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Research Article

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Fuzzy Stochastic Network Data Envelopment Analysis Models: An Approach to Banking Industry

H. Nasseri *, **M. Ahmadi khatir**
Department of Mathematics, University of Mazandaran, Babolsar, Iran.

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Abstract
The traditional DEA treats DMUs as black boxes and calculates their efficiencies by considering their initial inputs and their final outputs. As a result, some intermediate measures are lost in the process of changing the inputs to outputs. This paper organizes a three-stage DEA models by taking into account undesirable output with fuzzy stochastic data. To achieve this aim, an extended probability approach is applied on a reform of three-stage DEA models. A case study in the banking industry is presented to exhibit the efficacy of the procedures and demonstrate the applicability of the proposed model.

Keywords: Data envelopment analysis, Network DEA, Fuzzy random variable, Undesirable output.

* Corresponding author: Email: nasseri@umz.ac.ir

1. Introduction

Data Envelopment Analysis (DEA), initially introduced by Charnes et al. (1978), requires crisp input and output data, whereas real-life decisions are usually made in the state of uncertainty. In such situations, we often face the uncertain programming in DEA model, where in the data could possess randomness and fuzziness. On the other hand, in a production system, the input usually goes through several processes before it becomes the output. Traditional DEA models treat the system as awhile unit, disregarding the interactions of the processes in the system when calculating the efficiency. These two progresses in network and uncertainty DEA models need to handle together [1].

This paper solves a case of network DEA model in which the input and output data are assumed to be characterized by fuzzy random variables. We accomplish this task by converting the non-linear models formulated in these approaches to linear programming models. A case study for Bank Melli Iran (BMI) is presented to illustrate the features and applicability of the proposed DEA models.

The first paper discussing in network DEA was prepared by Charnes et al. (1986), which was founded in the army recruitment. Several models for measuring the efficiency of network systems have been proposed. The studies in this domain are classified in three groups: The first is an independent approach which recognizes the existence of the processes in the system, yet the efficiencies of the system and all processes are calculated independently. The second is a connected approach, in that interactions between processes are taken into account in calculating the system efficiency. The third is a relational approach; its underlying concept is that some kind of mathematical relationship exists between the system efficiency and the component process efficiencies [2]. Halkos et al. (2014) provided a unified classification of two-stage DEA model [3]. Kwon and Lee (2015) proposes a new approach to model a two-stage production process supported by using data from large U.S. banks [4]. Liu et al. (2015) proposed a two-stage DEA models with undesirable input–intermediate-outputs [5]. Carillo and Jorge (2016) give a new model for ranking alternatives that uses common weight DEA under a multi objective optimization approach. A serious problem that has been discussed frequently in the literature has been the lack of discrimination in DEA applications [6]. Nasseri et al. (2014) suggested a new ranking method based on the extension of PPS by virtual units named as relative similar units [7]. Wu et al. (2016) introduced a cross-efficiency approach based on pareto optimality which can be generated byvonly a common set of weights [8]. Hanafizade et al. (2014) used neuoral network DEA for measuring the efficiency of mutual funds [9]. Sahoo et al. (2016) discussed return to scale and most productivity scale size in DEA with negative data [10].

Hatami-Marbini et al. (2011a) classified the fuzzy DEA methods in the literature into five general groups, the tolerance approach [11-13], the α-level based approach, the fuzzy ranking approach [14,15], the possibility approach [16], and the fuzzy arithmetic approach [17]. Among these approaches, the α-level based approach is probably the most relevant fuzzy DEA model in the literature. Nevertheless, the possibility approach seems to be more efficient in hybrid uncertainty, especially with twofold fuzzy-random environment. Saati et al. (2002) proposed a fuzzy CCR model as a possibilistic programming problem by applying an alternative α-cut approach [18]. Puri and Yadav (2014) applied the suggested methodology by Saati et al. (2002) to solve fuzzy DEA model with undesirable outputs [19]. Khanjani et al. (2014a) proposed fuzzy free disposal hull models under possibility and credibility measures [20]. Khodabakhshi et al. (2016) proposed a fuzzy DEA model with an optimistic and pessimistic performance and congestion analysis in fuzzy DEA [21]. Hosseini et al. (2013) determined the return to scale situations in the presence of non-discretionary factors in DEA with interval data [22].

In random environments, the crips inputs and outputs of traditional DEA become random variable. Land et al. (1994) extended the chance constrained DEA model [23]. Olesen and Petersen (1995) developed the chance constrained programming model for efficiency evaluation using a piecewise linear envelopment of confidence regions for observed stochastic multiple-input multiple-output combinations in DEA [24]. Cooper et al. (1998) utilized joint chance constraints to extend the concept of stochastic efficiency [25]. Cooper et al. (2004) used chance-constrained programming for extending congestion DEA models [26]. Wu et al. (2013) proposed a stochastic DEA model by considering undesirable outputs with weak disposability [27]. Zhou et al. (2017) proposed a stochastic centralized two-stage network DEA model converted to linear models under some assumptions [28]. A review of stochastic DEA models can be found in a recent work by Olesen and Petersen (2015) [29].

In many cases, the stochastic programming techniques are not as suitable to cope with the optimization problems. If we impose the farfetched stochastic programming as the approach to the problem with randomness and fuzziness simultaneously, then we have to ignore the fuzziness to make an uncertainty reduction. Kwakernaak (1978, 1979) introduced the concept of fuzzy random variable [30,31], and then this idea enhanced by a number of researchers in the literature [32-34]. Qin and Liu (2010) developed a fuzzy random DEA (FRDEA) model where randomness and fuzziness exist simultaneously [35]. The authors characterized the fuzzy random data with known possibility and probability distributions. Tavana et al. (2012) also introduced three different fuzzy stochastic DEA models consisting of probability-possibility, probability-necessity and probability-credibility constraints in which input and output data entailed fuzziness and randomness at the same time [36]. Also, Tavana et al. (2013) provided a chance-constrained DEA model with random fuzzy inputs and outputs with Poisson, uniform and normal distributions [37]. After that, Tavana et al. (2014) proposed DEA models with birandom input-output [38]. Khanjani et al. (2014b) proposed fuzzy rough DEA models based on the expected value and possibility approaches [39]. Nasseri et al. (2016) proposed a fuzzy stochastic DEA model. They formulated a linear and feasible model with an extension of normal distribution to deal with fuzzy random data [40]. Ebrahimnejad et al. (2018) solved dual DEA problems with fuzzy stochastic data. This approach overcomes to the shortcomings of linearity and normal efficiency score relative to corresponding approaches [41].

This study tries to incorporate fuzzy random inputs and outputs in network model with undesirable output. We apply extended probability measure to deal with the fuzzy random environments. The achievement of the present study is threefold: (1) to formulate a new version of network DEA model equipped undesirable output (2) to formulate a linear model for solving fuzzy stochastic two-stage DEA model, (3) to demonstrate the applicability of the proposed model using a case study for the banking industry.

The reminder of the paper is organized as follows: Next section presents some approaches in two-stage model and proposes our proposed network model equipped with fuzzy stochastic input and output data. In section 3, the results of the case conducted for the banking industry to evaluate the efficiency of 13 branches. Section 4 presents our conclusions and future research directions.

2. The proposed model

2.1. Two- stage model

Consider two-stage process illustrated in Figure1. We have n DMUs that each DMU_j (j=1,2,…,n) has m inputs $X_j = (x_{1j}, x_{2j}, ..., x_{mj})$ and D outputs $Z_j = (z_{1j}, z_{2j}, ..., z_{Dj})$ to the first stage. These D outputs known as intermediate measure then are consumed to the second stage. The outputs from the second stage are $Y_j = (y_{1j}, y_{2j}, ..., y_{rj})$. Chen and Zhu (2004) developed an efficiency model that identified the efficient frontier of a two-stage production process linked by intermediate measures. They used a set of firms in the banking industry to illustrate how the new model could be utilized. Model (1) is the two-stage model proposed by Chen and Zhu.

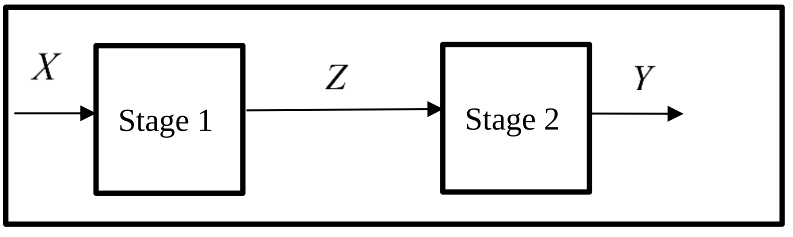

Figure 1. A two-stage DEA system

$$
\begin{aligned}
&\max \quad w_1\alpha - w_2\beta \\
&s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1)\\
&(\text{Stage}1) \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le \alpha x_{io},\ i = 1,2,...,m \qquad (1.1) \\
&\sum_{j=1}^{n} \lambda_j z_{dj} \ge \tilde{z}_{do},\ d = 1,2,...,D \qquad (1.2) \\
&\lambda_j \ge 0,\ j = 1,2,...,n \\
&(\text{Stage}\,2) \\
&\sum_{j=1}^{n} \mu_j z_{dj} \le \tilde{z}_{do},\ d = 1,2,...,D \qquad (1.3) \\
&\sum_{j=1}^{n} \mu_j y_{rj} \ge \beta y_{ro},\ r = 1,2,...,s \qquad (1.4) \\
&\mu_j \ge 0,\ j = 1,2,...,n
\end{aligned}
$$

Where α and β are the efficiency scores corresponding to Stage 1 and Stage 2, respectively. In addition, z_{dj} are the intermediary inputs which are outputs of Stage 1 and inputs of Stage 2 and the values of $\tilde{z}_{do}$ are unknown. Moreover, w_1 and w_2 are the weights reflecting the total preference over the two stages. The values of w_1 and w_2 will be equal when two stages 1 and 2 have the same importance, and they add up to 1. In this approach, DMUs that achieve efficiency score 1 in both stages are considered efficient.

Kao (2009) proposed a relational approach to model network systems. The underlying assumption is that the virtual multiplier associated with the same factor should be the same no matter whether it is the output of one process or the input of another. This approach requires that the aggregated output be less than or equal to the aggregated input for all processes in addition to the usual requirement for the system. A special case of the series system is the one in which all processes, except the first, are not allowed to utilize exogenous inputs and all processes, except the last, are not allowed to produce exogenous outputs [42]. Kao and Hwang (2008) have showed that, in this case, the system efficiency is the product of process efficiencies [43]. Chen et al. (2009) have showed that the model of Chen and Zhu (2004) is equivalent to Kao-Hwang’s model under constant returns to scale. The following, we adopt the last assumption to construct proposed network model [44,45].

2.2. Three-stage system

Let us consider the open-system depicted in Figure 2 and use Kao and Hwang (2008) approach to present the mathematical model (3) for this system as follows:

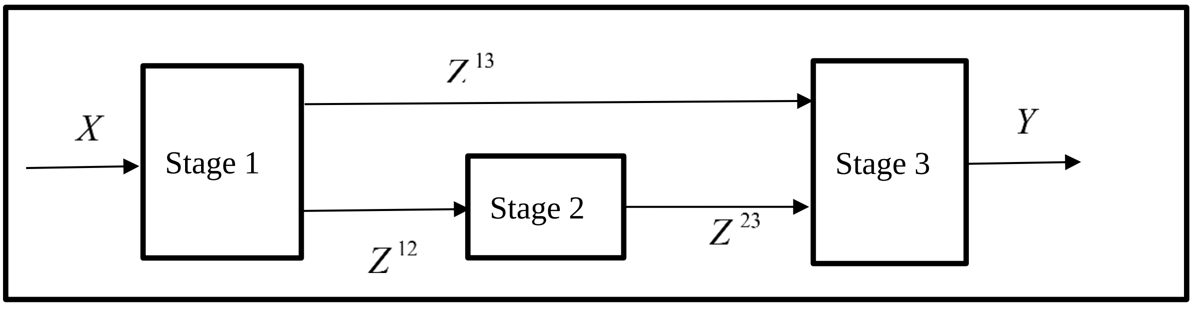

Figure 2. Network system of three Stages.

The constraint set (3.1) correspond to the system inputs, X, and final output, Y, which are the constraints for the conventional envelopment-form DEA model. The constraint set (3.2) correspond to intermediate products.

$$
\begin{aligned}
& \min \quad \theta \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io},\ i = 1,2,\dots,I \\
& \sum_{j=1}^{n} \lambda_j^3 y_{rj} \ge y_{ro},\ r = 1,2,\dots,R \qquad (3.1) \\
& \sum_{j=1}^{n} \lambda_j^1 z_{ej}^{13} - \sum_{j=1}^{n} \lambda_j^3 z_{ej}^{13} \ge 0,\ e = 1,2,\dots,E \\
& \sum_{j=1}^{n} \lambda_j^1 z_{sj}^{12} - \sum_{j=1}^{n} \lambda_j^2 z_{sj}^{12} \ge 0,\ s = 1,2,\dots,S \\
& \sum_{j=1}^{n} \lambda_j^1 z_{kj}^{23} - \sum_{j=1}^{n} \lambda_j^3 z_{kj}^{23} \ge 0, k = 1,2,\dots,K \qquad (3.2) \\
& \lambda_j^1,\ \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned}
$$

2.3. Fuzzy Stochastic model

The aim of this section is to equip proposed model (3) for evaluating the efficiencies of DMUs with fuzzy stochastic (intermediate) inputs and fuzzy stochastic (intermediate) outputs.

To this end, consider n DMUs, each unit consumes fuzzy stochastic inputs, denoted by $\tilde{\tilde{X}}_j = \left(\tilde{X}_j, X_j^{\alpha}, X_j^{\beta}\right)_{LR}$ and intermediate measure vectors $\tilde{\tilde{Z}}_j = \left(\tilde{Z}_j, Z_j^{\alpha}, Z_j^{\beta}\right)_{LR}$ to the first stage, and produces fuzzy stochastic outputs, denoted by $\tilde{\tilde{Y}}_j = \left(\tilde{Y}_j^g, Y_j^{g,\alpha}, Y_j^{g,\beta}\right)_{LR}$ as desirable outputs and $\tilde{\tilde{Y}}_j^b = (\tilde{y}_j^b, y_j^{b,\alpha}, y_j^{b,\beta})_{LR}$ as undesirable outputs. Let, each component of $\tilde{X}_j, \tilde{Z}_j, \tilde{Y}_j^g$, and $\tilde{Y}_j^b$ be normally distributed by $\tilde{X}_j \sim N\left(X_j, \sigma_j\right), \tilde{Z}_{dj} \sim N\left(Z_j, \sigma_j\right)$, $,\tilde{Y}_j^g \sim N\left(Y_j^g, \sigma_j^g\right)$ and $\tilde{Y}_j^b \sim N\left(Y_j^b, \sigma_j^b\right)$, respectively.

The chance-constrained programming (CCP) developed by Cooper et al. (2002) is a stochastic optimization approach suitable for solving optimization problems with uncertain parameters. Building on CCP and possibility theory as the principal techniques, the following $\overline{\text{Pr}} - \text{CCR}$ model is proposed:

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4) \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 x_{ij} \le \theta x_{io}\Big) \ge \gamma,\ i = 1,2,\dots,I \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^3 y_{rj}^g \ge y_{ro}^g\Big) \ge \gamma\ ,\ r = 1,2,\dots,R \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^3 y_{r'j}^b \le y_{r'o}^b\Big) \ge \gamma,\ r' = 1,2,\dots,R' \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{ej}^{13} - \sum_{j=1}^{n} \lambda_j^3 z_{ej}^{13} \ge 0\Big) \ge \gamma,\ e = 1,2,\dots,E \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{sj}^{12} - \sum_{j=1}^{n} \lambda_j^2 z_{sj}^{12} \ge 0\Big) \ge \gamma,\ s = 1,2,\dots,S \\
& \overline{\Pr}\Big(\sum_{j=1}^{n} \lambda_j^1 z_{kj}^{23} - \sum_{j=1}^{n} \lambda_j^3 z_{kj}^{23} \ge 0\Big) \ge \gamma, k = 1,2,\dots,K \\
& \lambda_j^1,\ \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned}
$$

Where $\gamma \in [0,1]$ is the predetermined thresholds defined by the DM and $\overline{\Pr}\,[\cdot]$ in Model (4) denote the fuzzy stochastic measure.

To get a linear form of solving model (4), we consider the following substitutions:

$$
\begin{aligned}
& \hat{x}_{ij} = \lambda_j^1 x_{ij},\ \hat{y}_{rj}^g = \lambda_j^3 y_{rj}^g,\ \hat{y}_{r'j}^b = \lambda_j^3 y_{r'j}^b \\
& \hat{z}_{ej}^{113} = \lambda_j^1 z_{ej}^{13},\ \hat{z}_{ej}^{313} = \lambda_j^3 z_{ej}^{13} \qquad\qquad (5) \\
& \hat{z}_{sj}^{112} = \lambda_j^1 z_{sj}^{12},\ \hat{z}_{sj}^{212} = \lambda_j^2 z_{sj}^{12} \\
& \hat{z}_{kj}^{223} = \lambda_j^2 z_{kj}^{23},\ \hat{z}_{kj}^{323} = \lambda_j^3 z_{kj}^{23}
\end{aligned}
$$

By substituting these variables, Model (4) reduces to the following model:

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (6) \\
& \sum_{j=1}^{n} \hat{x}_{ij} \le \theta x_{io}\,,\ i = 1,2,\dots,I \\
& \sum_{j=1}^{n} \hat{y}_{rj}^{g} \ge y_{ro}^{g}\ ,\ r = 1,2,\dots,R \\
& \sum_{j=1}^{n} y_{r'j}^{b} \le y_{r'o}^{b}\ ,\ r' = 1,2,\dots,R' \\
& \sum_{j=1}^{n} \hat{z}_{ej}^{113} - \sum_{j=1}^{n} \hat{z}_{ej}^{313} \ge 0,\ e = 1,2,\dots,E \\
& \sum_{j=1}^{n} \hat{z}_{sj}^{112} - \sum_{j=1}^{n} \hat{z}_{sj}^{212} \ge 0\ ,\ s = 1,2,\dots,S \\
& \sum_{j=1}^{n} \hat{z}_{kj}^{223} - \sum_{j=1}^{n} \hat{z}_{kj}^{323} \ge 0, k = 1,2,\dots,K \\
& \overline{\Pr}(\lambda_j^1 x_{ij} \le \hat{x}_{ij} \le \lambda_j^1 x_{ij}) \ge \gamma\ ;\ \overline{\Pr}(\lambda_j^3 z_{ej}^{13} \le \hat{z}_{ej}^{313} \le \lambda_j^3 z_{ej}^{13}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^3 y_{rj}^{g} \le \hat{y}_{rj}^{g} \le \lambda_j^3 y_{rj}^{g}) \ge \gamma\ ;\ \overline{\Pr}(\lambda_j^1 z_{sj}^{12} \le \hat{z}_{sj}^{112} \le \lambda_j^1 z_{sj}^{12}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^3 y_{r'j}^{b} \le \hat{y}_{r'j}^{b} \le \lambda_j^3 y_{r'j}^{b}) \ge \gamma\ ;\overline{\Pr}(\lambda_j^2 z_{sj}^{12} \le \hat{z}_{sj}^{212} \le \lambda_j^2 z_{sj}^{12}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^1 z_{ej}^{13} \le \hat{z}_{ej}^{113} \le \lambda_j^1 z_{ej}^{13}) \ge \gamma ;\overline{\Pr}(\lambda_j^2 z_{kj}^{23} \le \hat{z}_{kj}^{223} \le \lambda_j^2 z_{kj}^{23}) \ge \gamma \\
& \overline{\Pr}(\lambda_j^3 z_{kj}^{23} \le \hat{z}_{kj}^{323} \le \lambda_j^3 z_{kj}^{23}) \ge \gamma \\
& \lambda_j^1, \lambda_j^2, \lambda_j^3 \ge 0
\end{aligned}
$$

In order to solve the $\overline{\Pr}-$constrained programming model (6), we utilize Theorem1 and Definition 1.

Theorem 1 (Nasseri et al., 2016)**.** If $\tilde{\tilde{X}} \sim \overline{\mathrm{N}}(\bar{\mu},\sigma)$ with $\bar{\mu} = (\mu,\alpha,\beta)$ then

a**.** $\overline{\Pr}(\tilde{\tilde{X}} \le \mathrm{r}) > \gamma\ iff\ \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \ge (\Phi^{-1}(\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

b. $\overline{\Pr}(\tilde{\tilde{X}} \ge \mathrm{r}) > \gamma\ iff\ \dfrac{\mathrm{r}-\bar{\mu}}{\sigma} \le (\Phi^{-1}(1-\gamma), \dfrac{\alpha+\beta}{\sigma}, \dfrac{\alpha+\beta}{\sigma})$

And so

$$\overline{\Pr}(r \le \tilde{\tilde{X}} \le \mathrm{r}) > \gamma\ iff\ (\Phi^{-1}(\gamma), \frac{\alpha+\beta}{\sigma}, \frac{\alpha+\beta}{\sigma}) \le \frac{\mathrm{r}-\bar{\mu}}{\sigma} \le (\Phi^{-1}(1-\gamma), \frac{\alpha+\beta}{\sigma}, \frac{\alpha+\beta}{\sigma})$$

Definition 1 (Tanaka et al.,1984). Let $\tilde{a} = (m_1, \alpha_1, \beta_1)_{LR}$ *and* $\tilde{b} = (m_2, \alpha_2, \beta_2)_{LR}$ be two L-R fuzzy numbers and h a real number, $h \in [0,1)$. Then,if and only if, $(\tilde{a} \ge \tilde{b})_h$ $(h \in [0,1])$ the following two statement hold: $k \in [h,1]$

$$m_1 - L^{-1}(k)\alpha_1 \geq m_2 - L^{-1}(k)\alpha_2 \quad \forall k \in [h,1]$$
$$m_1 + R^{-1}(k)\beta_1 \geq m_2 + R^{-1}(k)\beta_2 \quad \forall k \in [h,1]$$

It is notable that the fuzzy ranking method adopted in this study is Tanaka (1984)'s approach at such threshold δ . Hence, we have:

$$
\begin{aligned}
& E_o(\delta,\gamma) = \min \quad \theta \\
& s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (7) \\
& \sum_{j=1}^{n} \hat{x}_{ij} \leq \theta(x_{io} + R^{-1}(\delta)x_{io}^{\beta} + \sigma_{ij}\Phi_{1-\gamma}^{-1}),\ i = 1,2,...,I \\
& \sum_{j=1}^{n} \hat{y}_{rj}^{g} \geq (y_{ro}^{g} - L^{-1}(\delta)y_{ro}^{g,\alpha} - \sigma_{rj}\Phi_{1-\gamma}^{-1}),\ r = 1,2,...,R \\
& \sum_{j=1}^{n} \hat{y}_{r'j}^{b} \leq (y_{r'o}^{g} + R^{-1}(\delta)y_{ro}^{g,\alpha} + \sigma_{r'j}\Phi_{1-\gamma}^{-1}),\ r' = 1,2,...,R' \\
& \sum_{j=1}^{n} \hat{z}_{ej}^{113} - \sum_{j=1}^{n} \hat{z}_{ej}^{313} \geq 0,\ e = 1,2,...,E \\
& \sum_{j=1}^{n} \hat{z}_{sj}^{112} - \sum_{j=1}^{n} \hat{z}_{sj}^{212} \geq 0,\ s = 1,2,...,S \\
& \sum_{j=1}^{n} \hat{z}_{kj}^{223} - \sum_{j=1}^{n} \hat{z}_{kj}^{323} \geq 0, k = 1,2,...,K \\
& \lambda_j^1(x_{ij} - L^{-1}(\delta)x_{ij}^{\alpha} - \sigma_{ij}\Phi_{1-\gamma}^{-1}) \leq \hat{x}_{ij} \leq \lambda_j^1(x_{ij} + R^{-1}(\delta)x_{ij}^{\beta} + \sigma_{ij}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^3(y_{rj} - L^{-1}(\delta)y_{rj}^{g,\alpha} - \sigma_{rj}\Phi_{1-\gamma}^{-1}) \leq \hat{y}_{rj}^{g} \leq \lambda_j^3(y_{rj}^{g} + R^{-1}(\delta)y_{rj}^{g,\beta} + \sigma_{rj}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^3(y_{r'j}^{b} - L^{-1}(\delta)y_{r'j}^{b,\alpha} - \sigma_{rj}\Phi_{1-\gamma}^{-1}) \leq \hat{y}_{r'j}^{b} \leq (\lambda_j^3 y_{r'j}^{b} + R^{-1}(\delta)y_{r'j}^{b,\beta} + \sigma_{r'j}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^1(z_{ej}^{13} - L^{-1}(\delta)z_{ej}^{13,\alpha} - \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \leq \hat{z}_{ej}^{113} \leq \lambda_j^1(z_{ej}^{13} + R^{-1}(\delta)z_{ej}^{13,\beta} + \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^3(z_{ej}^{13} - L^{-1}(\delta)z_{ej}^{13,\alpha} - \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \leq \hat{z}_{ej}^{313} \leq \lambda_j^3(z_{ej}^{13} + R^{-1}(\delta)z_{ej}^{13,\beta} + \sigma_{ej}^{13}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^1(z_{sj}^{12} - L^{-1}(\delta)z_{sj}^{12,\alpha} - \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \leq \hat{z}_{sj}^{112} \leq \lambda_j^1(z_{sj}^{12} + R^{-1}(\delta)z_{sj}^{12,\beta} + \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^2(z_{sj}^{12} - L^{-1}(\delta)z_{sj}^{12,\alpha} - \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \leq \hat{z}_{sj}^{212} \leq \lambda_j^2(z_{sj}^{12} + R^{-1}(\delta)z_{sj}^{12,\beta} + \sigma_{sj}^{12}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^2(z_{kj}^{23} - L^{-1}(\delta)z_{kj}^{23,\alpha} - \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \leq \hat{z}_{kj}^{223} \leq \lambda_j^2(z_{kj}^{23} + R^{-1}(\delta)z_{kj}^{23,\beta} + \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^3(z_{kj}^{23} - L^{-1}(\delta)z_{kj}^{23,\alpha} - \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \leq \hat{z}_{kj}^{323} \leq \lambda_j^3(z_{kj}^{23} + R^{-1}(\delta)z_{kj}^{23,\beta} + \sigma_{kj}^{23}\Phi_{1-\gamma}^{-1}) \\
& \lambda_j^1,\ \lambda_j^2, \lambda_j^3 \geq 0
\end{aligned}
$$

The above model is obviously a linear model. This model is an extension of Nasseri et al (2016)'s model to the proposed network CCR model when undesirable outputs are considered.

Definition 2. For the given level δ and γ , we define $E_o^T(\delta,\gamma) = E_o\ (\delta, \frac{\gamma}{2})$ as efficiency score of DMU_o in fuzzy random DEA Model.

3. Case Study

As the largest Iranian business bank, Bank Melli (BMI) Iran has a comprehensive network of over 3,300 branches and 37.000 employees in Iran. Countrywide coverage in Iran, service quality and an experienced multi-lingual staff are important factors of their success. In this section, we apply the proposed approach in this study to some commercial bank branches in Mazandaran province. Here the data sources consist of the reports of 13 branches. The inputs for the first stage are personnel score, cost, location and branch facilities with intermediate output service and total of deposits (TDs) (of current, short duration and long duration accounts). The second stage's input is TDs and loan are as intermediate output. Finally, in third stage with service and TDs as intermediate input and recovered loan as desirable outputs, and non-performing loans (delay in delivering loans and other facilities) as undesirable output. However, there always exist some degrees of uncertainty in the data which can be represented by fuzzy stochastic numbers. In banks, uncertainty occurs due to the difference between the actual data and the available data. For example, let actual cost for a bank be 1.688 billion (IRR) [2] and the possible data amounts of cost available at different places be 1.4 or 2 or 2.2 billion. Then the difference between actual data and possible data results into the occurrence of uncertainty in the data which further may affect. Therefore, in the present study, we fuzzify the data as TFNs. The collected crisp data in table 1 are considered as the mean of TFNs. The left and right spreads are calculated by 0.1% of mean value. On the other hand, the inputs and outputs are supposed as random variables. By using goodness of fit tests, normal distributions have been fit on the random variables. The corresponding expected value is the observed inputs (outputs) data and the standard deviation is one. Hence, each DMU is considered as a fuzzy variable with randomized mean. This fuzzy random input–intermediate-output data of each bank are available in Table 1. Each input, intermediate and output data are denoted by $(N(m,\sigma),\alpha)$, where m is the observed data as the expected value in normal distribution with $\sigma=1$, and α is the left and also the right spread.

Table 1. The fuzzy random input and output data[3]

DMU	Personnel Score	Cost	Branch Facilities	Location	Services	Interest Income	Loans	User fee income	Deposit	Non-performing loans
1	17,014,781	354,133	28,347	715	0.42	796,832	3,648,031	95,045	5,981,048	301,779
2	27,485,016	559,261	45,203	4,636	0.89	1,577,995	10,754,785	137,941	12,617,962	516,519
3	14,297,944	287,066	17,889	879	0.47	879,802	4,317,806	46,845	6,323,772	175,162
4	16,252,095	384,871	28,001	2,087	0.53	1,116,566	4,522,011	111,225	7,950,451	415,303
5	16,342,530	424,974	19,630	1,292	0.49	1,210,623	6,278,297	91,316	8,851,770	312,750
6	16,687,868	377,789	23,508	1,164	0.45	836,644	3,491,101	159,909	5,992,871	658,208
7	15,297,826	551,015	19,167	550	0.23	633,899	2,193,687	77,779	5,099,454	726,122
8	14,765,164	249,487	20,307	913	0.21	627,658	2,524,526	45,740	5,116,146	126,437
9	16,933,047	366,048	25,208	2,671	0.37	1,003,786	3,991,867	188,924	7,588,686	284,899
10	10,583,687	245,834	7,146	480	0.05	589,456	2,953,722	119,975	5,414,472	460,950
11	8,183,284	210,688	13,514	417	0.26	530,537	3,511,138	54,141	5,559,826	179,385
12	5,439,440	131,682	7,881	347	0.11	345,072	2,044,424	18,125	2,952,701	130,017
13	6,008,503	278,451	2,605	199	0.17	845,116	3,235,783	232,503	3,428,137	1,108,174

[2] This paper takes Iranian Rial as currency rate.

[3] The prices are in million Rials.

Four different (γ, δ)-threshold levels of (γ =0.9, δ =0.7), (γ =0.9, δ =0.4), (γ =0.7, δ =0.7), and (γ =0.5, δ =0.5) are considered based on the DMs' previous performance evaluation studies for model (7) proposed in this study.

In Table 2, we present the efficiency values associated with the bank branches for four specified threshold levels of (γ =0.9, δ =0.7), (γ =0.9, δ =0.4), (γ =0.7, δ =0.7), (γ =0.5, δ =0.5), and finally overall efficiency. The results of model (7) for probability-possibility levels are reported in Table 2. As shown in this table, DMU 13 is (γ, δ)-efficient at four given levels. Generally from Table 2, we can see the efficiency scores of the DMUs increase when the level δ decreases from (γ =0.9, δ =0.7) to (γ =0.9, δ =0.4) and the level γ decreases from (γ =0.9, δ =0.7) to (γ =0.7, δ =0.7) in model (7). Finally, Table 2 presents the average efficiency scores and the final rankings of the 13 bank branches. As shown in Figure 3 the proposed model (7) is stable in different levels. However, the average efficiency can be an appropriate overall index to indicate the efficiency variations.

Table 2. The fuzzy random efficiency scores and final ranking

	(γ=0.9, δ=0.7)	(γ=0.9, δ=0.4)	(γ=0.7, δ=0.7)	Overall efficiency	Ranking
1	0.353	0.353	0.353	0.353	11
2	0.446	0.446	0.446	0.446	5
3	0.597	0.585	0.597	0.593	2
4	0.456	0.455	0.456	0.456	4
5	0.514	0.513	0.514	0.514	3
6	0.34	0.339	0.34	0.340	12
7	0.271	0.27	0.272	0.271	13
8	0.406	0.405	0.406	0.406	9
9	0.437	0.436	0.437	0.437	7
10	0.441	0.44	0.441	0.441	6
11	0.398	0.397	0.398	0.398	10
12	0.42	0.419	0.42	0.420	8
13	1.000	1.000	1.000	1.000	1

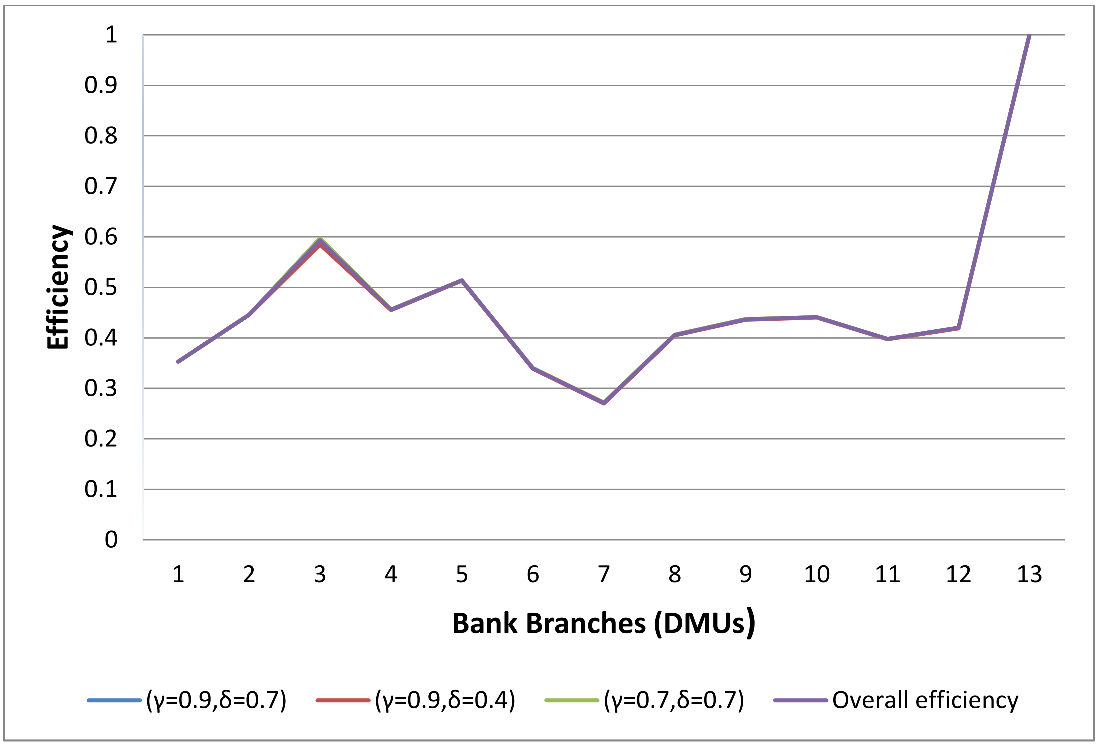

Figure 3. The final result

4. Conclusions

This paper formulated a DEA model handled three-stage process and undesirable outputs in fuzzy random environment. Actually, the extended model depicts the influence of the presence of fuzziness and randomness in the data over the efficiency values. To do this, we have firstly incorporated an undesirable output in three-stage DEA model. The resulting model converted into a new model with some variable substitutions. Then, to solve the uncertainty part of model, we applied the $\overline{\mathrm{Pr}}(.)$ measure led to a linear model. In addition, a case study for banking industry (BMI) is utilized to analyze the performance of some bank branches in Iran. In this way, a suitable network model is fitted with BMI on fuzzy stochastic environment.

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