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Research Article

A Mathematical Optimization Model for Integrating the Problems of Discrete Time-Cost Tradeoff (DTCTP) and Multi-Mode Resource-Constrained Project Scheduling (MRCPSP)

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Abstract

The problem of resource-constrained project scheduling(RCPSP) and the problem of time-cost tradeoff (DTCTP) are two mathematical optimization problems that have long mattered in project management in such a way that reducing the completion time of a project, which is achieved through increasing the resources required for executing the activities, usually turns into a necessity in practice. The existing methods and algorithms for solving this problem, considering the cost-slope of the activities as a pivotal index, have been defined differently to date. Yet, this paper aims to present a framework whereby project scheduling and time-cost tradeoff can be addressed under the circumstances that several modes of execution exist for each activity. Apart from the renewable resources, the non-renewable resources have also been taken into consideration for each activity. Hence, initially, a mathematical optimization model based on the assumptions of the problems is proposed, and then, via changing the variables and other mathematical modelling techniques, the problems are integrated and developed in form of a mixed-integer linear mathematical programming model. Eventually, the model is solved using the branch and bound method and the results are studied with sensitivity analysis.

Keywords : Reliability; Redundancy allocation problem; Series-parallel systems; Heuristic methods; Hybrid algorithm.

1 Introduction

Succinctly, 'spending the least possible' can be regarded as the overarching objective of any time-crashing model. As a result, it is a must to scrutinize the factors that inflict a cost or elevate the total cost, and to incorporate them into the model. Since the time-value of money is a very influential factor in the total cost of the projects, especially in the capital-intensive ones, incorporating this factor into the model is deemed necessary.

Hardly a project can be found where the need for expediting the completion exists not. As mentioned, such an expedition would entail higher costs and any wrong decision made in this regard would translate into a higher total cost for the

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project. Due to the fact that the total cost of the project plays a fundamental part in the revenues of the operators, the applied models of Discrete Time-Cost Tradeoff (DTCTP) seem appealing to them and have become popular. Therefore, it can be envisaged that the proposed model in this paper, which analyzes the costs of time crashing in a more feasible way, would be of significance to the employers of capital-intensive mega-projects in that it empowers them for today's competitive world.

The other aspect of the topic is the issue of scheduling. Over the past decades, resource-constrained project scheduling problem (RCPSP) has been a major research area as one of the optimization procedures with various types in objective functions, diverse assumptions for executing the activities, different resource constraints and the like. Generally, the prime focus in such problems is on the minimization of the makespan of the project which, in turn, has led to developing numerous exact, heuristic, and meta-heuristic methods for implementing projects with limited renewable resources among which the well-known RCPSP is a clear instance. This type of problem, regarded as an NP-hard one, attempts to minimize the overall duration of a project in light of precedence relations among the activities as well as the availability of constrained renewable resources. The extended version of this problem-with different objective functions (such as net present value optimization), different resource constraints (both renewable and non-renewable), multi-mode activities, etc.- can be often seen in complex optimization problems and has been reviewed by many authors in the related literature.

In many researches, RCPSP has been extended to multi-mode problems where each activity can be executed under a different duration of time and renewable/non-renewable resource use. Due to the complex nature of the problem, only a handful of algorithms have been proposed in the literature for solving such problems.

Through considering the execution of the activities under various technologies, this paper tends to deal with the procedure to select technologies for executing the activities. In doing so, awareness about the status quo and the processes of the project is the very first step in decision and policy making for an optimal management of the activities. Not being an exception, any decision-making about and investment on a project is tantamount to investing on equipment, manpower, and knowledge. The composition of the said factors, known as technologies, continually requires a well-planned management, which, without knowledge about the status and functions of the technologies in hand and evaluating them, would result in a low-efficient management, if not impossible. Technology evaluation models serve as a means for technology management whereby organizations can appropriately manage their technological details and, eventually, achieve a favorable organizational efficiency.

2 Literature Review

The problem of Discrete Time-Cost Tradeoff (DTCTP) has had prevailing applications to date and numerous researches have been conducted in this regard. On the one hand, the customers' need for receiving services in a shorter span of time, and the necessity to reduce the cost of project execution on the other have noticeably heightened the significance of such problems among the business owners and researchers as well.

2.1 Discrete Time-Cost Tradeoff Problem

DTCTP was first coined in 1979 by Hindelang and Muth [1] drew a lot of attention. Prabuddha et al. [2], and Vladimir et al. [3] showed that this problem is a non-deterministic polynomial-time hard one (NP-hard) and difficult to solve.

A fundamental assumption that is considered in many studies on such problems is that the cost of activities is a function of the duration of their execution. Here, duration is counted as a decision variable, with the lower and upper limit determined as crash duration, and normal duration, respectively. Of the linear mathematical models proposed in this respect, one can refer to the studies by Kelley and Walker [4], Fulkerson [5], Kelley [6], Ford and Fulkerson [7], Siemens [8], Goyal [9], and Elmaghraby and Salem [10]. Further to them, Moder et al. [11] considered the function of activities cost as a continuous one.

Hitherto, DTCTP has been solved via various methods such as exact methods like dynamic programming by Hindelang and Muth [1], enumeration algorithm by Patterson [12], or the branch and bound method by (Demeulemeester, Erenguc, [13-15]). However, none of the exact methods could solve the DTCTP in a large-scale problem. Hence, a tendency towards adopting other methods was formed among the researchers. Akkan [16]'s work is an instance of a heuristic method to solve such problems within an AoA network based on Lagrangian relaxation method. Liu et al. [17], and Peng et al. [18] made use of a genetic algorithm for solving DTCTP. Elmaghraby and Kamburowski [19] assigned a penalty or reward for the objective functions in DTCTP.

The crashing of the activities was put forth by Ann et al. [20] and Van Slyke [21] was the first to adopt Monte Carlo simulation in the field of crashing. Of his many achievements, one can touch on the project length estimation with more precision, the flexibility in a selecting distribution function for the time of the activities as well as the ability to calculate the criticality level of the path. Elsewhere, DTCTP was studied considering the time-switch constraint by Vanhoucke [22]. Nikoomaram et al. [23] presented a mathematical model for time cost trade-off Problem with considering concept of time value of money and budget limitation. Shahriari [24] developed a two-objective mathematical model for balancing compressing the project time with activities delay to prepare a suitable tool for decision makers caught in available facilities and due to the time of projects.

2.2 Resource-Constrained Project Scheduling Problem

In this section, we aim to provide a concise review of the related literature regarding project management and control so as to highlight the various aspects of the field as well as the research gaps. The problem of scheduling has been widely researched lately. In the following, a number of the studied topics and issues have been reviewed.

Brucker et al. [25] have studied the lower limits of multi-mode RCPSP and have used the concepts of minimum and maximum delays and also renewable and renewable resources in developing the scheduling. Wuliang and Chengen [26] have considered the possibility of executing the project activities in various modes with the end of studying the balance of cost and completion time. They have studied the impact of crashing on scheduling as well. In order to solve the model, they made use of a genetic algorithm whose results were compared with the corresponding results taken from the branch-and-limitation method.

Coelho and Vanhoucke [27], too, addressed the multi-mode resource-constrained project scheduling. With the aid of the SAT software, they initially studied the possible modes and then improved the feasible solutions with a genetic algorithm. In fact, the strongpoint of their study is in the concomitant application of SAT and a meta-heuristic algorithm. Lova et al. [28] used a hybrid genetic algorithm for solving the problem. To display, encode, and decode the solutions, they used the method of listing activities in forward, backward, serial, and parallel generation schemes. Also, they assigned a penalty in the fitness functions for those members of the population that violated the predefined resource amount. Besides, in their proposed method, the fitness function depicts a number between 0 and 1 for the least and most completion durations. Yet, this number exceeds 1 for those members that have surpassed the resource capacity. The final point about this approach is that the criterion of the least normalized resources has been used for selecting the mode of executing an activity in such a way that those modes that enjoy lesser resources are more likely to be selected so as to refrain from generating infeasible members.

Kyriakidis et al. [29] made use of mixed-integer linear programming (MILP) models to formulate single-mode and multi-mode project scheduling problems. They adopted a task-resource network approach which is used in process scheduling problems. Deblaere et al. [30] studied the conditions under which there were the likelihood of failure or disruption, for certain reasons, in the execution of the activity/activities. Accordingly, further to the taboo search method, they analyzed the performance of a number of exact scheduling procedures that are based on reaction to failure in activity execution.

Van Peteghem and Vanhoucke [31], in their proposed model, utilized a genetic algorithm to see if it was possible to simultaneously study the failure/non-failure of the activities. Mori et al. [32] dealt with a non-preemptive general model of multi-mode resource-constrained project scheduling problems where the duration of the activity depends upon the amount of renewable resources. To solve this problem, a genetic algorithm has been suggested and compared with the other existing models in the domain in terms of efficiency level.

To address the makespan minimization in MRCPSPs, Alcaraz et al. [33] innovatively propound novel operators for genetic models and a novel fitness function for the infeasible solutions. Upon comparison with the older models in the field, the efficiency of this model has been proven. Kolisch et al. [34] consider MRCPSP with a non-preemptive structure. There, the activity duration is a discrete function of the allocated renewable resources and the other resources. Presenting a zero and one formulation, they model the studied problem. This reveals how significant and applicable such a formulation is in the production and operations management models. In consideration of the difficulty in solving such problems in a large scale, a meta-heuristic method has been presented in the research for coming up with quality solutions. This method addresses the generation of feasible solutions wherever the problems have limited resources to a great extent. The proposed model initially seeks the feasible solutions via a local search and then moves to a neighborhood search in order to improve the objective function.

De Reyck [35] attends to the problem of scheduling the completion time of the project under generalized preemptive relations and several renewable and non-renewable resources. Kolisch [36] engages in a method to solve project scheduling problems with resource constraints and preemptive relations. In this model, concepts such as resource factors (depicting resources density matrix), problem constraints, and availability of the resources were taken into consideration. Reddy [37] studies the multi-mode, multi-activity RCPSP using PERT network charts for formulating a model. To attain better solutions, they applied an algorithm similar to genetic algorithms (GAs) as well as heuristic methods.

To date, several different methods have been put forth for solving multi-activity MRCPSPs that can be classified under three general categories of “exact”, “heuristic”, and “meta-heuristic”. Of the models that have been solved under the exact method category, one can mention the following scholars. The first method to solve the problem, which is a single and dual linear programming approach, was presented by Słowiński et al. [38]. Talbot [39] and Patterson et al. [40] proposed an enumeration scheme for solving the problem. Sprecher et al. [41] and Hartmann and Drexl [42] made use of branch-and-bound algorithms. Zhu et al. [43], Montoya [44], and Shirzadeh et al. [45] have put forward the methods of branch-and-bound, branch-and-price, and branch-and-cut, which lack efficiency in solving large-sized problems.

From among the models that used heuristic methods, Boctor [46] presented a heuristic for critical path, Kolisch and Drexl [36] utilized a three-phased local search method, Lova et al. [47] devised a multi-path method based on priority-rules.

There are many meta-heuristic methods used in the field from which the following ones have been shortlisted. Słowiński et al. [38] tried a simulated annealing method to solve a multi-objective, multi-resourced project scheduling. Bouleimen and Lecocq [48] and Józefowska et al. [49] applied the simulated annealing method for minimizing the project completion duration in MRCPSPs. Ozdamar [50] presents a genetic algorithm for a multi-activity, multi-mode RCPSP with the aim of bringing down the duration of project implementation to the least. Their initiative lies in proposing a novel method for encoding the chromosomes regarding the problem. Hartmann [51] and Alcaraz et al. [33] propounded a new crossover and mutation genetic algorithm for the same problem and end. Van Peteghem and Vanhoucke [31] presented a genetic algorithm for a RCPSP with and without prioritization among the activities. Of the recent researches in this domain, one can name Van Peteghem [52], Tavana [53], and Cheng [54].

3 Statement of the problem

Project delays are among the most conventional problems in project management. A delay in the project can stem from several factors which can be controlled with a proper management. One of the indices of a project success is the timely implementation of the schedule. At every organization, the capabilities and constraints in performing the project are continually and dynamically undergoing changes. Moreover, the environmental and technological changes that arise in the course of time might contribute to changes in the organizational strategies. Although numerous studies have been conducted to control the delays in the project management, it seems that the major reason behind the delays does not lie in the nature of the projects but in the organizational and managerial capabilities and maturity. Project management is regarded as adopting the necessary knowledge, skills, tools, and techniques for managing the implementation of the activities in order to meet the needs and expectations of the project's client via the materialization of the initiating, planning, executing, controlling, and closing processes.

Upon presenting a combinatorial (hybrid) mathematical model that considers various technologies for executing the activities, this paper attempts to simultaneously optimize the problems of project scheduling and time-and-cost tradeoff. The studied problem is analyzed in consideration of resource constraint. It goes without saying that the efficiency of the technologies at avail has an influential part in project scheduling and the leads and lags in the execution of the activities; hence, it is a must to heed the necessary requirements for identifying the efficiency of the to-be-used technologies prior to designing the model. The need for this research lies in the relationship between the concepts of scheduling, crashing and/or activities deferment. In a mathematical model, such a relationship would create a sort of correlation among the variables of the time-cost tradeoff and those of the scheduling that, in order to clarify them, we firstly need to explain the problem of time-cost tradeoff.

In time-cost tradeoff or crashing, the main question is that the duration of which activity needs to be shortened upon using extra resources. Crashing in CPM network is done to select one (or more) of the activities with the least cost for reducing the project duration. This process continues as long as the project meets the crashing objectives or the crashing cost does not exceed the benefits gained from it.

Another noteworthy point in this paper is considering the duration of the funds engagement in the crashing of the activities which is of great importance especially in the long-run projects. When the initial activities of a project are crashed, the required funds for crashing would be engaged to the end of the project. This is while crashing the finishing activities would entail a lesser duration of funds engagement. The same effect can be seen in connection with the savings attained from delaying the execution of activities as compared to implementing them at their normal time in the course of the project. Hence, the time-value of money would be an influential factor in this regard and this paper seeks to propose a model that considers this factor in the time-and-cost tradeoff problems. Accordingly, the cost of crashing is added to the function of total cost and the savings gained from delaying the activities would be deducted from it so that the DTCTP is accomplished in Program Evaluation Review Technique (PERT) Network charts in a more realistic way incorporating the time-value of money.

As stated earlier, we aim to combine the concepts of DTCTP and RCPSP for our formulation and, here, after giving an account of the former, a description of the latter will follow.

RCPSP, a major research topic for several decades, is known as an NP-hard problem that attempts to minimize the duration of the project completion in consideration of the precedence relations among the activities as well as the availability of the constrained, renewable resources. In many related studies, RCPSP has been extended to multi-mode activities where each activity can be executed at a different duration of time and renewable/non-renewable resource use.

In the extensive, recent researches conducted on the subject of multi-mode resource-constrained project scheduling problems, it is assumed that the activities are implemented under ideal conditions and simply a fixed finishing time is determined for the project. In other words, the project handover time is counted as a point in time.

In this paper, the proposed project scheduling model assumes two types of renewable and non-renewable resources. Non-renewable resources are allocated for the project only once, whereas the renewable ones can be renewed at any given time. Also, there is the likelihood of executing every activity via multiple modes with every mode representing a certain technology for carrying out the activity. Such an assumption would be practically conducive to flexibility in selecting the technologies and decision making.

Nowadays, technology is the golden key for unlocking the competition in the world of business and work, and a must for the economic growth of the organizations and nations. Scholars like Joseph Schumpeter and Robert Solow raised the concept of the need for investing on the application and expansion of technologies.

Modern technologies create more efficient methods for performing the affairs and, thus, introduce newer aspects in the activities of mankind. Accordingly, they bring up the possibility of improving the quality of products and services, boosting productivity, shortening the duration of supplying the new products, and satisfying the open-ended human needs. Implementation of the project activities, technological developments, changes in the methods to plan, execute, and control, as well as evaluation of technical changes each provide an opportunity for promoting the capabilities, competitiveness, and growth. As a result, proper application of technology is regarded as the suitable ground for reaping socio-economic benefits. Today, with the theories that consider the possibility of continuous growth and improvement on the basis of technology development, this need seems dire more than ever before. Therefore, the organization or company's strategies, decisions, and measures for attracting, adapting, and developing technologies form the core of its socio-economic growth process. Provided that an organization can steer this process in a systematic way, it would, undoubtedly, create the ground for its continuous growth and development, and hence, the hope to weather the tumultuous, competitive storm of the modern world. In this study, upon assuming the execution of the project activities using different technologies, we strive to enable the possibility of selecting the best technology to execute under multi-mode conditions.

Figure 1 shows the blending of the three concepts of DTCTP, RCPSP, and technology selection for the activities of a project.

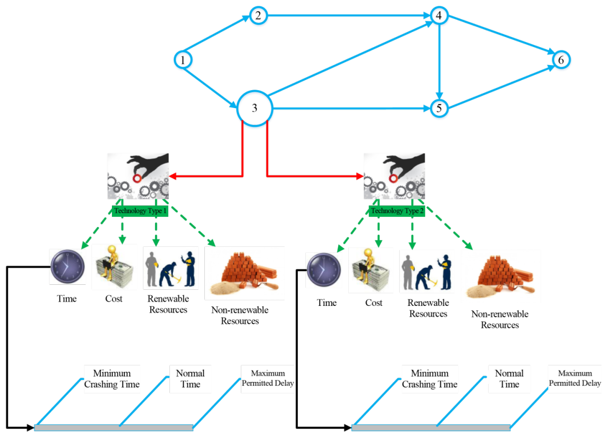

Figure 1: Blended DTCTP, RCPSP, and Technology Selection

4 Proposing a Non-linear Mathematical Model

At the outset, the indices, parameters, and variables used in the mathematical optimization model are presented.

Indices:

$\boldsymbol{i}$**:** activities (i=1,2,3,. . . ,n)

$\boldsymbol{m}$**:** modes (m=1,2,3,. . . ,Mi)

$\boldsymbol{t}$**:** time period (t=1,2,3, . . . , T)

Parameters:

$D_{f(im)}$: Minimum allowed time for executing activity i in mode m

$D_{n(im)}$: Normal time of activity i in mode m

$D_{m(im)}$: Maximum allowed time for executing activity i in mode m

H : Indirect (*overhead*) cost of the project

K_n : Direct cost of the project

C_{im} : Coefficient of crashing cost for activity i in mode m

C'_{im} : Coefficient of thrift attained from delaying activity i in mode m

C_{Max} : Maximum budget at avail

I_0 : Interest rate

r^r_{imk} : Amount of renewable resource k for activity i in mode m

r^n_{iml} : Amount of non-renewable resource l for activity i in mode m

a^r_k : Maximum allowed renewable resource k use

a^n_l : Maximum allowed non-renewable resource l use

Variables:

y_{im} :=1, if the activity i in mode m is crashed, 0, otherwise.

y_{im} : =1, if the activity i in mode m is executed with delay, 0, otherwise.

y''_{im} :=1, if the activity i in mode m is executed in normal time, 0, otherwise.

x_{imt} :=1, if the activity i in mode m started at the time t, 0, otherwise.

d_{im} : The (feasible) scheduled duration for the activity i in mode m

Now, here the assumptions governing the mathematical model are given.

- Both renewable and non-renewable resources are being used.
- The duration time of the activities is determined as fixed.
- The required resources for each activity are fixed.
- There exists the possibility of crashing the activities.
- There exists the possibility of delaying the activities.
- The duration time of the activities can be reduced to integers of time unit. In other words, where the unit of time is "day", reducing the time to decimals would not be possible.
- The multi-mode scheduling project has been taken into consideration.

Using the notations and assumptions above, the following mathematical model is proposed.

$$Min\ Z = \sum_{t=es_{n+1}}^{ls_{n+1}} t.x_{n+1,1,t} \tag{4.1}$$

$$y_{im}.D_{f(im)} \le y_{im}.d_{im} \le y_{im}.\left(D_{n(im)} - \varepsilon\right); \quad i = 1,..,n; m = 1,..,M_i. \tag{4.2}$$

$$y'_{im}.\left(D_{n(im)} + \varepsilon\right) \le y'_{im}.d_{im} \le y'_{im}.D_{m(im)}; \quad i = 1,..,n; m = 1,..,M_i. \tag{4.3}$$

$$y''_{im}.d_{im} = D_{n(im)}.y''_{im}; \quad i = 1,..,n; m = 1,..,M_i. \tag{4.4}$$

$$\sum_{m=1}^{M_i} y_{im} + y'_{im} + y''_{im} = 1; \quad \forall i = 1,..,n. \tag{4.5}$$

$$H\left(\sum_{t=es_{n+1}}^{ls_{n+1}} t.x_{n+1,1,t}\right) + K_n + \sum_{i=1}^{n}\sum_{m=1}^{M_i} C_{im}\left\{\left\{y_{im}.D_{n(im)} - y_{im}.d_{im}\right\} + \left(\sum_{t=es_{n+1}}^{ls_{n+1}} t.y_{im}.x_{n+1,1,t} - \sum_{t=es_{n+1}}^{ls_{n+1}} t.y_{im}.x_{imt}\right) I_0\right\} + \sum_{i=1}^{n}\sum_{m=1}^{M_i} C'_{im}\left\{\left\{y'_{im}.D_{n(im)} - y'_{im}.d_{im}\right\} + \left(\sum_{t=es_{n+1}}^{ls_{n+1}} t.y'_{im}.x_{n+1,1,t} - \sum_{t=es_{n+1}}^{ls_{n+1}} t.y'_{im}.x_{imt}\right) I_0\right\} \le C_{Max}. \tag{4.6}$$

$$d_{im} \le M \sum_{t=1}^{T} x_{imt} \quad i = 1,..,n; m = 1,..,M_i \tag{4.7}$$

$$\sum_{m=1}^{M_i}\sum_{t=es_i}^{ls_i} \left(t + d_{im}\right) x_{imt} \le \sum_{m=1}^{M_j}\sum_{t=es_j}^{ls_j} t.x_{jmt}; \quad \forall (i,j) \in A. \tag{4.8}$$

$$\sum_{m=1}^{M_i}\sum_{t=es_i}^{ls_i} x_{imt} = 1 \quad i = 1,...,n. \tag{4.9}$$

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i} r^r_{imk} \sum_{s=\max(t-D_{m(im)},es_i)}^{\min(t-1,ls_i)} x_{ims} \le a^r_k; \quad k = 1,..,R^k; t = 1,..,T. \tag{4.10}$$

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i} r_{iml}^{n} \sum_{t=es_i}^{ls_i} x_{imt} \leq a_l^n \quad l=1,..,R^n;$$
$$x_{imt}, y_{im}, y'_{im}, y'''_{im} \in \{0,1\};$$
$$\forall i, \forall m, \forall td_{im} \in Z; \forall i, \forall m. \quad (4.11)$$

In the mathematical model above, (4.1) expresses the objective function of the model. As stated, the objective function tends to minimize the total project makespan or the completion time of the project.
Constraint (4.2) is the controlling restriction for crashing the activities in such a way that, if an activity is crashed ($y_{im}=1$), then the lower limit of this crashing would be equal to $D_{f(im)}$. On the other hand, if an activity is crashed, owing to the existence of in the upper bound of constraint (4.2), then ($y_{im}=1$).
Constraint (4.3) is the controlling restriction for delaying the activities in such a way that, if an activity is delayed ($y'_{im}=1$), then the upper limit of this delay would be equal to$D_{m(im)}$. On the other hand, if an activity is delayed, owing to the existence of in the lower bound of constraint (4.3), then ($y'_{im}=1$).
Constraint (4.4) is the controlling restriction for executing an activity over the normal time in such a way that, if an activity is executed over the normal time ($y''_{im}=1$), then the time would be equal to $D_{n(im)}$On the other hand, with regard to constraint (4.4), the implementation of the activity over the time $D_{n(im)}$would definitely be ($y''_{im}=1$).
Constraint (4.5) guarantees that each activity is executed merely in one mode. It also expresses that, on the whole, there exist three modes for the duration of any activity, out of which one mode shall be selected:

- The activity is crashed
- The activity is delayed
- The activity is executed over the normal time.

Constraint (4.6) expresses the total amount of the project's direct and indirect costs, the costs inflicted by crashing the activities, and the savings gained from delaying the activities. The noteworthy point in this constraint is taking the time value of money into account in calculating the costs of crashing and the savings from delaying. All in all, this constraint can be broken down to the following segments, where their aggregate shall stand lower than the budget at hand.

- Costs inflicted by crashing the activities:

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i} C_{im}\Bigg\{\{y_{im}.D_{n(im)} - y_{im}.d_{im}\} + \Bigg(\sum_{t=es_{n+1}}^{ls_{n+1}} t.y_{im}.x_{n+1,1,t} - \sum_{t=es_{n+1}}^{ls_{n+1}} t.y_{im}.x_{imt}\Bigg) I_0\Bigg\} \quad (4.12)$$

- Savings gained from delaying the activities

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i} C'_{im}\Bigg\{\{y'_{im}.D_{n(im)} - y'_{im}.d_{im}\} + \Bigg(\sum_{t=es_{n+1}}^{ls_{n+1}} t.y'_{im}.x_{n+1,1,t} - \sum_{t=es_{n+1}}^{ls_{n+1}} t.y'_{im}.x_{imt}\Bigg) I_0\Bigg\} \quad (4.13)$$

- Direct and indirect costs of the projects

$$H\left(\sum_{t=es_{n+1}}^{ls_{n+1}} t.x_{n+1,1,t}\right) + K_n \quad (4.14)$$

Constraint (4.7) represents that the duration time of executing an activity in a certain mode can bear a numerical value only if that activity has started to be executed in that mode.
Constraint (4.8) has been suggested in order to observe the precedence relations among the activities in such a way that, if activity B precedes activity A, then the starting time of activity A equals the starting time of activity B plus the duration time of executing activity B.
Constraint (4.9) represents that an activity can be, only and in the least, implemented in one of the potential modes. Constraint (4.10) expresses the restrictions in the capacity of the renewable resources. Constraint (4.11) represents the restrictions in the capacity of the non-renewable resources.

5 Linearization of the Mathematical Model

In the mathematical model given in the previous section, a number of the raised constraints are non-linear. This would dramatically add to the intricacies of solving the problem in the model. Therefore, in this section, we attempt to linearize the non-linear constraints through using variable change and other mathematical modelling techniques so that we would eventually have a mathematical model under the classification of Mixed-Integer Linear Programming (MILP). To this end, let us firstly take constraint (4.2) into account wherein the expression $y_{im}.d_{im}$is a non-linear one. For Linearization, we make use of $f_{im} = y_{im}.d_{im}$variable change, in which $f_{im}$is a continuous variable, in a way that, if $y_{im} = 1$, then $f_{im} = d_{im}$, and if $y_{im} = 0$, then $f_{im} = 0$. Consequently, the following constraints replace the constraint (4.2).

$$y_{im}.D_{f(im)} \leq f_{im} \leq y_{im}.\left(D_{n(im)} - \varepsilon\right); \quad i = 1,..,n; m = 1,..,M_i \tag{5.15}$$

$$f_{im} \leq d_{im} + (1 - y_{im})\, M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.16}$$

$$f_{im} \geq d_{im} - (1 - y_{im})\, M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.17}$$

$$f_{im} \leq y_{im}.M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.18}$$

Similarly, for constraints (4.3) and (4.4), too, variable changes of $f'_{im} = y'_{im}.d_{im}$and $f''_{im} = y''_{im}.d_{im}$are used, and the following constraints are introduced for constraints (4.3) and (4.4).

$$y'_{im}.\left(D_{n(im)} + \varepsilon\right) \leq f'_{im} \leq y'_{im}.D_{m(im)}; \quad i = 1,..,n; m = 1,..,M_i \tag{5.19}$$

$$f'_{im} \leq d_{im} + \left(1 - y'_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.20}$$

$$f'_{im} \geq d_{im} - \left(1 - y'_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.21}$$

$$f'_{im} \leq y'_{im}.M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.22}$$

$$f''_{im} = D_{n(im)}.y''_{im}; \quad i = 1,..,n; m = 1,..,M_i \tag{5.23}$$

$$f''_{im} \leq d_{im} + \left(1 - y''_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.24}$$

$$f''_{im} \geq d_{im} - \left(1 - y''_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.25}$$

$$f''_{im} \leq y''_{im}.M; \quad i = 1,..,n; m = 1,..,M_i \tag{5.26}$$

Further, in constraint (4.6), where there are several non-linear expressions, we can apply the above-mentioned variable changes and alternative restrictions on the expressions $y_{im}.d_{im}$and $y'_{im}.d_{im}$in order to linearize them. However, for the first time, there arises a new non-linear expression like $y_{im}.x_{n+1,1,t}$in this constraint. Unlike the previous expressions which comprised the multiplication of a binary variable by a continuous variable, this expression consists of the multiplication of two binary variables. To linearize this expression, the $h_{imt} = y_{im}.x_{n+1,1,t}$variable change is applied. As a binary variable, h_{imt}equals 1 only if both variables of y_{im} and $x_{n+1,1,t}$ are 1, otherwise h_{imt} would be 0. Consequently, the following restrictions are added to the model.

$$h_{imt} \geq y_{im} + x_{n+1,1,t} - 1; \quad i = 1,..,n; m = 1,..,M_i; t = 1,..,T. \tag{5.27}$$

$$h_{imt} \leq y_{im}; \quad i = 1,..,n; m = 1,..,M_i; t = 1,..,T. \tag{5.28}$$

$$h_{imt} \leq x_{n+1,1,t}; \quad i = 1,..,n; m = 1,..,M_i; t = 1,..,T. \tag{5.29}$$

Similarly, we can do the same for linearizing the other existing non-linear expressions in constraint (4.6), i.e. $y_{im}.x_{imt}$, $y'_{im}.x_{n+1,1,t}$, and $y'_{im}.x_{imt}$since all of these expressions are multiplications of two binary variables. In the same vein, upon applying the variable changes of $g_{imt} = y_{im}.x_{imt}$, $h'_{imt} = y'_{im}.x_{n+1,1,t}$, and $g'_{imt} = y'_{im}.x_{imt}$on the expressions, the following restrictions are added to the model.

$$g_{imt} \geq y_{im} + x_{imt} - 1; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T. \tag{5.30}$$

$$g_{imt} \leq y_{im}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T. \tag{5.31}$$

$$g_{imt} \leq x_{imt}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T. \tag{5.32}$$

$$g'_{imt} \geq y'_{im} + x_{imt} - 1; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T. \tag{5.33}$$

$$g'_{imt} \leq y'_{im}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T. \tag{5.34}$$

$$g'_{imt} \leq x_{imt}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T. \tag{5.35}$$

$$h'_{imt} \geq y'_{im} + x_{n+1,1,t} - 1; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T. \tag{5.36}$$

$$h'_{imt} \leq y'_{im}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T. \tag{5.37}$$

$$h'_{imt} \leq x_{n+1,1,t}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T. \tag{5.38}$$

To continue with the linearization process of the model, in constraint (4.8), the multiplication of the continuous variable d_{im}in the binary variable of x_{imt}has resulted in another non-linear expression, which is similar to the formerly linearized expression $y_{im}.d_{im}$. Through the variable change of $B_{imt} = x_{imt}.d_{im}$as well as the replacement of constraint (4.8) with the following restrictions, we wrap up the process of linearizing our proposed model.

$$\sum_{m=1}^{M_i} \sum_{t=es_i}^{ls_i} (t.x_{imt} + B_{imt}) \leq \sum_{m=1}^{M_j} \sum_{t=es_j}^{ls_j} t.x_{jmt}; \quad \forall (i,j) \in A. \tag{5.39}$$

$$B_{imt} \leq d_{im} + (1 - x_{imt}) M; \quad i = 1,..,n; m = 1,..,M_i; t = 1,..,T \tag{5.40}$$

$$B_{imt} \geq d_{im} - (1 - x_{imt}) M; \quad i = 1,..,n; m = 1,..,M_i; t = 1,..,T \tag{5.41}$$

$$B_{imt} \leq x_{imt}.M; \quad i = 1,..,n; m = 1,..,M_i; t = 1,..,T \tag{5.42}$$

In the wake of applying the above modifications in the proposed mathematical model, we come up with a mixed-integer linear programming (MILP) which can be illustrated as follows.

$$Min\ Z = \sum_{t=es_{n+1}}^{ls_{n+1}} t.x_{n+1,1,t}$$

$$y_{im}.D_{f(im)} \leq f_{im} \leq y_{im}.\left(D_{n(im)} - \varepsilon\right); \quad i = 1,..,n; m = 1,..,M_i.$$

$$f_{im} \leq d_{im} + (1 - y_{im}) M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f_{im} \geq d_{im} - (1 - y_{im}) M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f_{im} \leq y_{im}.M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$y'_{im}.\left(D_{n(im)} + \varepsilon\right) \leq f'_{im} \leq y'_{im}.D_{m(im)}; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f'_{im} \leq d_{im} + \left(1 - y'_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f'_{im} \geq d_{im} - \left(1 - y'_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f'_{im} \leq y'_{im}.M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f''_{im} = D_{n(im)}.y''_{im}; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f''_{im} \leq d_{im} + \left(1 - y''_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f''_{im} \geq d_{im} - \left(1 - y''_{im}\right) M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$f''_{im} \leq y''_{im}.M; \quad i = 1,..,n; m = 1,..,M_i.$$

$$\sum_{m=1}^{M_i} y_{im} + y'_{im} + y''_{im} = 1; \qquad i = 1,..,n.$$

$$H\left(\sum_{t=es_{n+1}}^{ls_{n+1}} t.x_{n+1,1,t}\right) + K_n$$
$$+\sum_{i=1}^{n}\sum_{m=1}^{M_i} C_{im}\Bigg\{\{y_{im}.D_{n(im)} - f_{im}\}$$
$$+\left(\sum_{t=es_{n+1}}^{ls_{n+1}} t.h_{imt} - \sum_{t=es_{n+1}}^{ls_{n+1}} t.g_{imt}\right) I_0\Bigg\}$$
$$+\sum_{i=1}^{n}\sum_{m=1}^{M_i} C'_{im}\Bigg\{\{y'_{im}.D_{n(im)} - f'_{im}\}$$
$$+\left(\sum_{t=es_{n+1}}^{ls_{n+1}} t.h'_{imt} - \sum_{t=es_{n+1}}^{ls_{n+1}} t.g'_{imt}\right) I_0\Bigg\} \le C_{Max}$$

$$g_{imt} \ge y_{im} + x_{imt} - 1; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T.$$

$$g_{imt} \le y_{im}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T.$$

$$g_{imt} \le x_{imt}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T.$$

$$h_{imt} \ge y_{im} + x_{n+1,1,t} - 1; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T$$

$$h_{imt} \le y_{im}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T.$$

$$h_{imt} \le x_{n+1,1,t}; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T.$$

$$g'_{imt} \ge y'_{im} + x_{imt} - 1; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T.$$

$$g'_{imt} \le y'_{im}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T.$$

$$g'_{imt} \le x_{imt}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T.$$

$$h'_{imt} \ge y'_{im} + x_{n+1,1,t} - 1; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T.$$

$$h'_{imt} \le y'_{im}; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T.$$

$$h'_{imt} \le x_{n+1,1,t}; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T.$$

$$d_{im} \le M \sum_{t=1}^{T} x_{imt} \quad i = 1,..,n; m = 1,..,M_i$$

$$\sum_{m=1}^{M_i}\sum_{t=es_i}^{ls_i} (t.x_{imt} + B_{imt}) \le \sum_{m=1}^{M_j}\sum_{t=es_j}^{ls_j} t.x_{jmt}; \quad \forall (i,j) \in A$$

$$B_{imt} \le d_{im} + (1 - x_{imt}) M; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T$$

$$B_{imt} \ge d_{im} - (1 - x_{imt}) M; \quad i = 1,..,n; \quad m = 1,..,M_i; t = 1,..,T$$

$$B_{imt} \le x_{imt}.M; \quad i = 1,..,n; m = 1,..,M_i; \quad t = 1,..,T$$

$$\sum_{m=1}^{M_i}\sum_{t=es_i}^{ls_i} x_{imt} = 1 \quad i = 1,..,n$$

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i} r^r_{imk} \sum_{s=\max(t-D_{m(im)},es_i)}^{\min(t-1,ls_i)} x_{ims} \le a^r_k \quad k = 1,..,R^k; t = 1,..,T$$

$$\sum_{i=1}^{n}\sum_{m=1}^{M_i} r^n_{iml} \sum_{t=es_i}^{ls_i} x_{imt} \le a^n_l \quad l = 1,..,R^n$$

$$x_{imt}, y_{im}, y'_{im}, y'''_{im}, h_{imt}, h'_{imt}, g_{imt}, g'_{imt} \in \{0,1\};$$
$$d_{im} \in Z$$
$$f_{im}, f'_{im}, B_{imt} \ge 0$$

6 Numerical Illustration

To illustrate the model, here, a numerical example is given where the information about a sample project will be provided. In this line, Figure 2 displays the precedence relations among the project activities. As shown in Fig. 2, the sample project comprises ten activities, each of which can be executed under various modes.

Table 1 provides information about the implementation of the project activities including maximum crashing level, maximum delay in activity execution, as well as the coefficients for the costs inflicted and the savings gained from crashing and delaying the activities, respectively. Further, a type of renewable resources and a type of non-renewable resources have been considered in this numerical example.

Table 2 gives information about the usage amount of each activity from these resources. As shown, each activity can enjoy a different amount of resource use under different modes of execution. In this project, it shall be noted that activities 1 and 10 are hypothetical ones and, accordingly, the value of the parameters for them has been considered as zero.

The other bits of information needed for the sample project include the direct costs (10000), indirect costs (200), interest rate (0.1), and maximum budget at avail (15000 monetary units).

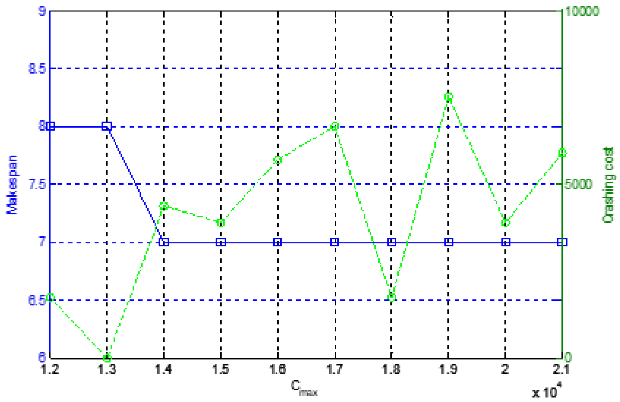

Figure 2: The precedence network of the numerical example

Table 1: Information on crashing and delaying activities and their respective costs and savings coefficients

Activity	*Number of mode*	D_f		D_n		D_m		C		C'	
1	1	0	0	0	0	0	0	0	0	0	0
2	2	3	4	4	6	5	8	1500	750	2525	2562
3	2	1	1	2	2	4	3	1200	1200	2506	2506
4	2	2	3	3	5	5	6	2000	1000	2560	2520
5	2	1	1	2	2	4	3	1450	1450	2547	2547
6	2	1	3	2	4	3	5	1800	1891	2575	2595
7	2	1	1	2	1	2	2	540	670	2560	2520
8	2	1	1	1	2	2	2	430	450	2525	2562
9	2	1	1	1	2	2	2	590	340	2575	2595
10	1	0	0	0	0	0	0	0	0	0	0

Now that the parameters of the model are known, we can attempt to solve the problem. With regard to the proposed optimized model and the follow-up linearization process which was described, the mathematical model has turned into a mixed–integer linear programming (MILP), where the branch and bound method can be used to get an optimal solution. In this study, LINGO software has been adopted for using the branch and bound method. Table 3 shows the results achieved for the variables of the model.

Table 2: Information on the coefficients of renewable and non-renewable resource use

Activity	r^r		r^n	
1	0	0	0	0
2	2	1	3	1
3	1	1	0	0
4	3	1	4	2
5	1	1	0	0
6	2	1	3	2
7	2	1	2	1
8	2	2	2	2
9	2	2	2	2
10	0	0	0	0
	$a^r = 10$		$a^n = 25$	

Since the activities 1 and 10 are hypothetical ones, information about them is being ignored.

Table 3: Results from branch and bound method for the numerical example

Activity	Mode	Duration	Crashing	Delay	Start time
2	1	4	No	No	1
3	2	2	No	No	1
4	1	3	No	No	1
5	2	1	Yes	No	5
6	1	1	Yes	No	5
7	2	1	No	No	6
8	1	2	No	Yes	4
9	1	1	No	No	6

As shown in the table, activities 5and 6 have undergone crashing and activity 8 has been delayed. Other activities of the project have been carried out within the normal span of time. In the end, the objective function of the sample project is equal to 10 which is the optimum makespan of the project.

7 Sensitivity Analysis

Here, we try to examine the impact of changing the parameter of maximum budget on the value of the objective function and the values of the costs inflicted by crashing as well as the savings from delaying the activities. In the first column of the following table, the decreasing values of the maximum budget at hand are given and the corresponding changes in the objective function and the triple factors in constraint 6 have been reported. These triple factors consist of direct and indirect costs of the project, the costs of crashing, and the savings gained from delaying the activities. How to calculate these factors is given in the relations 12, 13 and 14 and their linear from is given in the mixed-integer linear mathematical optimization.

As depicted in figure 3, with an increase in the maximum available budget, the value of the objective function or the completion time of the project is naturally ameliorated with the decrease. Concurrent with this drop, the direct and indirect costs, too, witness a downward trend which is readily justifiable in light of the higher speed in the completion time of the project.

In Figure 4, with a hike in the maximum budget at avail, in addition to a diminishing makespan, an upward trend in the costs of crashing the activities is noticed. Except for when the maximum available budget amounts to 6000 monetary units, such an upward trend can be seen in Fig.4. This hike, of course, has to do with the multiple properties of an optimum solution in such a way that we would witness the same makespan even if no activity is being crashed under these conditions. Apart from this exclusive case, an increment in the costs of crashing together with a decline in the completion time of the project is reasonable, meaning that the higher the available budget is, the higher the spending on crashing for reducing the makespan of the project would be.

Table 4: Results of sensitivity analysis

Maximum Available Budget (C_{Max})	Objective Function (makespan)	Direct and Indirect Costs	Crashing Costs	Saving from Delaying
21000	7	11400	5908	0
20000	7	11400	3900	0
19000	7	11400	7534	0
18000	7	11400	1740	0
17000	7	11400	6680	3062.4
16000	7	11400	5708	1767.5
15000	7	11400	3900	1767.5
14000	7	11400	4364	1767.5
13000	8	11600	0	0
12000	8	11600	1740	2519.3

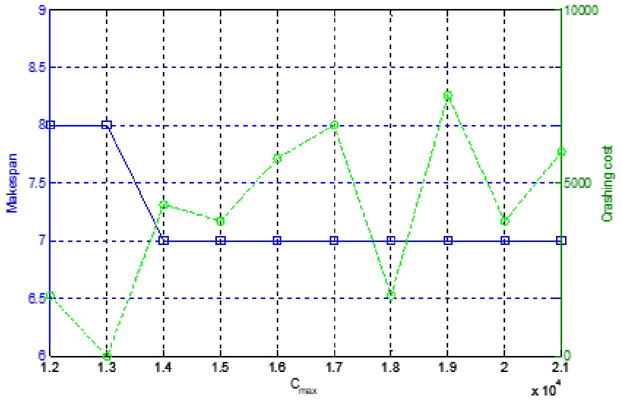

Figure 3: The relationship between the makespan and the direct and indirect costs of the project

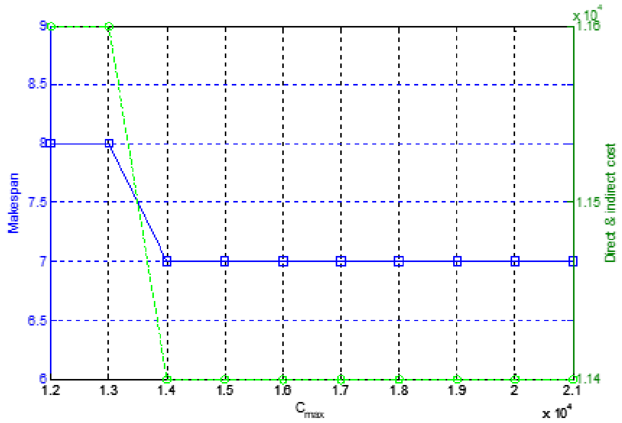

Figure 4: The relationship between the makespan and the costs of crashing the activities

In figure 5, the reduction in the value of the objective function is coupled with a decline in the savings from delaying the activities. As depicted in Figure, the saving gained from delaying the activities has decreased from a value of 2519.3 in the budget level of 12000 to zero in the budget level of 21,000. The reason is that, with a low budget in hand, the mathematical optimization model postpones some of the activities in order to compensate for the direct and indirect costs and to secure them through the savings gained. Obviously, delaying the execution of an activity entails a lengthier makespan. From another standpoint, the higher the budget at avail is, the lesser the need for saving and delaying the activities arises. Consequently, the amount of the savings diminishe.

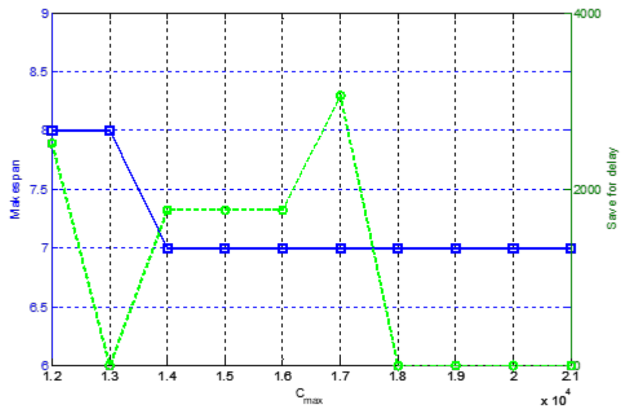

Figure 5: The relationship between the makespan and the savings gained from delaying the execution of the activities

8 Conclusion

Over the recent years, numerous researches have been conducted on resource constrained project scheduling largely focusing on a certain resource type, i.e. renewable ones. Rarely has a study considered both renewable and non-renewable resources. In the project scheduling model under study in this research, both types of resources have been assumed. Non-renewable resources are allocated for the whole project only once, whereas the renewable resources can be renewed at any given time span. Also, in the proposed model, the activities are defined in multiple modes. As stated in the literature review, MRCPSP has been studied by very few researchers. In such problems, there is a possibility for executing every activity via multiple modes. Every mode is a combination of duration and resource use, and, usually, the shorter the duration of a mode is, the more the need for resource use arises. Such an assumption practically creates flexibility in selecting the execution modes and making better decisions. One of the prime objectives in project scheduling is to carry out an activity over a favorable span of time using the least possible resource so as to be able to complete the project in a minimum makespan. Making use of various modes in implementing the activities is one of the strategies that would enable us to achieve such a goal. Adopting several modes, the model becomes flexible in selecting the most appropriate path in better performing the scheduling of the project with a higher quality, besides saving in the costs of implementing the project.

As a general rule, the more a model enjoys flexible assumptions, the higher the quality of the solutions would be. It was shown in this paper that we can make the problem conditions closer to the real world via adopting certain assumptions such as the time value of money (TVM) which is of great significance to the managers and decision-makers in deciding about the issue of time- and-cost tradeoff. This is so because a change in the value of money disrupts the scheduling of mega-projects whose implementation entails lengthy durations of time. Considering a non-linear function for the costs, too, is another noteworthy point in this vein.

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