Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 12, No. 4, 2020 Article ID IJIM-1368, 10 pages
DOR: http://dorl.net/dor/20.1001.1.20085621.2020.12.4.8.0
Research Article

Solving the Special Second Order Ozonation System by OHAM in Tackling the Liver Cancer

S. Pourjafar [*][†], Z. Ayati [‡][§], M. Shafiee [¶]

Received Date: 2019-09-03 Revised Date: 2020-03-21 Accepted Date: 2020-04-13

Abstract

Cyanobacteria (blue-green algae) such as microcystin, nodularins, saxitoxins, anatoxins, and cylindrospermopsin are caused a dangerous disease through drinking water. One of these toxins is Microcystin-LR that the existence of a tiny amount of it in drinking water leads to liver damage and cancer. Also, it makes a high cost of social and health spending. Even though Cyanobacteria are resistant against disinfectants but it has been shown that ozone has high ability to cyanobacteria detoxification. The dissolved ozone in aqueous leads to more disinfection and oxidation by OH radical simultaneously. In this paper, we have presented the solution of the second-order ozone decomposition system with OH radical and Microcystin-LR without dissolved organic content (DOC). It is achieved that one can tackle with such a large problem by the least cost and time. The governing differential system is solved by Optimal Homotopy Asymptotic Method (OHAM), and the obtained results are illustrated by some plots. It has been shown that with a small amount of ozone and a little time, a high dosage of Microcystin-LR can be altered to zero. For this purpose, the conditions PH=2, 4 and 7 have been evaluated at $10^\circ, 20^\circ, 30^\circ$C.

Keywords : Cyanobacteria (blue-green algae); Microcystin-LR; Second order ozonation; Optimal Homotopy Asymptotic Method.

1 Introduction

In recent years, ozone has attained a special importance as a strong oxidizing and disinfectant; and in human life, this element has become widely used including cancer prevention, medical treatment, disinfection of pools, sewage treatment, ozone layer repair, and so on. Evidently, to use ozone requires enough knowledge about the nature and chemical interactions of it. With more knowledge about application and combination of this element with the other elements will be determined its benefits and disadvantages. Re-

---

[*] Department of Mathematics, Guilan Science and Research Branch, Islamic Azad University, Rasht, Iran.

[†] Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran.

[‡] Corresponding author. Zainab.ayati@guilan.ac.ir, Tel:+98(911)1354622.

[§] Department of Engineering Sciences, Faculty of Technology and Engineering East of Guilan, University of Guilan, Rudsar-Vajargah, Iran.

[¶] Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran.

searchers have shown that the existence of more than 5 mg of ozone in our living environment increases the probability of cancer; so, we must consider the special situation and avoid consumption more than requirement for different applications. Notice that ozone production requires the expensive devices and more safe and disinfection does not always occur with increasing ozone dosage. So, determination of the amount effectiveness of ozone in projects must be important. Ozone tends strongly to have combination with other materials and its strong oxidant properties are formed various chemical interactions. Ozone has a short life and it is simply converted into oxygen. Because of ozone is a powerful oxidant, it posses to destroy the toxins produced by cyanobacteria and on this basis is the preferred disinfectant for water supplies [8]. Guarding our workplaces and life environment from contamination, and saving operating costs can be incentives to avoid unnecessary ozone production. for this purpose one can identify a suitable physicochemical model. The model has to determine the suitable values of parameters such as PH, temperature, and even duration of application time of ozone. To reach this goal, in this paper, a special system is used to determine the amount of soluble and time use ozone to eliminate Microcystin-LR under different condition PH, T and also OH radical.

2 The special second order ozonation system for disinfection of Algae in drinking water

In recent years, extensive researchs have been carried out on the use of ozone to disinfect water. It is natural that the factors such as pH, temperature, and organic and mineral substances in water can affect the decomposition of ozone and the amount of using.Ozone, as a strong oxidizing agent, has various processes for the treatment of minerals and organic matters in drinking water and sewage. According to the characteristics of ozone, self-decomposition also takes place. Therefore, different environmental conditions affect ozone consumption and determining the ratio concentrations of OH and O3. Ozone is more disinfectant than chlorine combinations towards most microorganism species [6]. But despite of ozone reaction with many of microbial materials and resulting disinfection them, ozone doesn't react with many inorganic and organic species [3, 4]. One of ozone's products, is the hydroxyl radical (OH) which is highly reactive and a powerful oxidant in reaction with ozone resistant micro pollutants. Because of high reactivity of OH radical towards the water matrix and ozone, measuring of OH radical during the process of ozonation is difficult and this speciallity causes that the steady-state of ozone concentration typically leads about 10^{-12} [1]. Basically, the oxidation of compounds by ozonation requires three elements, Ozone concentration, OH radical concentration, and second order rate constants for reaction of OH and O3 with special compounds. Cyanobacteria (blue-green algae) produce toxins such as microcystins, nodularins, saxitoxins, anatoxins and cylindrospermopsin that imperil the safety of drinking water. The important hazard of these toxins is liver damage including liver cancer [11]. Based on the WHO guideline, maximum allowed concentration of cyanobacteria for drinking water has been determined $1\mu g/l$ [2]. Microcystin-LR ozonation is easily with the rate constant $k = 3.4 \times 10^4 M^{-1} s^{-1}$[9], The experiments by ozone illustrate that only small reaction time and ozone doses are sufficient to oxidize Microcystin-LR [7]. Based on the chemical structure of Microcystin-LR (double bonds and /or amino groups) and similarity to nodularin, anatoxine-a, anatoxina(s) and saxitoxins, also react with ozone at a fast speed [11]. Under certain experimental conditions, i.e., agitation 100 rpm, temperature range from 10°c to $40°C$ and PH range from 2.5 to 9, the following ozone decomposition rate was determined

$$\begin{aligned} r_{O_3} = \\ 3.26 \times 10^5 exp(-4964/T)[O_3] \\ +5.69 \times 10^{18} exp(-10130/T) \times \\ [OH^-]^{1/2}[O_3]^{3/2}(mol/L \times min). \end{aligned} \tag{2.1}$$

The main causes of this decomposition are the direct ozone reaction and the hydroxide ion initiation of decomposition. Experimental and calculated ozone concentrations agree with in 10 % for 95 % of the experiments [10].

Due to the rapid and immediate reaction of ozone to Microcystin-LR, the role of OH only in O3 decomposition can be benefit and using the following system, the curve of ozone and the reduction of Microcystin-LR is obtained.

$$\begin{cases} r_M = -k_M[O_3][M], \\ r_{O_3} = -k_D[O_3] - k_R[OH^-]^{1/2} \\ \times[O_3]^{3/2} - k_M[O_3][M]. \end{cases} \tag{2.2}$$

where O3 and M denote ozone and Microcystin-LR variables, respectively. Also, kD, kR and kM illustrate, in turn, the rate constant of direct decomposition of ozone, the rate constant of ozone reaction by OH radical, and the rate constant of ozone with Microsystin-LR. In addition, [.] determines the concentration of a special matter. [OH] is dependent to concentration of O3 and Rct, that is a measure of the ratio of the concentrations of OH and O3, (i.e. $R_{ct} = [OH]\,[O3]$) typically varies $10^{-9} - 10^{-7}\,(M/M)$ for various water [11].

3 Basic Idea of OHAM

One of the numerical methods to solve a differential system is optimal homotopy asymptotic. Consider the following system,

$$\begin{cases} L_1\left[u_1(x)\right] + g_1(x) + \\ N_1\left[u_1(x), \ldots u_n(x)\right] = 0, \\ x \in D, B_1(u_1(x), \mathrm{d}u_1(x)/\mathrm{d}x = 0, \\ L_2\left[u_2(x)\right] + g_2(x) + \\ N_2\left[u_1(x), \ldots u_n(x)\right] = 0, \\ x \in D, B_2(u_2(x),\, \mathrm{d}u_2(x)/\mathrm{d}x) = 0, \\ \quad \vdots \\ L_n\left[u_n(x)\right] + g_n(x) + \\ N_n\left[u_1(x), \ldots u_n(x)\right] = 0, \\ x \in D, B_n(u_n(x), \mathrm{d}u_n(x)/\mathrm{d}x) = 0. \end{cases} \tag{3.3}$$

where x is an independent variable, u(x) an unknown function and g(x) a known function of x with domain D. Bi, Li and Ni are boundary, linear and nonlinear operators [5].

To illustrate idea of HAM, for each equation of the system, we construct a family of equations as follows:

$$\begin{aligned} &(1-p)[L_i(\varphi_i(x;p,C_{ij})) + g_i(x)] = \\ &H_i(x;p,C_{ij})[L_i(\varphi_i(x;p,C_{ij})) + \\ &g_i(x) + N_i(\varphi_i(x;p,C_{ij}))]. \end{aligned} \tag{3.4}$$

where$\varphi_i(x,p,C_{ij})$is an unknown function with convergence-control parameters $C_{ij}, i = 1,2,\ldots,n, \quad j = 1,2,\ldots,s$. $p \in [0,1]$ is the embedding parameter and $H_i(x,p,C_{ij})$ is a nonzero auxiliary function, except$H_i\left(x,0,C_{ij}\right) = 0$. If $p = 0$ then $\varphi_i\left(x,0,C_{ij}\right) = u_{i_0}\left(x\right)$ and with $p = 1$ one gets $\varphi_i\left(x,1,C_{ij}\right) = u_i\left(x,C_{ij}\right),\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,s$. thus, as changes from 0 to 1,
$\varphi_i\left(x,p,C_{ij}\right)$ deforms from $u_{i_0}\left(x\right)$ to $u_i\left(x,C_{ij}\right)$. Now, when $p = 0$, we obtain $u_{i_0}\left(x\right)$ by solving the following equations,

$$\begin{aligned} &L_i\left[u_{i_0}(x)\right] + g_i(x) = 0, \\ &B_i(u_{i_0}, \tfrac{du_{i_0}}{dx}) = 0. \quad i = 1,2,\ldots,n. \end{aligned} \tag{3.5}$$

Then, we select $H_i\left(x,p,C_{ij}\right)$ in the form as follows:

$$\begin{aligned} &H_i(x;p,C_{ij}) = pH_{i_1}(x;C_{ij}) + \\ &p^2 H_{i_2}(x;C_{ik}) + \ldots \\ &i = 1,2,\ldots,n, j = 1,2,\ldots,k = 1,2,\ldots, \end{aligned} \tag{3.6}$$

where $H_i\left(x;C_{ij}\right)$ are auxiliary functions in terms of independent variable x and convergence control parameters $C_{ij},\ i = 1,2,\ldots,n,\ j = 1,2,\ldots,s$. Suppose that:

$$\begin{aligned} &\varphi_i\left(x;p,C_{ij}\right) = u_{i_0}\left(x\right) + \sum_{k\geq 1}^{\infty} u_{i_k}\left(x;C_{ij}\right)p^k, \\ &i = 1,2,\ldots,n, \quad j = 1,2,\ldots,s. \end{aligned} \tag{3.7}$$

Now, by equating the coefficients with the same powers of p , one can obtain

$$\begin{aligned} &L_i(u_{i_1}(x,C_{ij})) = H_{i_1}(x,C_{ij})N_{i_0}(u_{i_0}(x)), \\ &B(u_{i_1}, \tfrac{du_{i_1}}{dx}) = 0. \end{aligned} \tag{3.8}$$

And

$$\begin{aligned} &L_i(u_{i_l}(x;C_{ij}) - u_{i_{l-1}}(x,C_{ij})) = \\ &H_{i_1}(x;C_{ij})N_{i_0}(u_{i_0}(x)) \\ &+\sum_{i=1}^{l-1} H_{i_l}(x;C_{ij})L(u_{i_{l-i}}(x;C_{ij})) \\ &+N_{i_{l-i}}(u_{i_0}(x),\ldots,u_{i_{l-i}}(x;C_{ij})), \\ &B(u_{i_l}, \tfrac{du_{i_l}}{\mathrm{d}x}) = 0, \quad l = 2,3,\ldots, \end{aligned} \tag{3.9}$$

where

$$\begin{aligned} &N_i(\varphi_i(x;p,C_{ij})) = N_{i_0}(u_{i_0}(x)) + \\ &N_{i_1}(u_{i0}(x), u_{i1}(x;C_{ij}))p + N_{i_2}(u_{i0}(x), \\ &u_{i1}(x;C_{ij}), u_{i2}(x;C_{ij}))p^2 + \ldots, \end{aligned} \tag{3.10}$$

and $N_{i_m}(u_{i_0}(x), u_{i_1}(x;C_{ij}), \ldots, u_{i_m}(x;C_{ij}))$, are the coefficients of p^m, in the expansion of $N_i(\varphi_i(x;p,C_{ij}))$, as a power series in terms of embedding parameter p . Noting that by basis of convergence, when $p=1$, then the following solution is exact

$$\begin{aligned}&\varphi_i(x;1,C_{ij}) = u_i(x;C_{ij}) = u_{i_0}(x)+\\ &\sum_{l\geq 1} u_{i_l}(x;C_{ij}), \qquad j=1,2,\ldots,s,\end{aligned} \tag{3.11}$$

and

$$\begin{aligned}&L_i\left[u_i(x,C_{ij})\right] + g_i(x) + N_i\left[u_i(x,C_{ij})\right] = 0,\\ &x\in D, \quad j=1,2,..s, \quad i=1,2,..n.\end{aligned} \tag{3.12}$$

Also, the m-th order approximate solution will be determined as follows

$$\begin{aligned}&\bar{u_i}(x;C_{ij}) = u_{i_0}(x) + \sum_{l\geq 1}^{m} \bar{u_{i}}_l(x,C_{ij}),\\ &j=1,2,\ldots s.\end{aligned} \tag{3.13}$$

Consequently, the following residual will obtain

$$\begin{aligned}&R_i(x,C_{ij}) =\\ &L_i\left[\bar{u_i}\,(x,C_{ij})\right] + g_i(x) + N_i\left[\bar{u_i}\,(x,C_{ij})\right],\\ &x\in D, \quad j=1,2,...,s, \quad i=1,2,..,n.\end{aligned} \tag{3.14}$$

Because the convergence-control parameters effect on the approximate solution, we use the least square method to determine the optimal values of these parameters.Thus, suppose

$$\begin{aligned}&J_i(C_{i_1}, C_{i_2}, \ldots, C_{i_s}) =\\ &\int_a^b {R_i}^2(x, C_{i_1}, C_{i_2}, \ldots, C_{i_s})\,\mathrm{d}x,\\ &i=1,2,\ldots,n,\end{aligned} \tag{3.15}$$

and solve the following equations

$$\begin{aligned}&\tfrac{\partial J_i}{\partial C_1} = \tfrac{\partial J_i}{\partial C_2} = \cdots = \tfrac{\partial J_i}{\partial Cs} = 0.\\ &i=1,2,\ldots,n.\end{aligned} \tag{3.16}$$

4 Application of OHAM to solve a second order decomposition of ozone system

Consider the following system of ozone decomposition in aqueous solution which illustrates the consumption of ozone for decaying of pure Microcystin-LR without NOM.

$$\begin{cases}\frac{\mathrm{d}M}{\mathrm{d}t} = -k_M O_3(t)\,M(t),\\ \\ \frac{\mathrm{d}O_3}{\mathrm{d}t} = -k_D\,O_3(t) - k_R[OH^-]^{1/2}O_3(t)^{3/2}\\ -k_M O_3(t)\,M(t),\end{cases}$$

where $O_3\,(t)$ and $M\,(t)$are concentration of dissolved ozone (O3), and Microcystin-LR at time $t(mgL^{-1})$, respectively. K_D is the rate constant of direct ozone decomposition (l/mg min) and K_R is the rate constant of ozone decomposition with OH radical (l/mg min). By OHAM, we have

$$\begin{cases}(1-p)(\frac{\mathrm{d}O_3}{\mathrm{d}t}) = H_1(x;p,c_i)\times\\ (\frac{\mathrm{d}O_3}{\mathrm{d}t} + k_D O_3(t) + k_R[OH^-]^{1/2}O_3(t)^{3/2}+\\ k_M O_3(t)\,M(t)),\\ \\ (1-p)(\frac{\mathrm{d}M}{\mathrm{d}t}) = H_2(x;p,c_i)[\frac{\mathrm{d}M}{\mathrm{d}t}+\\ k_M O_3(t)\,M(t)]\end{cases}$$

where

$$\begin{cases}H_1(x;p,c_i) = c_1 p + c_2 p^2,\\ H_2(x;p,c_j) = c_3 p + c_4 p^2,\end{cases}$$

$p\in[0,1]$, is the embedding parameter in the auxiliary functions H_1, H_2 and $c_i, i=1,2,3,4$ are convergence-control parameters.

We expand $O_3\,(t)$ and $M\,(t)$ by Taylor expansion, as follows:

$$\begin{aligned}O_3(t) &= O_{3_0}(t) + \sum_{i=1}^{\infty} p^i.O_{3_i}(t),\\ M(t) &= M_0(t) + \sum_{i=1}^{\infty} p^i.M_i(t).\end{aligned}$$

Then, after substituting the expansion of $O_3\,(t)$ and $M\,(t)$ in the system, we select the coefficient of the same powers in p , thus we have:

$$p^0:\begin{cases}\frac{\mathrm{d}}{\mathrm{d}t}O_{3_0}(t) = 0,\\ \\ \frac{\mathrm{d}}{\mathrm{d}t}M_0(t) = 0.\end{cases}$$

$$p^1:\begin{cases}\frac{\mathrm{d}}{\mathrm{d}t}O_{3_1}(t) = \frac{\mathrm{d}}{\mathrm{d}t}O_{3_0}(t) + c_1[\frac{\mathrm{d}}{\mathrm{d}t}O_{3_0}(t)\\ +K_D O_{3_0}(t) + K_R(OH)^{1/2}O_{3_0}(t)^{3/2}\\ +K_M M_0(t)O_{3_0}(t)],\\ \\ \frac{\mathrm{d}}{\mathrm{d}t}M_1(t) = \frac{\mathrm{d}}{\mathrm{d}t}M_0(t) + c_3[\frac{\mathrm{d}}{\mathrm{d}t}M_0(t)\\ +K_M M_0(t)O_{3_0}(t)].\end{cases}$$

$$p^2: \begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}O_{3_2}(t) = \frac{\mathrm{d}}{\mathrm{d}t}O_{3_1}(t) + c_1[\frac{\mathrm{d}}{\mathrm{d}t}O_{3_1}(t) + \\ K_D O_{3_1}(t) + K_R(OH)^{1/2}O_{3_1}(t)^{3/2} + \\ K_M(M_1(t)O_{3_0}(t) + M_0(t)O_{3_1}(t))] + \\ c_2[\frac{\mathrm{d}}{\mathrm{d}t}O_{3_0}(t) + K_D O_{3_0}(t) + K_R \times \\ (OH)^{1/2}O_{3_0}(t)^{3/2} + K_M(M_2(t)O_{3_0}(t) \\ +M_1(t)O_{3_1}(t) + M_0(t)O_{3_2}(t))], \\ \\ \frac{\mathrm{d}}{\mathrm{d}t}M_2(t) = \frac{\mathrm{d}}{\mathrm{d}t}M_1(t) + c_3[\frac{\mathrm{d}}{\mathrm{d}t}M_1(t) + \\ K_M M_0(t)O_{3_1}(t) + M_1(t)O_{3_0}(t)] + \\ c_4[\frac{\mathrm{d}}{\mathrm{d}t}M_0(t) + K_M(M_2(t)O_{3_0}(t) + \\ M_1(t)O_{3_1}(t) + M_0(t)O_{3_2}(t))]. \end{cases}$$

Note that, K_M is the rate constant of ozone with Microcystin-LR, also the relationship K_D and K_R are the rate constants of the direct ozone reaction and the reaction by ozone with OH radical, according to the values of the presented parameter T. We use the values of parameters and the initial values of ozone and Microcystin-LR in table 1. When the PH of water is lower than 3, OH reacts hardly with Ozone. So, the reaction ozone with OH radical is negligible and the process follows with the first order ozone reaction.The experimental research by Sotelo et al [10], determines the values of $K_D = .82\ 10^{-2}, 1.41\ 10^{-2}, 2.32\ 10^{-2}$, and $4.50\ 10^{-2} M^{-1}\mathrm{min}^{-1}$ in terms of T=$283, 293, 303, 313\ K$, respectively. Thus with this values and the following system at $PH = 2$, will have the convergence-control parameters by OHAM, as follows:

$$\begin{cases} \frac{\mathrm{d}M}{\mathrm{d}t} = -k_M\, O_3\,(t)\, M(t), \\ \\ \frac{\mathrm{d}O_3}{\mathrm{d}t} = -k_D[O_3\,(t)] - k_M[O_3\,(t)][M\,(t)]. \end{cases}$$

$$c_1 = 0.20323584521994994584,$$
$$c_2 = -0.53229930407629777170,$$
$$c_3 = -0.04867830818904467340 5,$$
$$c_4 = 0.03603266525414723463 8.$$

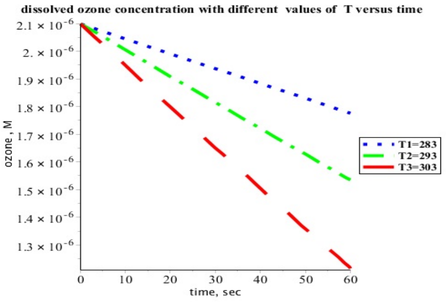

Figure 1: Concentration of dissolved ozone versus time with $PH = 2, O_3 = 0.1\, mgL^{-1}, T = 10°\mathrm{C}$ (dot),20°C(dash dot) and 30°C (space dash).

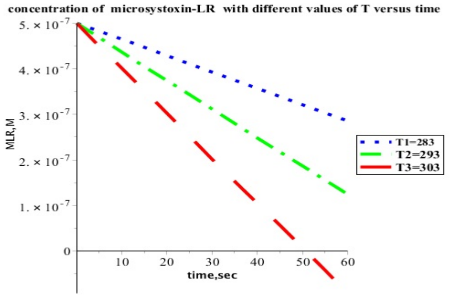

Figure 2: Concentration of Microcystin-LR versus time with $PH = 2, O_3 = 0.1\, mgL^{-1}, T = 10°\mathrm{C}$ (dot), 20°C (dash dot) and 30°C (space dash).

The results illustrate that at after 1 minute consume almost 16%, 27% ozone and as a result destructs almost 43%, 75% Microcystin-LR, in respectively. But when after almost 51 s, consume all of ozone and destruct all of Microcystin-LR.
Note that, by increasing the initial value of ozone from 0.1 mgL^{-1} to 1 mg/L, under conditions PH=2, T=283, 293 and 303K, the convergence control parameters determine as follows:

$$c_1 = 0.09340890655434991705,$$
$$c_2 = 0.23455525608503054 03,$$
$$c_3 = -0.00255815186610425749 3,$$
$$c_4 = 0.00010643385131522789 9,$$

and with 1 mgL^{-1} ozone, dont decrease the time of distraction of Microcystin-LR and after 61 seconds, which is in addition 10 s more than with 0.1 mgL^{-1} initial ozone, decay all of Microcystin-LR.
If PH varies to 4, and T = 283, 293, 303K, then KM changes to 4.41 10^4, 6.52 10^4, 9.50 10^4 , respectively [8]. Also, we use $K_D = 3.26 \times 10^5 exp(-4964/T)$, for all PH greater than 3, and apply the following system,

$$\begin{cases} \frac{\mathrm{d}M}{\mathrm{d}t} = -k_M O_3(t)M(t), \\ \\ \frac{\mathrm{d}M}{\mathrm{d}t} = -k_D O_3(t) \\ -k_R[OH^-]^{1/2}O_3(t)^{3/2} - k_M O_3(t)M(t), \end{cases}$$

Using OHAM to solve the above ozonation system, one can obtain the following convergence

Table 1: Parameters and initial values.

Parameter	Value
K_D	$3.26 * 10^5 e^{(-4964/T)}$
K_R	$5.69 * 10^{18} e^{(-10130/T)}$
K_M	$5.8 * 10^4, 1.02 * 10^5, 1.61 * 10^5$
$O_3(0), M$	$2.1 * 10^{-6} (0.1 mg/l)$
$M(0), M$	$5 * 10^{-7} (500 \mu g/l)$
OH, M	$2.1 * 10^{-15}$
T, K	283, 293, 303
PH	2
$R_{ct}, M/M$	10^{-9}

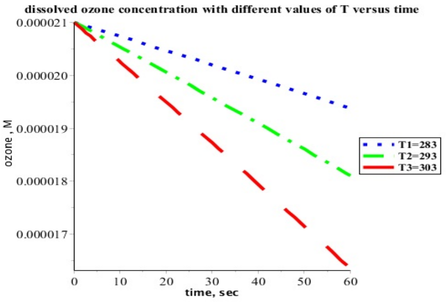

Figure 3: Concentration of dissolved ozone versus time with $PH = 2, O_3 = 1\ mgL^{-1}, T = 10°C$ (dot),20°C(dash dot) and 30°C (space dash).

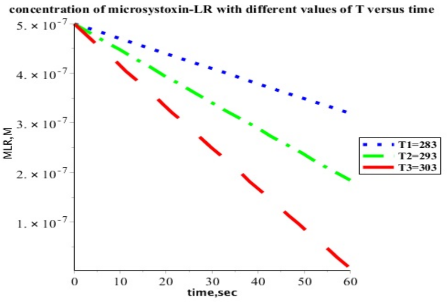

Figure 4: Concentration of Microcystin-LR versus time with $PH = 2, O_3 = 1\ mgL^{-1}, T = 10°C$ (dot), 20°C(dash dot) and30°C (space dash).

control parameters,

$$
\begin{aligned}
&T = 283,\\
&c_1 = 0.18824083172158575564,\\
&c_2 = 0.52539394964018297398,\\
&c_3 = -0.047346255885761162433,\\
&c_4 = -0.0076163769337997734232.
\end{aligned}
$$

$$
\begin{aligned}
&T = 293, 303,\\
&c_1 = 0.18915243839855876702,\\
&c_2 = -0.52511075336193013725,\\
&c_3 = -0.048554192855649067678,\\
&c_4 = -0.017840610983610175800.
\end{aligned}
$$

With the obtained control parameters, can observe after 1 minute consume almost 16% , 23% ozone for destruction of 55%, 91% Microcystin-LR in temperature 283K and 293K, respectively; but in temperature 303K, after 45 s destruct all of Microcystin-LR.

In condition PH=4, if one increases the initial value of ozone from 0.1 mgL^{-1} to 1 mgL^{-1}, with the following convergence control parameters determined by OHAM, destruction all of Microcystin-LR occur after 51 seconds at T=30 C, which destruction time will increase 6 seconds

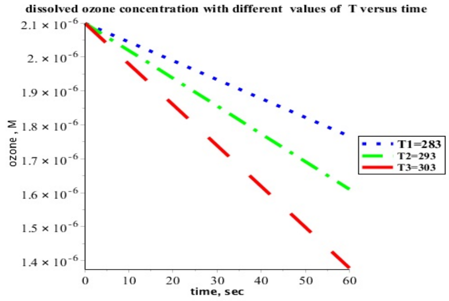

Figure 5: Concentration of dissolved ozone versus time with $PH = 4, O_3 = 0.1\ mgL^{-1}, T = 10°C$(dot),20°C(dash dot) and 30°C (space dash).

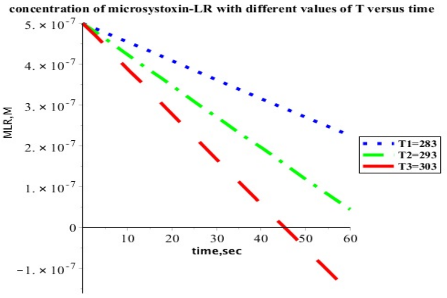

Figure 6: Concentration of Microcystin-LR versus time with $PH = 4, O_3 = 0.1\, mgL^{-1}, T = 10°C$ (dot), 20°C (dash dot) and 30°C (space dash).

more than first situation.

$$T = 283, 293, 303,$$
$$c_1 = 0.09024189614704221862,$$
$$c_2 = -0.2291530701380411301,$$
$$c_3 = -0.002685408144405715442,$$
$$c_4 = -0.004647606540651906 54.$$

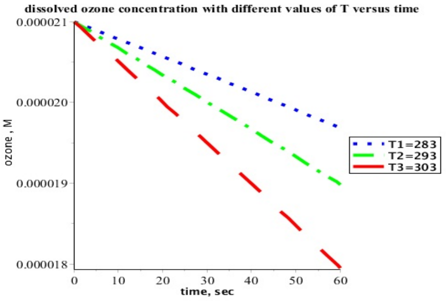

Figure 7: Concentration of dissolved ozone versus time with $PH = 4, O_3 = 1\ mgL^{-1}, T = 10°C$ (dot),20°C(dash dot) and 30°C(space dash).

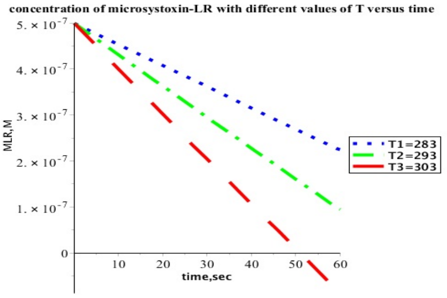

Figure 8: Concentration of Microcystin-LR versus time with $PH = 4, O_3 = 1\ mgL^{-1}, T = 10°C$ (dot), 20°C(dash dot) and 30°C(space dash).

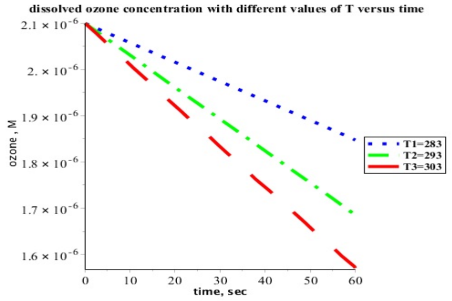

Figure 9: Concentration of dissolved ozone versus time with $PH = 4, O_3 = 0.1\ mgL^{-1}, T = 10°C$(dot),20°C(dash dot) and 30°C(space dash).

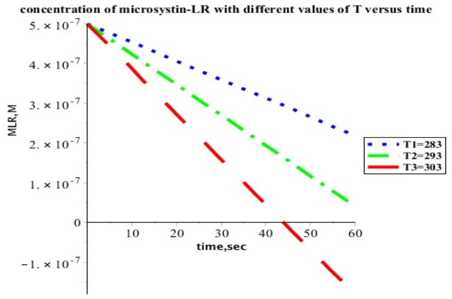

Figure 10: Concentration of Microcystin-LR versus time with $PH = 4, O_3 = 0.1\ mgL^{-1}, T = 10°C$(dot),20°C (dash dot) and 30°C(space dash).

With changing PH to 7, and temperatures 283,293 and 303, the KM presents 2.32 10^4, 3.80 10^4, 5.22 10^4 respectively [8]. We obtain the convergence control parameters as follows

$$T = 283, 293,$$
$$c_1 = 0.08548067305124722358 5,$$
$$c_2 = -0.3399395267544639575 1,$$
$$c_3 = -0.0268116227814568729 62,$$
$$c_4 = -0.1406096141483917303 4.$$

$$T = 303,$$
$$c_1 = 0.0238640702491404334 19,$$
$$c_2 = -0.1963615616265988533 9,$$
$$c_3 = -0.0128059496993781603 07,$$
$$c_4 = -0.1840498908965871942 5.$$

In PH=7, after 1 minute ozonation in temperature 283, is consumed 12% ozone and almost 56% of Microcystin-LR destructs and in temperature 293 with 20% ozone, 92% of Microcystin-LR

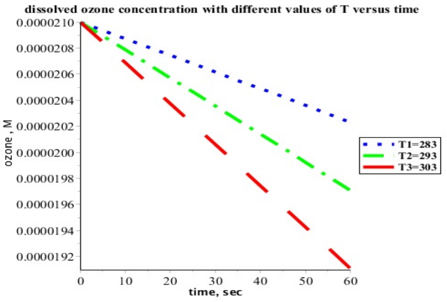

Figure 11: Concentration of dissolved ozone versus time with $PH = 7, O_3 = 1\ mgL^{-1}, T =$ 10°C(dot),20°C(dash dot) and 30°C (space dash).

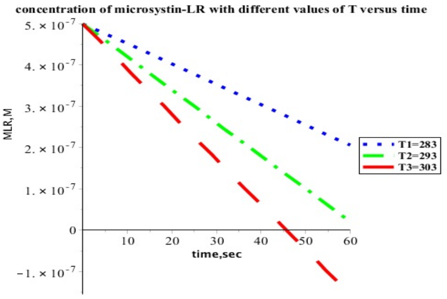

Figure 12: Concentration of Microcystin-LR versus time with $PH = 7, O_3 = 1\ mgL^{-1}, T = 10$°C (dot),20°C (dash dot) and 30°C (space dash).

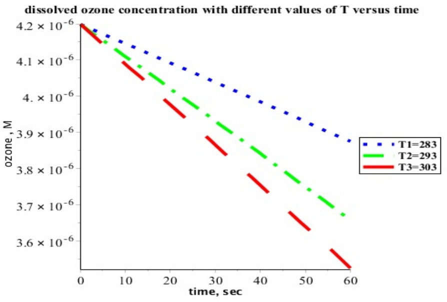

Figure 13: Concentration of dissolved ozone versus time with $PH = 7, O_3 = 0.2\ mgL^{-1}, T =$ 10°C(dot),20°C(dash dot) and 30°C (space dash).

destroys. But in temperature 303, after 44s all of ozone consumes and all of Microcystin-LR destroys.

In PH=7, with changing the initial value of ozone from 0.1 mgL^{-1} to 1 mgL^{-1}, in temperature 30°C by the following control parameters after 46 seconds the value of Microcystin-LR will be zero, which the time is more than before test.

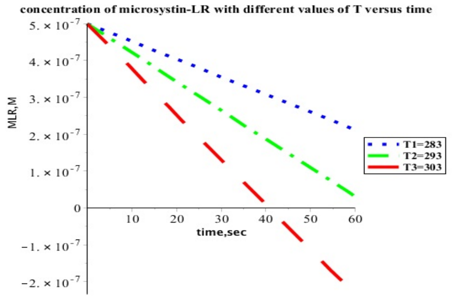

Figure 14: Concentration of Microcystin-LR versus time with $PH = 7, O_3 = 0.2\ mgL^{-1}, T =$ 10°C(dot), 20°C (dash dot) and 30°C (space dash).

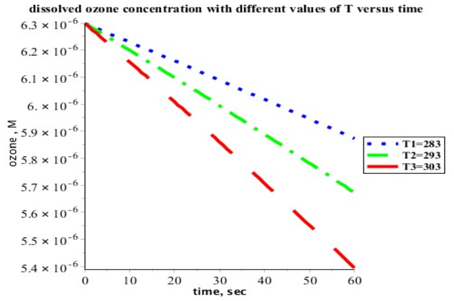

Figure 15: Concentration of dissolved ozone versus time with $PH = 7, O_3 = 0.3\ mgl^{-1}, T =$ 10°C(dot),20°C(dash dot) and 30°C (space dash).

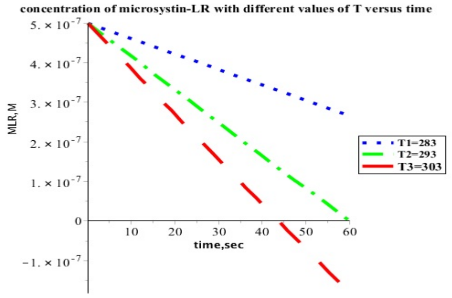

Figure 16: Concentration of Microcystin-LR versus time with $PH = 7, O_3 = 0.3mgL^{-1}, T = 10$°C (dot), 20°C (dash dot) and 30°C (space dash).

$$\begin{aligned}
&T = 283, 293, 303,\\
&c_1 = 0.0726278100719406268,\\
&c_2 = -0.1914638794293427372,\\
&c_3 = -0.002494544258988298589,\\
&c_4 = -0.01522467870102714533.
\end{aligned}$$

We calculate the problem with 0.2 mgL^{-1} ozone and the same other conditions. Microcystin-LR completely decay after 41 seconds, but consume almost 0.032 mgL^{-1} of ozone.

If we increase the value of initial ozone to 0.3 mgL^{-1}, the result is not better and after consumption 0.042 mgL^{-1} of ozone and after 44 seconds the value of Microcystin-LR gets to zero.

5 Conclusion

The spread of chronic diseases, including cancers, has led humans to think about the roots of diseases, including drinking water, which makes life impossible without it. Cyanobacteria (green-blue algae) are the cause of production of toxic substances and chronic diseases that have different resistance to disinfectants. One of these products is Microcystin-LR that can cause chronic liver disease, even liver cancer. Ozone is a strong oxidizing and disinfectant substance, and by applying a small amount of ozone, one can disinfect the Microcystin-LR. In this paper, Ozone decomposition was evaluated in water with PHs and temperatures ranging between 2, 4 and 7 and $10°C$, $20°C$, and $30°C$, respectively. A specific type of second-order ozone reaction with pure Microcystin-LR without dissolved organic content (DOC) has been considered and used the numeric method OHAM to solve the respective differential system. The result shows, with an initial value of $0.1 - 0.2$ mgL^{-1} ozone, and at $30°C$, it would be possible to eliminate all of Microcystin-LR in less than a minute. We also found that with an arbitrary increase in the initial amount of ozone, Microcystin-LR elimination time would not be reduced, and ozone levels should be optimized to avoid oversized ozone (which could be problematic itself). It has been shown that 500 mgL^{-1} Microcystin-LR can be eliminated with 0.2 mgL^{-1} of initial ozone in 41 seconds, while initial values of more than 0.2 mgL^{-1} will not decrease the duration of removal Microcystin-LR.

References

[1] M. S. Elovitz, U. von Gunten, H. P. Kaiser, Hydroxyl radical/ozone ratios during ozonation processes. II. The effect of temperature, pH, alkalinity, and DOM properties, *Ozone: science and engineering* 22 (2000) 123-150.

[2] S. Gupta, Cyanobacterial toxins: microcystin-LR, World Health Organisation *Geneva*, (1988).

[3] J. Hoigné, H. Bader, Rate constants of reactions of ozone with organic and inorganic compounds in water: non-dissociating organic compounds, *Water Research* 17 (1983) 173-183.

[4] J. Hoigné, H. Bader, W. R. Haag, J. Staehelin, Rate constants of reactions of ozone with organic and inorganic compounds in water III, Inorganic compounds and radicals, *Water Research* 19 (1985) 993-1004.

[5] V. Marinca, N. Herisanu, The Third Alternative of the Optimal Homotopy Asymptotic Method, Springer, *International Publishing Switzerland* 14 (2015) 391-465.

[6] J. M. Montgomery, Water treatment: principles and design, John Wiley & Sons, *New York*, (1985).

[7] J. Rositano, G. Newcombe, B. Nicholson, P. Sztajnbok, Ozonation of NOM and algal toxins in four treated waters, *Water Research* 35 (2001) 23-32.

[8] J. Rositano, B. C. Nicholson, P. Pieronne, Destruction of cyanobacterial toxins by ozone, *Ozone: Science and Engineering* 20 (1998) 223-238.

[9] AR. Shawwa, DW. Smith, Kinetics of microcystin-LR oxidation by ozone, *Ozone: Science and Engineering* 23 (2001) 161-170.

[10] J. L. Sotelo, F. J. Beltrán, F. J. Benitez, J. Beltran-Heredia, Ozone decomposition in water: kinetic study, *Industrial and Engineering Chemistry Research* 26 (1987) 39-43.

[11] U. Von Gunten, Ozonation of drinking water: Part I. Oxidation kinetics and product formation, *Water Research* 37 (2003) 1443-1467.

Sadegh Pourjafar is a PHD candidate of Applied Mathematics (Numerical Analysis) , in Rasht Branch, Islamic Azad University, Rasht,Iran and the faculty member at the Department of Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran. Interested in Numerical solution of Functional Equations and Numerical Linear Algebra.

Zainab Ayati is assistant professor of mathematics at university of Guilan. His research interest is numerical and exact methods for differential, integral equations, Numerical Linear Algebra and fractional calculus.

Mahmoud Shafiee is assistant professor of mathematics at Rasht branch, Islamic Azad University. His research interest is numerical methods for differential and integral equations and fractional calculus.