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Research Article

Determining Normalized Crisp Weights From Fuzzy Comparison Matrices Using an Improved Two-Stage Logarithmic Goal Programming Model

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Abstract

Fuzzy analytical hierarchy process (FAHP) is one of the most popular multi-criteria decision making (MCDM) techniques in which the weights of criteria and the evaluation measures of alternatives are generated based on fuzzy pairwise comparisons. There are many approaches for determining the weights from pairwise comparison matrices (PCM). Some of them have calculated the interval weights at all alphacuts. However, in some MCDM techniques (like technique for order preference by similarity to ideal solution (TOPSIS), vlsekriterijumska optimizacija i kompromisno resenje (VIKOR), and FAHP), it is needed to calculate normalized crisp weights. In this paper, the authors extend an index where interval weights obtained at all alphacuts are converted into the crisp weight. Finally, several illustrative examples in the literature are considered to show the verification of the proposed index and the obtained results are then compared with weights extracted by fuzzy preference programming (FPP). The results show that our approach is suitable for obtaining the crisp weights and also, has the similar ranking orders with those of the others.

Keywords : Fuzzy analytical hierarchy process; Alphacut; Multi-criteria decision making; Optimization; Fuzzy sets.

1 Introduction

A Multi-criteria decision making (MCDM) problem can be stated as the process of determining the best alternative among all the possible alternatives with respect to different criteria. Another significant advantage of MCDM techniques is that they can take into account both quantitative and qualitative criteria Bozbura er al. [1]. According to Guitoni and Martel's classification Guitoni and Martel [2], the methods of solving a MCDM problem can be decomposed into the four groups: (1) the elementary methods (e.g. weighted sum, maximin method); (2) the single synthesizing criterion (e.g. technique for order preference by similarity to ideal solution (TOPSIS), analytical hierarchy process

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Table 1: General reviews of the MCDM techniques.

Author (s)	Hybrid and improved methods of MCDM
Manouselis and Costopoulou [3]	Analysis and classification of multi-criteria recommender systems
El-Wahed [4]	Intelligent fuzzy MCDM
Chu and Lin [5]	An extension to fuzzy MCDM
Zavadskas and Turskis [6]	MCDM methods in economics
Bragge et al. [7]	Scholarly communities of research in MCDM: a bibliometric research profiling study
Liou and Tzeng [8]	MCDM methods in economics
Liou [9]	New concepts and trends of MCDM
Aruldoss et al. [10]	A survey on MCDM methods and its applications

(AHP)); (3) the outranking methods (e.g. elimination and et choice translating reality (ELECTRE), preference ranking organization method for enrichment evaluation (PROMETHEE)); and (4) the mixed methods (e.g. fuzzy conjunctive/disjunctive method). Tabel 1 presents the hybrid and improved techniques of MCDM.

Perhaps among all the MCDM techniques, the AHP approach is the simplest and the most popular approach which was first developed by saaty [11]. In this method, the MCDM problem is transformed into the hierarchical structure. The raw data of this method are presented as pairwise comparisons, which are the relative importance of a criterion (an alternative) over other criteria (alternatives). On the other hand, due to the different subjective judgements of managers when evaluating elements, there is this possibility that crisp values cannot state the decision-maker (DM)s uncertain preference level. Therefore, fuzzy concept was first proposed by Zadeh [12]. The fuzzy version of the AHP, i.e. fuzzy AHP (FAHP), was introduced by Van Laarhoven and Pedrycz [13], where fuzzy ratios as triangular membership functions are applied to compare criteria (alternatives). Buckley [14] derived priorities from pairwise comparison matrices (PCM) having trapezoidal membership functions. Boender et al. [15] proposed an improved approach to resolve priority normalization of Van Laarhoven and Pedryces method. Deng [16] extended an improved fuzzy method to implement the multi-criteria problems in a simple manner. Leung and Cao [17] investigated the consistency and ranking issues and suggested a fuzzy consistency definition by taking into account a tolerance deviation. There are several approaches for deriving priorities in FAHP. Some methods are as follows: the fuzzy least square method [18], method based on the fuzzy modification of the logarithmic least squares [15], the fuzzy geometric mean method [14], the direct fuzzification of the λ_max method [19], the synthetic extend analysis [20], the interval arithmetic [21], fuzzy preference programming (FPP) [22]. The last case applies to the two linear and nonlinear optimization models for generating weights from both consistent and inconsistent fuzzy comparison matrices (FCMs). The linear programming model is an optimization problem based on alpha-cuts decomposition where the fuzzy judgements are converted into a set of interval comparisons and a tolerance parameter which can be selected to one if all comparisons are symmetric. It transforms the interval prioritization into a fuzzy linear programming problem. Whereas, the nonlinear programming model is able to generate the consistency index and normalized crisp weights vector from FCMs concurrently. Accordingly, the AHP method has been applied by many researchers in the different decision-making scopes like management, manufacturing, industrial, etc.

Table 2 shows the different applications of the AHP method under the different environments (crisp, fuzzy, interval type-2 fuzzy numbers, etc.).

Unfortunately, the most of above approaches apply the consistency ratio (CR) after defuzzyfying the fuzzy pairwise comparisons. In addition, such an approach is time consuming when the number of alternatives or criteria is high.

Table 2: Hybrid and improved methods (applications) of AHP.

Author (s)	Specific area
nut et al. [23]	Shopping center sites
Yksel and Dadeviren [24]	Balanced Scorecard
Khademi-Zare et al. [25]	Ranking customer attributes in quality function deployment (QFD)
Sun [26]	Evaluating different notebook computer ODM companies
Taha and Rostam [27]	Machine tool selection
Yu et al. [28]	Ranking e-commerce websites in an e-alliance
Yang et al. [29]	Vessel selection for a particular cargo transfer
Aydogan [30]	Evaluating performance indicators
Dalalah et al. [31]	Supplier selection problem
Hatami-Marbini and Tavana [32]	Supplier selection problem for a high-technology manufacturing company
Jahanshahloo et al. [33]	Evaluating six cities for establishing a data factory
Jolai et al. [34]	Supplier selection and order allocation problem
Kara [35]	Supplier selection problem
Kuo and Liang [36]	Selecting the location of an international distribution center
Singh and Benyoucef [37]	Supplier selection for a sealed-bid reverse auction
Wang et al. [38]	Supplier selection problem
Kalantari et al. [39]	A decision support system for evaluating production policies
Vahdani et al. [40]	Robot selection problem
KarimiAzari et al. [41]	Risk assessment model selection
Vahdani et al. [42]	Assessing the performance of property responsibility insurance companies
Ghoseiri and Lessan [43]	Waste disposal site selection
Rostamzadeh et al. [44]	Entrepreneurial activity
Kabak et al. [45]	Building energy performance
Erdoan and Kaya [46]	Ranking private universities
Temur et al. [47]	Facility location selection
Mokhtarian et al. [48]	Facility location selection problem
Rashid et al. [49]	Robot selection problem
Krohling and Pacheco [50]	Ranking evolutionary algorithms
Dymova et al. [51]	System analysis engineer selection
Yang et al. [52]	Service quality assessment
Yong et al. [53]	Dynamic risk assessment of metro station
Chen [54]	Landfill site selection, supplier selection, and car evaluation
ebi and Otay [55]	Facility location selection problem
Kumar et al. [56]	Telecom sector
Uygun et al. [57]	Industry
Chang et al. [58]	e-book business model
Lupo [59]	International airport quality
Parameshwaran et al. [60]	Robot selection
Ghorabaee [61]	Robot selection problem
Abbasimehr and Tarokh [62]	Ranking reviewers in online communities
RazaviToosi and Samani [63]	Water management strategies
Wang et al. [64]	Green Supplier Selection
Efe [65]	ERP system selection
Chang et al. [66]	Government
Azadeh and Zadeh [67]	Maintenance policy selection
Dehghani et al. [68]	Evaluating service quality of public transportation
Putra et al. [69]	Determining quality of gemstones
alik et al. [70]	Evaluating territorial units
Ucal Sari et al. [71]	Developing an integrated discounting strategy
Jain et al. [72]	Supplier selection
TOPSIS	
Yazdi et al. [73]	Risk analysis of a spherical storage hydrocarbon tank
Beltro and Carvalho [74]	Prioritizing Construction Risks

Table 2. Continue

Other techniques integrated with AHP
TOPSIS
Analytic network process (ANP)
QFD, TOPSIS
TOPSIS
PROMETHEE
TOPSIS
Approximate interval TOPSIS
TOPSIS
Fuzzy decision-making trial and evaluation laboratory (DEMATEL) and fuzzy TOPSIS
Fuzzy TOPSIS and fuzzy elimination and choice translation reality (ELECTRE I)
Data envelopment analysis (DEA) and TOPSIS with interval data
Fuzzy TOPSIS and multi-period goal programming
Two-stage stochastic programming and fuzzy TOPSIS
DEMATEL, fuzzy ANP, fuzzy simple additive weighting (SAW) and fuzzy TOPSIS
Entropy method and fuzzy TOPSIS
Fractional programming, quadratic programming, and interval valued intuitionistic fuzzy TOPSIS
Mixed-integer mathematical programming, and fuzzy TOPSIS
Modified TOSIS with fuzzy approach
Fuzzy TOPSIS and nominal group technique
Fuzzy ANP, fuzzy TOPSIS, and fuzzy vlsekriterijumska optimizacija i kompromisno resenje (VIKOR)
ELECTRE
VIKOR, TOPSIS
ANP
Type-2 fuzzy TOPSIS
Type-2 fuzzy TOPSIS
Interval-valued fuzzy TOPSIS
Interval-valued fuzzy TOPSIS
TOPSIS
Interval type-2 fuzzy TOPSIS
Interval type-2 fuzzy TOPSIS
Type-2 fuzzy TOPSIS
Interval type-2 fuzzy TOPSIS
Interval type-2 fuzzy TOPSIS
DEA
ANP, DEMATEL
VIKOR, TOPSIS, GRA
ELECTRE
delphi, VIKOR
Interval type-2 fuzzy VIKOR
Interval type-2 fuzzy TOPSIS
TOPSIS, Max-Min
TOPSIS
TOPSIS
VIKOR, ANP
Fuzzy TOPSIS
Interval type-2 fuzzy TOPSIS
-
DEA
Goal programming
TOPSIS
AHP
-

Moreover, if CR is not satisfied, DMs should revise their judgements. Some researchers derived weights vector from ICMs. Saaty and Vargas [76] proposed interval numbers for the judgments in the AHP and then, obtained the priority vector from the given interval multiplicative preference relations using the Monte Carlo simulation method. They also put forward difficulties using this approach. Arbel [77] considered the interval judgements from the upper triangular part of the reciprocal interval matrix as linear constraints on the unknown priorities and then, formed the prioritization problem as a linear programming model. Kress [78] mentioned the ineffectiveness of Arbels method for solving $n(n-1)$ linear programs to deal with inconsistent interval comparison matrices (ICMs). Arbel and Vargas [79] constructed a hierarchical problem as a nonlinear programming model in which all local weights are considered as decision variables. In this technique, local weights optimum may not be global optimum because of the nonconvexity of the feasible region. Salo and Hmlinen [80] found the maximum and minimum feasible values for all interval priorities and incorporated the resulting extended intervals into further synthesis of global interval priorities. The main limitation of this approach is that it requires consistent interval preferences. Islam et al. [81] formulated a lexicographic goal programming model to find weights from inconsistent ICMs. They also provided an algorithm for identification and modification of inconsistent bounds. Haines [82] suggested a statistical method to generate priorities from ICMs. Two distributions on a feasible region are examined and their mean is then used to rank. Mikhailov [83] put forward a new approach to specify crisp priorities from interval comparison judgements. By introducing linear or nonlinear membership functions, the interval judgements are transformed into fuzzy inequality constraints. The two proposed linear or nonlinear mathematical programs are capable for deriving priorities from consistent and inconsistent interval judgements. The important features of method are that this opportunity exists for additional prioritization of the initial judgements and a consistency index that measures the level of inconsistency of interval judgements. Sugihara et al. [84] suggested an interval regression analysis approach. The proposed method is based on interval regression analysis [85, 86]. It involves the solution of lower and upper approximation models (the dual models) which are similar to rough approximations. These two models are based on the least upper and greatest lower bounds. Wang et al. [87] proposed a two-stage logarithmic goal programming (TLGP) model to generate weights from consistent or inconsistent ICMs. The first stage was devised to minimize the inconsistency of ICMs and the second one was extended to extract weights. The weights were assumed to be multiplicative rather than additive. In the hierarchical structures, a nonlinear programming method is used to aggregate local interval weights into global interval weights. The TLGP is also applicable to FCMs using levels of alpha-cuts and the extension principle. Wang et al. [88] also planned a method of test to check consistency of ICM. If ICM is consistent, Arbels linear programming model is used to extract interval weights and if ICM is inconsistent, the eigenvector method-based nonlinear programming approach is developed to attain interval weights. Wang and Elhag [89] introduced a goal programming approach to generate interval weights from ICM, which can be either consistent or inconsistent. Xu and Chen [90] defined the concepts of additive and multiplicative consistent interval fuzzy preference relations and then, some simple and practical linear programming models offered for achieving the priority weights from consistent or inconsistent interval fuzzy preference relations. Liu et al. [91] presented an incomplete interval multiplicative preference relation (where some of its elements cannot be given by the DM) and then, given the definitions of consistent and acceptable incomplete ones, respectively. Liu et al. [92] extended a new algorithm to obtain the priority weights from consistent or inconsistent interval fuzzy preference relations without the mathematical programming model to be solved.

Among all studies mentioned above, some of them [87, 22, 81, 88, 89, 91, 93] can only be applied to extract weights from consistent and inconsistent ICMs. According to Wang et al. [87], some of these approaches have the shortcomings. For example, Xu and Chen [93] obtained the degree of possibility instead of crisp values for criteria weights. To best of our knowledge, Mikhailov [22]s model is only the method which can derive the normalized crisp weights from FCMs, whereas the others generate interval weights. These approaches only represent the ranking order and degrees of preference criteria (alternatives) as $w_1 \overset{100}{\succ} w_2 \overset{100}{\succ} \ldots \overset{100}{\succ} w_N$ using alpha-cut transformations of FCMs.

Recently, Jaganathan et al. [94] applied the FAHP method to rank new manufacturing technologies. The hierarchy of their FAHP model is as follows: the overall decision goal (the best technologies) is at the top level, criteria lie in the middle level, and alternatives (technologies) are at the bottom level. Their methodology was in the general FAHP form with this difference that they adopted two approaches, namely TLGP [87] and FPP [22] when obtaining the weights of criteria (alternatives) from FCMs. They concluded that the FPP is more advantageous to solve a multi-level multi-person FAHP problem (as mentioned in Section 8 of their paper). The TLGP model for the case presented by Jaganathan et al. [94] cause to the following discrepancies: (1) it gives an interval-based priority vector; (2) the weights vector forms a complicated fuzzy number which are difficult to defuzzify and rank; (3) the application of nonlinear programming approach developed by Wang et al. [87] to obtain final composite priority is very unsuitable to rank and obtain final rankings (provides richer information about the uncertain nature of the priorities); and (4) it is not usable to the case of multi-level hierarchy. However, it is only suitable for cases in which one intends to specify the interval weights for criteria (alternatives) from consistent and inconsistent ICMs (see six numerical examples presented by Wang et al. [87]). Thus, they decided to use FPP instead of TLGP for ranking the manufacturing technologies. The goal of this paper is to resolve the discrepancies discussed by Jaganathan et al. [94] in order to derive normalized crisp weights from FCMs using the proposed index. It calculates the distance of the left and right limits from the reference limits at all alpha-cuts such that it calculates the unique crisp weight instead of interval weights at alpha-cuts.

In this section, the methodology of TLGP is explained. Assume that the triangular FCM, $R_n \times n$, is as follows:

$$R_{n\times n} = \begin{bmatrix} 1 & (a_{12},b_{12},c_{12}) & \cdots & (a_{1n},b_{1n},c_{1n}) \\ (a_{21},b_{21},c_{21}) & 1 & \cdots & (a_{2n},b_{2n},c_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (a_{n1},b_{n1},c_{n1}) & (a_{n2},b_{n2},c_{n2}) & \cdots & 1 \end{bmatrix}, \quad (1.1)$$

wherea_ij and c_ij are lower and upper bounds, respectively, and b_ij is the most likely value when evaluating or comparing the criterion (alternative) $i(i = 1, \ldots, n)$ with respect to criterion (alternative) $i(j = 1, \ldots, n)$. Since the interval numbers are used to TLGP, Jaganathan et al. [94] first applied alphacut representations to convert the above FCM into the following ICM, $R'_{n\times n}$:

$$R_{n\times n}(\alpha) = \begin{bmatrix} 1 & (l_{12}(\alpha),u_{12}(\alpha)) & \cdots & (l_{1n}(\alpha),u_{1n}(\alpha)) \\ (l_{21}(\alpha),u_{21}(\alpha)) & 1 & \cdots & (l_{2n}(\alpha),u_{2n}(\alpha)) \\ \vdots & \vdots & \ddots & \vdots \\ (l_{n1}(\alpha),u_{n1}(\alpha)) & (l_{n2}(\alpha),u_{n2}(\alpha)) & \cdots & 1 \end{bmatrix}, \quad (2.1)$$

where

$$l_{ij}(\alpha) = a_{ij}+\alpha(b_{ij}-a_{ij}), u_{ij}(\alpha) = c_{ij}-\alpha(c_{ij}-a_{ij}), \quad (3.1)$$

Next, these interval comparisons are used to generate the lower and upper weights at each alpha-cut using the following Model (1):

$$\begin{aligned} & Model(1) \\ & Max/Min \quad ln\ w_i = x_i - y_i \\ & x_i - y_i - x_j + y_j + p_ij \geq lnl_ij(\alpha), i = 1, \ldots, n; j = i+1, \ldots n, \\ & x_i - y_i - x_j + y_j - q_ij \leq lnu_ij(\alpha), i = 1, \ldots, n; j = i+1, \ldots n, \\ & \sum_{i=1}^{n}(x_i - y_i) = 0, \\ & \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(p_ij + q_ij) = J^*, \\ & x_i, y_i \geq 0, x_iy_i = 0, i = 1, \ldots, n, \\ & p_ij, q_ij \geq 0, p_ijq_ij = 0, i = 1, \ldots, n-1; j = i+1, \ldots, n, \end{aligned}$$

where w_i is the weight of criterion (alternative) $i(i = 1, \ldots, n)$, as well as p_ij and q_ij are nonnegative real numbers which are subtracted and added to lower and upper bound of constraints $l_ij(\alpha) \leq \frac{w_i}{w_j} \leq u_ij(\alpha)$ or $lnl_ij(\alpha) \leq lnw_i - lnw_j \leq lnu_ij(\alpha)$, respectively. Obviously, the consistent and inconsistent variables p_ij and q_ij should be as small as possible. Since $\ln lnw_i$ is nonnegative while $w_i \geq 0$ and negative while $w_i < 0$, the following nonnegative variables can be applied to the Model (1):

$$x_i = \frac{lnw_i + |lnw_j}{2}, \qquad i = 1, \ldots, n, \qquad (4.1)$$

$$y_i = \frac{-lnw_i + |lnw_j}{2}, \qquad i = 1, \ldots, n, \qquad (5.1)$$

In addition, J^* is the consistency index which is determined by solving the following linear programming Model (2) for all $\alpha \in [0, 1]$:

$$\begin{aligned} & Model(2) \\ & MinimizeJ = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(p_ij + q_ij) \\ & \quad x_i - y_i - x_j + y_j + p_ij \\ & \quad \geq lnl_ij(\alpha), i = 1, \ldots, n; j = i+1, \ldots n, \\ & \quad x_i - y_i - x_j + y_j - q_ij \\ & \quad \leq lnu_ij(\alpha), i = 1, \ldots, n; j = i+1, \ldots n, \\ & \quad \sum_{i=1}^{n}(x_i - y_i) = 0, \\ & \quad x_i, y_i \geq 0, x_iy_i = 0, i = 1, \ldots, n, \\ & \quad p_ij, q_ij \geq 0, p_ijq_ij = 0, \\ & \quad i = 1, \ldots, n-1; j = i+1, \ldots, n, \end{aligned}$$

If J is equal to zero, ICM is perfectly consistent and If J isnt equal to zero, then it is inconsistent. Interested readers can refer to Wang et al. [87] for investigating the more detailed analysis. As mentioned before, the above model can only determine the interval weights at each alpha-level from FCMs without a normalized crisp weight for each criterion (alternative) to be obtained. On the other hand, FPP is modeled by the following Model (3):

$$
\begin{aligned}
&Model(3)\\
&Maximize \mu\\
&\quad (m_{ru}-l_{ru})\mu w_u - w_r + l_{ru}w_u \leq 0,\\
&\quad (u_{ru}-m_{ru})\mu w_u - w_r + u_{ru}w_u \leq 0,\\
&\quad \sum_{i=1}^{N} w_i = 1, w_i > 0, i = 1, ..., N\\
&\quad r = 1, ..., N-1, r = 2, ..., N, u > r.
\end{aligned}
$$

where N is the number of criteria, $l_r u$, $u_r u$ and $m_r u$ are the lower, upper bounds and most likely value of triangular fuzzy numbers in FCMs, respectively, when experts compare rth criterion (alternative) with respect to uth criterion (alternative). w_i is the weight of criterion i and is the consistency index. If $= 1$, then the FCM is said to be consistent, and if is negative, then the matrix is said to be strongly inconsistent. The rest of this paper is organized as follows: In Section 2, the proposed index is presented. In Section 3, the proposed index is applied to four numerical examples as scientific evidences and finally, discussion and conclusion is summarized in Section 4.

2 The proposed index (limit distance mean (LDM))

In this section, the authors apply a ranking method in order to integrate the lower and upper weights of all alpha-cuts under each criterion (alternative) $i (i = 1, \ldots, n)$ to a normalized crisp weight. Let $\mu_{\hat{W}}(x)$ is partitioned into two MFs $\mu^l(x)$ and $\mu^r(x)$ (the left and right MF of $\hat{W}$). In addition, assume that $\mu^{min}(x)$ and $\mu^{max}(x)$ be minimum and maximum reference limits, respectively, as shown in Fig. 1. On the other hand, let w_α^{min}, w_α^{max}, w_α^l, and w_α^r be the measures generated by intersection of level with the $\mu^{min}(x)$, $\mu^{max}(x)$, $\mu_\alpha^l(x)$ and $\mu_\alpha^r(x)$. Hence, LDM can be calculated by using the following expression:

$$
LDM(\tilde{W}) = \frac{\sum_{\alpha=0.1}^{1}(\mu_\alpha^r(x) - \mu_\alpha^{min}(x))}{\sum_{\alpha=0.1}^{1}(\mu_\alpha^r(x) - \mu_\alpha^{min}(x)) - \sum_{\alpha=0.1}^{1}(\mu_\alpha^l(x) - \mu_\alpha^{max}(x))}, \quad (1.2)
$$

where α=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. Note that $(\mu_\alpha^r(x) - \mu_\alpha^{min}(x))$ and $(\mu_\alpha^l(x) - \mu_\alpha^{max}(x)$ are the positive and negative values. Thus, the negative sign is considered in the denominator. To simplify the calculations,

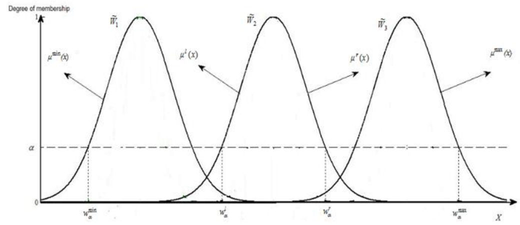

Figure 1: The left, right, minimum, and maximum reference limits.

Eq. 1.2 can be converted into the following equation:

$$
LDM(\tilde{W}) = \frac{\sum_{\alpha=0.1}^{1}|(w_\alpha^r - w_\alpha^{min})|}{\sum_{\alpha=0.1}^{1}|(w_\alpha^r - w_\alpha^{min}|) - \sum_{\alpha=0.1}^{1}|(w_\alpha^l - w_\alpha^{max}|)}, \quad (2.2)
$$

It is obvious that the bigger the measure of above index indicates the higher the importance of criterion (alternative).

Although the weights obtained by using Eq. 2.2 are crisp, in order to satisfy the normalization condition with $w_i \geq 0$ for $i = 1, \ldots, n$, they should be normalized. Therefore, the following normalized LDM (NLDM) is suggested:

$$
NLDM_i = (\tilde{W}) = \frac{\frac{\sum_{\alpha=0.1}^{1}|(w_\alpha^r - w_\alpha^{min})|}{\sum_{\alpha=0.1}^{1}|(w_\alpha^r - w_\alpha^{min}|) - \sum_{\alpha=0.1}^{1}|(w_\alpha^l - w_\alpha^{max}|)}}{\sum_{i=1}^{n}\frac{\sum_{i=1}^{n}|(w_\alpha^r - w_\alpha^{min})|}{\sum_{\alpha=0.1}^{1}|(w_\alpha^r - w_\alpha^{min}|) - \sum_{\alpha=0.1}^{1}|(w_\alpha^l - w_\alpha^{max}|)}},
\quad i = 1, ..., n, \quad (3.2)
$$

where the denominator guarantees normalization condition. Briefly, the following algorithm is proposed to generate the normalized crisp weights from FCMs using the TLGP model:

1. **1.** Convert the FCM (1) into ICM (2) by using Eq. 3.1 for all $i, j = 1, ..., n$.

2. **2.** Obtain J^* by using Model (2) for all alpha-cuts. If J^* is near to zero, the comparison matrix is perfectly consistent, then go to step 3, otherwise correct the comparisons in FCM (1) and go to the first step again.

3. **3.** Obtain the interval weights by using Model (1) for all α=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9,1 for each criterion i (alternative i) $(i = 1, ..., n)$.

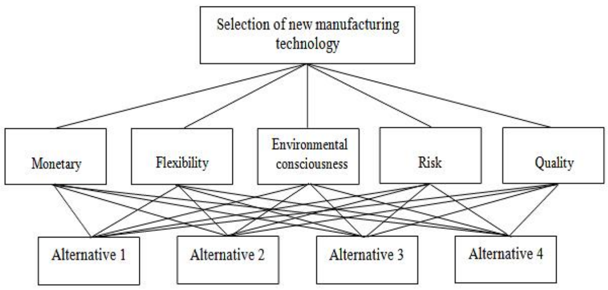

Figure 2: The hierarchical structure of FAHP for optimal manufacturing technology selection.

4. Convert the interval weights extracted by step 3 into a normalized crisp weight for each criterion i (alternative i)($i = 1, ..., n$) by using Eq. 3.2.

In the next section, application of our approach is presented to four real examples and the crisp weights are compared with the weights generated by using Model (3) [22]:

3 The illustrative examples

In order to show the applications of NLDM, the authors present four examples in the literature as scientific evidences for resolving the discrepancies of TLGP stated by Jaganathan et al. [94].

Example 3.1. *Consider the MCDM problem of new manufacturing technology selection presented by Jaganathan et al. [94]. The hierarchy is constructed in such a way that the overall decision goal is at the top level, five decision criteria are in the middle level, and four decision alternatives lie at the bottom level, as shown in Fig. 1. The DM is then asked to express his/her evaluation in terms of triangular fuzzy numerical values by comparing the decision elements (criteria or alternatives) at each level. Table presents the fuzzy scales used to DMs evaluation in order to construct FCM in FAHP. The value of can is selected 0.25, 0.5, 0.75, or 1. However, according to Jaganathan et al. [94], the DM is arbitrary for selecting the fuzzy scales in addition to the fuzzy scales presented in Table 3. This issue can reduce the failure rates in FAHP.*

Here, NLDM is only adopted to a FCM in which criteria (in middle level) are compared, as presented in Table 4. The interested reader can refer to Jaganathan et al. [94] for showing other FCMs, which compare alternatives as pairwise with respect to each criterion.

As shown in Table , five criteria: monetary, flexibility, environmental consciousness, risk, and quality are evaluated as pairwise according to DMs standpoints using the triangular fuzzy numbers. By transforming the triangular numbers in Table into ICM by using the alpha-cut representations created by using Eq. 3.1 (see Table), the lower and upper weights are obtained for each criterion by solving the Model (1) at eleven alpha-cuts, i.e.α=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9,1, as shown in Table .

In next step, in order to generate the normalized crisp weights for each criterion, $NLDM_i(i = 1, \ldots, n)$ is calculated by using Eq. 3.2. These measures are presented in Table . Moreover, the normalized crisp weights by FPP (the last row in Table) are determined in order to compare with weights derived by our approach.

Example 3.2. *Consider the following FCM, which has been examined by Wang et al. [87]:*

The above FCM can be transformed into the following ICM by using alpha-cut representation:

For recent ICM, the lower, and upper weights are obtained by solving the Model (1) at seven alpha-cuts, i.e. α=0.0, 0.2, 0.4, 0.6, 0.8, 1.0 and are written in Table 7. The weights extracted by the index and FPP are presented in Table 7 as well.

Example 3.3. *Consider FCM presented by Biswas et al. [95] as follows:*

According to Table 8, four criteria: profitability, initial investment, raw material availability, and buyer demand are assessed as pairwise according to DMs standpoints. By transforming the triangular numbers in Table 8 into the ICM (2) by using Eq. (1.3) (see Table 9), the lower and upper weights for each criterion are determined by solving the Model (1) at eleven alpha-cuts, as represented in Table 10. In addition, $NLDM_i(i = 1, \ldots, n)$ is calculated by using Eq. (2.3), as shown in Table 10. In order to compare our results with those than FPP, they have also been presented in this table.

Table 3: Fuzzy scales for constructing PCM in FAHP.

Fuzzy scale	Definition
$\tilde{1}=(\tilde{1}-\delta,\tilde{1}+\delta)$	Equal importance of one element over another
$\tilde{3}=(\tilde{3}-\delta,\tilde{3}+\delta)$	Weak importance of one element over another
$\tilde{5}=\tilde{5}-\delta,\tilde{5}+\delta)$	Strong importance of one element over another
$\tilde{7}=(\tilde{7}-\delta,\tilde{7}+\delta)$	Very strong importance of one element over another
$\tilde{9}=(\tilde{9}-\delta,\tilde{9}+\delta)$	Absolute importance of one element over another
$\tilde{2},\tilde{4},\tilde{6},\tilde{8}$	Intermediate values
Reciprocals	Reciprocals for inverse comparison

Table 4: FCM of criteria.

	Monetary	Flexibility	Environmental consciousness	Risk	Quality
Monetary	(1,1,1)	(1/6,1/5,1/4)	(1,2,3)	(2/5,1/2,2/3)	(2/7,1/3,2/5)
Flexibility		(1,1,1)	(7,8,9)	(5/2,3,7/2)	(1,2,3)
Environmental consciousness			(1,1,1)	(1/4,1/3,1/2)	(1/8,1/7,1/6)
Risk				(1,1,1)	(1/2,2/3,1)
Quality					(1,1,1)

Table 5: ICM.

	Monetary	Flexibility	Environmental consciousness	Risk	Quality
Monetary	1	$(1/6+0.03\alpha,\frac{1}{4}-0.05\alpha)$	$(1+\alpha,3-\alpha)$	$(2/5+0.1\alpha,2/3-0.16\alpha)$	$(2/7+0.04\alpha,2/5-0.06\alpha)$
Flexibility		1	$(7+\alpha,9-\alpha)$	$(5/2+0.5\alpha,7/2-0.5\alpha)$	$(1+\alpha,3-\alpha)$
Environmental consciousness			1	$(1/4+0.08\alpha,1/2-0.16\alpha)$	$(1/8+0.01\alpha,1/6-0.02\alpha)$
Risk				1	$(1/2+0.16\alpha,1-0.33\alpha)$
Quality					1

Table 5 Continue

Example 3.4. *Consider the following FCM constructed by Ayhan [96]:*

where five criteria: quality, origin, cost, delivery, and after sales are assessed as pairwise according to DMs preferences through the triangular fuzzy numbers. Table 12 presents ICM (2) at all the alpha-cuts by using Eq. 3.1. Moreover, the lower and upper weights for each criterion are obtained by solving the Model (1) at eleven alpha-cut levels, as presented in Table 13. Now,$NLDM_i(i=1,\ldots,n)$ is calculated by using Eq. 3.2. These measures are presented in Table 13. In order to evaluate the effectiveness of the proposed approach, the normalized crisp weights

Table 6: Weights obtained by Wang et al. [87], our approach, and comparison with Mikhailov-model [22].

Alpha	$[w_1^L, w_1^U]$	$[w_2^L, w_2^U]$	$[w_3^L, w_3^U]$	$[w_4^L, w_4^U]$	$[w_5^L, w_5^U]$
0.0	[0.527,0.675]	[2.365,2.920]	[0.276,0.341]	[0.944,1.024]	[1.748,2.165]
0.1	[0.544,0.658]	[2.450,2.840]	[0.285,0.330]	[0.976,1.150]	[1.798,2.086]
0.2	[0.573,0.642]	[2.542,2.738]	[0.294,0.319]	[1.007,1.101]	[1.850,2.012]
0.3	[0.580,0.621]	[2.637,2.690]	[0.303,0.309]	[1.048,1.070]	[1.904,1.942]
0.4	[0.572,0.627]	[2.644,2.775]	[0.297,0.319]	[1.003,1.085]	[1.840,1.940]
0.5	[0.561,0.616]	[2.708,2.843]	[0.305,0.330]	[1.032,1.079]	[1.781,1.872]
0.6	[0.552,0.599]	[2.788,2.910]	[0.314,0.341]	[1.033,1.074]	[1.729,1.804]
0.7	[0.559,0.581]	[2.885,2.963]	[0.324,0.338]	[1.041,1.067]	[1.693,1.739]
0.8	[0.575,0.582]	[2.979,3.016]	[0.322,0.333]	[1.045,1.060]	[1.659,1.678]
0.9	[0.551,0.604]	[2.859,3.189]	[0.300,0.349]	[1.014,1.111]	[1.592,1.743]
1.0	[0.529,0.625]	[2.782,3.365]	[0.280,0.361]	[0.984,1.163]	[1.531,1.810]
NI_i	0.0651	0.4749	0.0110	0.1528	0.2963
Weights derived by FPP	0.0871	0.4141	0.0472	0.1582	0.2934

Table 7: Weights obtained by Wang et al. [87], our approach, and comparison with Mikhailov-model [22].

Alpha	$[w_1^L, w_1^U]$	$[w_2^L, w_2^U]$	$[w_3^L, w_3^U]$	$[w_4^L, w_4^U]$
0.0	[2.328,3.408]	[1.000,1.809]	[0.553,1.000]	[0.293,0.429]
0.2	[2.424,3.282]	[1.087,1.730]	[0.578,0.919]	[0.304,0.412]
0.4	[2.522,3.162]	[1.171,1.651]	[0.605,0.853]	[0.316,0.396]
0.5	[2.571,3.104]	[1.212,1.611]	[0.620,0.824]	[0.322,0.388]
0.6	[2.621,3.047]	[1.253,1.572]	[0.636,0.797]	[0.328,0.381]
0.8	[2.723,2.936]	[1.334,1.493]	[0.669,0.749]	[0.340,0.367]
1.0	[2.828,2.828]	[1.414,1.414]	[0.707,0.707]	[0.353,0.353]
NI_i	0.4688	0.2229	0.0970	0.0180
Weights derived by FPP	0.5233	0.2666	0.1333	0.0666

Table 8: Weights obtained by Wang et al. [87], our approach, and comparison with Mikhailov-model [22].

	Profitability	Initial investment	Raw material availability	Buyer demand
Profitability	(1,1,1)	(3/2,2,5/2)	(2/3,1,3/2)	(1,1,1)
Initial investment	(2/5,1/2,2/3)	(1,1,1)	(2/3,1,3/2)	(2/3,1,3/2)
Raw material availability	(2/3,1,3/2)	(2/3,1,3/2)	(1,1,1)	(3/2,2,5/2)
Buyer demand	(1,1,1)	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)

Table 9: Weights obtained by Wang et al. [87], our approach, and comparison with Mikhailov-model [22].

	Profitability	Initial investment	Raw material availability	Buyer demand
Profitability	1	$(3/2+0.5\alpha, 5/2-0.5\alpha)$	$(2/3+0.33\alpha, 3/2-0.5\alpha)$	(1,1)
Initial investment		1	$(2/3+0.33\alpha, 3/2-0.5\alpha)$	$(2/3+0.33\alpha, 3/2-0.5\alpha)$
Raw material availability			1	$(3/2+0.5\alpha, 5/2-0.5\alpha)$
Buyer demand				1

by FPP (the last row in Table 13) and the weights obtained by Ayhan [96] approach have also been presented in the above table. Obviously, the similar ranking order have obtained based on all approaches.

4 Discussion and conclusion

In this paper, NLDM was proposed to resolve the discrepancies stated by Jaganathan et al. [94] when they adopted TLGP for generating the crisp

Table 10: Weights obtained by Wang et al. [87], our approach, and comparison with Mikhailov-model [22].

Alpha	$[w_1^L, w_1^U]$	$[w_2^L, w_2^U]$	$[w_3^L, w_3^U]$	$[w_4^L, w_4^U]$
0.0	[0.907,1.180]	[0.689,0.898]	[1.123,1.395]	[0.872,1.100]
0.1	[0.915,1.228]	[0.698,0.916]	[1.080,1.337]	[0.845,1.089]
0.2	[0.926,1.289]	[0.729,0.930]	[1.061,1.312]	[0.816,1.072]
0.3	[0.936,1.348]	[0.696,0.944]	[1.036,1.340]	[0.785,1.065]
0.4	[0.951,1.410]	[0.718,0.961]	[1.034,1.317]	[0.758,1.052]
0.5	[0.968,1.450]	[0.706,0.977]	[1.018,1.211]	[0.743,1.043]
0.6	[0.985,1.316]	[0.716,0.999]	[1.001,1.211]	[0.718,1.023]
0.7	[1.006,1.371]	[0.721,0.995]	[1.002,1.336]	[0.717,1.006]
0.8	[1.019,1.158]	[0.771,0.988]	[1.012,1.345]	[0.765,0.992]
0.9	[1.032,1.106]	[0.888,0.975]	[1.028,1.084]	[0.904,0.973]
1.0	[1.052,1.069]	[0.938,0.946]	[1.055,1.066]	[0.938,0.949]
NI_i	0.3110	0.1577	0.3332	0.1978
Weights derived by FPP	0.2552	0.1850	0.3272	0.2303

Table 11: FCM.

	Quality	Origin	Cost	Delivery
After sales				
Quality	(1,1,1)	(1,1,1)	(4,5,6)	(6,7,8)
(4,5,6)				
Origin	(1,1,1)	(1,1,1)	(4,5,6)	(6,7,8)
(6,7,8)				
Cost	(1/6,1/5,1/4)	(1/6,1/5,1/4)	(1,1,1)	(1/4,1/3,1/2)
(2,3,4)				
Delivery	(1/8,1/7,1/6)	(1/8,1/7,1/6)	(2,3,4)	(1,1,1)
(1/6,1/5,1/4)				
After sales	(1/6,1/5,1/4)	(1/8,1/7,1/6)	(1/4,1/3,1/2)	(4,5,6)
(1,1,1)				

Table 12: ICM.

	Quality	Origin	Cost	Delivery
After sales				
Quality	1	(1,1)	$(4+\alpha,6-\alpha)$	$(6+\alpha,8-\alpha)$
$(4+\alpha,6-\alpha)$				
Origin		1	$(4+\alpha,6-\alpha)$	$(6+\alpha,8-\alpha)$
$(6+\alpha,8-\alpha)$				
Cost			1	$(1/4+0.083\alpha,1/2-0.166\alpha)$
$(2+\alpha,4-\alpha)$				
Delivery				1
$(1/6+0.033\alpha,1/4-0.05\alpha)$				
After sales				
1				

weights from FCMs. They couldnt obtain the crisp weights for the pairwise comparisons presented in Table 4 using Model (1) and proposed the Mikhailov nonlinear programming model [22]. In order to show the effectiveness of the suggested approach, our NLDM was then applied to four real examples in the literature.

In example 3.1, the five normalized crisp weights obtained by our approach are 0.0651, 0.4749, 0.0110, 0.1528 and 0.2963, respectively, as shown in Table 6. By comparing the crisp weights determined by our NLDM with those obtained by

Table 13: Weights obtained by the different approaches.

Alpha	$[w_1^L, w_1^U]$	$[w_2^L, w_2^U]$	$[w_3^L, w_3^U]$	$[w_4^L, w_4^U]$	$[w_5^L, w_5^U]$
0.0	[2.704,2.718]	[2.718,2.718]	[0.535,0.557]	[0.405,0.413]	[0.595,0.605]
0.1	[2.704,2.718]	[2.718,2.718]	[0.535,0.557]	[0.405,0.413]	[0.595,0.605]
0.2	[2.705,2.718]	[2.715,2.718]	[0.509,0.525]	[0.409,0.413]	[0.626,0.629]
0.3	[2.707,2.718]	[2.715,2.718]	[0.518,0.520]	[0.412,0.414]	[0.628,0.631]
0.4	[2.718,2.718]	[2.717,2.718]	[0.519,0.521]	[0.415,0.416]	[0.623,0.625]
0.5	[2.715,2.718]	[2.716,2.718]	[0.519,0.521]	[0.417,0.418]	[0.620,0.621]
0.6	[2.718,2.718]	[2.718,2.718]	[0.519,0.521]	[0.418,0.420]	[0.616,0.618]
0.7	[2.718,2.718]	[2.718,2.718]	[0.518,0.520]	[0.421,0.422]	[0.612,0.614]
0.8	[2.713,2.718]	[2.718,2.718]	[0.518,0.519]	[0.424,0.426]	[0.610,0.612]
0.9	[2.711,2.718]	[2.707,2.714]	[0.516,0.518]	[0.428,0.429]	[0.619,0.611]
1.0	[2.692,2.717]	[2.651,2.707]	[0.514,0.519]	[0.435,0.436]	[0.610,0.613]
NI_i	0.4640	0.4650	0.0245	0.0030	0.0425
FPP	0.3922	0.3998	0.0646	0.0517	0.0841
Ayhan [96]	0.3830	0.4090	0.0740	0.0570	0.0760

Table 14: Weights obtained by the different approaches.

The difference between two consecutive weights by NLDM	The difference between two consecutive weights by FPP
w˙1-w˙2=0.2459	w˙1-w˙2=0.2567
w˙2-w˙3=0.1259	w˙2-w˙3=0.1333
w˙3-w˙4=0.0790	w˙3-w˙4=0.0677

Mikhailov-model [22], the authors note that the crisp weights of criteria 1 and 3 have the variations based on the reasons discussed below, but, crisp weights of criteria 2, 4 and 5 have the partial differences. Moreover, in example , the normalized crisp weights generated by our index are 0.4688, 0.2229, 0.0970, and 0.0180, respectively, as shown in Table 7. Although, these weights have the variations with those given by FPP (which are 0.5233, 0.2666, 0.1333, and 0.0666, respectively), the ranking order by three models are identical (the ranking order given by TLGP is as $w_1 \overset{100}{\succ} w_2 \overset{100}{\succ} \ldots \overset{100}{\succ} w_N$, where $\overset{100}{\succ}$ shows the degree of preference a weight over next weight). Obviously, the ranking order by TLGP provides the information about the degrees of preference of weights but it cannot determine normalized crisp weights for each element. On the other hand, the difference between each two consecutive weights based on the above ranking order by NLDM and FPP are as follows: As shown in above table, the above variations are almost similar based on both models. There are similar arguments for examples example and . In other words, although these differences of weights influence total score of a criterion (alternative), it does not change the ranking order of criteria (alternatives) when application to MCDM techniques. FPP generates the weights of criteria using a nonlinear optimization model. It determines the crisp weights and the consistency index from FCMs simultaneously. But, the second approach (TLGP) is based on a discrete multiple stage methodology as follows: (1) transforming FCMs into the ICMs using alphacut transformations; (2) attaining J^* by incorporating the interval numbers into Model (2); (3) obtaining the interval weights by using Model (1); and (4) converting the interval weights into the crisp weights by NLDM. Unlike the first approach, the normalized crisp weights are then determined in several stages. Thus, this point can be concluded which differences in the number of stages, the nature of generating the crisp weights of two methodologies and type of computational tool or software result in errors arisen from rounding the decimal part of weights. It proves the variations of two weights obtained for each criterion as explained above. In spite of such differences, the ranking order of criteria and the degree of importance between them for both models are almost the same. This argument verifies the effectiveness of the proposed approach for obtaining the crisp weights. The advantage of the proposed index is that it determines crisp weights from the interval weight generated by TLGP. Many approaches and optimization models exist where the weights of criteria and/or the evaluation measures of alternatives with respect to criteria are determined in the interval form at alpha cuts. According to our index, these weights can be transformed into the crisp measures. It is usable to each number alternatives and criteria in the different decision-making problems and also, it calculates the similar ranking order for criteria weights.

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