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Research Article

Resolution of Fuzzy Complex Systems of Linear Equations Via Wu's Method

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Abstract

The aim of this paper is to present an algebraic method which is called Wu's method for solving fuzzy complex systems of linear equations. Wu's method is used as a solution procedure for solving the crisp polynomial equations system. This algorithm leads to solving characteristic sets that are amenable to easy solution. First, the system of fuzzy complex systems of linear equations is transformed to an equivalent crisp polynomial equations system. Then, using Wu's method an algorithm for finding fuzzy solutions of original system is introduced. One of the great benefits of our approach is that gives all solutions at a time. To illustrate the easy application of the proposed method, numerical examples are given and the obtained results are discussed.

Keywords : Triangular fuzzy numbers; Fuzzy complex numbers; Fuzzy complex systems of linear equations; Characteristic sets; Wu's algorithm.

1 Introduction

Solving systems of fuzzy equations has been challenge to researcher in many fields. The initial work focused on solving a fuzzy linear equation [5]. Many approaches have been proposed to solve fuzzy linear systems in different form [1, 11, 12, 24, 28, 37]. However, such techniques cannot easily be applied to complex systems (involving many variables, implicit variables, and high nonlinear degrees), and results are difficult to evaluate. In these method, it is necessary to know that solutions are negative or positive. Otherwise, we cannot use the methods. The determination of a suitable initial point for the methods is not easy. In these methods, we can only find some of the approximate solutions. In the methods, we have no criteria or necessary and sufficient conditions to recognize the solution existence of the fuzzy systems. The methods do not give information about the number of solutions of the fuzzy systems and if the fuzzy systems have no solutions, then the methods can be misleading. To overcome the drawbacks, we are motivated to propose efficient algebraic approaches for the resolution of fuzzy polynomial equations and fuzzy polynomial equations systems [13, 14, 15, 16, 30]. Complex system of linear equations plays a vital role in real life problems such as optimization, current flow, economics and engineering. Hence,

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its resolution has an essential role in the fields. A general complex system of linear equations may be written as $CZ = W$, where C and W are standard complex matrices and Z is the unknown complex vector. For the sake of simplicity, variables and parameters of these systems are defined exactly in the modeling. But in actual practice, the parameters and variables may be uncertain or vague and those are found, in general, by some experiment or experience. Hence, to overcome the uncertainty, one may use the complex numbers as fuzzy. Accordingly, fuzzy complex system of linear equations is considered as

$$[C]\{\tilde{Z}\} = \{\tilde{W}\},$$

where the coefficient matrix $[C]$ is an usual complex matrix, $\{\tilde{W}\}$ is a column vector of fuzzy complex number and $\{\tilde{Z}\}$ is the vector of fuzzy complex unknown.

Fuzzy complex number was first proposed by Buckley [4]. Qiu et al. [26, 27] discussed the sequence and series of fuzzy complex numbers and their convergence. Solution of fuzzy complex system of linear equations was described by Rahgooy et al. [29] and applied to circuit analysis problem. Jahantigh et al. [21] developed a numerical procedure for complex fuzzy linear systems. Behera and Chakraverty [2] proposed a new and simple center and width based method for solving fuzzy real and complex system of linear equations. Solution sets of complex linear interval systems were investigated by Hladik [20]. Householder method was used by Djanybekov [10] for the solution of interval complex linear systems. In interval complex linear systems, the coefficient matrix was also taken as interval by the authors [10]. Further, Candau et al. [6] analyzed the complex interval arithmetic using polar form. Fuzzy modeling and identification procedure was implemented by Cao et al. [7] for the analysis and design of complex control systems. Filev [18] applied fuzzy approach to the control of nonlinear systems. Further Petkovic and Petkovic [25] investigated the complex interval arithmetic and applied it to various example problems. The authors [25], presented a circular form of the interval complex number. Complex interval arithmetic was also studied by Rokne and Lancaster [31]. Recently, Behera and Chakraverty [3] proposed a new method for solving general fuzzy complex system of linear equations. In the original system, the elements of unknown variable vector and right hand side vector are considered as complex fuzzy number. Initially the general system is solved by adding and subtracting the left and right bounds of the fuzzy complex unknown and right hand side fuzzy complex vector respectively. Subsequently above obtained solutions are used to get the final solution of the general fuzzy complex system of linear equations. Recently, Farahani et al. [17] proposed an approach to solve the fuzzy complex system of linear equations based on the eigenvalue method. In the eigenvalue method, the computation of solutions of a system is done independently with respect to each other. In the proposed method, the computation of solutions of a fuzzy complex system of linear equations leads to finding the eigenvalues of a matrix. Hence, the useful tools can be used in the linear algebra such as converting the matrix into a triangular matrix via elementary row operations and applying the determinant properties.

Our method is based on the Wu's algorithm to solve fuzzy complex systems of linear equations. Since 1980, Wu Wen-Tsun has considerably improved Ritt's theory and developed some efficient algorithms for zero decomposition of arbitrary polynomial systems [32, 34]. Ritt-Wu's method successfully was applied to many problems in science and engineering [35]. This method is more efficient than approach for solving real polynomial equation systems in some cases such as [8, 19, 22]. Using Wu's algorithm to solve polynomial equation systems leads to solving characteristic sets. As these sets have triangular structure, finding the variety of these sets can be simply done by a forward substituting. The main idea of the proposed approach is based on converting the fuzzy complex system of linear equations into a crisp system and getting a polynomial system of $8n$ equations and $6n$ unknowns such that the solutions of the new system may be obtained by a successful scheme of solving systems. Thus solving a system of fuzzy polynomial equations is converted to a system of univariate polynomial equations where finding the roots of these polynomials is easier than solving the original system.

The rest of the paper is set out as follows. In Section 2, preliminaries on fuzzy arithmetic, fuzzy complex arithmetic, polynomials and fuzzy complex system of linear equations are introduced. The main algorithm to find solutions of the fuzzy complex system of linear equations is presented in Section 3. The proposed method is illustrated by solving some examples in Section 4. Last section includes the conclusion.

2 Preliminaries

This section contains two subsections. Preliminaries on fuzzy arithmetic and fuzzy complex arithmetic are introduced in the first subsection. In the second subsection, fuzzy complex system of linear equations is explained.

2.1 Fuzzy arithmetic and fuzzy complex arithmetic

In this subsection, fuzzy arithmetic and fuzzy complex arithmetic are reviewed.

Definition 2.1 *A fuzzy subset $\tilde{u}$ of $\mathbb{R}$ is defined by its membership function*

$$\mu_{\tilde{u}} : \mathbb{R} \longrightarrow [0, 1],$$

which assigns a real number in the interval $[0, 1]$ to each element $x \in \mathbb{R}$ and the value $\mu_{\tilde{u}}(x)$ shows the grade of membership of x in $\tilde{u}$.

Definition 2.2 *A fuzzy number $\tilde{u}$ is a fuzzy set as $\mu_{\tilde{u}} : \mathbb{R} \longrightarrow [0, 1]$ which satisfies:*

1. *$\mu_{\tilde{u}}$ is normal, i.e., there exists an element t_0 such that $\mu_{\tilde{u}}(t_0) = 1$;*

2. *$\mu_{\tilde{u}}$ is quasi-concave, i.e., $\mu_{\tilde{u}}(\lambda t_1+(1-\lambda)t_2) \geq min\{\mu_{\tilde{u}}(t_1), \mu_{\tilde{u}}(t_2)\}, \forall t_1, t_2 \in \mathbb{R}, \forall\lambda \in [0, 1]$;*

3. *$\mu_{\tilde{u}}$ is upper semi-continuous, i.e., $\tilde{u}(r) = \{x \in \mathbb{R} : \mu_{\tilde{u}}(x) \geq r\}$ is closed subset for each $r \in (0, 1]$;*

4. *The 0-level set of $\tilde{u}_0$ is compact subset of $\mathbb{R}$.*

The set of all fuzzy numbers is denoted by E^1.

Definition 2.3 *A popular fuzzy number is the triangular fuzzy number that is presented by $\tilde{u} = (a_1, a_2, a_3)$, where, $a_1 \leq a_2 \leq a_3$ and its membership function is as follows:*

$$\mu_{\tilde{u}}(x) = \begin{cases} 0 & x \leq a_1, \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3, \\ 0 & x > a_3. \end{cases}$$

With regard to Definition 2.2 and the above points, it is concluded that the r-cut $\tilde{u}(r)$ of $\tilde{u}$ is a compact and convex subset of $\mathbb{R}$ for each $r \in [0, 1]$, i.e., $\tilde{u}(r)$ is a closed interval in $\mathbb{R}$ for each $r \in [0, 1]$.
The r-cut of fuzzy number (2.3) is as follows:

$$\tilde{u}(r) = [\underline{u}(r), \overline{u}(r)] = [a_1+(a_2-a_1)r, a_3+(a_2-a_3)r]. \tag{2.1}$$

Using the concept of r-level sets, the relationship between ordinary sets and fuzzy sets can be characterized by the following theorem.

Theorem 2.1 *[23] (Decomposition Theorem) A fuzzy set $\tilde{u}$ can be represented by $\tilde{u} = \bigcup_{r\in[0,1]} r.\tilde{u}(r)$ denotes the algebraic product of a scaler of r with the r-level set of $\tilde{u}(r)$.*

The arithmetical operations on fuzzy numbers, based on Zadeh's extension principle [36] and decomposition theorem [23] are numerically performed on level sets, i.e., r-cuts. From the interval arithmetic [23], the arithmetical operations on fuzzy numbers are written for their r-level sets as follows.

Definition 2.4 *For arbitrary fuzzy numbers $\tilde{u}, \tilde{v}$, and real number k, we have:*

1. $\tilde{u} = \tilde{v}$ if and only if $\underline{u}(r) = \underline{v}(r)$ and $\overline{u}(r) = \overline{u}(r)$, for all $r \in [0, 1]$,

2. $u(r) + v(r) = [\underline{u}(r), \overline{u}(r)] + [\underline{v}(r), \overline{v}(r)] = [\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)]$,

3.

$$k.u(r) = \begin{cases} [k.\underline{u}(r), k.\overline{u}(r)] & k \geq 0, \\ [k.\overline{u}(r), k.\underline{u}(r)] & k < 0. \end{cases}$$

Definition 2.5 *According to [2], an arbitrary fuzzy complex number may be represented as* $\tilde{z} = p + iq$, *where* $p = [\underline{p}(r), \overline{p}(r)]$ *and* $q = [\underline{q}(r), \overline{q}(r)]$, *for all* $0 \leq r \leq 1$.

As such the above can be written as

$$\tilde{z} = [\underline{p}(r), \overline{p}(r)] + i[\underline{q}(r), \overline{q}(r)].$$

Definition 2.6 *For any two arbitrary fuzzy complex number* $\tilde{z}_1 = x_1 + iy_1$ *and* $\tilde{z}_2 = x_2 + iy_2$ *(where,* x_1, y_1, x_2 *and* y_2 *are fuzzy numbers), the fuzzy complex arithmetic is written for addition and multiplication as*

1. $\tilde{z}_1 + \tilde{z}_2 = (x_1 + x_2) + i(y_1 + y_2)$,
2. $\tilde{z}_1 \times \tilde{z}_2 = \{(x_1 \times x_2) - (y_1 \times y_2)\} + i\{(x_1 \times y_2) + (y_1 \times x_2)\}$.

2.2 Fuzzy complex system of linear equations

The $n \times n$ fuzzy complex system of linear equations is written as

$$\begin{cases} c_{11}\tilde{z}_1 + c_{12}\tilde{z}_2 + ... + c_{1n}\tilde{z}_n = \tilde{w}_1, \\ c_{21}\tilde{z}_1 + c_{22}\tilde{z}_2 + ... + c_{2n}\tilde{z}_n = \tilde{w}_2, \\ \vdots \\ c_{n1}\tilde{z}_1 + c_{n2}\tilde{z}_2 + ... + c_{nn}\tilde{z}_n = \tilde{w}_n. \end{cases} \quad (2.2)$$

In matrix notation we may then write the above as $[C]\{\tilde{Z}\} = \{\tilde{W}\}$, where the coefficient matrix $[C] = (c_{kj}), 1 \leq k \leq n, j \leq n$ is a complex $n \times n$ matrix, $\{\tilde{W}\} = \{\tilde{w}_k\}$, $1 \leq k$ is a column vector of fuzzy complex number and $\{\tilde{Z}\} = \{\tilde{z}_j\}$ is the vector of fuzzy complex unknown. System (2.2) can be represented as

$$\sum_{j=1}^{n} c_{kj}\tilde{z}_j = \tilde{w}_k, \quad k = 1, 2, ..., n. \quad (2.3)$$

The complex coefficient matrix, fuzzy complex unknown and the right hand fuzzy complex number vector may be written respectively as

$$c_{kj} = a_{kj} + ib_{kj},$$

$$\tilde{Z} = \tilde{z}_j = p_j + iq_j = [\underline{p_j}(r), \overline{p_j}(r)] + i[\underline{q_j}(r), \overline{q_j}(r)]$$

and

$$\tilde{W} = \tilde{w}_k = u_j + iv_j = [\underline{u_k}(r), \overline{u_k}(r)] + i[\underline{v_k}(r), \overline{v_k}(r)].$$

Next, the following equation is obtained by substituting $\tilde{Z}$ and $\tilde{W}$ in Eq. (2.3)

$$\begin{aligned} &\textstyle\sum_{j=1}^{n}(a_{kj} + ib_{kj})([\underline{p_j}(r), \overline{p_j}(r)] + i[\underline{q_j}(r), \overline{q_j}(r)]) \\ &= [\underline{u_k}(r), \overline{u_k}(r)] + i[\underline{v_k}(r), \overline{v_k}(r)], \quad k = 1, 2, ..., n. \end{aligned} \quad (2.4)$$

Eq. (2.4) can now be written as

$$\begin{aligned} &\textstyle\sum_{j=1}^{n} a_{kj}([\underline{p_j}(r), \overline{p_j}(r)] + i[\underline{q_j}(r), \overline{q_j}(r)]) \\ &+i\textstyle\sum_{j=1}^{n} b_{kj}([\underline{p_j}(r), \overline{p_j}(r)] + i[\underline{q_j}(r), \overline{q_j}(r)]) \\ &= [\underline{u_k}(r), \overline{u_k}(r)] + i[\underline{v_k}(r), \overline{v_k}(r)], \end{aligned} \quad (2.5)$$

for $k = 1, 2, ..., n$. Then, Eq. (3.9) is equivalent to the following equation:

$$\begin{aligned} &\textstyle\sum_{j=1}^{n} a_{kj}[\underline{p_j}(r), \overline{p_j}(r)] + \sum_{j=1}^{n} -b_{kj}[\underline{q_j}(r), \overline{q_j}(r)] \\ &+i\textstyle\sum_{j=1}^{n} a_{kj}[\underline{p_j}(r), \overline{p_j}(r)] + \sum_{j=1}^{n} b_{kj}[\underline{q_j}(r), \overline{q_j}(r)] \\ &= [\underline{u_k}(r), \overline{u_k}(r)] + i[\underline{v_k}(r), \overline{v_k}(r)] \end{aligned} \quad (2.6)$$

for $k = 1, 2, ..., n$. Here, a_{kj} and b_{kj} both may be positive and/or negative. To handle the positive and negative values of a_{kj} and b_{kj} the above equation is written as below

$$\begin{aligned} &\textstyle\sum_{a_{kj} \geq 0} a_{kj}[\underline{p_j}(r), \overline{p_j}(r)] + \\ &\textstyle\sum_{b_{kj} < 0} -b_{kj}[\underline{q_j}(r), \overline{q_j}(r)] + \\ &\textstyle\sum_{a_{kj} < 0} a_{kj}[\overline{p_j}(r), \underline{p_j}(r)] + \\ &\textstyle\sum_{b_{kj} \geq 0} -b_{kj}[\overline{q_j}(r), \underline{q_j}(r)] + i \\ &\textstyle\sum_{a_{kj} \geq 0} a_{kj}[\underline{p_j}(r), \overline{p_j}(r)] + \\ &\textstyle\sum_{b_{kj} \geq 0} -b_{kj}[\underline{q_j}(r), \overline{q_j}(r)] + \\ &\textstyle\sum_{a_{kj} < 0} a_{kj}[\overline{p_j}(r), \underline{p_j}(r)] + \\ &\textstyle\sum_{b_{kj} < 0} b_{kj}[\overline{q_j}(r), \underline{q_j}(r)] \\ &= [\underline{u_k}(r), \overline{u_k}(r)] + i[\underline{v_k}(r), \overline{v_k}(r)], \end{aligned} \quad (2.7)$$

for $k = 1, 2, ..., n$. So, with regard to Definition 2.4, the parametric form of system (2.2) can be presented in the following form:

$$\begin{aligned}\underline{u_k}(r) &= \sum_{a_{kj}\geq 0} a_{kj}\underline{p_j}(r) + \sum_{b_{kj}<0} -b_{kj}\underline{q_j}(r)\\ &+ \sum_{a_{kj}<0} a_{kj}\overline{p_j}(r) + \sum_{b_{kj}\geq 0} -b_{kj}\overline{q_j}(r),\\ \overline{u_k}(r) &= \sum_{a_{kj}\geq 0} a_{kj}\overline{p_j}(r) + \sum_{b_{kj}<0} -b_{kj}\overline{q_j}(r)\\ &+ \sum_{a_{kj}<0} a_{kj}\underline{p_j}(r) + \sum_{b_{kj}\geq 0} -b_{kj}\underline{q_j}(r),\\ \underline{v_k}(r) &= \sum_{a_{kj}\geq 0} a_{kj}\underline{p_j}(r) + \sum_{b_{kj}\geq 0} -b_{kj}\underline{q_j}(r)\\ &+ \sum_{a_{kj}<0} a_{kj}\overline{p_j}(r) + \sum_{b_{kj}<0} b_{kj}\overline{q_j}(r),\\ \overline{v_k}(r) &= \sum_{a_{kj}\geq 0} a_{kj}\overline{p_j}(r) + \sum_{b_{kj}\geq 0} -b_{kj}\overline{q_j}(r)\\ &+ \sum_{a_{kj}<0} a_{kj}\underline{p_j}(r) + \sum_{b_{kj}<0} b_{kj}\underline{q_j}(r).\end{aligned} \tag{2.8}$$

Theorem 2.2 *The systems of (2.2) and (2.7) have the same solutions.*

Proof. Since all of the coefficients and all of the unknowns in the polynomial functions of $f_k(\tilde{z}_1, ..., \tilde{z}_n) = \sum_{j=1}^{n} c_{kj}\tilde{z}_j$, for $k = 1, ..., n$, are fuzzy complex numbers, according Definition (2.6), the values of $f_k(\tilde{z}_1, ..., \tilde{z}_n)$, for $k = 1, ..., n$, are fuzzy complex numbers. With regard to Definition (2.4), two fuzzy complex numbers of $f_k(\tilde{z}_1, ..., \tilde{z}_n)$ and $\tilde{w}_k$, for $k = 1, ..., n$, are equal when Eq. (2.8) hold for $1 \leq k \leq n, j \leq n$.

3 Resolution of fuzzy complex systems of linear equations via Wu's method

In this section, an approach based on Wu's algorithm is presented for solving a system of fully fuzzy polynomial equations.

3.1 Wu's Algorithm and Varieties

In this subsection, the characteristic sets are introduced at first. Then, Wu's Algorithm and its relation with varieties will be explained. Let $\Gamma = \mathbb{K}[x_1, \ldots, x_n]$ be the polynomial ring in n variables over a field $\mathbb{K}$ of characteristic zero. We assume the variables $x_1, \ldots, x_n$ ordered so that $x_i < x_j$ for $i < j$. If the variable x_m is selected, then a polynomial $f \in \Gamma$ can be written as a univariate polynomial in variable x_m as

$$f = I_t x_m^t + I_{t-1}x_m^{t-1} + \cdots + I_0,$$

where t, the degree of f with respect to x_m, is denoted by $\deg_{x_m}(f)$, and

$$I_i \in \mathbb{K}[x_1, \ldots, x_{m-1}, x_{m+1}, \ldots, x_n],$$

for $0 \leq i \leq t$. We denote by $lc(f, x_m)$ the leading coefficient I_t. The class of f is defined as the greatest subscript c of x appearing in f and is denoted by $class(f)$. The class of a constant is defined to be zero. The variable x_c and $lc(f, x_c)$ are called leading variable and initial of f and are denoted by $lv(f)$ and $ini(f)$, respectively. A polynomial $g \in \Gamma$ is said to be reduced with respect to f if $deg_{x_c}(g) < deg_{x_c}(f)$ where $c = class(f) \neq 0$. The polynomial g is reduced with respect to $F \subset \Gamma$ if g is reduced with respect to any $f \in F$. We define a partial order on polynomials as follows. Let $f,\ g \in \Gamma$. The polynomial g has a higher rank than f and is denoted by $f < g$ if one of the following hold;

1. $class(f) < class(g)$.
2. $class(f) = class(g) = c$ and $deg_{x_c}(f) < deg_{x_c}(g)$.

If $class(f) = class(g) = c$ and $deg_{x_c}(f) = deg_{x_c}(g)$ or both polynomials are constant, then we say f and g are equivalent, denoted by $f \sim g$. An ordered polynomial set $F = \{f_1, f_2, \ldots, f_r\}$ is a triangular set if either $r = 1$ or $class(f_1) < class(f_2) < \cdots < class(f_r)$. The triangular set F is called an ascending set if f_j is reduced with respect to f_i for $i < j$. Now we extend the partial order on polynomials to provide a partial order for ascending sets. Let $F = \{f_1, \ldots, f_r\}$ and $G = \{g_1, \ldots, g_k\}$ be ascending sets. We say $F < G$ if one of the following two cases holds;

1. There is $j \leqslant \min\{r, k\}$ such that $f_i \sim g_i$ for $i < j$, but $f_j < g_j$.
2. $r > k$ and $f_i \sim g_i$ for all $i \leqslant k$.

For incomparable ascending sets we write $F \sim G$. When $F < G$ we say that F has a lower rank than G. An ascending set of lowest rank consisting of polynomials chosen from F is called basic set of F. Now, we introduce an interesting division for multivariable polynomials that is known as the pseudo division.

Proposition 3.1 *[9] Let $f, g \in \Gamma$ and $class(f) = c$. Then there is an equation*

$$I_c^m g = qf + r,$$

where $q, r \in \Gamma$, $I_c = ini(f)$, $m \geqslant 0$, and either $r = 0$ or r is reduced with respect to f.

The polynomial r in Proposition 3.1 is known as a pseudo remainder of g on pseudo division by f is denoted by $prem(g, f)$. Given an ascending set $F = \{f_1, \ldots, f_r\}$ and $g \in \Gamma$. By successive pseudo divisions, we get the following remainder formula

$$I_1^{s_1} I_2^{s_2} \cdots I_r^{s_r} g = \sum q_i f_i + R \tag{3.9}$$

where $I_i = ini(f_i)$, $s_i \geqslant 0$, $q_i \in \Gamma$ and R is reduced with respect to F. If we choose each s_i to be as small as possible, then R is unique and is denoted by $prem(g, F)$. For a finite subset G from Γ we put

$$prem(G, F) = \{prem(g, F) \mid g \in G\}.$$

The ideal generated in Γ by F is denoted by $\langle F \rangle$.

Definition 3.1 *An ascending set B in Γ is called a characteristic set of a non-empty polynomial set $F \subset \Gamma$ if $B \subset \langle F \rangle$ and $prem(F, B) = \{0\}$.*

Let $F \subset \Gamma$. The set

$$V(F) = \{(a_1, \ldots, a_n) \in \mathbb{K}^n \mid f(a_1, \ldots, a_n) = 0, \forall f \in F\}$$

is the variety defined by F. For a polynomial set $G \subset \Gamma$, we define $V(F/G) = V(F) \backslash V(G)$, called quasi-algebraic variety. The main properties of characteristic sets are summarized in the following theorem.

Theorem 3.1 *[32] (Wu's Well-ordering Principle) Let B be a characteristic set of $F \subset \Gamma$. Then*

$$V(F) = V(B/I_B) \bigcup \cup_{b \in B} V(F \cup B \cup \{ini(b)\})$$

where $I_B = \prod_{b \in B} ini(b)$.

Based on Wu's Well-ordering Principle Theorem, Wu's algorithm is presented to give all characteristic sets that are needed for computing $V(F)$.

Algorithm [33] *(Wu' s Method)*
Input: $F \subset \Gamma$, a non-empty set
Output: Z, a set of characteristic sets such that $V(F) = \bigcup_{B \in Z} V(B/I_B)$, where $I_B = \prod_{b \in B} ini(b)$.

1. $Z := \emptyset$, $D := \{F\}$
2. While $D \neq \emptyset$ Do
   2.1. Pick an element F' from D
   2.2. $D := D \backslash \{F'\}$
   2.3. Choose a characteristic set B of F'
   2.4. If $B \neq \{1\}$ then
      2.4.1. $Z := Z \cup \{B\}$
      2.4.2. $D := D \bigcup \cup_{b \in B} \{F' \cup B \cup \{ini(b) \mid ini(b) \neq 1\}\}$
3. Return Z

By Wu's algorithm, we can write the $V(F)$ as a union of quasi-algebraic variety of characteristic sets. Therefore, we can find $V(F)$ easily because these sets are easy to solve.

Example 3.1 *Apply Wu's algorithm to $F = \{xy + x + y, xy^2 + x + y\}$ with $y < x$. Put $F' := F$, then $D = \emptyset$. The set $B = \{y^3 - y^2, xy + x + y\}$ is a characteristic set. Thus $Z := \{B\}$. We have $ini(xy + x + y) = y + 1$ and $ini(y^2 - y^3) = 1$, therefore, $D := \{F' \cup \{y + 1\}\}$. Now put $F' := \{xy + x + y, xy^2 + x + y, y + 1\}$. The set $\{1\}$ is a characteristic set of F' and $D = \emptyset$. Therefore, the output is $Z = \{\{y^3 - y^2, xy + x + y\}\}$ and*

$$\begin{aligned} V(F) &= V(\{y^3 - y^2, xy + x + y\}) \backslash V(y + 1) \\ &= \{(x = 0, y = 0), (x = -\tfrac{1}{2}, y = 1)\}. \end{aligned}$$

3.2 Main idea

In this section we propose a new method to solve a fuzzy complex system of linear equations. Considering the fuzzy complex system of linear equations as (1), where p_j and q_j in the fuzzy complex unknown $\tilde{z}_j = p_j + iq_j$ and u_k and v_k in the right hand fuzzy complex number vector $\tilde{w}_k = u_k + iv_k$, $1 \leq k \leq n$, $j \leq n$ are represented by triangular fuzzy numbers (p_{j1}, p_{j2}, p_{j3}), (q_{j1}, q_{j2}, q_{j3}), (u_{k1}, u_{k2}, u_{k3}) and (v_{k1}, v_{k2}, v_{k3}). Then, with regard to Eq. (2.1), we have

$$\begin{aligned} \underline{p_j}(r) &= p_{j1} + (p_{j2} - p_{j1})r, \\ \overline{p_j}(r) &= p_{j3} + (p_{j2} - p_{j3})r, \\ \underline{q_j}(r) &= q_{j1} + ((p_{j2} - p_{j1})r, \\ \overline{q_j}(r) &= q_{j3} + (q_{j2} - q_{j3})r, \end{aligned}$$

and

$$\begin{aligned} \underline{u_j}(r) &= u_{j1} + (u_{j2} - u_{j1})r, \\ \overline{u_j}(r) &= u_{j3} + (u_{j2} - u_{j3})r, \\ \underline{v_j}(r) &= v_{j1} + ((v_{j2} - v_{j1})r, \\ \overline{v_j}(r) &= v_{j3} + (v_{j2} - v_{j3})r. \end{aligned}$$

Now, if we substitute

$$\underline{p_j}(r), \overline{p_j}(r), \underline{q_j}(r), \overline{q_j}(r), \underline{u_j}(r), \overline{u_j}(r), \underline{v_j}(r)$$

and $\overline{v_j}(r)$ in (9), with attention to Definition 2.4, system (1), may be converted into a crisp system which is a polynomial system with $8n$ equations and $6n$ unknowns. Now, Wu's algorithm gives all characteristic sets that are needed for computing the variety defined by crisp system in the ring

$$\mathbf{R} = \mathbb{R}[p_{11}, p_{12}, p_{13}, p_{21}, p_{22}, p_{32}, \ldots p_{n1}, \\ p_{n2}, p_{n3}, q_{11}, q_{12}, q_{13}, q_{21}, q_{22}, q_{32}, \ldots q_{n1}, q_{n2}, q_{n3}],$$

So, we can find the solution of the crisp system by a forward substitution. On the other hand, according to Theorem 2.2, the original system and the crisp system have the same solutions. So, all of the solutions of the original system is obtained.

Now, we express the resolution processes of a fuzzy complex system of linear equations using the basis in terms of the following algorithm.

Algorithm *(Main Algorithm)*

Input: The fuzzy complex system of linear equations $\mathbb{F}$

Output: The set of solutions, i.e., S for $\mathbb{F}$

1. Compute the parametric form of $\mathbb{F}$
2. Compute the crisp form of system $\mathbb{F}$ from the parametric system, i.e., $\mathbb{F}'$
3. $S := WuMethod(\mathbb{F}')$
4. End

4 Applications and numerical examples

In this section, we study some examples of the fuzzy complex systems of linear equations and solve them with the proposed approach in this paper. Then, their results are compared with other methods. The applied fuzzy numbers in the following examples are the triangular fuzzy numbers with the linear membership function (see Definition 2.3 in Subsection *2.1*).

Example 4.1 *[21] Let us consider* 2×2 *complex fuzzy linear system as*

$$\begin{cases} \tilde{z}_1 - \tilde{z}_2 = [r, 2-r] + i[1+r, 3-r], \\ \tilde{z}_1 + 3\tilde{z}_2 = [4+r, 7-2r] + i[r-4, -1-2r]. \end{cases}$$

We solve this system by our algorithm. Let $\tilde{z}_1 = p_1 + iq_1 = (p_{11}, p_{12}, p_{13}) + i(q_{11}, q_{12}, q_{13})$ and $\tilde{z}_2 = p_2 + iq_2 = (p_{21}, p_{22}, p_{23}) + i(q_{21}, q_{22}, q_{33})$. Now, the above system may be converted into the following equivalent system that is called parametric system:

$$\begin{cases} ([p_{11} + (p_{12} - p_{11})r, p_{13} + (p_{12} - p_{13})r]+ \\ i[q_{11} + (q_{12} - q_{11})r, q_{13} + (q_{12} - q_{13})r])- \\ ([p_{21} + (p_{22} - p_{21})r, p_{23} + (p_{22} - p_{23})r]+ \\ i[q_{21} + (q_{22} - q_{21})r, q_{23} + (q_{22} - q_{23})r]) \\ = [r, 2-r] + i[1+r, 3-r], \\ \\ ([p_{11} + (p_{12} - p_{11})r, p_{13} + (p_{12} - p_{13})r]+ \\ i[q_{11} + (q_{12} - q_{11})r, q_{13} + (q_{12} - q_{13})r])+ \\ 3([p_{21} + (p_{22} - p_{21})r, p_{23} + (p_{22} - p_{23})r]+ \\ i[q_{21} + (q_{22} - q_{21})r, q_{23} + (q_{22} - q_{23})r]) \\ = [4+r, 7-2r] + i[r-4, -1-2r]. \end{cases}$$

The above system can be rewritten as follows:

$$\begin{cases} ([p_{11} + (p_{12} - p_{11})r, p_{13} + (p_{12} - p_{13})r]+ \\ i[q_{11} + (q_{12} - q_{11})r, q_{13} + (q_{12} - q_{13})r])+ \\ ([-p_{23} + (-p_{22} + p_{23})r, -p_{21} + (-p_{22}+ \\ p_{21})r] + i[-q_{23} + (-q_{22} + q_{23})r, -q_{21}+ \\ (-q_{22} + q_{21})r]) \\ = [r, 2-r] + i[1+r, 3-r], \\ \\ ([p_{11} + (p_{12} - p_{11})r, p_{13} + (p_{12} - p_{13})r]+ \\ i[q_{11} + (q_{12} - q_{11})r, q_{13} + (q_{12} - q_{13})r]) \\ ([3p_{21} + (3p_{22} - 3p_{21})r, 3p_{23}+ \\ (3p_{22} - 3p_{23})r] + i[3q_{21} + (3q_{22} - 3q_{21})r, \\ 3q_{23} + (3q_{22} - 3q_{23})r]) \\ = [4+r, 7-2r] + i[r-4, -1-2r]. \end{cases}$$

The above system can be rewritten as follows:

$$\begin{cases} ([p_{11} - p_{23} + (p_{12} - p_{11} - p_{22} + p_{23})r, \\ p_{13} - p_{21} + (p_{12} - p_{13} - p_{22} + p_{21})r] \\ +i[q_{11} - q_{23} + (q_{12} - q_{11} - q_{22} + q_{23})r, \\ q_{13} - q_{21} + (q_{12} - q_{13} - q_{22} + q_{21})r])+ \\ = [r, 2-r] + i[1+r, 3-r], \\ \\ ([p_{11} + 3p_{21} + (p_{12} - p_{11} + 3p_{22} - 3p_{21})r, \\ p_{13} + 3p_{23} + (p_{12} - p_{13} + 3p_{22} - 3p_{23})r] \\ +i[q_{11} + 3q_{21} + (q_{12} - q_{11} + 3q_{22} - 3q_{21})r, \\ q_{13} + 3q_{23} + (q_{12} - q_{13} + 3q_{22} - 3q_{23})r]) \\ = [4+r, 7-2r] + i[r-4, -1-2r]. \end{cases}$$

The equivalent crisp polynomial equation system is obtained from the above system as follows:

$$F^{'}:\begin{cases} p_{11} - p_{23} = 0, \\ p_{12} - p_{11} - p_{22} + p_{23} = 1, \\ p_{13} - p_{21} = 2, \\ p_{12} - p_{13} - p_{22} + p_{21} = -1, \\ q_{11} - q_{23} = 1, \\ q_{12} - q_{11} - q_{22} + q_{23} = 1, \\ q_{13} - q_{21} = 3, \\ q_{12} - q_{13} - q_{22} + q_{21} = -1, \\ p_{11} + 3p_{21} = 4, \\ p_{12} - p_{11} + 3p_{22} - 3p_{21} = 1, \\ p_{13} + 3p_{23} = 7, \\ p_{12} - p_{13} + 3p_{22} - 3p_{23} = -2, \\ q_{11} + 3q_{21} = -4, \\ q_{12} - q_{11} + 3q_{22} - 3q_{21} = 1, \\ q_{13} + 3q_{23} = -1, \\ q_{12} - q_{13} + 3q_{22} - 3q_{23} = -2. \end{cases}$$

Using Wu's algorithm, the set of characteristic sets for $F^{'}$ is

$$Z = \Big\{ z_1 = \{-11 + 8p_{11}, p_{12} - 2, 8p_{13} - 23, \\ -1 + 8q_{11}, 4q_{12} - 3, 8q_{13} - 13, 8p_{21} - 7, \\ p_{22} - 1, 8p_{23} - 11, 8q_{21} + 11, 4q_{22} + 5, 8q_{23} + 7\} \Big\}.$$

By Wu's Well-ordering Principle Theorem, we have

$$V(F^{'}) = (V(z_1) \backslash V(a)) = (V(z_1) \backslash V(268435456)).$$

Therefore,

$$V(F^{'}) =$$

$$\{p_{11} = \tfrac{11}{8}, p_{12} = 2, p_{13} = \tfrac{23}{8}, p_{21} = \tfrac{7}{8}, p_{22} = 1, \\ p_{23} = \tfrac{11}{8}, q_{11} = \tfrac{1}{8}, q_{12} = \tfrac{3}{4}, q_{13} = \tfrac{13}{8}, q_{21} = -\tfrac{11}{8}, \\ q_{22} = -\tfrac{5}{4}, q_{23} = -\tfrac{7}{8}\}.$$

Finally, the equations corresponding to this basis are solved, and the solution set of the system is as:

$$\begin{aligned} \tilde{z}_1 &= p_1 + iq_1 \\ &= (1.375, 2, 2.875) + i(0.125, 0.75, 1.625) \\ &= [1.375 + 0.625r, 2.875 - 0.875r] + \\ & i[0.125 + 0.625r, 1.625 - 0.875r] \end{aligned}$$

and

$$\begin{aligned} \tilde{z}_2 &= p_2 + iq_2 \\ &= (0.875, 1, 1.375) + i(-1.375, -1.25, -0.875) \\ &= [0.875 + 0.125r, 1.375 - 0.375r] + \\ & i[-1.375 + 0.125r, -0.875 - 0.375r]. \end{aligned}$$

Finally, we should mention that the proposed method and the presented method in [3, 17] find the same solution. But, the presented method in [21] finds the solution

$$\tilde{z}_1 = [1.375 + 0.625r, 2.875 - 0.875r]$$
$$+i[0.125 + 0.625r, 1.625 - 0.875r]$$

and

$$\tilde{z}_2 = [0.875 + 0.125r, 1.375 - 0.375r]$$
$$+i[1.375 + 0.125r, 0.875 - 0.375r]$$

.

Example 4.2 *A linear time invariant electric circuit with complex coefficient and fuzzy complex sources may be modeled as fuzzy complex system of linear equations (Eq. (1)). Here, (c_{kj}) is complex coefficient matrix, $\{\tilde{w}_k\}$ is a fuzzy complex source and $\{\tilde{z}_j\}$ may be current or voltage in the system.*

An example problem of a simple RLC circuit [29] with fuzzy current and fuzzy source as shown in Fig. 1 is considered. Corresponding fuzzy complex system of linear equations for this circuit problem may be represented as

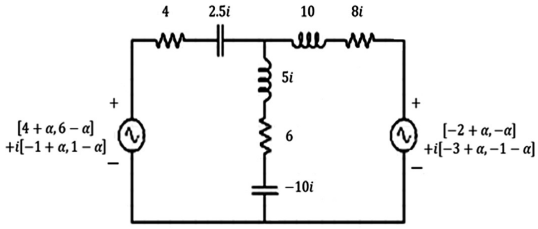

Figure 1: A RLC circuit with fuzzy current and fuzzy source.

$$\begin{cases} (10-i7.5)\tilde{z}_1-(6-i5)\tilde{z}_2 \\ =[4+r,6-r]+i[-1+r,1-r], \\ \\ -(6-i5)\tilde{z}_1+(16+i3)\tilde{z}_2 \\ =[-2+r,-r]+i[-3+r,-1-r]. \end{cases}$$

Let

$$\tilde{z}_1=p_1+iq_1=(p_{11},p_{12},p_{13})+i(q_{11},q_{12},q_{13})$$

and

$$\tilde{z}_2=p_2+iq_2=(p_{21},p_{22},p_{23})+i(q_{21},q_{22},q_{33}).$$

If we apply the proposed method in this paper, the parametric form of the original system is as follows:

$$\begin{cases} (10-i7.5)([p_{11}+(p_{12}-p_{11})r,p_{13}+ \\ (p_{12}-p_{13})r]+i[q_{11}+(q_{12}-q_{11})r,q_{13}+ \\ (q_{12}-q_{13})r])-(6-i5)([p_{21}+ \\ (p_{22}-p_{21})r,p_{23}+(p_{22}-p_{23})r]+i[q_{21}+ \\ (q_{22}-q_{21})r,q_{23}+(q_{22}-q_{23})r]) \\ =[4+r,6-r]+i[-1+r,1-r], \\ \\ -(6-i5)([p_{11}+(p_{12}-p_{11})r,p_{13}+ \\ (p_{12}-p_{13})r]+i[q_{11}+(q_{12}-q_{11})r,q_{13}+ \\ (q_{12}-q_{13})r])+(16+i3)([p_{21}+ \\ (p_{22}-p_{21})r,p_{23}+(p_{22}-p_{23})r]+i[q_{21}+ \\ (q_{22}-q_{21})r,q_{23}+(q_{22}-q_{23})r]) \\ =[4+r,6-r]+i[-1+r,1-r]. \end{cases}$$

As mentioned in Subsection 2.2, the crisp polynomial system can be written as

$$\begin{cases} 10p_{11}+7.5q_{11}-6p_{23}-5q_{23}=4, \\ 10p_{12}-10p_{11}+7.5q_{12}-7.5q_{11}+6p_{23} \\ -6p_{22}+5q_{23}-5q_{22}=1, \\ 10p_{13}+7.5q_{13}-6p_{21}-5q_{21}=6, \\ 10p_{12}-10p_{13}+7.5q_{12}-7.5q_{13}+ \\ 6p_{21}-6p_{22}+5q_{21}-5q_{22}=-1, \\ 10q_{11}-7.5p_{13}-6q_{23}+5p_{21}=-1, \\ 10q_{12}-10q_{11}+7.5p_{13}-7.5p_{12}+ \\ 6q_{23}-6q_{22}+5p_{22}-5p_{21}=1, \\ 10q_{13}-7.5p_{11}-6q_{21}+5p_{23}=1, \\ 10q_{12}-10q_{13}+7.5p_{11}-7.5p_{12}+ \\ 6q_{21}-6q_{22}+5p_{22}-5p_{23}=-1, \\ -6p_{13}-5q_{13}+16p_{21}-3q_{23}=-2, \\ 6p_{13}-6p_{12}+5q_{13}-5q_{12}+16p_{22} \\ -16p_{21}+3q_{23}-3q_{22}=1, \\ -6p_{11}-5q_{11}+16p_{23}-3q_{21}=0, \\ 6p_{11}-6p_{12}+5q_{11}-5q_{12}+16p_{22} \\ -16p_{23}+3q_{21}-3q_{22}=-1, \\ -6q_{13}+5p_{11}+16q_{21}+3p_{21}=-3, \\ 6q_{13}-6q_{12}+5p_{12}-5p_{11}+16q_{22} \\ -16q_{21}+3p_{22}-3p_{21}=1, \\ -6q_{11}+5p_{13}+16q_{23}+3p_{23}=-1, \\ 6q_{11}-6q_{12}+5p_{12}-5p_{13}+16q_{22} \\ -16q_{23}+3p_{22}-3p_{23}=-1. \end{cases}$$

Using Wu's algorithm, the set of characteristic sets for F' is

$$Z=\Big\{z_1=\{51288327p_{11}-16225328,121249p_{12}-42944,-20105296+51288327p_{13},-3630080+51288327q_{11},121249q_{12}-13168,-7510048+51288327q_{13},-1778153+51288327p_{21},121249p_{22}-7930,51288327p_{23}-4930627,12204112+51288327q_{21},$$

$$25125+121249q_{22},9051638+51288327q_{23}\}\Big\}.$$

By Wu's Well-ordering Principle Theorem, we have

$$V(F')=(V(z_1)\backslash V(a))=(V(z_1)\backslash V(10349)).$$

Therefore,

$$V(F')=\{p_{11}=0.3164,p_{12}=0.3542,$$
$$p_{13}=0.3920,p_{21}=0.0347,p_{22}=0.0654,$$
$$p_{23}=0.0961,q_{11}=0.0708,q_{12}=0.1086,$$

$q_{13} = 0.1464, q_{21} = -0.2379, q_{22} = -0.2072,$
$q_{23} = -0.1765\}.$

Finally, the equations corresponding to this basis are solved, and the solution set of the system is as:

$$\begin{aligned}\tilde{z}_1 &= p_1 + iq_1\\ &= (0.3164, 0.3542, 0.3920)\\ &+ i(0.0708, 0.1086, 0.1464)\\ &= [0.3164 + 0.0378r, 0.3920 - 0.0378r]\\ &+ i[0.0708 + 0.0378r, 0.1464 - 0.0378r]\end{aligned}$$

and

$$\begin{aligned}\tilde{z}_2 &= p_2 + iq_2\\ &= (0.0347, 0.0654, 0.0961)\\ &+ i(-0.2379, -0.2072, -0.1765,)\\ &= [0.0347 + 0.0307r, 0.0961 - 0.0307r]\\ &+ i[-0.2379 + 0.0307r, -0.1765 - 0.0307r].\end{aligned}$$

It follows that the proposed method and the presented methods in [3, 17, 29] find the same solution.

5 Conclusion

In this paper, the fuzzy complex systems of linear equations are investigated and a new approach was proposed to find all the solutions of these system based on the Wu's method. In this approach, the fuzzy complex systems of linear equation is converted to an equivalent crisp polynomial equations system and the crisp system is solved using Wu's method. Wu's algorithm leads us to solve triangular systems that are easy to solve. The proposed method is independent from a suitable initial point and all solutions can be obtained simultaneously. Example problems are solved by the present method and are compared with known solutions to show efficiency and effectiveness of the methodology.

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