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Research Article

Effects of Nonstationary and Uncertain Demand on an Inventory System Under Belief Structure Condition

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Abstract

This paper presents a new inventory model for deteriorating items, allowable shortages, and stochastic inflationary conditions, considering a belief-rule-base inventory control (BRB-IC) method. This is a new insight in comparison with the previous research, which considers a belief-rule-based inventory control under nonstationary demand. The Genetic Algorithm and exact methods have been considered for minimizing the objective function. The numerical example and a sensitivity analysis provided to illustrate the theoretical results.

Keywords : Inventory; Nonstationary Demand; Uncertainty; Evidential Reasoning; Belief Rule Base.

1 Introduction

From the emergence of classical studies in the field of scientific management which dates back to the first and second decades of last century, the topic of inventory controlling and how to decide on that, was one of the most significant concerns of the managers and until now, extensive efforts have been made in this field and various models for inventory controlling have been presented (Fareghian & Rahimzadeh, 2017)[1]. Most of the models which were presented at early stages, were relatively simple and as far as possible, they tried to simplify the facts and also they assumed the parameters of the models as deterministic quantities but the real world is complicated and attaining the certain and deterministic data, is a daunting task.

The majority of inventory controlling problems, take place in industrial, distribution and service areas, where the demand is dynamic and also nondeterministic (Treharne & Sox, 2002) [2]. The reasons for dynamic demand are: (1.1) a multistage of product life-cycle (Metan & Thiele, 2008 [3]; Song & Zipkin, 1993 [4]), (1.2) technological advances and breakthroughs and short products life (Bertsimas & Thiele, 2006a [5]; Giannoccaro, Pontrandolfo [6], & Scozzi, 2003 [7]; Song & Zipkin, 1993) [4], (1.3) impacts related to the seasons (Zipkin, 1989 [40]), (1.4) unstable desires of customers (Bertsimas & Thiele, 2006a [5]), (1.5) un-

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steady economic conditions (Song & Zipkin, 1993 [4]) and (1.6) exchange rate changes (Scheller-Wolf & Tayur, 1997) [8]. It is impossible to avoid, that the future demand will origin from a distribution that is different from what governs historical demand (Scarf, 1958) [9].

In the past, researches and studies in the inventory controlling with dynamic and unsteady demand merely concentrated on stochastic methodologies with specific demand models. For example, in the sequential periods, demands were specified by various well-known distributions (Bollapragada & Morton, 1999 [10]; Gavirneni & Tayur, 2001 [11]; Karlin, 1960 [12]; Morton, 1978 [13]; Tarim & Kingsman, 2006[14]; Veinott, 1966 [15]), dynamic Markov decision processes (Iida, 1999 [16]; Song & Zipkin, 1993 [4]; Treharne & Sox, 2002 [2]) and autoregressive, moving average or mixed autoregressive-moving average processes (Johnson & Thompson, 1975 [17]; Lee, Padmanabhan, & Whang, 1997 [18]; Lee, So, Tang, 2000 [19]; Raghunathan, 2001 [20]. Also, an innovative merged framework for forecasting and inventory management for short-cycle products was provided by Kurawarwala and Matsuo (1996) [21] for the first time. Also by using stochastic methodologies, demand-price related problems, were modeled and formulated (Federgruen & Heching, 1999 [22]; Gallego & van Ryzin, 1994 [23]). (Ghoreishi, Mirzazadeh, Weber& Nakhai-Kamalabadi, 2015 [24]) developed an economic order quantity model for non-instantaneous deteriorating items with selling price and inflation induced demand under the effect of inflation and customer returns. In which the customer returns are assumed as a function of demand and price. Also, (Gholami, Mirzazadeh, 2018 [25]) studied the inventory management literature regarding the models with controllable lead time, and observed that an accurate demand distribution is often skewed to the right for many items and fitting the normal distribution to the random demand may cause a great financial loss for an inventory system. (Kouki, Zied Babai, Jemai, and Minner, 2018 [26]) proposed a continuous-review full-lost sale base stock inventory model in which uncertain demand follows the compound Poisson distribution. Also, (Kim and Sarkar, 2017 [27] proposed a joint replenishment inventory model with multistage quality improvement and lead time-dependent ordering cost.

Although, in practical and real inventory problems, attaining the precise knowledge about the demand such as stochastic distribution and time series data, is really hard and not realistic (Bertsimas & Thiele, 2006a[5]; Petrovic, Petrovic, & Vujosevic, 1996 [28]). It is obvious that, on the basis of unrealistic assumptions, inventory controlling strategies, do not have any result except poor and not effective performances. Therefore, there is a strong need and desire to devise new alternative non-probabilistic inventory control strategies with slight and limited data. By completely ignoring the stochastic hypotheses, Fuzzy mathematical programming (Dey & Chakraborty, 2009 [29]; Li, Kabadi, & Nair, 2002 [30]; Petrovic et al., 1996 [28]; Roy & Maiti, 1997 [31]; Yao & Su, 2000 [32]) and Robust counterpart optimization (RCO) (Bertsimas & Thiele, 2006a [5], 2006b [33]) have been studied to deal with uncertain inventory problems. In these studies, with neglecting historical demand, a set of forecasting demands is modeled in forms of fuzzy sets or intervals and so optimal decisions and policies, are acquired on the foundation of the finite future planning periods. Also, systems based on fuzzy logic are suggested to work out inventory control problems with fuzzy forecasting demand (Hung Fang, Nuttle, & King, 1997 [34]; Kamal & Sculfort, 2007 [35]; Leung, Lau, & Kwong, 2003 [36]), but the fuzzy rules mentioned in the related literature have a big problem. They just considered qualitative expert knowledge without surveying quantitative expert knowledge and historical demand. Another way of conventional approximation approach is to change completely the indistinct and non-deterministic forecasting demand into single-point value based on the data and preferences of the manager (Gen, Tsujimura, & Zheng, 1997 [7]), then use stochastic approximation approach to create and develop "optimal" inventory control policies. But this approach has a drawback, it does not take account of demand uncertainty fully and therefore, mentioned optimal policies are not very reliable and trustworthy. Also (A. Mirzazadeh, 2011 [37]) stated that the inventory models, generally, are derived with considering two methods: minimizing the average annual cost, and minimizing the discounted cost. Then he compares the optimal ordering policies determined by these methods under uncertain inflationary situations. (Sheng, Zhu & Wang, 2018 [38]) considered a production-inventory problem involving uncertain data with constraints that the production rates are restricted to an appropriate interval. (Prak and Teunter, 2019 [39]) proposed a framework for addressing forecasting uncertainty that is applicable to any inventory model, demand distribution, and parameter estimator. (Ganyakmaz, Ozekici & Fikri, 2019 [40]) proposed and investigate a multi-period, single item, periodic-review inventory control model where they explicitly model a continuous-time stochastic input price process which determines both purchase and selling prices and consequently influences the customer demand.

Moreover, except dynamic and non-deterministic demand, the majority of the models in the literature, are based on the definite planning horizon into the future which leads to that, the quantity of order for the forthcoming period can be considerably affected by forecasts for far periods and it is obvious that, for the distant periods, forecasting is hardly and rarely reliable (Mellichamp & Love, 1978 [41]). So based on these drawbacks and considerations, we propose a belief-rule-based inventory control (BRB-IC) method according to the current inventory, historical demand data, and necessary and obligatory short-term forecasting demand. The method is developed and formed from the decision support mechanism of belief-rule-based inference methodology –RIMER (Yang, Liu, Wang, Sii, & Wang, 2006 [42]; Yang, Liu, Xu, Wang, & Wang, 2007 [43]) which is built and derived based on the evidential reasoning (ER) approach (Yang, 2001; Yang & Sen, 1994 [44]; Yang& Singh, 1994 [44]; Yang & Xu, 2002a [45]; Yang & Xu, 2002b [46]) and rule-based expert system. RIMER is a modeling and inference program under uncertain and non-deterministic conditions which is combined with a belief-rule structure. It has been applied in many cases such as graphite content detection (Yang et al., 2006, 2007 [42]), pipeline leak detection (Chen, Yang, Xu, Zhou, & Tang, 2011 [47]; Xu et al., 2007 [43]; Zhou, Hu, Xu, Yang, & Zhou, 2011 [48]; Zhou, Hu, Yang, Xu Zhou, 2009 [49]), clinical guideline (Kong, Xu, Liu, & Yang, 2009 [18]), nuclear safeguards assessment (Liu, Ruan, Wang, & Martinez, 2009 [50]), new product development and evolvement (Tang, Yang, Chin, Wong, & Liu, 2011 [51]), system sustainability and reliability prediction (Hu, Si, & Yang, 2010 [52]), and gyroscopic drift prediction (Si, Hu, Yang, & Zhang, 2011 [38]). It is noteworthy that the BRB-IC method can build a knowledge-based framework for inventory control and also can deal with different kinds of uncertain data. It can help experts and decision-makers to change the construction of belief-rule-base and also update and improve it by using their judgmental knowledge. Also, this method is easy for implementation and understanding while it does not need heavy computational efforts. (Nodoust, Mirzazadeh & Weber, 2017 [53] assumed this kind of inflation in an inventory system which is a production inspection system with returning unsatisfying items for being reworked. (Zhou, Dou, Sun, Jiang& Tan, 2017 [54]) proposed a sustainable decision-making model is proposed for the evaluation of engine manufacturing. (Zichang He& Wen Jiang, 2018 [55]) propose a new belief Markov chain model combining Dempster-Shafer evidence theory and The discrete-time Markov chain. In their model, the uncertain data are allowed to be handled in the form of interval number, and the basic probability assignment is generated by an optimization method based on the distance between interval numbers. (Liu, Deng& Chan, 2018 [56]) propose a systematic method to select a supplier. ANP and entropy weight is adapted to calculate subjective and objective weight separately. In order to get a comprehensive weight, DEMATEL and game theory are combined. They used Dempster-Shafer theory to deal with the uncertainties of input data and get the best supplier. (Zichang He& Wen Jiang,2018 [55]) proposed an evidential Markov (EM) decision-making model based on Dempster- Shafer evidence theory and Markov modeling to address this issue and to model the real human decision-making process maker. (Mi Zhou, Xin-Bao Liu, Yu-Wang Chen, Jian-Bo Yang, 2018 [57]) suggested the ER rule is generalized to deal with Multiple attribute decision-making problems in group decision-making circumstance where the weights and reliabilities of both experts and attributes are considered. (Asoke Kumar Bhunia et al, 2017 [58]) Developed an economic production lot size model for a production-inventory system of manufacturing firm which consists of three departments (production, Marketing, and Research& Development). They assumed that demand is to be dependent on both the selling price and marketing cost of the product. (AbuDahab, Xu& Chen, 2016 [59]) They extended the original belief rule-based inference methodology using the evidential reasoning approach by introducing generalized belief rules as knowledge representation scheme and using the evidential reasoning rule for evidence combination in the rule-based inference methodology instead of the evidential reasoning approach. In this paper, we have assumed nonstationary and uncertain demand. Also, a detailed analysis has been done for surveying the effect of uncertain inflationary conditions on the optimal ordering policy under stochastic inflationary conditions and arbitrary probability density functions for the internal and external inflation rates. Deteriorating items and shortages have been considered. Since the definitive method was a defect, we used the genetic method. Because the demand values obtained from the model are in the interval, to solve a definite method, the value of the function is also calculated as an interval. In order to obtain our definitive number, we have calculated the average demand and based on that the value of the function was calculated. For this reason, we used the genetic method to solve this problem and after calculations, it turned out that the results obtained from the genetic method are better than the definitive method, which is shown in the following sections. A numerical example and a sensitivity analysis are used to illustrate the model. The rest of the paper is organized as follows. The assumptions and notations of the inventory model considered are described in section 2. The concept of belief structure and ER approach are presented in section 3. Section 4 represents the formulation and description of the proposed inventory model. Section 5 represents the solution procedure. The results of parameter tuning and numerical examples are presented than in section 6. In section 7 we presented the sensitivity analysis in order to show the validity of the algorithm.

1. **Assumptions and Notations**

   (a) **Assumptions:**

The mathematical model in this paper is developed based on the following assumptions:

1. The demand rate is uncertain as a belief structure.
2. Lead time is negligible. Also, the initial and final inventory level is zero.
3. The Shortage is allowed and fully backlogged except for the final cycle.
4. The replenishment is instantaneous and lead time is zero.
5. The initial inventory level is zero.
6. The system operates for prescribed time-horizon of length H.
7. A constant fraction of the on-hand inventory deteriorates per unit time, as soon as the item is received into inventory.

1. (a) **Notations:**

Also, the following notations are used:
$ETVC(n,k)$: The total present value of costs over the time horizon;
$TI(n,k)$: The total quantity of goods in warehouse over time horizon;
T: The interval of time between replenishment;
k: The proportion of time in any given inventory cycle which orders can be filled from the existing stock;

n: The number of replenishments during time horizon;
i_m: Internal (for m=1) and external (for m=2) inflation rates;
$f(i_m)$: The pdf of i_m;
r: The discount rate;
D: The demand rate per unit time;
A: The ordering cost per order at time zero;
c_{Im}: The internal (for m=1) and external (for m=2) inventory carrying cost (for I=1) and shortage cost (for I=2) per unit time at time zero;
p: The external purchase cost at time zero;
θ: The constant deterioration rate;
$M_{im}(Y)$: The moment generating function of i_m for m=1 and m=2;
H: The fixed time horizon;
Other notations will be introduced later. It is assumed that the length of the planning horizon is$H$=nT, Where, n is an integer for the number of replenishments to be made during period$H$and $T$is an interval of time between replenishments. The unit of time can be considered as a year, a month, a week, etc. and $k(0 \prec k \leq 1)$ is the proportion of time in any given inventory cycle which orders can be filled from the existing stock. Thus, during the time interval$[(j-1)T, jT]$, the inventory level leads to zero and shortages occur at the time$(j+k-1)T$. Shortages are accumulated until jTbefore they are back ordered and are not allowed in the last replenishment cycle. The optimal inventory policy yields the ordering and shortage points, which minimize the total expected inventory cost over the time horizon.

1. Belief- rule- based inventory control method

Most of the traditional models for inventory control are based on a definite planning horizon into the future. These methods consistently incorporate the tacit assumption that demand forecasts are relatively accurate, which results in that the order quantity for the future period can be significantly affected by forecasts for distant periods. However, the forecasting for distant periods is hardly reliable. Inventory strategies should well be based on the current inventory, historical demand information and necessary short- term forecasting demand. Besides, there is a need to develop an inventory control model that does not rely on the stochastic hypothesis and meanwhile can handle various types of uncertain information common in real life inventory control processes. With those considerations, we propose a belief-rule-based inventory control method which inherits the information processing ability of RIMER and allows the incorporation of human knowledge and historical demand information to derive reliable inventory control strategy. The belief-rule-based expert system is a deployment of traditional rule-based systems and is capable of representing more complicated causal relationship and handling different types of uncertain information.

3.1. Demand assessment

In this paper, the case of having an evaluation of many decision makers about the demand rate is studied. Each member mentions an idea with a belief degree. For better understanding, define a set of M decision makers as follows:

$$D = \{D_1 D_2 D_3 ... D_n ... D_M\}. \qquad (1.1)$$

Suppose the relative weights of the decision makers wherew_i is the relative weight for$D_i$and is normalized which$0 \prec w_i \prec 1$ an equation (1.2) is fulfilled.

$$\sum_{i=1}^{M} w_i = 1. \qquad (1.2)$$

TABLE 1. Belief structure.

	D_M	...	D_i	...	D_1
H_1	$\beta_{1,1}$	...	$\beta_{1,i}$	...	$\beta_{1,M}$
...	...	...	...	...	...
H_n	$\beta_{n,1}$	...	$\beta_{n,i}$	...	$\beta_{n,M}$
...	...	...	...	...	...
H_N	$\beta_{N,1}$	...	$\beta_{N,i}$	...	$\beta_{N,M}$

Define N unique values H_i as probable values of inflation, presented by decision makers.

$$H = \{H_1 H_2 H_3 ... H_i ... H_N\} \qquad (1.3)$$

$\beta_{n,i}$ Shows the probable degree that nth decision maker assigns to the ith value of inflation. So $\beta_{n,i}$ is a belief degree that:

$$\sum_{n=1}^{N} \beta_{n,i} \leq 1, \beta_{n,i} \geq 0. \qquad (1.4)$$

The evaluation is considered to be complete if:

$$\sum_{n=1}^{N} \beta_{n,i} = 1. \quad (1.5)$$

And it is called to be incomplete if:

$$\sum_{n=1}^{N} \beta_{n,i} \prec 1 \quad (1.6)$$

For example, decision maker D_n shows that the rate of inflation is $H_i$by the belief degree of $\beta_{n,i}$ and so on each member gives a slight degree for each value of a set H. Table 1 has been built to show this fact. The components of the first column are probable values of inflation and the first row has the members of the decision maker team. $\beta_{n,i}$ denotes the probable degree that the nth decision maker assigns to the$i$th value of inflation. Each member can specify his numerical opinion in any way, complete or incomplete. The team members can also say that they do not have any specific idea about someone. This condition leads to an incomplete evaluation since in this condition the sum of the belief degrees for the member is not equal to 1. In the cases which sum of all columns is equal to 1, we have the complete assessment. Decision making under such condition is with a high uncertainty because it is so hard to use inflation in solving the model with this structure. To deal with and solve this problem, a method based on Dempster-Shafer evidence theory is presented called Evidential Reasoning (ER) approach with aggregating these belief degrees with using the relative weights of decision makers and the probable values of inflation. The method is demonstrated in the next section.

3.2. Evidential Reasoning method

Let$m_{n,i}$ be a basic probability mass which is calculated as follows:

$$m_{n,i} = w_i \beta_{n,i}. \quad (1.7)$$

Let $m_{H,i}$ be the remaining probability mass unallocated to each decision makerD_i, $m_{H,i}$ is calculated as follows:

$$m_{H,i} = 1 - \sum_{n=1}^{N} m_{n,i} = 1 - w_i \sum_{n=1}^{N} \beta_{n,i}. \quad (1.8)$$

Decompose$m_{H,i}$, into$\tilde{m}_{H,i}$ and$\bar{m}_{H,i}$ as follows:

$$\bar{m}_{H,i} = 1 - w_i \quad (1.9)$$

$$\tilde{m}_{H,i} = w_i (1 - \sum_{n=1}^{N} \beta_{n,i}) \quad (1.10)$$

With:

$$m_{H,i} = \bar{m}_{H,i} + \tilde{m}_{H,i}. \quad (1.11)$$

The evaluation of decision makers that make the general property is aggregated to build a single assessment of a general property. The probability masses allocated to the different assessments as well as the probability mass left unallocated are denoted by:

$$m_{n,Q(L)}, (n = 1, ..., N), \bar{m}_{H,Q(L)}, \tilde{m}_{H,Q(L)}.$$

and:

$$m_{H,Q(L)}.$$

Let$Q(1.1) = 1$, then we get

$$m_{n,Q(1)} = m_{n,1}, (n = 1, ..., N), \bar{m}_{H,Q(1)} = \bar{m}_{H,1}, \tilde{m}_{H,Q(1)} = \tilde{m}_{H,1} \quad (1.12)$$

$$m_{H,Q(1)} = m_{H,1}. \quad (1.13)$$

The merged probability masses can be generated by aggregating all the probability assignments using the following recursive ER algorithm:

$$\{H_n\} : (n = 1, ..., N) m_{n,Q(i+1)} = v_{Q(i+1)} \left[m_{n,Q(i)} m_{n,i+1} + m_{H,Q(i)} m_{n,i+1} + m_{n,Q(i)} m_{H,i+1} \right]. \quad (1.14)$$

$$n = 1, ..., N. \quad (1.15)$$

We continue to let$i = 1$, which results in (1.15):

$$\{H\} :$$

$$m_{H,Q(i+1)} = \bar{m}_{H,Q(i+1)} + \tilde{m}_{H,Q(i+1)} \quad (1.16)$$

$$\tilde{m}_{H,Q(i+1)} = v_{Q(i+1)} \left[\tilde{m}_{H,Q(i)} \tilde{m}_{H,i+1} + \bar{m}_{H,Q(i)} \tilde{m}_{H,i+1} + \tilde{m}_{H,Q(i)} \bar{m}_{H,i+1} \right] \quad (1.17)$$

$$\bar{m}_{H,Q(i+1)} = v_{Q(i+1)} \left[\bar{m}_{H,Q(i)} \bar{m}_{H,i+1} \right] \quad (1.18)$$

$$v_{Q(i+1)} = \left[1 - \sum_{s=1}^{N} \sum_{j=1, j\neq s}^{N} m_{s,I(i)} m_{j,i+1}\right]^{-1}, i = \{1, 2, ..., M-1\} \quad (1.19)$$

$v_{Q(1.2)}$as calculated by (1.19) is used to normalize$m_{n,Q(1.2)}$, $m_{H,Q(1.2)}$so that

$$\sum_{n=1}^{N} m_{n,Q(2)} + m_{H,Q(2)} = 1. \quad (1.20)$$

Letβ_n denotes the combined degree of belief evaluated toH_n which is produced by merging the assessments for all the decision makers.$\beta_n$is then calculated by:

$$\{H_n\} : \quad \beta_n = \frac{m_{n,Q(M)}}{1 - \bar{m}_{H,Q(M)}}, (n = 1, ..., N) \quad (1.21)$$

$$\{H\} : \quad \beta_H = \frac{\tilde{m}_{H,Q(M)}}{1 - \bar{m}_{H,Q(M)}}. \quad (1.22)$$

Consider some relative importance for probable values of inflation which is called Utility functions. For instance,$u(H_n)(n = 1, ..., N)$ is the relative importance of$H_n$which is a number between 0 and 1.
Now, the utilities of $u(H_n)(n = 1, ..., N)$ are estimated via utility functions$u(H_n)$provided we have some belief$\beta_H$left unallocated in the assessments we can somehow arbitrarily calculate an utility interval for the quantities of$H_n$being evaluated. This interval is calculated as follows:

$$u_{\max} = (\beta_N + \beta_H)u(H_N) + \sum_{n=1}^{N-1} \beta_n u(H_n). \quad (1.23)$$

$$u_{\min} = (\beta_1 + \beta_H)u(H_1) + \sum_{n=2}^{N} \beta_n u(H_n). \quad (1.24)$$

$$u_{avg} = \frac{u_{\min} + u_{\max}}{2} \quad (1.25)$$

The inflation value after aggregating the decision makers ideas by ER approach is led to become an interval number called $[u_{\min}, u_{\max}]$.

Demonstrating the inflation by an interval number and using the interval arithmetic, the objective function for profit is altered to related multi-objective functions. These functions are minimized and solved by Maple software which is represented in the next section.

Under the hypotheses of complete assessment, the utility of values of H_n is calculated as a single point according to

$$u = \sum_{n=1}^{N} \beta_n u(H_n). \quad (1.26)$$

But the aim of this research is the incomplete assessment which deals with the interval inflation value. The inventory model, considered in this paper is shown in the next section.

1. **The mathematical formulation:**

The objectives of the problem can be explained as follows (A.Mirzazadeh, 2013):

1. Minimization of the expected present value of costs over the time horizon

$$MinZ_1 = ETVC(n, k) \quad (1.27)$$

Let ECP, ECH, ECS and ECR denote the expected present value of the purchasing, carrying, shortage and replenishment costs, respectively. The detailed analysis of each cost function is given as follows:

4.1. Expected present value of the purchasing cost

During any given period, the order quantity is consisting of both demand and deterioration for the relevant period excluding shortage part of the period and the amount required to satisfy the demand during the shortage period in the preceding time interval. For the jth cycle, $(j = 1, 2, ..., n-1)$the expected present value of the purchase cost can be formulated as follows:

$$ECP_{j-1} = E\left[pe^{-R_2(j-1)T}\int_{(j-1)T}^{(k+j-1)T} De^{\theta(t-(j-1)T)}dt + pe^{-R_2 jT}\int_{(k+j-1)T}^{jT} Ddt\right]$$
$$= E\left[pD\left[e^{-R_2(j-1)T}(e^{\theta kT}-1)/ + e^{-R_2 jT}(1-k)T\right]\right]$$
$$for: j = 1...n-1, R_2 = r - i_2. \quad (1.28)$$

The above equation can be rewritten as:

$$ECP_{j-1} = (pD/\theta)e^{(1-j)rT}(e^{\theta kT}-1)M_{i_2}((j-1)T) + pDT(1-k)e^{-rjT}M_{i_2}(jT)$$
$$for: j = 1, ..., n-1 \quad (1.29)$$

In the last period shortages are not allowable, therefore the expected present value of the purchase cost is:

$$ECP_{n-1} = E\left[pe^{-R_2(n-1)T}\int_{(n-1)T}^{nT} De^{\theta(t-(n-1)T)}dt\right]$$
$$= E\left[(pD/\theta)e^{-R_2(n-1)T}(e^{\theta T}-1)\right] \quad (1.30)$$

where$R_2 = r - i_2$. It can similarly be rewritten as

$$ECP_{n-1} = (pD/\theta)e^{-r(n-1)T}(e^{\theta T}-1)M_{i_2}((n-1)T) \quad (1.31)$$

Therefore, the total purchase cost for all cycles can be written as follows:

$$ECP = ECP_{n-1} + \sum_{j=1}^{n-1} ECP_{j-1} \quad (1.32)$$

4.2. Expected present value of the inventory cost

The inventory carrying cost is divided into internal (for m=1) and external (for m=2) classes. The carrying cost for the jth cycle ($j = 1, 2, ..., n-1$) for the mth class (m=1,2) is:

$$ECH_{jm} = E[c_{1m}\int_{(j-1)T}^{(k+j-1)T}(t-(j-1)T)De^{-R_m t}e^{\theta(t-(j-1)T)}dt]$$
$$= c_{1m}DE\left[(t - \frac{1}{\theta - R_m})\frac{e^{-R_m t+\theta(t-(j-1)T)}}{\theta - R_m}\Big|_{(j-1)T}^{(k+j-1)T} - \frac{(j-1)T}{\theta - R_m}e^{-Rmt+\theta(t-(j-1)T)}\Big|_{(j-1)T}^{(k+j-1)T}\right]$$
$$= c_{1m}DE\left[\frac{e^{-R_m(k+j-1)T+\theta kT}}{(\theta - R_m)^2}(k(\theta - R_m)-1) + \frac{e^{-R_m(j-1)T}}{(\theta - R_m)^2}\right] =$$
$$c_{1m}DE[\frac{e^{-R_m(j-1)T}(1+e^{(\theta - R_m)kT}(kT(\theta - R_m)-1))}{(\theta - R_m)^2}]$$
$$for: j = 1, ..., (n-1), R_m = r - i_m, m = 1, 2. \quad (1.33)$$

In the last period for the mth class (m=1,2) from similar machinations we have:

$$ECH_{nm} = E\left[c_{1m}\int_{(n-1)T}^{nT}(t-(n-1)T)De^{-R_m t}e^{\theta(t-(n-1)T)}dt\right]$$

$$ECH_{nm} = c_{1m}DE(\frac{e^{-R_m(n-1)T}(1+e^{(\theta - R_m)T}((\theta - R_m)T-1))}{(\theta - R_m)^2} \quad (1.34)$$

$$for: R_m = r - i_m, m = 1, 2 \quad (1.35)$$

In the last period the inventory level comes to zero at the end of period. The total internal and external carrying costs for all cycles can be given as follows:

$$ECH = \sum_{m=1}^{2}\sum_{j=1}^{n-1} ECH_{jm} + \sum_{m=1}^{2} ECH_{nm} \quad (1.36)$$

4.3. Expected present value of the shortages cost

The expected present value of the shortages cost for the j-th cycle ($j = 1, 2, ..., n-1$) for the m-th class (m=1,2) can be computed as:

$$ECS_{jm} = E\left[c_{2m}\int_{(k+j-1)T}^{jT}(jT-t)De^{-R_m t}dt\right]$$
$$= c_{2m}DE\left[\left[\frac{e^{-R_m t}}{-R_m}(jT-t-\frac{1}{R_m})\right]_{(k+j-1)T}^{jT}\right]$$
$$= c_{2m}DE$$
$$\left[\frac{e^{-R_m jT}}{{R_m}^2} - \frac{e^{-R_m(k+j-1)T}}{-R_m}((1-k)T - \frac{1}{-R_m})\right]$$
$$= c_{2m}DE$$
$$\left[\frac{e^{-R_m jT}(1+((1-k)R_m T-1)e^{-R_m T(k-1)})}{{R_m}^2}\right]$$

Where$R_m = r - i_m$. It can be rewritten as

$$ECS_{jm} = c_{2m}DE(\frac{e^{-R_m jT} + ((1-k)R_m T - 1)e^{-R_m T(k+j-1)}}{{R_m}^2}) \tag{1.37}$$

$$for: j = 1, ..., (n-1), R_m = r - i_m, m = 1, 2 \tag{1.38}$$

The total shortages cost during the entire planning horizon H can be written as follows:

$$ECS = \sum_{m=1}^{2}\sum_{j=1}^{n-1} ECS_{jm} \tag{1.39}$$

4.4. Expected present value of the ordering cost

The expected present value of the ordering cost for replenishment at time (j-1)T for the j-th cycle is:

$$ECR_j = Ae^{-rjT}m_{i1}(jT) for: j = 1, ..., n-1 \tag{1.40}$$

The total replenishment cost can be given as follows:

$$ECR = \sum_{j=0}^{n-1} ECR_j \tag{1.41}$$

Hence, the total expected inventory cost of the system during the entire planning horizon H is given by:

$$ETVC(n, k) = ECP + ECH + ECS + ECR \tag{1.42}$$

The objective is to determine the optimal values of T and $k$to minimize $ETVC(n, k)$

1. SOLUTION PROCEDURE

This section discusses the solution procedure of optimization problems for the expected present value of costs over the time horizon. Many researchers have successfully used metaheuristic methods to solve complex optimization problems in different fields of scientific and engineering disciplines. Some of these algorithms are simulated annealing, tabu search, genetic algorithm, particle swarm optimization, ant colony optimization, differential evolution, among others. Among these algorithms, the widely used efficient algorithms genetic algorithm (GA) and particle swarm optimization (PSO) have been applied to solve the optimization problem. In this paper, the genetic algorithm (GA) is employed. On the other hand, we used the definitive method. That means: The problem is determining$n$and$k$to lead the minimum of the total expected inventory system cost. For a given value of$n$, the necessary condition of optimality is as follows:

$$\frac{dETVC(n,k)}{dk} = PDT\sum_{j=1}^{n-1}\left[e^{((1-j)r+\theta k)T}M_{i_2}((j-1)T) - e^{-rjT}M_{i_2}(jT)\right] + \sum_{m=1}^{2}\left[c_{1m}DKT^2e^{-r(k-1)T+\theta kT}\sum_{j=1}^{n-1}\left[e^{-rjT}M_{i_m}((k+j-1)T)\right]\right] + \sum_{m=1}^{2}\left[c_{2_m}DT^2(k-1)\sum_{j=1}^{n-1}\left[e^{-rT(k+j-1)}M_{i_m}(T(k+j-1))\right]\right] = 0 \tag{1.43}$$

The iterative methods such as newton method can be used for calculating$k$. The second- order condition for a minimum is:

$$\frac{d^2ETVC(n,k)}{dk^2} = pD\theta T^2\sum_{j=1}^{n-1}\left[e^{((1-j)r+\theta k)T}M_{i_2}((j-1)T)\right] + \sum_{m=1}^{2}\left[c_{1m}DT^2\sum_{j=1}^{n-1}E\left[(1+kT(\theta-R_m))e^{(-R_m(k+j-1)+\theta k)T)}\right]\right] + \sum_{m=1}^{2}\left[c_{2m}DT^2\sum_{j=1}^{n-1}\left[E\left[e^{-R_m(k+j-1)T}(TR_m(1-k)+1)\right]\right]\right] \succ 0 \tag{1.44}$$

1. NUMERICAL EXAMPLE

Following example is providing according to the results. The internal and external inflation rates have the normal distribution function with means of$\mu_1 = 0.08$and$\mu_2 = 0.14$, standard deviations of $\sigma_1 = 0.04$and$\sigma_2 = 0.06$, respectively. From the above formulations and the demand assessment which was represented as an interval number$D = \{5362.705, 6115.185\}$. In the previous section, the average demand rate is 5738.945. The company interest rate is 20% per annum, the deterioration rate of the on-hand inventory per unit time is 0.01 and the length of time horizon is 10 years.

$$r = 0.2; H = 10years; \theta = 0.01$$

The system costs at the beginning of time horizon are$c_{11} = 0.2$; $c_{12} = 0.4$; $c_{21} = 0.8$; $c_{22} = 0.6$; $p = 5$; $A = 100$.

6.1. Definitive methods

Using these parameter values, the optimal solution of the models is obtained and the results are shown in Table 2.

Table 2. Optimal solution for numerical example

N	K	ETVC(n,k)
5	0.683993	495623.89
10	0.684074	474264.84
20	0.684190	453965.20
30	0.684317	447413.12
40	0.684440	444777.96
50	0.684590	443763.56
55	0.684689	443592.14
56	0.684761	443576.89
57	0.684814	443567.67
58*	0.684824	443563.49
59	0.684842	443564.76
60	0.684893	443571.16
80	0.684923	444433.14
100	0.684951	446133.56

The minimum cost over the time horizon is 443563.49 for $n^* = 58$and$k^* = 0.684824$. Optimal interval of time between replenishment,T^*, equals to$H/_{n^*} = 0.175$year. The shortages occur after elapsing 68.5% of the cycle time.

6.2. Genetic method

The minimum cost over the time horizon is 418276.81 for $n^* = 66$and$k^* = 0.6843$. Optimal interval of time between replenishment,T^*, equals to $H/_{n^*} = 0.151$year. The shortages occur after elapsing 69.9% of the cycle time.

1. SENSITIVITY ANALYSIS

To study the effects of system parameters changes H,θ,D,r, μ_1, μ_2, σ_1, σ_2, A, p, c_{11}, c_{12}, c_{21} and c_{22}on the optimal cost, the replenishment time and k^* which is derived by the proposed method, a sensitivity analysis was performed. This fact is done by increasing the parameters by 20, 50 , 100% and decreasing the parameters to 20, 50 , 90%, taking each one at a time and keeping the remaining parameters at their original values. The following conclusion in definitive methods can be derived from the sensitivity analysis based on Table 3.

Table 3. Effects of changes in model parameters on n, k and optimal expected system cost in definitive method.

1. As the mean of the internal inflation rate increases, the number of replenishments (n) decrease and k increases. By increasing the mean of the external inflation rate, increase the number of replenishments (n) and k. The optimal expected present value of cost (ETVC) increases whenμ_1 and$\mu_2$increase but highly sensitive toμ_2. Induction of this result is the purchase cost increasing by external inflation rate is more than other cost components.

2. Table 2 shows that the optimal value of k and the number of replenishments (n) are insensitive to changes in the standard deviations of inflation rates.

3. The number of replenishments (n) is highly sensitive to the change of the parameters$D$,$A$and H, is little sensitive to changes in c_{12} and insensitive to changes in r,p,θ, c_{11}, c_{12}, c_{21} andc_{22}.

4. The optimal value of k is highly sensitive to the change of the parameters c_{12}, c_{21} and c_{22} is moderately sensitive to r and c_{11} is sensitive to$p$,θ,D,A and$H$.

Table 1: Results.

		-90%	**-50%**	**-20%**	**0%**	**20%**	**50%**	**100%**
D	N	18	41	52	58	63	72	85
	K	0.683736	0.684672	0.684785	0.684824	0.684856	0.684889	0.684920
	ETVC	48074.75	225380.13	356492.20	443563.49	530456.23	660551.84	876931.68
r	N	15	44	45	58	61	64	67
	K	0.785769	0.759842	0.735691	0.684824	0.667734	0.643124	0.591374
	ETVC	847245.73	782135.62	599785.19	443563.49	354192.56	266804.83	170526.81
1	N	60	59	58	58	58	57	55
	K	0.669269	0.675729	0.680031	0.684824	0.688929	0.694403	0.701356
	ETVC	443257.35	443259.41	443462.61	443563.49	443644.33	443728.91	443911.42
2	N	47	49	53	58	61	65	69
	K	0.670108	0.674419	0.679781	0.684824	0.690695	0.702305	0.719432
	ETVC	227898.19	333779.60	439668.73	443563.49	503585.13	613682.32	784054.18
1	N	58	58	58	58	58	58	58
	K	0.684834	0.684829	0.684824	0.684824	0.684824	0.684824	0.684814
	ETVC	443542.25	443546.53	443555.73	443563.49	443576.15	443609.31	443639.43
2	N	58	58	58	58	58	58	58
	K	0.678875	0.680179	0682935	0.684824	0.687511	0.693437	0.709420
	ETVC	443389.73	443416.91	443489.36	443563.49	443681.12	443752.93	443864.21
θG	N	58	58	58	58	58	59	60
	K	0.688705	0.686877	0.685284	0.684824	0.683963	0.682178	0.681337
	ETVC	443030.21	443272.58	443398.95	443563.49	443694.42	443833.23	444082.91
H	N	12	22	37	58	75	99	120
	K	0.695427	0.691205	0.687206	0.684824	0.683019	0.681574	0.679892
	ETVC	64123.59	224397.89	352286.57	443563.49	517942.78	600969.70	682578.66
p	N	29	46	54	58	62	66	71
	K	0.704788	0.695807	0.689513	0.684824	0.680197	0.675629	0.665841
	ETVC	51298.62	226795.08	357059.62	443563.49	529869.34	659032.50	873698.44
A	N	185	85	66	58	53	47	41
	K	0.685032	0.684974	0.684883	0.684824	0.684795	0.684713	0.684675
	ETVC	431639.77	438466.31	441726.49	443563.49	445223.55	447472.74	450760.21
C11	N	56	57	58	58	59	60	61
	K	0.761019	0.724817	0.696380	0.684824	0.673266	0.656935	0.631448
	ETVC	442847.34	443183.42	443416. 63	443563.49	443705.23	443907.45	444219.14
C12	N	50	56	57	58	59	61	64
	K	0.853212	0.764884	0705783	0.684824	0.656142	0.607722	0.563087
	ETVC	441193.93	443037.49	443148.58	443563.49	443935.31	443924.18	445101.16
C21	N	58	58	58	58	58	58	59
	K	0.531705	0.604053	0.641868	0.684824	0.706413	0.739442	0.771060
	ETVC	443440.35	443505.70	443542.94	443563.49	443582.74	443604.97	443637.71
C22	N	58	58	58	58	58	58	59
	K	0.539767	0.617053	0.670509	0.684824	0.705723	0.732830	0.768933
	ETVC	443438.13	443504.29	443541.90	443563.49	443582.80	443605.90	443637.98

5. The total expected inventory cost of the system is highly sensitive to the changes in the parameters$D$,r,$H$and$p$insensitive to θ, c_{11}, c_{12}, c_{21}, c_{22} and A.

The following conclusion in Genetic methods can be derived from the sensitivity analysis based on Table 4.
Table 4. Effects of changes in model parameters on n, k and optimal expected system cost in Genetic method
In this table, we only look at changes in the critical parameters that were obtained from the previous table. And we found that the values obtained through the genetic method are more accurate and appropriate than the previous one.

1. SOME PARTICULAR CASES

In this section, an attempt has been made to study some important special cases of the model.
Case 1: If the internal and external inflation rates have the same pdf, the expected present value of the total cost$ETVC(n,k)$ can be obtained by deleting $\sum_{m=1}^{2}$in Eq. and substituting:

$$c_{1m} = c_1, R_m = R, i_m = i, for : I; 1, 2 and m = 1, 2 \tag{1.45}$$

The previous numerical example assumes that the inflation rate has the normal distribution function with the mean of $\mu = 0.11$and the standard deviation of$\sigma = 0.05$. The optimal solution in this case as follows:$n^* = 49$, $k^* = 0.685109$, $ETVC(n,k) = 39195.46$and$T^* = 0.204$year. The number of replenishment and inventory system cost decrease and k increases.
Case 2: If shortages are not allowed, k=1 and the expected present worth of the total variable cost$ETVC(n)$ can be obtained. The minimum solution of $ETVC(n)$ for the discrete variable of n must satisfy the following equation:

$$ETVC(n) \le 0 \le ETVC(n+1) \tag{1.46}$$

Where$ETVC(n) = ETVC(n) - ETVC(n-1)$. In the numerical example, using the above inequality, the following solution is obtained:$n^* = 69$,$ETVC(n) = 45596.73$and $T^* = 0.144$year. It shows that $n$and $ETVC(n)$increase in the without shortages case.
Case 3: If available inventory has no deterioration ($\theta = 0$) over time the cost function, after modeling, may be rewritten as follows:

$$\begin{aligned}
&ETVC(n,k,\theta=0) = \\
&A\sum_{j=0}^{n-1}\Big[e^{-r_jT}M_{i_1}(jT)\Big] \\
&+ pDT\Big[e^{-rT(n-1)}M_{i_2}((n-1)T) \\
&+\sum_{j=1}^{n-1}\Big[ke^{-rT(j-1)}M_{i_2}((j-1)T)-(k-1)e^{-r_jT}M_{i_2}(jT)\Big]\Big] \\
&+\sum_{m=1}^{2}\Big[c_{1m}D\Big[\sum_{j=1}^{n-1}E\Big[\frac{e^{-R_m(j-1)T}(1-e^{-R_mkT}(kTR_m+1)}{{R_m}^2}\Big] \\
&+E\Big[\frac{e^{-R_m(n-1)T}(1-e^{-R_mT}(TR_m+1))}{{R_m}^2}\Big]\Big]\Big]+ \\
&\sum_{m=1}^{2}\sum_{j=1}^{n-1} \\
&\Big[c_{2m}DE\Big[\frac{e^{-R_mjT}+((1-k))R_mT-1)e^{-R_mT(k+j-1)}}{{R_m}^2}\Big]\Big]
\end{aligned} \tag{1.47}$$

The cost function can be minimized by the methods indicated in this study. For a given value of n, the necessary condition of optimality is:

$$\begin{aligned}
&\frac{dETVC(n,k,\theta=0)}{dk} \\
&= pDT\sum_{j=1}^{n-1}\Big[e^{-rT(j-1)}M_{i_2}((j-1)T)-e^{-r_jT}M_{i_2}(jT)\Big]+ \\
&\sum_{m=1}^{2}\Big[c_{1m}DkT^2e^{-rT(k-1)} \\
&\sum_{j=1}^{n-1}\Big[e^{-r_jT}M_{i_m}((k+j-1)T\Big]\Big]+ \\
&\sum_{m=1}^{2}\Big[c_{2m}DT^2(k-1) \\
&\sum_{j=1}^{n-1}\Big[e^{-rT(k+j-1)}M_{i_m}(T(k+j-1))\Big]\Big] \\
&= 0
\end{aligned} \tag{1.48}$$

Table 2: Results.

		-90%	-50%	-20%	0%	20%	50%	100%
r	N	50	57	64	66	68	69	70
	K	0.69	0.69	0.68	0.68	0.68	0.67	0.64
	ETVC	1225596.17	740342.61	520664.35	418276.81	337227.55	250139.22	159786.90
H	N	2	30	47	66	76	109	138
	K	0.69	0.69	0.68	0.68	0.68	0.66	0.65
	ETVC	9235.94	184391.43	333463.40	418276.81	485626.32	563043.18	638878.12
A	N	146	89	68	66	55	49	44
	K	0.66	0.66	0.68	0.68	0.68	0.68	0.69
	ETVC	404391.58	411428.27	414837.62	418276.81	418724.88	421063.70	424730.84

And the sufficient condition is the second derivative is positive. In this case, the numerical result is obtained as follows:$n^* = 58$,$K^* = 0.685146$,$ETVC(n,k) = 44319.44$and $T^* = 0.172$ year. Thus $ETVC(n)$ is decreased, k is increased and n is not changed in comparison to the main model.

Case 4: Now assume that the internal and external inflation rates have the same pdf, no shortages allowed and$\theta = 0$. This may be solved by using Eq. (1.47) and (1.48), substituting $k = 1$ and considering (1.45).therefore, the optimal solution is as follows:$n^* = 62$,$ETVC(n,k) = 41389.45$and $T^* = 0.161$year.

1. **Conclusions**

In order to deal with the inventory control problem under nonstationary and uncertain demand, a belief-rule-base inventory control method was suggested in this paper. Unlike traditional methods that make ordering policies based on a definite planning horizon into the future whose resulting order quantity for the forthcoming period can be importantly affected by inoperative forecasts for distant periods, the belief-rule-base inventory control method makes policies according to current inventory, historical demand data and necessary short-term forecasting demand. Also, this model considers an inventory model with stochastic internal and external inflation rates for deteriorating items, allowable shortages, and stochastic inflationary conditions. Usually, in the inventory systems under inflationary conditions, it has been assumed that inflation rates are constant over the planning horizon. But, many economic, political, social and cultural variables may also affect the future changes in the inflations rate. Therefore, assuming constant inflation rates is not valid, especially when the time horizon is more than two years. As shown, Since the definitive method was a defect, we used the genetic method. Because the demand values obtained from the model are in the interval, to solve a definite method, the value of the function is also calculated as an interval. In order to obtain our definitive number, we have calculated the average demand and based on that the value of the function was calculated. For this reason, we used the genetic method to solve this problem and after calculations, it turned out that the results obtained from the genetic method are better than the definitive method. The numerical examples have been given and sensitivity analysis has been conducted to illustrate the theoretical results. The results of sensitivity analysis indicate that which of the parameters are more sensitive to the optimal solution. Also, the results indicate the importance of taking into account stochastic inflation, especially when there is considerable uncertainty associated with inflation rates. Finally, four special cases have been discussed identical inflation rates, no shortages situation, no deterioration and considering all the three cases simultaneously. These cases are compared with the main model through the numerical example.

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