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Research Article

Fuzzy Assessment of Heavy Metal Pollution

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Abstract

The present work is aimed to extend the common pollution indices into the fuzzy environment. For this purpose, a method was developed for converting the heavy metal contamination in soil by fuzzy numbers. Then, the most commonly used pollution indices are defined as fuzzy numbers by applying the α-cuts approach. To evaluate the degree of heavy metal contamination in a specific level, a degree of belonging was also suggested. The feasibility and effectiveness of the proposed methods were also examined via an applied example.

Keywords : Fuzzy contamination; Triangular fuzzy number; Fuzzy pollution criterion; Degree of belonging.

1 Introduction

Heavy metal pollution of surface soils has become a serious concern for human health. Since it can enter drinking water and food chain due to the result of economic activities or increasing agriculture, industrialization, and urbanization. Therefore, the evaluation of heavy metals in the environment is of crucial importance in environmental pollution studies. In this regard, the pollution indices are crucial tools for evaluating ecological geochemistry assessment (for more information, see [2, 5, 7, 8, 10, 14, 17, 19, 26, 29, 34]). To quantify metal accumulation and their contamination degree, some common criteria have been employed by authors. The heavy metal enrichment factor (EF) [28] is defined as $EF = c/(c_M + 2c_{\text{MAD}})$ where c is the concentration of a given metal at contaminated sites. c_M is the median concentration of an element in the background soil sample while c_{MAD} is the median absolute deviation from the median. Enrichment factor categories are interpreted as $EF \leq 2$: deficiently to minimal enrichment, $2 < EF \leq 5$: moderate enrichment, $5 < EF \leq 20$: significant enrichment, $20 < EF \leq 40$: very high enrichment and $EF \geq 40$: extremely high enrichment. The contamination factor (CF) [1] can be calculated by $CF = c/C_M$. The degree of mean contamination of soil by k metal is defined as $MCF = 1/k \sum_{j=1}^{k} CF_i$. The CF (MCF) of each metal can be classified as either: low ($CF < 1$), moderate ($1 \leq CF < 3$), considerable ($3 \leq CF < 6$), or very high ($6 \leq CF$)

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contaminations. The pollution load index estimates the metal contamination status and the necessary action of k heavy metals can be calculated by $PLI = (\prod_{j=1}^{k} CF_J)^{1/k}$ [30]. $PLI \geq 1$ indicates an immediate intervention to ameliorate pollution; whereas $0.5 \leq PLI < 1$ suggests that more detailed study is required to monitor the site, $0 \leq PLI < 0.5$ is indicative of the need for drastic rectification measures to be taken, while $PLI < 0$ suggests that the metal contamination is perfect. The value of the geoaccumulation index (I_{geo}) can be determined by $I_{geo} = \log_2(\frac{c}{1.5c_M})$ [22]. The contamination levels evaluated by I_{geo} can be classified as follows: unpolluted ($I_{geo} \leq 0$), unpolluted to moderately polluted ($0 < I_{geo} \leq 1$), moderately polluted ($1 < I_{geo} \leq 2$), moderately to strongly polluted ($2 < I_{geo} \leq 3$), strongly polluted ($4 < I_{geo} \leq 5$), extremely polluted ($I_{geo} > 5$).

Notably, the heavy metals accumulation in surface soils is under the influence of many environmental variables such as parent material, soil properties, and human activities such as industrial areas, traffic, farming, wastewater irrigation, and mine tailings. Moreover, the heavy metals accumulation level could be different in surface soils of an environment. To evaluate the degree of heavy metal pollution of surface soils, the classical procedure usually makes a rigorous report as a mean or median of some central quantities based on the random soil samples. In such a case, it is hard to determine whether the accumulations of heavy metals in an environment is an exact value or not. On the other hand, the fuzzy set theory does not make rigorous descriptions for uncertain situations like heavy metals accumulation. Fuzzy accumulation such as fuzzy mean or fuzzy median seems more suitable when evaluating the degree of pollution of an environment. Therefore, there is a need to extend the conventional pollination indices as well as their interpreters in a fuzzy environment. Since Zadeh [33] introduced the notion of fuzzy sets to evaluate the uncertainty as an imprecise number, the fuzzy set theory has been successfully applied in various fields of decision making as a suitable tool for handling vague information [3,5,6,9,12-14,16,20,21,23-25,27,31,32,35]. During the last decades, fuzzy sets have been largely explored for a wide diversity of real-world applications. Regarding the modeling uncertainty and imprecision of soil heavy metal pollution, some common fuzzy pollution indices have been extended into the fuzzy environment. A degree of belonging was also proposed to verify the conditions of the degree of pollution of the proposed fuzzy pollution indices. For practical reasons, the proposed fuzzy pollution indices are illustrated using an applied study.

The rest of this paper is organized as follows: Section 2 reviews some basic concepts of fuzzy numbers. In this section, the degree of belonging of a fuzzy number to an interval is also introduced. Section 3 extends the classical common pollution criteria based on the fuzzy heavy metals contamination. A numerical example is also illustrated in this section to clarify the discussions in this paper. Finally, a brief conclusion is provided in Section 4.

2 Preliminaries

This section briefly reviews several concepts and terminology related to fuzzy numbers used throughout this paper. A fuzzy set $\widetilde{A}$ of $\mathbb{X}$ (the universal set) is defined by its membership function $\widetilde{A} : \mathbb{R} \to [0,1]$. The set $\widetilde{A}[\alpha] := \{x \in \mathbb{X} : \widetilde{A}(x) \geq \alpha\}$ is called the α-level set (or α-cut) of the fuzzy set $\widetilde{A}$, for each $\alpha \in (0,1]$ [18]. The set $supp(\widetilde{A}) = \widetilde{A}[0]$ is also defined equal to the closure of the set $\{x \in \mathbb{X} : \widetilde{A}(x) > 0\}$. A fuzzy set $\widetilde{A}$ of $\mathbb{R}$ (the real line) is called a fuzzy number (**FN**) if it is normal, i.e. there exists a unique $x^*_{\widetilde{A}} \in \mathbb{R}$ with $\widetilde{A}(x^*_{\widetilde{A}}) = 1$, and for every $\alpha \in [0,1]$, the set $\widetilde{A}[\alpha]$ is a non-empty compact interval in $\mathbb{R}$. This interval will be denoted by $\widetilde{A}[\alpha] = [\widetilde{A}^L_\alpha, \widetilde{A}^U_\alpha]$, where $\widetilde{A}^L_\alpha = \inf\{x : x \in \widetilde{A}[\alpha]\}$ and $\widetilde{A}^U_\alpha = \sup\{x : x \in \widetilde{A}[\alpha]\}$. It said that $\widetilde{A}$ is a positive fuzzy number if $\inf supp(\widetilde{A}) \geq 0$. It is worth noting that, having a sequence of α-cuts $\{\widetilde{A}[\alpha]\}_{\alpha=0}^1$ of a fuzzy number $\widetilde{A}$, the membership function of $\widetilde{A}$ at $x \in \mathbb{R}$ can be calculated by $\widetilde{A}(x) = \sup\{\alpha \in [0,1] : x \in \widetilde{A}[\alpha]\}$ [18]. The triangular fuzzy numbers (**TFN**s) denoted by $\widetilde{A} = (a; l, r)_T$ are the most common fuzzy numbers used in real applications. The membership function of $\widetilde{A}=(a;l,r)_T$ can be written as:

$$\widetilde{A}(x)=$$

$$\begin{cases} \frac{x-a+l}{l} & a-l\leq x<a, \\ \frac{a+r-x}{r} & a\leq x\leq a^r, \\ 0 & x\in\mathbb{R}-[a-l,a+r]. \end{cases} \quad (2.1)$$

Specifically, a symmetric triangular fuzzy number is denoted by $\widetilde{A}=(a;l)_T$. Moreover, for two **FN**s of $\widetilde{A}$ and $\widetilde{B}$ and any $\alpha\in[0,1]$, some common arithmetic operations can be defined as [18]:

$$(\widetilde{A}\oplus\widetilde{B})[\alpha]=[\widetilde{A}^L_\alpha+\widetilde{B}^L_\alpha,\widetilde{A}^U_\alpha+\widetilde{B}^U_\alpha],$$
$$(\widetilde{A}\otimes\widetilde{B})[\alpha]=[(\widetilde{A}\otimes\widetilde{B})^L_\alpha,(\widetilde{A}\otimes\widetilde{B})^U_\alpha],$$

where

$$(\widetilde{A}\otimes\widetilde{B})^L_\alpha=$$
$$[\min\{\widetilde{A}^L_\alpha\widetilde{B}^L_\alpha,\widetilde{A}^L_\alpha\widetilde{B}^U_\alpha,\widetilde{A}^U_\alpha\widetilde{B}^L_\alpha,\widetilde{A}^U_\alpha\widetilde{B}^U_\alpha\},$$
$$(\widetilde{A}\otimes\widetilde{B})^L_\alpha=$$
$$[\max\{\widetilde{A}^L_\alpha\widetilde{B}^L_\alpha,\widetilde{A}^L_\alpha\widetilde{B}^U_\alpha,\widetilde{A}^U_\alpha\widetilde{B}^L_\alpha,\widetilde{A}^U_\alpha\widetilde{B}^U_\alpha\},$$

$\oplus$ and $\otimes$ denote the addition and multiplication operations, respectively [18]. It should be noted that if $\widetilde{A}$ and $\widetilde{B}$ are two positive **FN**s, then:

$$(\widetilde{A}\otimes\widetilde{B})[\alpha]=[\widetilde{A}^L_\alpha\widetilde{B}^L_\alpha,\widetilde{A}^U_\alpha\widetilde{B}^U_\alpha].$$

The rest of this section is devoted to define and discuss a criterion to evaluate the degree to which a fuzzy number belongs to an interval. This criterion can be then applied to evaluate the pollution of heavy metal in an environment.

Definition 2.1. *Let $\widetilde{A}$ be an **FN** and $I\subseteq\mathbb{R}$ be an interval. Then, the degree to which $\widetilde{A}$ belongs to I is defined by*

$$d(\widetilde{A}\in I)=\frac{\int_I\widetilde{A}(x)dx}{\int_{\mathbb{R}}\widetilde{A}(x)dx}. \quad (2.2)$$

Lemma 2.1. *Assume that $\widetilde{A}$ is an **FN**.*

1) If $\{I_j\}_{j=1}^k$ is a sequence of disjoint intervals on $\mathbb{R}$ such that $\cup_{j=1}^k I_j=\mathbb{R}$ then $\sum_{j=1}^k d(\widetilde{A}\in I_j)=1$.

2) For $I\subseteq\mathbb{R}$ $d(\widetilde{A}\in I)=1$ if and only if $supp(\widetilde{A})\subseteq I$.

Proof. If $\{I_j\}_{j=1}^k$ is a sequence of disjoint intervals on $\mathbb{R}$ then:

$$\sum_{j=1}^k d(\widetilde{A}\in I_j)=\sum_{j=1}^k\frac{\int_{I_k}\widetilde{A}(x)dx}{\int_{\mathbb{R}}\widetilde{A}(x)dx}=\frac{\int_{\cup_{j=1}^k I_j}\widetilde{A}(x)dx}{\int_{\mathbb{R}}\widetilde{A}(x)dx}=\frac{\int_{\mathbb{R}}\widetilde{A}(x)dx}{\int_{\mathbb{R}}\widetilde{A}(x)dx}=1.$$

Also, $d(\widetilde{A}\in I)=1$ if and only if $\frac{\int_I\widetilde{A}(x)dx}{\int_{supp(\widetilde{A})}\widetilde{A}(x)dx}=1$ if and only if $supp(\widetilde{A})\subseteq I$. □

Remark 2.1. *It is worth noting that $d(\widetilde{A}\in I)$ may be interpreted as the probability that $\widetilde{A}$ belongs to I. Moreover, based on a given sequence of disjoint intervals $\{I_j\}_{j=1}^k$, from the aforementioned lemma, it can be concluded that $\widetilde{A}\in I_{j*}$ if $d(\widetilde{A}\in I_{j*})=\max_{j=1}^k d(\widetilde{A}\in I_j)$. The possible interpretations of the proposed belonging degree d are listed in Table 1.*

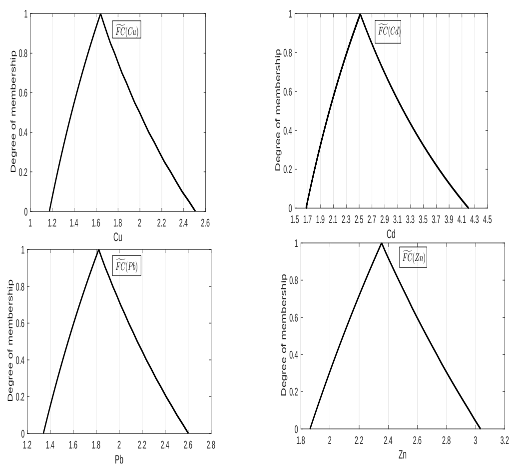

Figure 1: Membership functions of $\widetilde{CF}$s for heavy metals in Example.

Table 1

No.	range of d	interpretation
1	$d \in [0.0, 0.05)$	$\widetilde{A}$ is completely out of I
2	$d \in [0.05, 0.15)$	$\widetilde{A}$ is absolutely out of I
3	$d \in [0.15, 0.25)$	$\widetilde{A}$ is strongly out of I
4	$d \in [0.25, 0.35)$	$\widetilde{A}$ is more or less out of I
5	$d \in [0.35, 0.45)$	$\widetilde{A}$ is weakly out of I
6	$d \in [0.45, 0.55]$	is not decisive
7	$d \in (0.55, 0.65]$	$\widetilde{A}$ weakly belongs to I
8	$d \in (0.65, 0.75]$	$\widetilde{A}$ more or less belongs to I
9	$d \in (0.75, 0.85]$	$\widetilde{A}$ strongly belongs to I
10	$d \in (0.85, 0.95]$	$\widetilde{A}$ absolutely belongs to I
11	$d \in (0.95, 1]$	$\widetilde{A}$ completely belongs to I

Table 2: Degrees to which $\widetilde{CF}(A)$ belongs to I_i, $i = 1, 2, 3, 4$ in Example.

	$I_1 = (-\infty, 1)$	$I_2 = [1, 3)$	$I_3 = [3, 6)$	$I_4 = [6, \infty)$
$d(\widetilde{CF}(Zn) \in I_j)$	0	0.995	0.005	0
$d(\widetilde{CF}(Pb) \in I_j)$	0	1	0	0
$d(\widetilde{CF}(Cd) \in I_j)$	0	0.66	0.34	0
$d(\widetilde{CF}(Cu) \in I_j)$	0	1	0	0

Table 3: Degrees to which $\widetilde{I}_{geo}(A)$ belongs to I_i, $i = 1, 2, \ldots, 7$ in Example.

	$I_1 = (-\infty, 0)$	$I_2 = [0, 1)$	$I_3 = [1, 2)$	$I_4 = [2, 3)$	$I_5 = [3, 4)$	$I_6 = [4, 5)$	$I_7 = [5, \infty)$
$d(\widetilde{I}_{geo}(Zn) \in I_j)$	0	0.91	0.09	0	0	0	0
$d(\widetilde{I}_{geo}(Pb) \in I_j)$	0.1	0.90	0	0	0	0	0
$d(\widetilde{I}_{geo}(Cd) \in I_j)$	0	0.74	0.26	0	0	0	0
$d(\widetilde{I}_{geo}(Cu) \in I_j)$	0.27	0.73	0	0	0	0	0

Table 4: Degrees to which $\widetilde{PLI}$ belongs to I_i, $i = 1, 2, 3, 4$ in Example.

	$I_1 = (-\infty, 0)$	$I_2 = [0, 0.5)$	$I_3 = [0.5, 1)$	$I_4 = [1, \infty)$
$d(\widetilde{PLI} \in I_j)$	0	0.03	0.97	0

Table 5: Degrees to which $\widetilde{MCF}$ belongs to I_i, $i = 1, 2, 3, 4$ in Example.

	$I_1 = (-\infty, 1)$	$I_2 = [1, 3)$	$I_3 = [3, 6)$	$I_4 = [6, \infty)$
$d(\widetilde{MCF} \in I_j)$	0	1	0	0

3 Pollution criteria based on fuzzy information

To evaluate heavy metal enrichment and degree of contamination in soils, the fuzzy set theory was used for fuzzy pollution criteria aiming to derive contamination degree in this section. In this regard, this paper is focused on the most commonly used indices including enrichment factor, geo-accumulation index, and pollution load

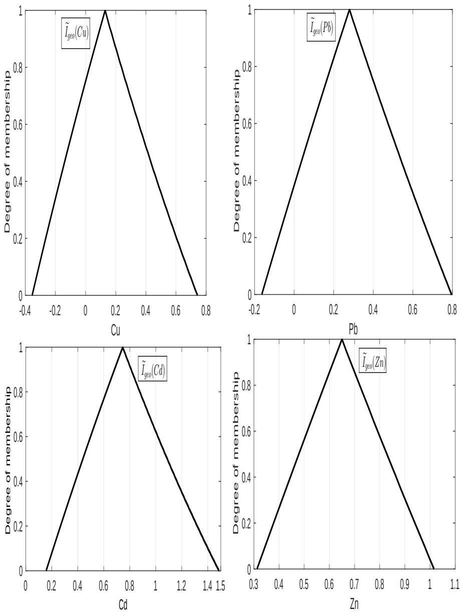

Figure 2: Membership functions of $\widetilde{I}_{geo}$s for heavy metals in Example.

index. For this purpose, we suggest a method inspired by Buckley [6]. He introduced a fuzzy method based on the confidence interval for estimating the mean of a population. In this paper, instead of obtaining a fuzzy-valued estimation of a mean or a median, we define a triangular fuzzy number using the standard confidence interval for mean or median at a given significance level. The procedure is illustrated by the following definitions.

Definition 3.1. *Let $\boldsymbol{x}^A = (x_1^A, x_2^A, \ldots, x_n^A)$ and $\boldsymbol{x}^{AB} = (x_1^{AB}, x_2^{AB}, \ldots, x_m^{AB})$ be two random samples of concentrations of located sites and their background soil of heavy metal of A. The fuzzy enrichment factor of a heavy metal A (**FEF**(A)) is defined to be a fuzzy number with the following α-cuts:*

$$\widetilde{EF}(A)[\alpha] = [(\widetilde{EF}(A))_\alpha^L, (\widetilde{EF}(A))_\alpha^U],$$

where

$$(\widetilde{EF}(A))_\alpha^L = \inf_{(c^A, c_M^{AB}, c_{MAD}^{AB}) \in \mathcal{K}^{AB}[\alpha]} \frac{c^A}{(c_M^{AB} + 2c_{MAD}^{AB})},$$

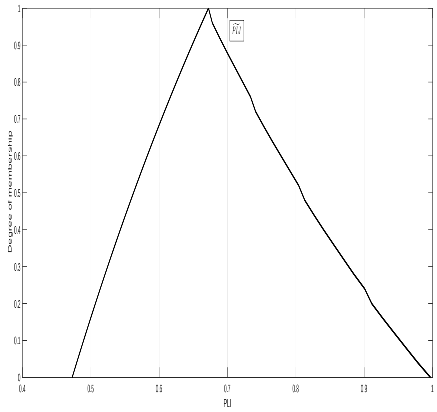

Figure 3: Membership function of $\widetilde{PLI}$ in Example.

$$(\widetilde{EF}(A))_\alpha^U = \sup_{(c^A, c_M^{AB}, c_{MAD}^{AB}) \in \mathcal{K}^{AB}[\alpha]} \frac{c^A}{(c_M^{AB} + 2c_{MAD}^{AB})},$$

in which

1. $\mathcal{K}^{AB}[\alpha] = \widetilde{c}^A[\alpha] \times \widetilde{c}_M^{AB}[\alpha] \times \widetilde{c}_{MAD}^{AB}[\alpha]$,

2. $$\widetilde{c}^A = \begin{cases} (\overline{x}^A; t_{0.025,n-1}\frac{S^A}{\sqrt{n}})_T & \text{if } n \text{ is small,} \\ (\overline{x}^A; z_{0.025}\frac{S^A}{\sqrt{n}})_T & \text{if } n \text{ is large,} \end{cases}$$
   *where $\overline{x}^A = \frac{1}{n}\sum_{i=1}^n x_i^A$, $S^A = \sqrt{\frac{1}{n-1}\sum_{i=1}^n (x_i^A - \overline{x}^A)^2}$ is the fuzzy sample mean concentration (**SFC**) of a given metal at contaminated sites. z_α is also the α^{th} percentile of the standard normal distribution, and $t_{\nu,\alpha}$ stands for α^{th} percentile of the t-distribution with ν degrees of freedom.*

3. $\widetilde{c}_M^{AB} = (M_{\boldsymbol{x}^{AB}}; 1.57\frac{IQR_{\boldsymbol{x}^{AB}}}{\sqrt{m}})_T$ *is the fuzzy sample median concentration (**SFMC**) of an element in the background soil sample,*

4. $\widetilde{c}_{MAD}^{AB} = (M_{\boldsymbol{y}^{AB}}; 1.57\frac{IQR_{\boldsymbol{y}^{AB}}}{\sqrt{m}})_T$ *is the fuzzy sample median absolute deviation (**SF-MAD**) from the median of an element A in the background soil sample,*

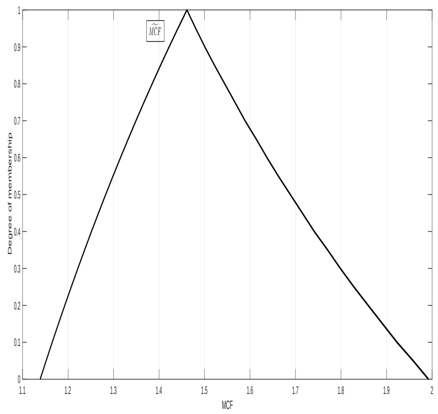

Figure 4: Membership function of $\widetilde{MCF}$ in Example.

in which

a) $M_{\boldsymbol{x}^{AB}}$ *is the sample median based on the random sample of* $\boldsymbol{x}^{AB}$*,*

b) $\boldsymbol{y}^{AB} = (|x_1^{AB} - M_{\boldsymbol{y}^{AB}}|, |x_2^{AB} - M_{\boldsymbol{y}^{AB}}|, \ldots, |x_m^{AB} - M_{\boldsymbol{y}^{AB}}|)$,

c) $M_{\boldsymbol{y}^{AB}}$ *is the sample median based on the random sample of* $\boldsymbol{y}^{AB}$*,*

d) $IQR_{\boldsymbol{x}^{AB}}$ *and* $IQR_{\boldsymbol{y}^{AB}}$ *denote the interquartile range based on the random sample* $\boldsymbol{x}^{AB}$ *and* $\boldsymbol{y}^{AB}$*, respectively.*

Definition 3.2. *Let* $\boldsymbol{x}^A = (x_1^A, x_2^A, \ldots, x_n^A)$ *and* $\boldsymbol{x}^{AB} = (x_1^{AB}, x_2^{AB}, \ldots, x_m^{AB})$ *be two random samples of concentrations of located sites and their background soil of heavy metal of A. The fuzzy contamination factor of a heavy metal A (**FCF**(A)) is defined to be a fuzzy number with the following* α*-cuts:*

$$\widetilde{CF}(A)[\alpha] = [(\widetilde{CF}(A))_\alpha^L, (\widetilde{CF}(A))_\alpha^U], \qquad (3.3)$$

where

$$(\widetilde{CF}(A))_\alpha^L = \inf_{(c^A, c_M^{AB}) \in \mathcal{K}^{AB}[\alpha]} \frac{c^A}{c_M^{AB}},$$

$$(\widetilde{CF}(A))_\alpha^U = \sup_{(c^A, c_M^{AB}) \in \mathcal{K}^{AB}[\alpha]} \frac{c^A}{c_M^{AB}},$$

in which $\mathcal{K}^{AB}[\alpha] = \widetilde{c}^A[\alpha] \times \widetilde{c}_M^{AB}[\alpha]$ *and* $\widetilde{c}^A$*,* $\widetilde{c}_M^{AB}$ *are defined in Definition 3.4.*

Definition 3.3. *Let* $\boldsymbol{x}^A = (x_1^A, x_2^A, \ldots, x_n^A)$ *and* $\boldsymbol{x}^{AB} = (x_1^{AB}, x_2^{AB}, \ldots, x_m^{AB})$ *be two random samples of concentrations of located sites and their background soil of heavy metal of A. The fuzzy geo-accumulation index of a heavy metal A (**FIGEO**(A)) is defined to be a fuzzy number* $\widetilde{I}_{geo}$ *with the following* α*-cuts:*

$$\widetilde{I}_{geo}(A)[\alpha] = [(\widetilde{I}_{geo}(A))_\alpha^L, (\widetilde{I}_{geo}(A))_\alpha^U], \qquad (3.4)$$

where

$$(\widetilde{I}_{geo}(A))_\alpha^L = \inf_{(c^A, c_M^{AB}) \in \mathcal{K}^{AB}[\alpha]} \log_2\left(\frac{c^A}{1.5 c_M^{AB}}\right),$$

$$(\widetilde{I}_{geo}(A))_\alpha^U = \sup_{(c^A, c_M^{AB}) \in \mathcal{K}^{AB}[\alpha]} \log_2\left(\frac{c^A}{1.5 c_M^{AB}}\right),$$

in which $\mathcal{K}^{AB}[\alpha] = \widetilde{c}^A[\alpha] \times \widetilde{c}_M^{AB}[\alpha]$.

Definition 3.4. *The fuzzy mean contamination in soil (**FMCF**) by all metals* $A_1, A_2, \ldots, A_k$ *is defined to be a fuzzy number as* $\widetilde{MCF} = \frac{1}{k} \oplus_{l=1}^k \widetilde{CF}(A_l)$*. Furthermore, the fuzzy pollution load index (**FPLI**) is defined as a fuzzy number:*

$$\widetilde{PLI} = (\otimes_{l=1}^k \widetilde{CF}(A_l))^{\frac{1}{k}}. \qquad (3.5)$$

It is also noticeable that, based on the arithmetic operations on α-cuts of fuzzy numbers, the α-cuts of **FMCF** and **FPLI** can be evaluated as follows:

$$\widetilde{MCF}[\alpha] = [\frac{1}{k}\sum_{l=1}^k (\widetilde{CF}(A_l))_\alpha^L, \frac{1}{k}\sum_{l=1}^k (\widetilde{CF}(A_l))_\alpha^U], \qquad (3.6)$$

and

$$\widetilde{FPLI}[\alpha] = [(\prod_{l=1}^k (\widetilde{CF}(A_l))_\alpha^L)^{\frac{1}{k}}, (\prod_{l=1}^k (\widetilde{CF}(A_l))_\alpha^U)^{\frac{1}{k}}]. \qquad (3.7)$$

In the following, the feasibility and effectiveness of the extended fuzzy pollution criteria are examined via a numerical example presented by Grzebisz et al. [11].

Example 3.1. *This example considers the city of Poznan (Poland) to identify its dangerous heavy metals load and define areas of their environmental impact. In this regard, four heavy metals of Pb, Cd, Zn, and Cu were studied in 350 sites to assess heavy metals contamination. The descriptive statistics of basic surface soil properties in the surface horizon are listed in Table 1, pp. 495 of [11]. Soil samples were collected from the depth of 0-20 cm. The proposed fuzzy pollution indices were applied in this study to discover possible sources that might influence the different distribution of elements over the study area.*

Note that the random sample of background values corresponding to each heavy metal Pb, Cd, Zn, and Cu are not given in the Grzebisz et al.'s paper. However, they evaluated the background mean values Pb, Cd, Zn, and Cu as 16.8, 0.3, 31.7, and 10, respectively. As discussed in the Introduction section, to evaluate heavy metals pollution of urban soils, it is better to model such quantities as fuzzy numbers instead of exact values. In this regard, without the loos of generality, the artificial background values are provided as symmetric **TFN**s including: $\widetilde{c}_M^{PbB} = (16.8; 4)_T$, $\widetilde{c}_M^{CdB} = (0.3; 0.1)_T$, $\widetilde{c}_M^{ZnB} = (31.7; 5)_T$, and $\widetilde{c}_M^{CuB} = (10; 3)_T$. Based on the proposed method, the fuzzy contamination corresponding to each heavy metal are then evaluated as symmetric **TFN**s as $\widetilde{c}^{Pb} = (30.58; 2.74)_T$, $\widetilde{c}^{Cd} = (0.755; 0.086)_T$, $\widetilde{c}^{Zn} = (72.98; 5.92)_T$, and $\widetilde{c}^{Cu} = (16.41; 1.16)_T$. The **FCF** of the studied heavy metals (Pb, Cd, Zn, and Cu) are plotted in Fig. 1. Moreover, $d(\widetilde{CF} \in I_j)$ values of the aforementioned metals are listed in Table 2 for each corresponding contaminated level. As can be, $d(\widetilde{CF}(A) \in I_1) = \max_{i=1}^{4} d(\widetilde{CF}(A) \in I_j)$ for each heavy metal $(A = Cu, Pb, Cd, Zn)$. According to Definition 2.1, it can be said that the **CF**-values for all heavy metals fall in the class I_2 which indicates the moderate contamination of the soil. Simultaneously, we may conclude that the Poznan soil is 1) moderately polluted by Zn and Pb, 2) polluted more or less moderately by Cu and Cd. Based on **FCF**s listed in Table 2, the concentration of heavy metals in the soil varied in the following increasing trend: $Cu > Pb > Zn > Cd$. This suggests that Cu and Pb pollution is relatively serious compared to other metals. Based on **FIGEO**-values in Table 3, it can be also concluded that the Poznan soil is moderately polluted with these metals. The plot of **FIGEO**s for all heavy metals (Cu, Pb, Cd, Cr, and Ni) are also depicted in Fig. 2. Furthermore, the results indicated that the Parzen soil is moderately polluted by Pb, Cd, Zn, and Cu. In this regard, the Poznan soil is 1) polluted fully moderately by Zn and Pb, 2) polluted more or less moderately by Cu and Cd. According to the proposed **FIGEO**, the heavy metal contamination of the soil declined in the following order: $Zn > Pb > Cu > Cd$. Moreover, the plots of **FPLI** and **FMCF** for mentioned metals are shown in Figures 3 and 4, respectively. The degrees to which **FIGEO**s of the underlying heavy metals belong to each contaminated levels $I_1 - I_4$ are also presented in Table 4. The results suggest the need for more detailed study to monitor the Poznan soil. Moreover, from Table 5, it can be concluded that Poznan soil is moderately polluted by Pb, Cd, Zn, and Cu.

4 Conclusion

Pollution criteria plays a crucial role in monitoring heavy metal contamination in real applications. The classical procedures exploit exact indices to describe the degree of pollution with a heavy metal in the environment. However, the heavy metals contamination is often a non-exact value due to different reasons such as soil's features. Regarding the nature of such quantities, it is better to model the heavy metals contamination by fuzzy sets. This paper extends some common pollution criteria based on the fuzzy contamination of heavy metals. For this purpose, the α-cuts approaches were employed to construct fuzzy pollution indices. A criterion is also suggested to evaluate the degree to which a fuzzy pollution index belongs to its relevant pollution levels. The possible effectiveness and advantages of the proposed method are also illustrated using a real data set. Results show that the proposed method performs quite well in providing fuzzy pollution indices in real-world applications. However, the proposed method is general and should be explored for other pollution indices.

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