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Research Article

An Inverse Two-Stage FDH Model in the Presence of Shared Resources

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Abstract

In this research, two-stage networks are considered in which there are common resources between both stages. Also, there are links between the first and second stages that are undesirable for the first stage. Two models are rendered, the first is a model for calculating the efficiency of the mentioned two-stage processes under the non-convex technology and the second model is an inverse model for calculating output values through input changes. The changes are such that efficiency values remain constant. Actually, at first, a non-radial network free disposal hull (FDH) model is planned to evaluate the entire and stage efficiencies of two-stage processes with shared resources. The introduced framework is a mixed integer nonlinear programming plan and is computed using a heuristic approach. Then, an inverse two-stage FDH approach is presented to determine outputs related to two stages for perturbing inputs of each stage while the efficiency values remain unchanged. To clarify the proposed models, an application from the literature is used.

Keywords : Data envelopment analysis; Two-stage system; Inverse DEA; Free disposal hull (FDH); Shared resources.

1 Introduction

Data Envelopment Analysis (DEA) is a non-parametric means for analyzing the relative efficiency of decision making units (DMUs) that all have several input and output factors. Today, DEA found many applications in different fields, some of which are: finance, health, education, production, transportation, etc. [52, 53, 54, 32, 20, 19].

In classical DEA, the DMUs are considered in the form of black boxes, i.e. only inputs and outputs are included and internal processes are not considered. But as processes became more complex, the need to analyze the system's internal processes became inevitable. Thus, network DEA, including different models to investigate disparate structures was created [55, 56, 47, 42, 41, 40, 39, 38, 37, 36, 35, 34].

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Seiford and Zhu [57] addressed processes with two stages and employed the ordinary DEA approach to each stage. Golani et al. [58] developed a technique for measuring the efficiency level in a system consisting of two successively interconnected subsystems. Kao [59] proposed a method to calculate the efficiency of organizations which consisted of several parallel subsystems. Other researchers then proposed models with more complex structures, containing a combination of series and parallel processes or measures such as undesirable outputs [50, 51, 49, 48, 46, 44, 47]. Amirteimoori [47] presented a DEA two-stage approach to assess the efficiency of two-stage network systems with series frameworks as long as there are shared resources and perfect and imperfect outputs. The proposed approach by Amirteimoori [47] was radial and under the convex technology. Tavakoli and Mostafaee [4] extended network DEA approaches to FDH techniques while common resources were not included and the changes of measures were not investigated. Therefore, this research tackles these issues.

Actually, the estimation of changes of some outputs (inputs) for changes of some inputs (outputs) when the efficiency value is maintained is a significant aspect for decision makers. Accordingly, in the DEA literature, it can be found some studies such as [13, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34] to consider this topic. To more illustrate, Wei et al. [28] firstly developed an inverse DEA approach to consider inputs (outputs). Lertworasirikul et al. [21] provided the inverse BCC (Banker, Charnes and Cooper) model to deal with a problem of resource allocation. Jahanshahloo et al. [24] presented inverse enhanced Russell models to assess inputs, outputs and both of them. An et al. [25] provided an inverse two-stage DEA model with undesirable outputs to address resource planning. Their approach was radial and under the convex technology. Kalantary et al. [22] assessed the sustainability of chains of supply using an inverse network dynamic range adjusted measure technique. Kalantary and Farzipoor Saen [26] developed a network dynamic slacks based measure (SBM) model to estimate the sustainability of supply chains. Zhang and Cui [23] provided the inverse non-radial DEA model. As the consideration shows and we are aware, the majority of inverse DEA models are under the convex technology. Furthermore, few researchers have addressed inverse two-stage DEA models. However, in many real applications, the convexity property is violated and also shared resources are presented in many two-stage processes. Therefore, in this study, at first a non-radial free disposal hull (FDH) model is suggested to evaluate the general and stage efficiency scores of two-stage processes with common resources and undesirable intermediate measures. The introduced model is a mixed integer nonlinear programming problem and it is computed using a heuristic method. Then, an inverse FDH two-stage model with shared resources is proposed to assess outputs for perturbations of inputs while the efficiency value of each stage is preserved. The developed technique is multi-objective and can be solved applying the weighted sum approach. Finally, a case study from the literature is used to explicate the models designed.

The remaining of this study is planned as follows: In Section 2, the prerequisites are given. In Section 3, the models proposed, including the two-stage FDH model with common resources and its inverse problem are presented. In Section 4, a real case study from the literature is used to explain the approaches introduced. Lastly, conclusions and suggestions are provided in Section 5.

2 Preliminaries

In this section, some the prerequisites, covering the FDH model, inverse DEA, and general problem statement are described.

2.1 Free disposal hull (FDH) model

The production possibility set (PPS) in DEA is made using several principles, one of which is convexity. This principle states that a convex combination of DMUs also belongs to the PPS. But in many real world situations, this principle is not satisfied, so it is removed. By relaxing this principle, an alternative PPS was created. The FDH model was presented by Deprins et al. [33]. and extended by Lovell et al. [45]. An appealing characteristic of the FDH model is that it submits only one of the existing efficient DMUs for each inefficient DMU as a reference unit. Consequently, the FDH reference set is more compatible with many practical applications. By considering the observations (X_j, Y_j) $j = 1, 2, ..., n$, the PPS of the FDH model is made based on DEA principles as follows:

$$T = \Big\{(X, Y) \ : \ \sum_{j=1}^{n} \lambda_j \omega X_j \leq X \\ \sum_{j=1}^{n} \lambda_j \omega Y_j \geq Y, \quad \omega \in \mathbb{R}_+ \\ \lambda_j \in \{0, 1\}, \sum_{j=1}^{n} \lambda_j = 1\Big\}$$

where $X_j \in \mathbb{R}^m_+$ are inputs, $Y_j \in \mathbb{R}^s_+$ are outputs for DMU j and ω is a non-negative real numbers. Also, λ_j is intensity variables. By taking the technology T into account, the FDH model becomes as follows:

$$\begin{aligned} min \ \ & \theta && (2.1) \\ s.t. \ \ & \sum_{j=1}^{n} \lambda_j \omega x_{ij} \leq \theta x_{io}, \quad i = 1, 2, ..., m \\ & \sum_{j=1}^{n} \lambda_j \omega y_{rj} \geq y_{ro}, \quad r = 1, 2, ..., s \\ & \lambda_j \in \{0, 1\}, \quad j = 1, 2, ..., n \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \omega \geq 0. \end{aligned}$$

Problem (2.1) is a mixed-integer nonlinear programming problem, using the big-M method, this problem can be transformed to a mixed-integer linear programming in this way:

$$\begin{aligned} min \ \ & \theta && (2.2) \\ s.t. \ \ & \sum_{j=1}^{n} \Lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, 2, ..., m \\ & \sum_{j=1}^{n} \Lambda_j y_{rj} \geq y_{ro}, \quad r = 1, 2, ..., s \\ & 0 \leq \Lambda_j \leq M \lambda_j, \quad j = 1, 2, ..., n \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \in \{0, 1\}, \quad j = 1, 2, ..., n \end{aligned}$$

where M is a large enough number.

2.2 Inverse DEA

Unlike traditional optimization, which seeks to calculate optimal decisions concerning goals and constraints, inverse optimization takes decisions as inputs and sets goals and / or constraints that optimally or precisely optimize those decisions. Inverse DEA is a branch of inverse optimization in which changes are made to input (output) and appropriate outputs (inputs) for those changes are sought. In this stage, we briefly review the inverse DEA approach presented by Wei et al. [28]. The CCR model (2.3) originally presented by Charnes et al. [18]. Suppose that the optimal value is θ^*.

$$\begin{aligned} min \ \ & \theta && (2.3) \\ s.t. \ \ & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, 2, ..., m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, 2, ..., s \\ & \lambda_j \geq 0, \quad j = 1, 2, ..., n \end{aligned}$$

x_{ij} $(i = 1, 2, ..., m, \ j = 1, 2, ..., n)$ shows the i th input related to j th DMU, and y_{rj} $(r = 1, 2, ..., S,$ $j = 1, 2, ..., n)$ is the r th output of j th unit. Then the values to Y_o are added (subtracted) $(\beta_o = Y_o \pm \Delta Y)$ and the new input values α are estimated that are greater (less) than X_o so that (α, β) has the same efficiency as θ^*. Thus the following model is computed,

$$
\begin{aligned}
min \quad & (\alpha_{1o}, \alpha_{2o}, ..., \alpha_{mo}) \qquad (2.4)\\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta^* \alpha_{io}, \quad i = 1, 2, ..., m\\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \beta_{ro}, \quad r = 1, 2, ..., s\\
& x_{io} \leq \alpha_{io}, (x_{io} \geq \alpha_{io}), \quad i = 1, 2, ..., m\\
& \lambda_j \geq 0, \quad j = 1, 2, ..., n
\end{aligned}
$$

To solve the problem (2.4), the weighted sum method can be applied. By considering the weights $\omega_i > 0$, a single objective function $\sum_{i=1}^{m} \omega_i \alpha_{io}$ can be resulted. The following linear programming model is computed for all positive weights [29]:

$$
\begin{aligned}
min \quad & \sum_{i=1}^{m} \omega_i \alpha_{io} \qquad (2.5)\\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta^* \alpha_{io}, \quad i = 1, 2, ..., m\\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \beta_{ro}, \quad r = 1, 2, ..., s\\
& x_{io} \leq \alpha_{io}, (x_{io} \geq \alpha_{io}) \quad i = 1, 2, ..., m\\
& \lambda_j \geq 0, \quad j = 1, 2, ..., n
\end{aligned}
$$

An important point in the inverse DEA model is that after adding the new DMU, the efficiency of the others DMUs remains constant. In other words, these new DMUs do not alter the efficiency frontier and the efficiency of these new DMUs is the same as the efficiency of the previous DMUs.

2.3 Problem statement

Consider Figure 1, initially given in [47], this system has two subsystems or stages. Each subsystem has independent inputs (x_p and h_p) and outputs (y_p and q_p), and also inputs that are shared between the two subsystems (k_p). There is, moreover, a link between the two systems, which is the undesirable output of the first stage and is considered as the input of the second stage (z_p). That link is products that are incompletely produced from Stage 1 and need to be repaired in Stage

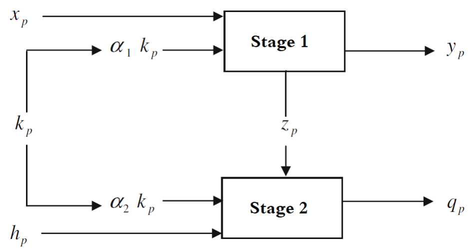

Figure 1: subsystems [47]

2, for example, a lathe can be considered that some products do not cut properly. α_1 and α_2 are the coefficient convex combination of k_p i.e. $0 \leq \alpha_1 \leq 1$, $0 \leq \alpha_2 \leq 1$ and $\alpha_1 + \alpha_2 = 1$.

Amirteimoori [47] advanced a two-stage DEA approach based upon the convex technology (the envelopment form and radial form) to estimate the entire efficiency of the system shown in Figure 1. Nevertheless, a problem is how the performance of systems with the structure shown in Figure 1 under the non-convex technology can be estimated. Also, the following questions arise: How outputs of each subsystem for changes of inputs can be determined while the efficiency values related to each stage is preserved?
Can a network FDH model with shared resources be presented that estimates overall and stage efficiency of systems with the design shown in Figure 1?

In the next section, these problems are investigated and a on-radial two-stage FDH model is offered to evaluate the entire and stages efficiencies of two-stage processes with shared resources under the non-convex technology. Also, its inverse model is developed to estimate outputs.

3 Proposed method

In this section, we focus on two main models: First, we present a model for evaluating the efficiency of two-stage processes based upon the FDH technology while shared resources present between two stages. To illustrate in detail, a non-radial input-oriented model based on network FDH with constant returns to scale is provided. Then we propose an inverse network FDH model so that new output values can be measured considering the changes given on inputs. These changes are such that preserve the efficiency scores achieved from the introduced non-radial network FDH model. Also, the models developed in this section are mixed integer nonlinear programming problems and we try to extract them from the nonlinear form.

3.1 Non-radial network FDH model with shared resources

To evaluate the overall and stage efficiencies of these dealerships, model (3.8) is calculated. The findings are shown in Table 3. Notice that the importance of each stage has been considered to be equal, i.e. $v_1 = v_2 = \frac{1}{2}$. As can be seen, two dealerships, 7 and 13 are overall and stage efficiencies with the score one. Furthermore, three dealerships 1, 7 and 13 are efficient in the sale representative stage and four dealerships 3, 6, 7 and 13 are efficient in the repair shop stage as can be seen in columns 2 and 3.

Among the units, the unit 18 has the least overall efficiency as you can see in column 4. Also, it can be found that it shows weaker performance in the sale representative stage. Now using model (3.18), the output values are estimated for the increase of inputs by 10 percent while the efficiency values of two stages remain at the same levels.

Table 4 shows the results that are the values added to the outputs. For illustration, consider the dealership 1 for the aforementioned increase of inputs, the increase of faultless cars is 17.1 and customer satisfaction is 9.3 in the sale representative stage. Furthermore, the extension of repaired care and net income related to the repair shop stage is 0.26 and 176.2428, respectively. In the same way, it can be considered the changes of outputs connected with other dealerships.

Suppose that we have n two-stage systems with the design exhibited in Figure 1. Each the unit under consideration, DMU o, has the independent inputs related to stage 1 denoted by $X_o = (x_{1o}, x_{2o}, ..., x_{Io})$ which produce outputs, $Y_o = (y_{1o}, y_{2o}, ..., y_{Do})$ and $Z_o = (z_{1o}, z_{2o}, ..., z_{Ro})$, the output vector Y_o is the perfect output and the output vector Z_o is the vector of undesirable output, Z_o is also considered as the inputs of stage 2. $H_o = (h_{1o}, h_{2o}, ..., h_{Lo})$ and $Q_o = (q_{1o}, q_{2o}, ..., q_{Ao})$ moreover show the independent inputs and outputs of stage 2, respectively. Finally, $K_o = (k_{1o}, k_{2o}, ..., k_{Uo})$ is the common resources between two stages. We propose the following model to evaluate the overall and stages efficiency values of two-stage processes with the structure beforementioned:

$$
\begin{aligned}
min \quad & v_1E_1 + v_2E_2 && (3.6)\\
s.t. \quad & E_1 = 1 - \frac{1}{I+R+U}\left(\Sigma_{i=1}^{I}\frac{s_i^1}{x_{io}} + \Sigma_{r=1}^{R}\frac{s_r^3}{z_{ro}} + \Sigma_{u=1}^{U}\frac{s_u^4}{k_{uo}}\right)\\
& E_2 = 1 - \frac{1}{U+R+L}\left(\Sigma_{u=1}^{U}\frac{s_i^8}{k_{uo}} + \Sigma_{r=1}^{R}\frac{s_r^5}{z_{ro}} + \Sigma_{l=1}^{L}\frac{s_l^6}{h_{lo}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& stage\ \ 1\\
& \sum_{j=1}^{n}\omega^1\lambda_j^1 x_{ij} + s_i^1 = x_{io} \qquad i = 1,2,...,I\\
& \sum_{j=1}^{n}\omega^1\lambda_j^1 y_{dj} - s_d^2 = y_{do} \qquad d = 1,2,...,D\\
& \sum_{j=1}^{n}\omega^1\lambda_j^1 z_{rj} + s_r^3 = z_{ro} \qquad r = 1,2,...,R
\end{aligned}
\tag{3.7}
$$

$$
\mu_u\sum_{j=1}^{n}\omega^1\lambda_j^1 k_{uj} + s_u^4 = \mu_u k_{uo} \qquad u = 1,2,...,U
$$

$$
\begin{aligned}
& stage\ \ 2\\
& \sum_{j=1}^{n}\omega^2\lambda_j^2 z_{rj} + s_r^5 = z_{ro} \qquad r = 1,2,...,R\\
& \sum_{j=1}^{n}\omega^2\lambda_j^2 h_{lj} + s_l^6 = h_{lo} \qquad l = 1,2,...,L\\
& \sum_{j=1}^{n}\omega^2\lambda_j^2 q_{aj} - s_a^7 = q_{ao} \qquad a = 1,2,...,A\\
& (1-\mu_u)\sum_{j=1}^{n}\omega^2\lambda_j^2 k_{uj} + s_u^8 = (1-\mu_u)k_{uo} \qquad u = 1,2,...,U
\end{aligned}
$$

Table 1: Performance measures [47]

	Inputs	Outputs	Common inputs
Sale agents	Personnel, number of cars	Faultless cars, user satisfaction	Rewards and operation expenses
Fixing shop	Wind-screen wiper, Personnel, wind-shields, defective cars	Fixed cars, net earnings	Rewards and operation expenses

Table 2: Data [47]

	Stage 1				Link	Stage 2					Shared inputs
Dealer-ship	Perso-nnels	Cars	Faultl-ess cars	User sati-sfaction	Defect-ive car	Windscr-een wiper	Winds-hields	perso-nnel	Fixed cars	Net ea-rnings	Reward operation expenses
1	11	190	171	93	19	24	1	31	18	473	235
2	13	206	185	64	21	26	2	30	20	635	310
3	10	176	158	71	18	23	1	22	17	412	206
4	9	149	133	76	16	20	1	21	16	410	208
5	14	191	171	89	20	27	1	28	18	629	316
6	10	163	146	74	17	21	1	22	17	411	201
7	8	151	137	91	14	17	1	19	14	401	198
8	12	169	151	93	18	23	2	33	17	399	204
9	15	193	172	87	21	29	3	38	20	670	331
10	14	188	168	89	20	28	2	35	20	650	328
11	16	199	176	91	23	31	3	41	21	780	349
12	11	161	142	93	19	23	1	29	19	601	299
13	10	158	140	98	18	19	1	24	18	430	211
14	13	171	154	86	17	23	2	28	17	620	312
15	14	173	154	89	19	24	1	31	18	640	328
16	15	185	164	91	21	26	3	32	20	703	342
17	12	159	142	95	17	27	1	30	17	513	261
18	19	207	182	88	25	33	3	45	22	841	419
19	12	197	174	83	23	26	2	31	22	591	283
20	17	201	179	79	22	30	3	41	21	841	408

Common condition

$$\sum_{j=1}^{n} \lambda_j^1 = 1, \quad \lambda_j^1 \in \{0,1\}$$
$$\sum_{j=1}^{n} \lambda_j^2 = 1, \quad \lambda_j^2 \in \{0,1\}$$
$$LB_u \leq \mu_u \leq UB_u \qquad u = 1, 2, ..., U$$
$$s_i^1, s_d^2, s_r^3, s_u^4, s_r^5, s_l^6, s_a^7, s_u^8, \omega^1, \omega^2 \geq 0$$

where v_1 and v_2 are positive weights which are determined based on the importance of each stage, also LB_u and UB_u are the lower bound and upper bound for μ_u with $0 \leq LB_u < UB_u \leq 1$, respectively.

Theorem 3.1. *Model (3.6) is always feasible.*

Proof. Set $\lambda_o^1 = 1, \lambda_o^2 = 1$ and the other λ_j^1 and λ_j^2 be zero. Also, set $\omega^1 = 1, \omega^2 = 1$, and μ_u be a number between LB_u and UB_u. The slack variables also be zero. With this, all conditions hold, so model (3.6) is feasible. □

Similar to [17, 16, 15], in model (3.6), undesirable outputs of the stage 1 are dealt with considering them as inputs. As can be seen, this model is a nonlinear integer programming problem. Using the big-M method, we have the fol-

Table 3: The efficiency scores

DMU	Eff. of stage 1	Eff. of stage 2	Total Eff.
1	1	0.9378	0.9689
2	0.8959	0.9725	0.9342
3	0.9523	1	0.9761
4	0.9051	0.9848	0.9450
5	0.8390	0.9564	0.8977
6	0.9294	1	0.9647
7	1	1	1
8	0.8942	0.7767	0.8355
9	0.8100	0.8575	0.8337
10	0.8211	0.9481	0.8846
11	0.7820	0.8580	0.8200
12	0.7940	0.9373	0.8656
13	1	1	1
14	0.8309	0.9427	0.8868
15	0.7826	0.9357	0.8591
16	0.7767	0.9386	0.8576
17	0.8348	0.8628	0.8488
18	0.7250	0.7851	0.7551
19	0.8705	0.9777	0.9241
20	0.7655	0.9268	0.8461

Table 4: The purtubution of outputs

DMU	Faultless cars	User satisfaction	Fixed cars	Net earnings
1	17.1000	9.3000	0.2600	176.2428
2	18.5000	71.1715	2	63.5000
3	15.8000	44.4438	1.7000	41.2000
4	13.3000	21.1774	1.6000	41
5	17.1000	35.9423	1.8000	62.9000
6	14.6000	32.6759	1.7000	41.1000
7	13.7000	9.1000	1.4000	40.1000
8	15.1000	17.3292	1.7000	47.7222
9	17.2000	38.6730	2	132.3529
10	16.8000	33.7504	2	65
11	17.6000	37.5956	2.5258	78
12	14.2000	10.7533	1.9000	60.1000
13	14	9.8000	1.8000	43
14	15.4000	26.5212	1.7000	62
15	15.4000	23.5212	1.8000	64
16	16.4000	28.8277	2	70.3000
17	15.3242	9.5000	0.2610	100.7253
18	18.2000	44.9796	4.0184	84.1001
19	17.4000	44.1343	2.2000	59.1000
20	17.9000	51.7876	2.1000	84.1000

lowing model:

$$min \quad E^* = v_1 E_1 + v_2 E_2 \tag{3.8}$$

$$s.t. \quad E_1 = 1 - \frac{1}{I+R+U}\left(\sum_{i=1}^{I} \frac{s_i^1}{x_{io}} + \sum_{r=1}^{R} \frac{s_r^3}{z_{ro}} + \sum_{u=1}^{U} \frac{s_u^4}{k_{uo}}\right)$$

$$E_2 = 1 - \frac{1}{U+R+L}\left(\sum_{u=1}^{U} \frac{s_i^8}{k_{uo}} + \sum_{r=1}^{R} \frac{s_r^5}{z_{ro}} + \sum_{l=1}^{L} \frac{s_l^6}{h_{lo}}\right) \tag{3.9}$$

$stage \;\; 1$

$$\sum_{j=1}^{n}\Lambda_j^1 x_{ij} + s_i^1 = x_{io} \quad i = 1,2,...,I \tag{3.10}$$

$$\sum_{j=1}^{n}\Lambda_j^1 y_{dj} - s_d^2 = y_{do} \quad d = 1,2,...,D \tag{3.11}$$

$$\sum_{j=1}^{n}\Lambda_j^1 z_{rj} + s_r^3 = z_{ro} \quad r = 1,2,...,R \tag{3.12}$$

$$\mu_u \sum_{j=1}^{n}\Lambda_j^1 k_{uj} + s_u^4 = \mu_u k_{uo} \tag{3.13}$$

$$u = 1,2,...,U$$

$stage \;\; 2$

$$\sum_{j=1}^{n}\Lambda_j^2 z_{rj} + s_r^5 = z_{ro} \quad r = 1,2,...,R \tag{3.14}$$

$$\sum_{j=1}^{n}\Lambda_j^2 h_{lj} + s_l^6 = h_{lo} \quad l = 1,2,...,L \tag{3.15}$$

$$\sum_{j=1}^{n}\Lambda_j^2 q_{aj} - s_a^7 = q_{ao} \quad a = 1,2,...,A$$

$$(1-\mu_u)\sum_{j=1}^{n}\Lambda_j^2 k_{uj} + s_u^8 = (1-\mu_u)k_{uo} \tag{3.16}$$

$$u = 1,2,...,U$$

$Common \;\; condition$

$$\sum_{j=1}^{n}\lambda_j^1 = 1, \quad \lambda_j^1 \in \{0,1\}, \quad 0 \le \Lambda_j^1 \le M\lambda_j^1 \tag{3.17}$$

$$\sum_{j=1}^{n}\lambda_j^2 = 1, \quad \lambda_j^2 \in \{0,1\}, \quad 0 \le \Lambda_j^2 \le M\lambda_j^2$$

$$LB_u \le \mu_u \le UB_u \quad u = 1,2,...,U$$

$$s_i^1, s_d^2, s_r^3, s_u^4, s_r^5, s_l^6, s_a^7, s_u^8 \ge 0$$

that M is a positive large number. The problem is still nonlinear due to constraints (3.16), but using a parametric linear approach this problem goes out a mixed-integer linear programming form. For solving this problem, we use a heuristic approach following [11, 12, 14, 27]. We set $\mu_u = 1-k\times\epsilon$, $k = 0,1,...,[1/\epsilon]+1$, $\epsilon = 0.01$. Thus, the problem is solved for $\mu_u \in \{0, 0.01, 0.02, ..., 1\}$.

Assume in model (3.8), the optimal objective function and the optimal values E_1 ad E_2 obtained for $\mu_u \in \{0, 0.01, 0.02, ..., 1\}$ are denoted by $E^{*\mu}$, $E^{*\mu_1}$ and $E^{*\mu_2}$, respectively.

Accordingly, $\min_\mu\{E^{*\mu}\} = E^*$ and the corresponding $E^{*\mu_1}$ and $E^{*\mu_2}$ are treated as the overall efficiency (E^*) and the efficiencies of stage 1 (E_1^*) and stage 2 (E_2^*). The efficiency values are defined between zero and one, i.e. $0 \le E^* \le 1$, $0 \le E_1^* \le 1$ and $0 \le E_2^* \le 1$. The two-stage system is called efficient if and only if $E^* = 1$, in other words $E_1^* = E_2^* = 1$. Otherwise, it is inefficient.

In the next subsection, the inverse two-stage FDH problem is proposed to estimate outputs of both stages.

3.2 Inverse network FDH model with shared resources

The intention in this subsection is to measure the amount of output changes corresponded to two stages for making changes in the input values related to stages 1 and 2 so that the stages efficiency values remain without change. After solving the problem (3.8) and considering E_1^* and E_2^* as the optimal efficiency values of stages 1 and 2 resulted from model (3.8), we propose the following model to assess outputs y'_{do} and q'_{ao} for the increase of inputs by $\Delta x_{io}, \Delta z_{ro}, \Delta k_{uo}$ and Δh_{lo}. We have the following model:

$$max \;\; (y'_{1o}, y'_{2o}, ..., y'_{Do}, q'_{1o}, q'_{2o}, ..., q'_{Ao}) \tag{3.18}$$

$$s.t. \;\; E_1^* = 1 - \frac{1}{I+R+U}\Big(\sum_{i=1}^{I}\frac{s_i^1}{x_{io}+\Delta x_{io}} + \sum_{r=1}^{R}\frac{s_r^3}{z_{ro}+\Delta z_{ro}} + \sum_{u=1}^{U}\frac{s_u^4}{k_{uo}+\Delta k_{uo}}\Big) \tag{3.19}$$

$$E_2^* = 1 - \frac{1}{U+R+L}\left(\sum_{u=1}^{U} \frac{s_i^8}{k_{uo}+\Delta k_{uo}} + \sum_{r=1}^{R} \frac{s_r^5}{z_{ro}+\Delta z_{ro}} + \sum_{l=1}^{L} \frac{s_l^6}{h_{lo}+\Delta h_{lo}}\right)$$

$$\begin{aligned}
& stage \;\; 1 \\
& \sum_{j=1}^{n} \Lambda_j^1 x_{ij} + s_i^1 = x_{io} + \Delta x_{io} \qquad i = 1,2,...,I \\
& \sum_{j=1}^{n} \Lambda_j^1 y_{dj} \geq y'_{do} \qquad d = 1,2,...,D \\
& \sum_{j=1}^{n} \Lambda_j^1 z_{rj} + s_r^3 = z_{ro} + \Delta z_{ro} \qquad r = 1,2,...,R \\
\mu_u & \sum_{j=1}^{n} \Lambda_j^1 k_{uj} + s_u^4 = \mu_u(k_{uo} + \Delta k_{uo}) \qquad u = 1,2,...,U \\
& y_{do} \leq y'_{do} \qquad d = 1,2,...,D \\
& stage \;\; 2 \\
& \sum_{j=1}^{n} \Lambda_j^2 z_{rj} + s_r^5 = z_{ro} + \Delta z_{ro} \qquad r = 1,2,...,R \\
& \sum_{j=1}^{n} \Lambda_j^2 h_{lj} + s_l^6 = h_{lo} + \Delta h_{lo} \qquad l = 1,2,...,L \\
& \sum_{j=1}^{n} \Lambda_j^2 q_{aj} \geq q'_{ao} \qquad a = 1,2,...,A \\
& (1-\mu_u)\sum_{j=1}^{n} \Lambda_j^2 k_{uj} + s_u^8 = (1-\mu_u)(k_{uo} + \Delta k_{uo}) \\
& \qquad\qquad u = 1,2,...,U \\
& q_{ao} \leq q'_{ao} \qquad a = 1,2,...,A \\
& \\
& Common \;\; condition \\
& \sum_{j=1}^{n} \lambda_j^1 = 1, \quad \lambda_j^1 \in \{0,1\}, \quad 0 \leq \Lambda_j^1 \leq M\lambda_j^1 \\
& \sum_{j=1}^{n} \lambda_j^2 = 1, \quad \lambda_j^2 \in \{0,1\}, \quad 0 \leq \Lambda_j^2 \leq M\lambda_j^2 \\
& LB_u \leq \mu_u \leq UB_u \quad u = 1,2,...,U \\
& s_i^1, s_d^2, s_r^3, s_u^4, s_r^5, s_l^6, s_a^7, s_u^8 \geq 0
\end{aligned}$$

Also, by considering this fact that a coefficient of inputs can be added to inputs, we have the following model:

$$max \;\; (y'_{1o}, y'_{2o}, ..., y'_{Do}, q'_{1o}, q'_{2o}, ..., q'_{Ao}) \tag{3.20}$$

$$\begin{aligned}
s.t. \;\; & E_1^* = 1 - \frac{1}{I+R+U}\left(\sum_{i=1}^{I} \frac{s_i^1}{\alpha x_{io}} + \sum_{r=1}^{R} \frac{s_r^3}{\alpha z_{ro}} + \sum_{u=1}^{U} \frac{s_u^4}{\alpha k_{uo}}\right) \\
& E_2^* = 1 - \frac{1}{U+R+L}\left(\sum_{u=1}^{U} \frac{s_i^8}{\alpha k_{uo}} + \sum_{r=1}^{R} \frac{s_r^5}{\alpha z_{ro}} + \sum_{l=1}^{L} \frac{s_l^6}{\alpha h_{lo}}\right)
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
& stage \;\; 1 \\
& \sum_{j=1}^{n} \Lambda_j^1 x_{ij} + s_i^1 = \alpha x_{io} \quad i = 1,2,...,I \\
& \sum_{j=1}^{n} \Lambda_j^1 y_{dj} \geq y'_{do} \quad d = 1,2,...,D \\
& \sum_{j=1}^{n} \Lambda_j^1 z_{rj} + s_r^3 = \alpha z_{ro} \quad r = 1,2,...,R \\
\mu_u & \sum_{j=1}^{n} \Lambda_j^1 k_{uj} + s_u^4 = \mu_u \alpha k_{uo} \quad u = 1,2,...,U \\
& y_{do} \leq y'_{do} \quad d = 1,2,...,D \\
& stage \;\; 2 \\
& \sum_{j=1}^{n} \Lambda_j^2 z_{rj} + s_r^5 = \alpha z_{ro} \quad r = 1,2,...,R \\
& \sum_{j=1}^{n} \Lambda_j^2 h_{lj} + s_l^6 = \alpha h_{lo} \quad l = 1,2,...,L \\
& \sum_{j=1}^{n} \Lambda_j^2 q_{aj} \geq q'_{ao} \quad a = 1,2,...,A \\
& (1-\mu_u)\sum_{j=1}^{n} \Lambda_j^2 k_{uj} + s_u^8 = (1-\mu_u)\alpha k_{uo} \\
& \qquad\qquad u = 1,2,...,U \\
& q_{ao} \leq q'_{ao} \quad a = 1,2,...,A
\end{aligned}$$

Common condition

$$\sum_{j=1}^{n} \lambda_j^1 = 1, \quad \lambda_j^1 \in \{0,1\}, \quad 0 \leq \Lambda_j^1 \leq M\lambda_j^1$$
$$\sum_{j=1}^{n} \lambda_j^2 = 1, \quad \lambda_j^2 \in \{0,1\}, \quad 0 \leq \Lambda_j^2 \leq M\lambda_j^2$$
$$LB_u \leq \mu_u \leq UB_u \quad u = 1,2,...,U$$
$$s_i^1, s_d^2, s_r^3, s_u^4, s_r^5, s_l^6, s_a^7, s_u^8 \geq 0$$

where α is a number greater than 1. Notice that models (3.18) and (3.20) are multi-objective and can be solved using the weighted sum methods. For more illustration, as mentioned in Hwang and Masud [2] and Mavrotas [3], approaches for solving multi-objective problems can be categorized into three classes, including priori, interactive and posteriori techniques. In this research, one of the priori methods that is among the most popular is used.

In the next section, an example is presented to illustrate the models developed in this research.

4 Typical example

In this section, we study a real case study from the automotive industry that firstly provided by Amirteimoori [47]. A car factory sends the manufactured products to 20 of its agents and then these representatives sell products, but cars may have some minor problems that need to be fixed before delivery to the customer. Dealerships that are considered as two-stage processes have inputs and outputs. Dealerships contain two stages, sales representative and repair shop. The inputs of sales representative are personnel and number of cars and their outputs incorporate user satisfaction and some vehicles without defects. Repair shops use personnel to repair defective vehicles, windshields and wind-screen wipers to fix the defective cars. Note that the defective cars are undesirable outputs of the sale agents stage. The outputs of the fixing shop are the number of fixed cars and net earnings. Employee rewards and operation expenses are two common resources that must be shared between the sales agent and the fixing shop. Table 1 describes performance measures, and Table 2 provides their values.

5 Conclusion

In this study, a non-radial network FDH model has been offered to assess the entire and stage efficiencies of two-stage systems with undesirable intermediate measures and shared resources. This proposed approach is under the non-convex technology and is solved using a heuristic approach. Furthermore, an inverse network FDH model has been introduced to estimate outputs of stages 1 and 2 for the augment of inputs related to stages 1 ad 2 while the stages efficiency is maintained. An application from the literature has also been provided to describe the models proposed. One of the main challenges in these models has been solving models because they are nonlinear and contain binary values. Therefore, a heuristic method has been used in order to address this matter.

In this research, undesirable intermediate measures and desirable inputs and outputs have been considered. The extension of the proposed approach when negative measures, undesirable inputs and outputs are presented is an area of focus for further study. Also, the generalization of models introduced including integer measures is an absorbing work to examine. The suggested technique can, furthermore, be developed to analyze the performance and changes of other network structures.

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