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Developing an Efficiency Based Target Setting Model for K-Stage Serial Structures

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Abstract

Data envelopment analysis as an effective approach to evaluate the relative efficiency of decision-making units, recently applied for developing production plans in terms of resource allocation and target setting in many production and operation systems. Most of these systems are of network structures where the outputs from a given stage are intermediate measures which form the inputs for the next stage. This paper proposes a new k-stage DEA based multi objective linear programming model for target setting in the serial network structures, considering undesirable outputs. Since the proposed model takes the size of under evaluation units into account when developing the inputs/outputs arrangements, the new idea ensures the possibility of the results. An empirical example of a production system is used to illustrate the applicability of the proposed model. The results show that based on the new plan, all of the efficiency scores are improved.

Keywords : K-Stage Network; Data Envelopment Analysis (DEA); Production Planning; Undesirable Outputs; Efficiency Evaluation.

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1 Introduction

Data envelopment analysis first introduced by Charnes et al. [7] is a nonparametric approach used extensively to evaluate the efficiencies of a set of homogenous units with multiple inputs and multiple outputs. Because of its development and widespread applications, DEA has

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attracted essential attention among academics and practitioners. In the past few years, DEA has become increasingly popular in efficiency analysis with more than 5000 articles published (Emrouznejad et al. [11]). The application of DEA as a non parametric quantitative tool is not restricted to assess the relative efficiency of peer units. Many extensions of the original work of Charnes et al. [7] have been proposed and used successfully in a wide variety of applications especially in production planning.

Production planning has been considered as one of the most important parts of production and operation systems due to its role in arrangement of inputs and outputs and set the production mission. During recent years, more and more

attentions have been paid to applying different approach to production planning problem, which brings a different viewpoint on finding the preferred production plans. Recently, data envelopment analysis (DEA) as an effective approach applied for developing production plans in terms of resource allocation and target setting, from the efficiency point of view.

Researches on resource allocation and target setting have been published by many authors. Golany and Tamir [13] presented a DEA based resource allocation model which simultaneously determines input and output targets based on maximizing total output. Fare et al. [12] used DEA for modeling the possibility of reallocation of a fixed input. Their paper considers the problem of input target setting when there is an equality relationship between the levels of input devoted to various uses. Based on the invariance and pareto-minimality principals, Cook and Kress [8] developed an approach to equitable allocation of fixed /common cost which is imposed on all decision making units. Beasley [6] based on the idea that DEA can be used to maximize the average efficiency of DMUs, developed some DEA based models for allocating fixed costs and allocating inputs resources and output targets to DMUs, simultaneously. Based on the Cook and Kress [8] Jahanshahloo et al. [15] suggested an alternative approach for equitable allocation of shared costs using invariance principle and without solving any LP problem. Korhonen and Syrjanen [16] formulated a combined MOLP-DEA model to find the most preferred resource allocation plan in a central decision making environment. Lozano and Villa [19] considered a centralized DMU who owns or supervises all the operating units. In such intra-organizational scenario the DM has an interest in maximizing the efficiency of individual units at the same time that total input consumption is minimized or total output production is maximized. Lozano et al. [18] used DEA based centralized target setting for regional recycling operations. They applied DEA for analyzing the performance of a number of Spanish municipalities. Cook and Zhu [9] extended the Cook and Kress [8] approach, and provided a practical equitable approach to the cost allocation problem under the condition of variable returns to scale.

Lozano and Villa [20] proposed two different multi-objective DEA target-setting approaches for a port logistics problem. The first one is an interactive approach and the second, uses a lexicographic multi-objective approach. The weights of inputs and outputs in both approaches are derived using AHP. Lin [17] showed that under a certain conditions, Cook and Zhus approach doesnt provide feasible solution. Based on this concept he improved the work of Cook and Zhu [9] and developed a new fixed costs / resources allocation approach to set fixed targets according to the amount of fixed resources shared by individual DMUs. Asmild et al. [5] based on the concept of centralized DEA models, reconsidered the resource allocation models proposed by Lozano and villa [19] and Lozano et al. [18] in a centralized decision making environment and suggest to only consider adjustments of previously inefficient units. Amirteimoori and Kordrostami [1] developed an efficiency based model to equitably allocate imposed fixed costs/ inputs and setting fixed targets to DMUs. Du et al. [10] developed two DEA-based production planning approaches to (a) maximizing the average or overall production performance of the entire organization and (b) simultaneously maximizing total outputs produced and minimizing total inputs consumed by all units.

Amirteimoori and Mohaghegh Tabar [3] presented a DEA-based approach for allocating experts among gas companies and setting targets for them and showed that how output targets can be decided at the same time as decisions is made about resource allocation. Amirteimoori and Kordrostami [2] introduced a DEA approach based on the Du et al. [10] to making future production plans in a centralized decision making environment when demand changes can be forecasted in the next production season. Zhang et al. [23] in their study takes into account the factors that influence units production abilities to avoid new plans beyond units abilities or not fully exploring their potentials. Assuming demand changes in the next production season can be predicted they developed two DEA-based approaches for production planning to optimize the total resource utilization. Hatami-Marbini et al. [14] used a common set of weights method based on the goal

programming concept, proposed a DEA model to determine the amount of input and output reduction needed for each DMU to increase the efficiency score across the reference set. Wu et al. [22] developed a DEA based centralized resource allocation and target setting model based on the concepts of satisfaction degree of DMUs for reallocating the DMUs discretionary inputs and setting their output targets in order to improve the environmental performance of the 30 regional highway transportation systems in China.

While conventional DEA considers a single process that consumes all the inputs and produces all the outputs, network DEA considers several processes each of them consumes its own set of inputs and produce its own set of outputs, in addition to consuming and producing intermediate products. These intermediate products are considered as inputs for some stages and outputs for others (Lozano et al. [18]). To achieve a proper production plan for such interrelated systems, this paper aims to develop a network DEA based multi objective linear programming model in presence of bad (undesirable) factors, in such a way that the overall and corresponding sub-component performance of the all DMUs be improved after implementation of the new plan.

The present paper is organized into four major sections. Section 1 briefly reviews the existing literature on target setting for outputs and inputs in several researches. Section 2, extends the work of Amirteimoori and Kordrostami [2] into a k-stage serial structure. Section 3, illustrates the proposed modal by a data set from a chain of poultry farms in Guilan province, Iran. Finally, section 4 develops discussion and conclusion.

2 The proposed model

Production in large organizations with a centralized decision-making environment involves the participation of more than one unit, each contributing a part of the total production. Several DEA-based studies conducted in such an environment, and a few of them dealt with production planning problem. On the other hand, there are many multi stage network processes in which some outputs from a given stage do not become inputs to the next stage, and the intermediate
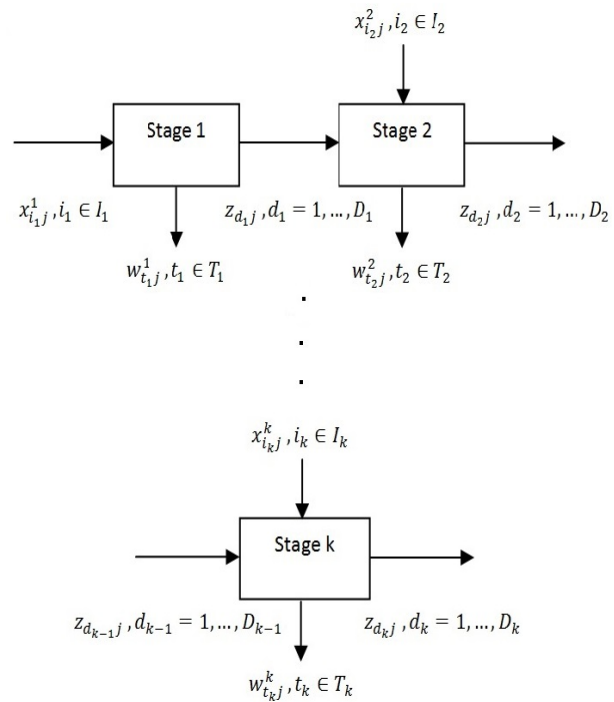

Figure 1: A k-stage serial production system

stage has its own inputs and outputs. In addition, each stage may produce undesirable outputs. This study aims to propose a DEA based model for production planning in the network structures with centralized decision-making environment.

Suppose, there are a set of n DMUs denoted by DMU_j $(j = 1, 2, ..., n)$, each of them consists of k sub-DMUs $(k = 1, 2, ..., K)$. Suppose, DMU_j consumes m_1 inputs in the first stage $x_{i_1 j}^1$ $(i_1 = 1, 2, ..., m_1)$, m_2 inputs in the second stage $x_{i_2 j}^2$ $(i_2 = 1, 2, ..., m_2)$ and m_k inputs in the k-th stage $x_{i_k j}^k$ $(i_k = 1, 2, ..., m_k)$. Suppose also, DMU_j produces D_1 outputs in the first stage which uses as inputs of the second stage $z_{d_1 j}^1$ $(d_1 = 1, 2, ..., D_1)$ and referred to as intermediate measures. The intermediate measures of the second and (k-1)-th stages denoted by $z_{d_2 j}^2$ $(d_2 = 1, 2, ..., D_2)$ and $z_{d_{k-1} j}^{k-1}$ $(d_{k-1} = 1, 2, ..., D_{k-1})$, respectively. Note that $z_{d_k j}^k$ $(d_k = 1, 2, ..., D_k)$ known as the outputs of the k-th stage which leave the system to satisfy customers demands. In addition to the above measures, each stage may produce undesirable outputs as by-products of the pro-
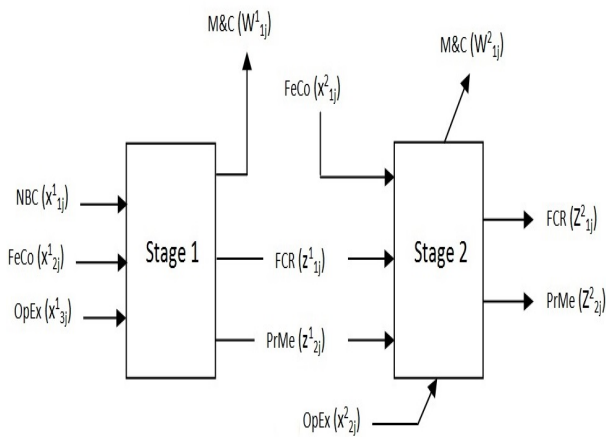
Figure 2: A two stage serial production system

duction process. We define the undesirable outputs of the first, second and k-th stage as $w^1_{t_1 j}$ $(t_1 = 1, 2, ..., T_1)$, $w^2_{t_2 j}$ $(t_2 = 1, 2, ..., T_2)$ and $w^k_{t_k j}$ $(t_k = 1, 2, ..., T_k)$, respectively. Figure 1 shows a conventional multi stage network process.

The relative efficiencies of a set of n DMUs in their first stage and k-th stage $(k \neq 1)$, can be calculated using the following linear fractional models:

$$\max \quad \frac{\sum\limits_{d_1=1}^{D_1} \mu_{d_1} z^1_{d_1 o}}{\sum\limits_{i_1=1}^{I_1} \nu_{i_1} x^1_{i_1 o} + \sum\limits_{t_1=1}^{T_1} \beta_{t_1} w^1_{t_1 o}} = e^1_o$$

$$s.t. \quad \frac{\sum\limits_{d_1=1}^{D_1} \mu_{d_1} z^1_{d_1 j}}{\sum\limits_{i_1=1}^{I_1} \nu_{i_1} x^1_{i_1 j} + \sum\limits_{t_1=1}^{T_1} \beta_{t_1} w^1_{t_1 j}} \leq 1 \ , \ \forall \ i_1, t_1 \qquad (2.1)$$

$$\nu_{i_1}, \mu_{d_1}, \beta_{t_1} \geq \varepsilon \ , \quad \forall \ j$$

and,

$$\max \quad \frac{\sum\limits_{d_k=1}^{D_k} \mu_{d_k} z^k_{d_k o}}{\sum\limits_{d_{k-1}=1}^{D_{k-1}} \mu_{d_{k-1}} z^{k-1}_{d_{k-1} o} + \sum\limits_{i_k=1}^{I_k} \nu_{i_k} x^k_{i_k o} + \sum\limits_{t_k=1}^{T_k} \beta_{t_k} w^k_{t_k o}}$$

$$= e^k_o$$

$$s.t. \quad \frac{\sum\limits_{d_k=1}^{D_k} \mu_{d_k} z^k_{d_k j}}{\sum\limits_{d_{k-1}=1}^{D_{k-1}} \mu_{d_{k-1}} z^{k-1}_{d_{k-1} j} + \sum\limits_{i_k=1}^{I_k} \nu_{i_k} x^k_{i_k j} + \sum\limits_{t_k=1}^{T_k} \beta_{t_k} w^k_{t_k j}}$$

$$\leq 1 \ , \ (k \neq 1)$$

$$\nu_{i_k}, \mu_{d_k}, \beta_{t_k} \geq \varepsilon \ , \quad \forall \ j,$$

(2.2)

where ν_{i_k}, μ_{d_k} and β_{t_k} are the factor weights assigned to the i-th input, d-th desirable output and the t-th undesirableoutput, in k-th stage. ν_{i_k}, μ_{d_k} and β_{t_k} are a set of the most favorable weights for the DMU_o in the sense of maximizing the ratio scale.

Based on the concept of target setting for inputs and outputs, central DMU should plan the new arrangement of the inputs/outputs for next production season. Suppose, the change of output d in stage k for the next production season, denoted by $F_{d_k}(d_k = 1, 2, , D_k)$ is predictable, all can be either positive or negative, corresponding to an increase or a decrease in the demand for output d in stage k. The amount of change in input i and undesirable output t for the stage k referred to as $C_{i_k}(i_k = 1, 2, , I_k)$ and $G_{t_k}(t_k = 1, 2, , T_k)$, respectively. We introduce variable $c_{i_k j}$ to represent the change in input i of DMU_j in the k-th stage for the next production season, $g_{t_k j}$ to represent the change in undesirable output t of DMU_j in the k-th stage for the next production season and $f_{d_k j}$ to represent the change in desirable output d of DMU_j in the k-th stage for the next production season. Therefore, $\overline{x}^k_{i_k j} = x^k_{i_k j} + c^k_{i_k j}$, $\overline{z}^k_{d_k j} = z^k_{d_k j} + f^k_{d_k j}$ and $\overline{w}^k_{t_k j} = w^k_{t_k j} + g^k_{t_k j}$ represent the amount of i-th input, d-th desirable output and t-th undesirable output of DMU_j in the k-th stage for the next production season, respectively. Clearly, $\sum_{j=1}^{n} c^k_{i_k j} = C^k_{i_k}$, $\sum_{j=1}^{n} f^k_{d_k j} = F^k_{d_k}$ and $\sum_{j=1}^{n} g^k_{t_k j} = G^k_{t_k}$.

The main assumption of the modeling process is that the produced outputs can be increased in the next production period when input usages are increased. In other words, it's assumed that the decision making units have the authority to increase their outputs when their inputs usages are increased. In the proposed approach, we believe that the change of inputs and undesirable outputs in the next season should be planned in such a way that each DMU_j have an efficiency score (e_{j-ns}) greater than or equal to its original efficiency in the current period. Hence, we must have the following model. As it can be seen, first inequality ensures the new efficiency of stage 1 in DMU_j be greater than or equal to its old efficiency. Similarly, second inequality ensures the new efficiency of stage 1 in DMU_j be greater than or equal to its old efficiency of stage

k ($k \neq 1$) in DMU_j. These inequalities are subjected to the constraints which limit share of all DMUs from the inputs/ outputs changes to the predicted (outputs) or planned (inputs) changes.

$$e^1_{j-ns} = \frac{\sum\limits_{d_1=1}^{D_1} \mu_{d_1} (z^1_{d_1 j} + f^1_{d_1 j})}{\sum\limits_{i_1=1}^{I_1} \nu_{i_1} (x^1_{i_1 j} + c^1_{i_1 j}) + \sum\limits_{t_1=1}^{T_1} \beta_{t_1} (w^1_{t_1 j} + g^1_{t_1 j})} \geq e^1_j$$

$$e^k_{j-ns} = \frac{\sum\limits_{d_k=1}^{D_k} \mu_{d_k} (z^k_{d_k j} + f^k_{d_k j})}{\sum\limits_{d_{k-1}=1}^{D_{k-1}} \mu_{d_{k-1}} (z^{k-1}_{d_{k-1} j} + f^{k-1}_{d_{k-1} j}) + \sum\limits_{i_k=1}^{I_k} \nu_{i_k} (x^k_{i_k j}}$$

$$\cdots \frac{}{+ c^k_{i_k j}) + \sum\limits_{t_k=1}^{T_k} \beta_{t_k} (w^k_{t_k j} + g^k_{t_k j})} \geq e^k_j \,, (k \neq 1)$$

s.t.

$$\sum_{j=1}^{n} c^k_{i_k j} = C^k_{i_k} \,, \ \forall \ i_k$$

$$\sum_{j=1}^{n} f^k_{d_k j} = F^k_{d_k} \,, \ \forall \ d_k$$

$$\sum_{j=1}^{n} g^k_{t_k j} = G^k_{t_k} \,, \ \forall \ t_k$$

$$\nu_{i_k}, \mu_{d_k}, \beta_{t_k} \geq \varepsilon \,, \ \forall \ j$$

$$c^k_{i_k j} \geq 0 \ if \ C^k_{i_k} \geq 0 \,, \ c^k_{i_k j} \leq 0 \ if \ C^k_{i_k} \leq 0,$$
$$f^k_{d_k j} \geq 0 \ if \ F^k_{d_k} \geq 0 \,, \ f^k_{d_k j} \leq 0 \ if \ F^k_{d_k} \leq 0,$$
$$g^k_{t_k j} \geq 0 \ if \ G^k_{t_k} \geq 0 \,, \ g^k_{t_k j} \leq 0 \ if \ G^k_{t_k} \leq 0.$$

$$(2.3)$$

Obviously, the above system of equations is non-linear. If we make the change of variable $\bar{c}^k_{i_k j} = \nu_{i_k} c^k_{i_k j}$, $\bar{f}^k_{d_k j} = \mu_{d_k} f^k_{d_k j}$ and $\bar{g}^k_{t_k j} = \beta_{t_k} g^k_{t_k j}$, the

following linear model will derive:

$$e^1_{j-ns} = \frac{\sum\limits_{d_1=1}^{D_1} \mu_{d_1} z^1_{d_1 j} + \sum\limits_{d_1=1}^{D_1} \bar{f}^1_{d_1 j}}{\sum\limits_{i_1=1}^{I_1} \nu_{i_1} x^1_{i_1 j} + \sum\limits_{i_1=1}^{I_1} \bar{c}^1_{i_1 j} + \sum\limits_{t_1=1}^{T_1} \beta_{t_1} w^1_{t_1 j} + \sum\limits_{t_1=1}^{T_1} \bar{g}^1_{t_1 j}}$$
$$\geq e^1_j$$

$$e^k_{j-ns} = \frac{\sum\limits_{d_k=1}^{D_k} \mu_{d_k} z^k_{d_k j} + \sum\limits_{d_k=1}^{D_k} \bar{f}^k_{d_k j}}{\sum\limits_{d_{k-1}=1}^{D_{k-1}} \mu_{d_{k-1}} z^{k-1}_{d_{k-1} j} + \sum\limits_{d_{k-1}=1}^{D_{k-1}} \bar{f}^{k-1}_{d_{k-1} j} + \sum\limits_{i_k=1}^{I_k} \nu_{i_k} x^k_{i_k j}}$$

$$\cdots \frac{}{+ \sum\limits_{i_k=1}^{I_k} \bar{c}^k_{i_k j} + \sum\limits_{t_k=1}^{T_k} \beta_{t_k} w^k_{t_k j} + \sum\limits_{t_k=1}^{T_k} \bar{g}^k_{t_k j}} \geq e^k_j \,, \ (k \neq 1)$$

$$(2.4)$$

s.t.

$$\sum_{j=1}^{n} \bar{c}^k_{i_k j} = \nu_{i_k} C^k_{i_k} \,, \ \forall \ i_k, k$$

$$\sum_{j=1}^{n} \bar{f}^k_{d_k j} = \mu_{d_k} F^k_{d_k} \,, \ \forall \ d_k, k$$

$$\sum_{j=1}^{n} \bar{g}^k_{t_k j} = \beta_{t_k} G^k_{t_k} \,, \ \forall \ t_k, k \qquad (2.5)$$

$$\nu_{i_k}, \mu_{d_k}, \beta_{t_k} \geq \varepsilon \,, \ \forall \ j$$

$$\bar{c}^k_{i_k j} \geq 0 \ if \ C^k_{i_k} \geq 0 \,, \ \bar{c}^k_{i_k j} \leq 0 \ if \ C^k_{i_k} \leq 0,$$
$$\bar{f}^k_{d_k j} \geq 0 \ if \ F^k_{d_k} \geq 0 \,, \ \bar{f}^k_{d_k j} \leq 0 \ if \ F^k_{d_k} \leq 0,$$
$$\bar{g}^k_{t_k j} \geq 0 \ if \ G^k_{t_k} \geq 0 \,, \ \bar{g}^k_{t_k j} \leq 0 \ if \ G^k_{t_k} \leq 0.$$

Suppose the change of d-th desirable output, t-th undesirable output and the change of input consumption for i-th (external) input of DMU_j in k-th stage are denoted by $\varphi_{kj} F^k_{d_k}, \delta_{kj} G^k_{t_k}$ and $\alpha_{kj} C^k_{ik}$, respectively. To develop a rational plan, $\varphi_{kj}, \delta_{kj}$ and α_{kj} should be selected proportionately to the size of DMU_j. In order to develop a feasible production planning model, first we determine the potential of each of the DMUs in term of the magnitude size of the input and output:

Definition 2.1 *The magnitude size of DMU_o on the input side in stage k, denoted by MSI_{ko}, is defined as the optimal objective value of the following models for stages 1 and k ($k \neq 1$), respectively:*

$$MSI_{1o} = \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 o}^1$$

$$s.t. \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j}^1 \le 1 , \ \forall i_1 \qquad (2.6)$$

$$\nu_{i_1} \ge \varepsilon , \ \forall j$$

and,

$$MSI_{ko} = \sum_{i_k=1}^{I_k} \nu_{i_k} x_{i_k o}^k + \sum_{d_k=1}^{D_k} \mu_{d_k} z_{d_k o}^k$$

$$s.t. \sum_{i_k=1}^{I_k} \nu_{i_k} x_{i_k j}^k + \sum_{d_k=1}^{D_k} \mu_{d_k} z_{d_k j}^k \le 1 , \forall j \qquad (2.7)$$

$$\nu_{i_k}, \mu_{d_k} \ge \varepsilon , \ \forall i_k, d_k$$

MSI_o reflects the magnitude of DMU_o in size in stage k and DMU_o is said to be greater than DMU_l in the input side if and only if $MSI_{ko} > MSI_{kl}$.

Definition 2.2 *The magnitude size of DMU_o on the output side in stage k, denoted by MSO_{ko}, is defined as the optimal objective value of the following models for stages 1 and k ($k \ne 1$), respectively:*

$$MSO_{1o} = \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 o}^1 + \sum_{t_1=1}^{T_1} \beta_{t_1} w_{t_1 o}^1$$

$$s.t. \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j}^1 + \sum_{t_1=1}^{T_1} \beta_{t_1} w_{t_1 j}^1 \le 1 , \ \forall j \qquad (2.8)$$

$$\mu_{d_1}, \beta_{t_1} \ge \varepsilon , \ \forall d_1, t_1$$

and,

$$MSO_{ko} = \sum_{d_k=1}^{D_k} \mu_{d_k} z_{d_k o}^k + \sum_{t_k=1}^{T_k} \beta_{t_k} w_{t_k o}^k$$

$$s.t. \sum_{d_k=1}^{D_k} \mu_{d_k} z_{d_k j}^k + \sum_{t_k=1}^{T_k} \beta_{t_k} w_{t_k j}^k \le 1 , \ \forall j \qquad (2.9)$$

$$\mu_{d_k}, \beta_{t_k} \ge \varepsilon , \ \forall d_k, t_k$$

Similarly, DMU_o is said to be greater than DMU_l in the output side of stage k, if and only if $MSO_{ko} > MSO_{kl}$. For each DMU_o, let $\alpha_{ko} = MSI_{ko} \sum_{j=1}^n MSI_{kj}$ and $\varphi_{ko} = \delta_{ko} = MSO_{ko}/\sum_{j=1}^n MSO_{kj}$ with $\sum_{j=1}^n \alpha_{kj} = \sum_{j=1}^n \varphi_{kj} = \sum_{j=1}^n \delta_{kj} = 1$.

With these proportions, we take the input usage and output production of all DMUs into consideration. Note that the difficulty with these values with respect to $\bar{c}_{i_k j}^k$, $\bar{f}_{d_k j}^k$ and $\bar{g}_{t_k j}^k$ is that there is no guarantee that they satisfy model (2.4). In the absence of such a production plan, a rational objective is to introduce goal achievement variables for efficiency level and inputs and outputs levels.

Let, $\bar{c}_{i_k j}^k - \alpha_{kj} \nu_{i_k} C_{i_k}^k = T_{i_k j}^{k+} - T_{i_k j}^{k-}$, $\bar{f}_{d_k j}^k - \varphi_{kj} \mu_{d_k} F_{d_k}^k = T_{d_k j}^{k+} - T_{d_k j}^{k-}$ and $\bar{g}_{t_k j}^k - \delta_{kj} \beta_{t_k} G_{t_k}^k = T_{t_k j}^{k+} - T_{t_k j}^{k-}$ in which $T_{i_k j}^{k+}, T_{i_k j}^{k-}, T_{d_k j}^{k+}, T_{d_k j}^{k-}, T_{t_k j}^{k+}$ and $T_{t_k j}^{k-}$ are non negative deviation variables, above and below the goals. Variables s_{kj}^+ and s_{kj}^- are also goal deviation variables for efficiency measure in stage k. To guarantee the feasibility and to ensure that each DMU_j can preserve its efficiency level, we consider the following model. The functional inequalities of the model guarantee that each sub-DMU preserves its efficiency levels and the goal constraint insures that the new efficiencies shift toward one, as well as possible.

Minimizing the sum of $T_{i_k j}^{k+}$ and $T_{i_k j}^{k-}$ means that we minimize the deviation of $\alpha_{kj} \nu_{i_k} C_{i_k}^k$. Hence, if it is possible for DMU_j to have $c_{i_k j}^k = \alpha_{kj} \nu_{i_k} C_{i_k}^k$, i.e.to have $T_{i_k j}^{k+} - T_{i_k j}^{k-} = 0$, then, it will be automatically found when we come to solve the above program. Similar augmentations can be put forward for $T_{d_k j}^{k+} - T_{d_k j}^{k-}$, $T_{t_k j}^{k+} - T_{t_k j}^{k-}$ and s_{kj}^+, s_{kj}^-. Now, developing an objective function for minimizing the mentioned above deviations, a multi-objective linear programming (MOLP)

model is developed for an optimal target setting:

$$Min \sum_{k=1}^{K} (\sum_{j=1}^{n} [s_{kj}^{+} + s_{kj}^{-}] + \sum_{j=1}^{n} \sum_{i_k=1}^{I_k} [T_{i_k j}^{k+} + T_{i_k j}^{k-}]$$
$$+ \sum_{j=1}^{n} \sum_{d_k=1}^{D_k} [T_{d_k j}^{k+} + T_{d_k j}^{k-}] + \sum_{j=1}^{n} \sum_{t_k=1}^{T_k} [T_{t_k j}^{k+} + T_{t_k j}^{k-}])$$

$$s.t. \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j}^{1} + \sum_{d_1=1}^{D_1} \overline{f}_{d_1 j}^{1} - ([\sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j}^{1}$$
$$+ \sum_{i_1=1}^{I_1} \overline{c}_{i_1 j}^{1}] + [\sum_{t_1=1}^{T_1} \beta_{t_1} w_{t_1 j}^{1} + \sum_{t_1=1}^{T_1} \overline{g}_{t_1 j}^{1}])$$
$$= s_{1j}^{+} - s_{1j}^{-} , \ \forall \ j$$

$$\sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j}^{1} + \sum_{j=1}^{n} \overline{f}_{d_1 j}^{1} - e_{j}^{1}([\sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j}^{1}$$
$$+ \sum_{j=1}^{n} \overline{c}_{i_1 j}^{1}] + [\sum_{t_1=1}^{T_1} \beta_{t_1} w_{t_1 j}^{1} + \sum_{j=1}^{n} \overline{g}_{t_1 j}^{1}]) \geq 0, \forall \ j$$

$$\sum_{d_k=1}^{D_k} \mu_{d_k} z_{d_k j}^{k} + \sum_{d_k=1}^{D_k} \overline{f}_{d_k j}^{k} - ([\sum_{d_{k-1}=1}^{D_{k-1}} \mu_{d_{k-1}}$$
$$z_{d_{k-1} j}^{k-1} + \sum_{d_{k-1}=1}^{D_{k-1}} \overline{f}_{d_{k-1} j}^{k-1}] + [\sum_{i_k=1}^{I_k} \nu_{i_k} x_{i_k j}^{k}$$
$$+ \sum_{i_k=1}^{i_k} \overline{c}_{i_k j}^{k}] + [\sum_{t_k=1}^{T_k} \beta_{t_k} w_{t_k j}^{k} + \sum_{t_k=1}^{T_k} \overline{g}_{t_k j}^{k}])$$
$$= s_{kj}^{+} - s_{kj}^{-} , \ (k \neq 1) \ , \ \forall \ j$$

$$\sum_{d_k=1}^{D_k} \mu_{d_k} z_{d_k j}^{k} + \sum_{d_k=1}^{D_k} \overline{f}_{d_k j}^{k} - e_{j}^{k}([\sum_{d_{k-1}=1}^{D_{k-1}} \mu_{d_{k-1}}$$
$$z_{d_{k-1} j}^{k-1} + \sum_{d_{k-1}=1}^{D_{k-1}} \overline{f}_{d_{k-1} j}^{k-1}] + [\sum_{i_k=1}^{I_k} \nu_{i_k} x_{i_k j}^{k}$$
$$+ \sum_{i_k=1}^{i_k} \overline{c}_{i_k j}^{k}] + [\sum_{t_k=1}^{T_k} \beta_{t_k} w_{t_k j}^{k} + \sum_{t_k=1}^{T_k} \overline{g}_{t_k j}^{k}]) \geq 0, \forall \ j$$

$$\sum_{j=1}^{n} \overline{c}_{i_k j}^{k} = \nu_{i_k} C_{i_k}^{k} , \ \forall \ i_k, k$$
$$\sum_{j=1}^{n} \overline{f}_{d_k j}^{k} = \mu_{d_k} F_{d_k}^{k} , \ \forall \ d_k, k$$
$$\sum_{j=1}^{n} \overline{g}_{t_k j}^{k} = \beta_{t_k} G_{t_k}^{k} , \ \forall \ t_k, k$$

(2.10)

$$\overline{c}_{i_k j}^{k} - \alpha_{kj} \nu_{i_k} C_{i_k}^{k} = T_{i_k j}^{k+} - T_{i_k j}^{k-} , \ \forall \ i_k, j$$
$$\overline{f}_{d_k j}^{k} - \varphi_{kj} \mu_{d_k} F_{d_k}^{k} = T_{d_k j}^{k+} - T_{d_k j}^{k-} , \ \forall \ d_k, j$$
$$\overline{g}_{t_k j}^{k} - \delta_{kj} \beta_{t_k} G_{t_k}^{k} = T_{t_k j}^{k+} - T_{t_k j}^{k-} , \ \forall \ t_k, j$$

$$\nu_{i_k}, \mu_{d_k}, \beta_{t_k}, T_{i_k j}^{k+}, T_{i_k j}^{k-}, T_{d_k j}^{k+}, T_{d_k j}^{k-}, T_{t_k j}^{k+}, T_{t_k j}^{k-},$$
$$s_{kj}^{+}, s_{kj}^{-} \geq \varepsilon$$
$$\overline{c}_{i_k j}^{k} \geq 0 \ if \ C_{i_k}^{k} \geq 0 \ , \ \overline{c}_{i_k j}^{k} \leq 0 \ if \ C_{i_k}^{k} \leq 0,$$
$$\overline{f}_{d_k j}^{k} \geq 0 \ if \ F_{d_k}^{k} \geq 0 \ , \ \overline{f}_{d_k j}^{k} \leq 0 \ if \ F_{d_k}^{k} \leq 0,$$
$$\overline{g}_{t_k j}^{k} \geq 0 \ if \ G_{t_k}^{k} \geq 0 \ , \ \overline{g}_{t_k j}^{k} \leq 0 \ if \ G_{t_k}^{k} \leq 0.$$

(2.11)

We should point out that based upon the pro-posed production plan, the inputs and outputs changes in the next season should be allocated to all DMUs, in such a way that any efficiency score does not reduce. The new PPS describes a reliable reference for future decisions.

3 Empirical Example

In this section, a real data set (consisted of 13 farms of Green Hen poultry chain which are located in Guilan Province, Iran) is used to illustrate the proposed approach. Each farm employs new born chicks, feed and other inputs to produce maximum meat while attempts to minimize consumed inputs and undesirable outputs. The proposed two-stage model is illustrated in Figure 2.

As an empirical example, Table 1 illustrated a data set for 13 poultry farms (DMUs), where New Born Chicks (x_{1j}^{1}), Feed Cost (x_{2j}^{1}) and Operational Expenses (x_{3j}^{1}) are three inputs to the first stage, Feed Conversion Ratio (z_{1j}^{1}) and produced Meat (z_{2j}^{1}) are two intermediate measures, i.e. the outputs of the first stage and the inputs to the second stage, Mortality and Condemn (w_{1j}^{1}) is undesirable output of the first stage which leaves the system, Feed Cost (x_{1j}^{2}) and Operational Expenses (x_{2j}^{2}) are two external inputs to the second stage and Feed Conversion Raito (z_{1j}^{2}), Produced Meat (z_{2j}^{2}) and Mortality and Condemn (w_{1j}^{2}) are two desirable and one undesirable output from the second stage, respectively.

The input/output values of all DMUs (farms), are presented in columns 1-12. In addition, Columns 13-15 presented the relative efficiency scores of all of the 13 DMUs and their sub-DMUs from a Russell extended two stage model (Amirteimoori et al. [4]). The results show that DMUs 4, 11 and 12 are overall efficient while DMUs 1, 2, 3, 5, 6, 7, 8, 9, 10 and 13 are inefficient. The inefficiencies of DMUs 2, 3, 5, 8 and 10 root in stage one, while DMUs 1, 6, 7, 9 and 13 are inefficient in both stages.

Suppose, the central decision making unit forecasts the demand changes for system outputs

[Produced Meat (PrMe) and Feed Conversion Ratio (FCR)] as $F_1^2 = 28000$ and $F_2^2 = 0$, while we expect an increase in undesirable output [Mortality and Condemn ($M\&C$)] equal to 100 ($G_1^2 = 28000$). Note, G_1^2 is a non-controllable variable and its value determines by the system. To meet these changes the central DMU also determine the possible change of the total inputs [New Born Chicks (NBC), Feed Cost (FeCo) and Operational Expenses (OpEx)] in the first stage as $C_1^1 = 10000$, $C_2^1 = 150000$ and $C_3^1 = 55000$, respectively. The undesirable output of the first stage [Mortality and Condemn ($M\&C$)] and two intermediate measures [Produced Meat (PrMe) and Feed Conversion Ratio (FCR)] are $G_1^1 = 350$, $F_1^1 = 6600$ and $F_2^1 = 0$, respectively. In addition, Two external inputs for the second stage, include Feed Cost (FeCo) $C_1^2 = 400000$ and Operational Expenses (OpEx) $C_2^2 = 110000$.

In order to develop the new arrangements of inputs/ outputs for DMU_j, the magnitude size the all DMUs on the input and output sides are computed in Table 2. Then, using the proposed planning idea, the new input/output targets for all poultries are presented in Table 3 (columns 2 to 12), with the new CCR efficiencies in columns 13 to 15.

The results also show that based on the new plan, all of the efficiency scores are improved, while six out of thirteen poultries are DEA efficient with three of them also efficient before applying the model. As can be seen in Table 3, based on the new planning model the DMUs 1, 5 and 7 known as three newly DEA efficient units. For example, the efficiency score of DMU_1 In stage 1, 2 and whole process was 0.9480, 0.9905 and 0.9692, respectively, whereas the new efficiency scores have increased to 1.0000, 1.0000 and 1.0000. The new input/output arrangement of DMU_1 which lies it onto the efficient frontier would land at the point in which its first stage inputs (New Born Chicks, Feed Cost and Operational Expenses) and undesirable output are 13395, 158925, 61192.5 and 495, respectively; while its intermediate measures are 1.69 and 7147.5. To lie on the efficient frontier in the second stage, DMU_1 needs to arrange its external inputs to 470270 and 105333, while the

values of its outputs (Feed Conservation Ratio, produced meat and Mortality and condemn) are estimated to be as 1.98, 30578.6 and 180. As can be seen in Table 3, based on the new planning model the DMUs 1, 5 and 7 known as three newly DEA efficient units. Note, the new plan takes into account the potentiality (magnitude size) of units when developing the inputs/outputs arrangements, which ensures the results feasibility.

4 Conclusion

In recent years, many mathematical models have been developed to make a contribution to the problem of production planning. The current paper developed a DEA-based approach for production planning in network production systems, considering undesirable factors. Using a data set of 13 poultry farms the corresponding efficiency scores of the new production plan have been computed and compared with those of the original plan. As shown in results, the proposed model improved the efficiencies of all units significantly. The proposed approach, also allows the modeler to set priorities on objectives. It is widely applicable and fit well with real world examples.

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Table 1: The inputs and outputs values of the under consideration units

DMU	Stage 1						Stage 2					eff. 1	eff. 2	EFF.
	Input			Output	Intermediate		Input		Output					
	NBC	FeCo	OpEx	M&C	FCR	PrMe	FeCo	OpEx	FCR	PrMe	M&C			
1	12700	148500	57370	467	1.69	6691.5	438500	97920	1.98	28582.2	173	0.9480	0.9905	0.9692
2	14670	171740	63900	513	1.65	7871.3	491760	110160	1.93	32387.2	197	0.9421	1	0.9710
3	13300	154930	63220	1263	1.72	6921.3	435410	106150	2.00	28506.3	306	0.7974	1	0.8987
4	15000	182880	66590	421	1.71	8280.9	518560	126650	1.95	34075.0	79	1	1.0000	1
5	12000	147490	57030	758	1.68	6340.5	415130	100700	1.98	26256.5	256	0.8495	1	0.9248
6	14000	165080	63640	1098	1.70	7134.8	449710	113700	1.97	29828.0	263	0.7962	0.9721	0.8841
7	13000	168930	62020	646	1.75	7202.4	468450	110550	2.03	30158.7	144	0.8902	0.9513	0.9207
8	14900	175430	71680	821	1.62	7475.9	532190	119100	2.04	33414.6	214	0.7956	1	0.8978
9	13500	169520	62300	518	1.71	7399.7	480800	106770	1.94	30439.0	246	0.9282	0.9872	0.9577
10	12800	144130	60930	623	1.63	6356.4	433090	105240	2.03	28223.5	167	0.8480	1	0.9240
11	19800	235970	80960	1042	1.67	10373.2	685800	144430	2.01	44581.2	336	1	1	1
12	11000	133540	51340	385	1.68	5933.8	378100	86880	2.00	25683.4	89	1	1	1
13	12600	148870	57210	479	1.63	6521.1	440730	102420	1.88	28405.3	186	0.9075	0.9880	0.9478

Table 2: The magnitude size of the inputs and outputs in stage k (k=1,2)

DMU	MSI_{1j}	MSI_{2j}	MSO_{1j}	MSO_{2j}	α_{1j}	α_{2j}	$\varphi_{1j} = \delta_{1j}$	$'_{2j} = ffi_{2j}$
1	0.6491	0.6461	0.6272	0.6402	0.0695	0.0706	0.0691	0.0713
2	0.7434	0.7254	0.7345	0.7254	0.0796	0.0793	0.0810	0.0808
3	0.6873	0.6525	0.7170	0.6415	0.0736	0.0713	0.0790	0.0715
4	0.7854	0.7774	0.7623	0.7604	0.0841	0.0849	0.0840	0.0847
5	0.643	0.6212	0.6219	0.5903	0.0689	0.0679	0.0686	0.0658
6	0.7208	0.6787	0.7213	0.6699	0.0772	0.0742	0.0795	0.0746
7	0.7245	0.6974	0.6876	0.6746	0.0776	0.0762	0.0758	0.0752
8	0.7781	0.7837	0.7269	0.7487	0.0833	0.0856	0.0801	0.0834
9	0.7285	0.7078	0.6936	0.6832	0.0780	0.0773	0.0765	0.0761
10	0.6470	0.6480	0.6115	0.6321	0.0693	0.0708	0.0674	0.0704
11	1	1	1	1	0.1071	0.1093	0.1102	0.1114
12	0.5817	0.5602	0.5536	0.5738	0.0623	0.0612	0.0610	0.0639
13	0.6494	0.6539	0.6133	0.6365	0.0695	0.0714	0.0676	0.0709

Table 3: New arrangements of the inputs and outputs

DMU	Stage 1						Stage 2					eff. 1	eff. 2	EFF.
	Input			Output	Intermediate		Input		Output					
	NBC	FeCo	OpEx	M&C	FCR	PrMe	FeCo	OpEx	FCR	PrMe	M&C			
1	13395	158925	61192.5	495	1.69	7147.5	470270	105333	1.98	30578.6	180	1	1	1
2	15466	183680	68278	545	1.65	8407.2	527445	118486.5	1.93	34649.6	205	1	1	1
3	14036	165970	67268	1295	1.72	7442.7	467495	113636.5	2.00	30508.3	313	1	1	1
4	15841	195495	71215.5	455	1.71	8835.3	556765	135564.5	1.95	36446.6	88	1	1	1
5	12689	157825	60819.5	785	1.68	6793.3	445685	107829.5	1.98	28098.9	263	0.9860	1	0.9930
6	14772	176660	67886	1130	1.7	7659.5	483100	121491	1.97	31916.8	270	0.9821	1	0.9910
7	13776	180570	66288	676	1.75	7702.7	502740	118551	2.03	32264.3	152	1	0.9536	0.9768
8	15733	187925	76261.5	853	1.62	8004.6	570710	128088	2.04	35749.8	222	0.9423	1	0.9712
9	14280	181220	66590	549	1.71	7904.6	515585	114886.5	1.94	32569.8	254	0.9926	1	0.9963
10	13493	154525	64741.5	650	1.63	6801.2	464950	112674	2.03	30194.7	174	0.9554	1	0.9777
11	20871	252035	86850.5	1086	1.67	11100.5	734985	155906.5	2.01	47700.4	347	1	1	1
12	11623	142885	54766.5	409	1.68	6336.4	405640	93306	2.00	27472.6	95	1	1	1
13	13295	159295	61032.5	506	1.63	6967.3	472860	109917	1.88	30390.5	193	0.9708	0.9902	0.9805

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