cr^* when the number of customers is 30.

Figure 7: The changes of costs with different of Cr^* when the number of customers is 100.

equation (4.21), in which $(x_{(C)}, y_{(C)})$ is the coordinates of the gravity center of cluster C, (x_j, y_j) is the coordinates of customer j, and n_C is the number of customers assigned to cluster C.

$$\left(x_{(C)}, y_{(C)}\right) = \left(\frac{\sum_{j \in C} x_j}{n_C}, \frac{\sum_{j \in C} y_j}{n_C}\right) \quad (4.21)$$

Secondly, the sum of distances of each potential site from all gravity centers of the clusters is calculated. The potential sites are sorted in an ascending order and ranked from 1 to M according to their Euclidean distance from the gravity centers of clusters. The Euclidean distance is calculated by equation (4.22). In this equation, (x^*, y^*) is the coordinates of the desired potential site among all potential sites. Moreover, w_i is the total Euclidean distance of the potential site j from all gravity centers of the clusters, (x_i, y_i) is the coordinates of the potential site i, $(x_{(C)}, y_{(C)})$ is the coordinates of the gravity center of cluster C, m is the number of clusters, and M is the

number of potential sites.

$$(x^*, y^*): \quad Mininmize \ w_i$$
$$= \sum_{C=1}^{m} \left[(x_i - x_{(C)})^2 + (y_i - y_{(C)})^2\right]^{1/2}$$
$$\forall \ i = 1, ..., M \quad (4.22)$$

The top-ranked potential site is selected to be established. As it will be mentioned in next step, if the capacity of the current opened depot is unable to fulfill all clusters considering total demand of each cluster and the Cr^{**} value, the next potential site in the sorted list is selected to serve the remaining clusters. This procedure (i.e., establishing the depot(s)) is repeated until all clusters can be served.

4.1.3 Allocating clusters to depots

In third step of GCM, the clusters are respectively allocated to the ranked depots. Each depot serves clusters as many as possible, based on the value of Cr^{**} and the credibility that the next cluster demand does not exceed the remaining capacity of the depot. To allocate the clusters, the Euclidian distance of the gravity center of each cluster to the top-ranked depot is calculated. Afterwards, the clusters are ranked in an ascending order based on the distance of their gravity centers to the top-ranked depot. The top-ranked cluster is allocated to the top-ranked depot, if the relation $Cr = Cr^{**}$ is fulfilled. If there is an empty capacity for the top-ranked depot, the second-ranked cluster is allocated to the depot considering the above relation. The allocation of clusters to the top-ranked depot is finished when there is not enough capacity to serve another cluster. In this situation, the allocation procedure is repeated for next-ranked depots until all clusters are allocated.

4.1.4 Routing

In this step of GCM, the routing problem is solved for each cluster that starts and ends to the relevant depot. The routing problem in CLRP-FD is the same as traveling salesman problem (TSP), which is solved by using ant colony system (ACS). ACS is referred to ants' treatment to find food [11]. The ants spread a material called pheromone and put it on their ways in such a way that other ants can pass the same route. The

pheromone of shorter route increases and therefore, more ants move along that way. Artificial ants construct a solution by selecting a customer to visit sequentially until all customers are visited. In fact, ants select the next customer to visit using a combination of heuristic and pheromone information. A local updating rule is applied to modify the pheromone on the selected route, during the construction of a route. When all ants construct their tours, the amount of pheromone of the best selected route and the global best solution, are updated according to the global updating rule. More details on ACS can be found in [7, 49, 17].

As mentioned before, in this paper the demand of each customer is a triangular fuzzy number, so it cannot be directly considered as a deterministic number like other methods that tackle the deterministic CLRP. Since the real value of demand is identified as the vehicle reaches the customer, the simulation experiment is used to estimate the deterministic value of each customer's demand. For each feasible planned route that the solution of the HHA stands for, additional distances (f_k) due to route failures are obtained by a stochastic simulation algorithm. The steps of stochastic simulation are summarized as follows:

Step 1: Estimate the "actual" demand of each customer by following processes: (2.1) randomly generate a real number D in the interval between the left and right bounds of the triangular fuzzy demand of the customer, and compute its membership m; (2.2) generate a random number r in the interval of [0,1]; (2.3) compare r and m, if $r = m$, then "actual" demand of the customer is adopted to D; otherwise, it is not accepted that the demand of the customer is considered D. In this case, random numbers D and r are generated repeatedly until random numbers D and r are found such that relation $r = m$ is satisfied; (3.4) check and repeat (2.1) till (2.3), and terminate the process when each customer has a simulated "actual" demand quantity.

Step 2: Move vehicles along the routes designed using credibility theory. Apply ACS to the routes and calculate the additional distance due to route failures in terms of "actual" demand.

Step 3: Repeat Steps 1 and 2 M times. In this work, the proper value of M was considered 300 after some computational experiments.

Step 4: Compute the average additional dis-

tances that come out of stochastic simulation, and return it as the additional distance (f_k).

Note that, the routing cost in the CLRP-FD, denoted by $f(S)$ in which S is a solution among *pop-size* solutions, consists of two parts: additional distances and planned route distances. The additional distances are calculated by the stochastic simulation. Instead, prior to calculating the additional distance the planned route distances between the depots and allocated clusters are computed by ACS without considering the fuzzy demands of customers.

4.2 Improving the solutions

In the second phase of the HHA, a genetic algorithm (GA), firstly introduced by Holland [21], is used to improve the solutions that come out of the first phase. As mentioned before, the CLRP belongs to the class of NP-hard problems. For this reason, the exact solution methods become highly time-consuming as the problem instances increase in size. Therefore, due to the combinatorial nature of the CLRP-FD and the efficiency of population-based algorithms in solving combinatorial problems, GA is applied to improve the initial solutions. In fact, the main motivation for this selection is that in recent years, a large number of GAs have developed by researchers with considerable success in solving routing-like problems [18, 19, 20, 51].

The idea behind a GA is to model the natural evolution by using genetic inheritance together with Darwin's theory. A population of individuals representing tentative solutions is maintained over many generations. New individuals are produced by combining members of the population via crossover and mutation operators, and then replace existing individuals based on their fitness functions [35].

The critical issues in developing a GA are chromosome representation, population initialization, fitness function, evaluation process, selection process, and crossover and mutation operators. Furthermore, the GA parameters such as number of generation, population size, crossover and mutation probability needs to be determined before the execution due to their great impacts on the performance [2]. At first, the representation scheme of the solutions created in the first phase is described. Afterwards, the selection operation of two individuals is explained and then crossover

and mutation methods are shown by an example. Finally, the evaluation process to produce the next generation is described.

4.2.1 Chromosome representation

The GA requires a string representation scheme (chromosome) to encode solutions of the problem. In this work, each chromosome consists of several segments directly related to the number of clusters. Moreover, each segment is a string of binary values. An example of solution obtained from the first phase along with its encoded chromosome is shown in Fig. 2. Each segment of the chromosome is made up of some genes such that it displays the opened depot and the customers who assigned to the depot. The value of the first gene of a segment represents the depot's number (integers between 1 and M). The position of other genes of a segment represents customers' numbers in which binary value of each gene determines whether the customer assigned to the depot or not.

4.2.2 Selection operator

The selection operation is a process to select two individuals from the current population. In this study, the roulette wheel selection procedure is employed in such a way that the selection probability of each chromosome is proportional to its fitness value. Moreover, based on the fitness values, a tentative set of solutions (selected among current population) is stored and passed directly to the new population for elite protection. Other chromosomes of the new population are generated through an evolutionary loop applying the crossover and mutation operators. More precisely, the selection probability of chromosome S denoted by $p_{selection}(S)$ is expressed by Eq. (4.23) in which $fit(S)$ is the fitness of chromosome S. Recall that, S is a given solution among *pop-size* solutions obtained from first phase of HHA.

$$p_{selection}(S) = \frac{fit(S)}{\sum_{S=1}^{pop-size} fit(S)} \quad (4.23)$$

Fitness of chromosome S is defined by Eq. (4.24) as follows:

$$fit(S) = \frac{1}{f(S)} \quad (4.24)$$

In which $f(S)$ is routing cost associated with the solution S. Since, the routing cost should be minimized, the inverse of routing cost of a solution is considered as fitness value. Meanwhile, the value of $f(S)$ for each solution is obtained by the method explained in section 4.1.4.

4.2.3 Crossover operator

In the proposed GA, crossover operator is only used in the customers' genes. To do so, a random number, say n, is generated based on a discrete uniform distribution in $\{1,2,3, \ldots, N\}$. In the crossover operator, in children, the content of the customers' genes until n will remain the same as the first parent. The genes from $n+1$ to N are filled by the order that remaining genes appear in the second parent. This work continues for remaining segments of the chromosome while the crossover point is fixed. In this step, regarding the crossover rate, say p_c, the new solutions (offspring) are generated. Fig. 3 exemplifies the crossover operator applied in the second phase.

4.2.4 Mutation operator

To make the search more diversified and to escape from local optima, mutation operator is used on the depots' genes. To do so, a random number, say m, is generated based on a discrete uniform distribution in $\{1, 2, 3, \ldots, M\}$. In the mutation operator, in children, the content of the depot's gene takes the value of m. Note that, the mutation of depots' genes on remaining segments of the chromosome can be changed by selecting another random number in $\{1, 2, 3, \ldots, M\}$. Fig. 4 shows a swap mutation done at the depots' gene of each segment. In this step, p_m is considered as mutation rate such that new mutated solutions (offspring) are generated based on this value.

4.2.5 Reproduction

After producing offspring, their feasibility is checked. The generated offspring is infeasible if the capacity of depot or vehicle or both of them are not met. For example, after the crossover operator, if total demand of all the customers who assigned to a depot be more than its capacity, the offspring is infeasible. When this is the case, the value of $f(S)$ will increase by adding the penalization. After considering the feasibility and the penalization of infeasible offspring, the new

generation is selected from among the old generation and the new generated offspring. Two mechanisms, namely elitism policy and roulette wheel selection, are used in the proposed algorithm to select the new generation. Indeed, the former empowers the intensification capability of the algorithm, and the latter enhances its diversification. To build the new generation, a predetermined percentage, say p_e, of the chromosomes are selected based on the elitism policy and the roulette wheel selection technique is used to select the rest. It is notable that after some computational experiments, the proper values of p_c, p_m and p_e were considered 0.8, 0.2 and 0.6, respectively. The general structure of the HHA is given by Fig. 5.

5　Computational results

5.1　Sensitivity analysis of the parameters of the model CLRP-FD

In this section, some numerical experiments are given to show the performance of the CLRP-FD's model and the efficiency of the HHA. In the first experiment, to evaluate the sensitivity of the parameters of the model, different size of instances is considered to conduct computational experiments. It is assumed that there are 30 customers and 5 candidate depots for a small-size instance, and 100 customers and 7 candidate depots for a large-size instance. In each instance, the coordinates of all customers and depots are generated randomly in [100×100]. Moreover, the fuzzy demands of customers, which are triangular fuzzy numbers like $\tilde{d} = (d_1,\ d_2,\ d_3)$, are selected randomly. More precisely, d_1, d_2 and d_3 are randomly generated within [10,35], [36,60] and [61,110], respectively. The relative data for two test instances are listed in Table 1. Note that, the generated test instances are similar to the real-world cases and entirely consistent with the real data. Thus, the obtained results can be applied for real-world application.

The HHA is coded in MATLAB 7.10.0 on a computer, holding Intel Core^{TM} Duo CPU T2450 2.00 GHz. To find the proper value of dispatcher preference index (Cr^*), its value is changed in the interval of 0.1 to 1 by a step of 0.1. In this work, the value of assignment preference index (Cr^{**}) is set to 1 due to convenience and reducing the number of different investigative statues. The average

computational results of 10 times are given in Tables 2 and 3 for small and large-size instances, respectively. The columns of the tables respectively labeled: the dispatcher preference index denoted by Cr^*, the cost of planned routes, the cost of additional distances, routing cost that include the planned routes and additional distances, depot cost, vehicle cost, total cost that consist of routing cost as well as depot and vehicle's costs and finally, the CPU time of solutions. For more convenience, summary of the results are depicted in Figs. 6 and 7. As it is shown in Tables 2-3 and also in Figs. 6-7, when the value of dispatcher preference index equals 0.6, the total cost has its own minimum value.

According to Figs. 6 and 7, lower values of parameter Cr^* denote a tendency to use total vehicle capacity. These values are associated with the routes with the shorter planned distances. On the other hand, lower values of parameter Cr^* increase the number of cases in which vehicles visit customers but are unable to serve them, thereby lead to increase the total additional distance due to the "failure". Higher values of parameter Cr^* are characterized by less utilization of vehicle capacity along the planned routes and fewer additional distances. Therefore, the proper value of Cr^* is approximately around 0.6 based on the total cost.

5.2　Performance of the proposed hybrid heuristic algorithm

In this section, the efficiency of the HHA is compared with the conventional approach developed for solving the CLRP-FD. To evaluate the efficiency of the proposed HHA, the results of Tables 2-3 are compared with the greedy clustering method (GCM) that is similar to the first phase of the HHA. Table 4 indicates the summary of computations on instances described in section 5.1. First column of the table shows the dispatcher preference index (Cr^*). The next three columns along with the last three columns show the comparison of results between the GCM proposed by Zare Mehrjerdi and Nadizadeh [59] and the HHA developed in this work for small and large-size instances, respectively. Note that, the column labeled "Gap" reports the gap percentage that is computed for each instance as $100 \times [(\text{OFV}_{HHA} - \text{OFV}_{GCM})/ \text{OFV}_{GCM}]$ where OFV_{HHA} and OFV_{GCM} are denoted as objective function value

of HHA and GCM, respectively. As shown in Table 4, the HHA has improved solutions of two instances for all Cr^* values. So, the HHA is more efficient than the GCM in terms of solution quality. Further performance assessment of the HHA is the comparison of the results with the lower bound of solutions that obtained from an exact algorithm. A simple method for creating a lower bound is through the relaxation of some assumptions that the problem is based on or the constraint(s) used in modeling the problem. To obtain the lower bound of CLRP-FD, the assumption of fuzzy demand of customers is ignored and the demand of each customer is set to its left boundary. In other words, the problem of CLRP-FD changes to CLRP in such a way that the left boundary of the triangular fuzzy demand is considered as the demand of each customer (i.e., d_{1j} instead of $\tilde{d}_j = (d_{1j}, d_{2j}, d_{3j})$). To do so, the constraints (3.10) in the model changes as follows:

$$\sum_{i \in V} \sum_{j \in J} d_{1j} x_{ijk} \le Q \qquad \forall \ k \in K \quad (5.25)$$

Clearly, when the left boundary of fuzzy demands are considered as the deterministic demands of customers, the total demand of customers is decreased in compare to a case in which the demands are fuzzy. Moreover, fewer total demands result in less utilization of depots and vehicles and then less total cost. Thus, considering the d_{1j} as the demand of customer j provides a lower bound for the CLRP-FD's solution. Tables 5 and 6 show the lower bound of the instances described in section *5.1* as well as a summary of the results in terms of the quality of solutions and the computational solutions times.

Table 5 indicates the lower bound of two instances explained in section *5.1*. The first column of the table is the name of instances and the next three columns summarize the quality of solutions consists of HHA solution, lower bound and the gap between them. Note that, the column named HHA solution is obtained from the total cost of Table 2 and 3 with credibility of 0.6, which have the minimum total costs among other credibility values. The gap percentage for each instance is also computed as $100 \times [($HHA solution – Lower bound$)/$ Lower bound$]$. Three last columns of the Table 5 indicate the CPU time of HHA solutions and lower bound and the gap between them. Regarding the results presented in Table 5, fixing

the demand of all customers to their left boundaries has significant effects on both quality and CPU time of solutions. As shown in Table 5, if the assumption of the fuzzy demands is omitted, the quality of solutions will be improved by 62.9% and 37.9% for small and large-size instances, respectively. Argue is that, the less utilization of vehicles and depots cause the additional distance omission when the customer demand is deterministic. This is the sort of situation that reduces the lower bound values to 620.4 and 969 for small and large-size instances, respectively. Similarly, if the assumption of fuzzy demand of customers is relaxed, the average CPU time is reduced by 13.6% and 25.8% for small and large-size instances, respectively.

It is of interest to notice that, to obtain the lower bound, commercial solver of LINGO 11 was also applied in addition to the HHA. However, it was unable to solve large-size instance and the solution was infeasible after 8 hours of running time. Table 6 shows the lower bound and CPU time of the instances that solved by both LINGO 11 and HHA. Consequently, as seen in Table 6, the proposed HHA is more efficient in compare to LINGO solver in terms of not only quality of lower bounds, but also CPU times of solutions.

Further results of the numerical experiment for evaluating the efficiency of HHA and the performance of CLRP-FD's model is shown in Table 7. Table 7 consists of 9 standard test instances that exist in the literature of CLRP [41]. To use the standard test instances of CLRP in case of CLRP-FD, some changes on CLRP instances are required. The following heuristic steps show the process of changing a CLRP instance to a CLRP-FD one.

Step 1: At first, the determinist demand of each customer transforms to a triangular fuzzy demand as follows: (2.1) consider the deterministic demand of each customer in the CLRP as the left boundary of fuzzy demand in CLRP-FD; (2.2) calculate the right boundary of fuzzy demand by triple the left boundary obtained from (2.1); (2.3) consider a random number in the range of 1.5 to 2.5 times of the left boundary as the middle number of triangular fuzzy demand.

Step 2: Consider the maximum right boundaries of fuzzy demands of all customers as D_{3j} (i.e. $D_{3j} = \max_{j \in J}\{d_{3j}\}$. Compare D_{3j} with the vehicle capacity of the CLRP instance denoted by

Q. If the relation $D_{3j} \leq Q$ is satisfied, then consider Q as the vehicle capacity of the CLRP-FD instance. Otherwise, consider D_{3j} as the vehicle capacity of the CLRP-FD instance.

Step 3: Triple the depot's capacity of the LRP instance and consider that as the depot's capacity of CLRP-FD instance.

According to above steps, it is clear when the left boundaries of fuzzy demands of CLRP-FD instance are equal with the demands of CLRP instance, the solution of CLRP instance will be as a lower bound for the solution of CLRP-FD instance. Table 7 shows the summary of results on several standard test instances. First column of the table indicates the name of standard test instances on CLRP that can be found in [5, 37]. Three next columns show the optimal solutions obtained from the algorithms in the literature of CLRP. The solutions obtained by HHA with Cr^* value of 0.6 are shown at the next three columns. The last column of Table 7 indicates the gap of costs between HHA and optimal solutions. As a result of Table 7, it is concluded that the solution of CLRP for each instance is a lower bound for the solution of CLRP-FD.

6 Conclusion and future research

Logistics costs often represent a large portion of the expenses of companies. In order to reduce them, facility location and vehicle routing are crucial. In the management decision of the logistics, facility location problems and vehicle routing problems are interdependent. But most of time, they are considered separately. This cannot reduce company's total cost in reality and will also increase the total cost sometimes. The location-routing problem (LRP) overcomes above drawback by simultaneously tackling facility location problem and vehicle routing problem. Nowadays, there are many literatures to research LRP, but most of them have only studied the LRPs with deterministic demands. This paper contributes to the capacitated location-routing problem with fuzzy demands that is closer to reality in the real-world. A fuzzy chance-constrained programming formulation has been developed for the problem. To solve the problem, a hybrid heuristic algorithm (HHA) with two main phases was proposed

in which greedy search algorithm was applied to generate initial population of solutions at the first phase and genetic algorithm was used for further improvement of the solutions at the second. The additional distances due to fuzzy demands and route failures were estimated by stochastic simulation for each planned route. For performance evaluation of the model and the proposed HHA, different numerical experiments were carried out. The computational experiments have shown that the dispatcher preference index (Cr^*) greatly influences the planned routes' length, additional distance, and fixed cost of depots and vehicles. So, the best value of this sensitive parameter of the model was obtained via conducting computational experiments. Comparison of the results between HHA and GCM (i.e., the developed algorithm in the literature of CLRP-FD) has indicated that the HHA is more efficient than GCM. Moreover, the lower bound of solutions was computed by the HHA and the commercial solver of LINGO 11. Finally, numerical experiments with standard test instances of CLRP have carried out to show the performance of the CLRP-FD's model.

This paper has some limitation that is capable for future researches: (a) investigating the effect of another major parameter of the model, named assignment preference index (Cr^{**}), on the length of the planned routes and the additional distances traveled by vehicles due to route failures by statistical methods like design of experiments, (b) considering the CLRP-FD with pickup and delivery, (c) given the demand of customers as trapezoidal fuzzy instead of triangular fuzzy, (d) developing other solution algorithms, e.g. hybrid evolutionary algorithms, (e) proposing new stochastic simulation for estimating both actual demand of customers and additional distances, and (f) developing the model under more realistic assumptions, e.g. heterogeneous fleet.

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