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Research Article

Window Network Data Envelopment Analysis: An Application to Investment Companies

P. Peykani *, E. Mohammadi †‡

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Abstract

Network data envelopment analysis (NDEA) is one of the most important branches of data envelopment analysis (DEA) that is employed in order to performance measurement of decision making units (DMUs) with internal or network structures. In this study, the window network data envelopment analysis (WNDEA) model will be proposed, that is capable to be used in the presence of panel data. Additionally, the proposed model is applied to evaluate the dynamic efficiency of 5 investment companies in Tehran stock exchange during the period from 2013 to 2017. Experimental results show that the proposed window network DEA model is effective and employing this model increases the reliability of the results.

Keywords : Network Data Envelopment Analysis; Two-Stage Structure; Window Analysis; Dynamic Efficiency; Investment Company.

1 Introduction

Window analysis, as presented by Charnes et al. [5] is a non-parametric panel approach that can be used to handle cross-sectional and time-varying data investigate the dynamic efficiency. Applying the window data envelopment analysis (WDEA) can help the decision maker (DM) to examine the dynamic changes of the efficiency of each decision making unit (DMU) comprehensively over time. Under the WDEA approach, the performance of a DMU in a period can be contrasted with the performance of other DMUs as well as with its own performance in other periods. In other words, by employing this method, DM can assess the efficiency of different DMUs in different periods through a sequence of overlapping windows. Significantly, the number of DMUs is increased thus using this approach enhances the discriminating power by increasing the number of decision making units when a limited number of DMUs is available. With respect to these features and advantages, WDEA is used by many researchers. In following, some practical studies that apply the window DEA approach to dynamic performance assessment of DMUs are introduced. Webb [24] used window DEA model for measuring the relative efficiency levels of large UK retail banks in the period 1982-1995. Yang and Chang [26] employed DEA window analysis technique for efficiency measurement of Taiwans

*Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran.
†Corresponding author. e_ mohammadi@iust.ac.ir, Tel:+98(21)73225075.
‡Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran.

integrated telecommunication firms over the period 2001–2005. Since there were only three firms, window data envelopment analysis approach was utilized by researchers to increase the number of decision-making units so that the discriminating power can be increased. Pulina et al. [21] examined the relationship between size and efficiency of hotels across all of the 20 regions in Italy applying a window DEA approach. Salem Al-Eraqi et al. [22] applied DEA window analysis in order to investigate the efficiency of 22 cargo seaports situated in the regions of East Africa and Middle East based on the panel data for the period from 2000 to 2005. Hemmasi et al. [11] evaluated the performance of Iranian wood panels industry using window DEA approach based on a free oriented slack-based measure (SBM) model. Pjevevi et al. [20] utilized DEA window analysis to measure the efficiency of ports and to investigate the possibility of changes in the port efficiency over time. Wang and Zhang [23] applied window DEA model to determine the energy and environmental efficiency of 29 Chinas administrative regions in the period 2000-2008. Wu et al. [25] used super-efficiency DEA and window analysis approaches to dynamically evaluate circular economy efficiency of 30 regions in China during the period of 2005-2010. Arefrad and Alipoor [2] assessed the performance of Guilan Refah bank branches using window data envelopment analysis for the period from 2011 to 2013. Al-Refaie et al. [1] estimated the efficiencies of blowing machines in plastics industry using window data envelopment analysis. Ohe and Peypoch [15] measured the efficiency of Japanese ryokans applying the window DEA covering the period 2005-2012. Jia and Yuan [12] used DEA window analysis model for measuring the operational efficiencies of multi-branched public hospitals in China. Flokou et al. [9] evaluated the efficiency of the public hospital sectors in Greece for the period from 2009 to 2013 applying window DEA. Chen et al. [7] applied window data envelopment analysis for measuring the energy efficiency of Chinese Yangtze River Deltas 15 cities in the period 2009–2013. Halkos and Polemis [10] integrated radial and non-radial efficiency measurements in a window data envelopment analysis framework for estimating the efficiency of the power generation sector in the USA states. One of the drawbacks of classical window DEA models is the neglect of internal or linking activities. In other words, the DMU is considered as black-box system and the operations and interrelations of the processes within the system are neglected. For eliminating this issue, the window network DEA approach must be used instead of window DEA approach. Because, network DEA models can measure the efficiencies of system and process at the same time, and derive mathematical relationships between them, based on which the most effective way to improve the efficiency of a DMU can be identified. The rest of this paper is organized as follows. The modeling of network data envelopment analysis for two-stage process with added inputs to the second stage will be explained in Section 2. Then, by applying the window analysis technique, the window network data envelopment analysis model will be proposed in Section 3. The proposed window two-stage DEA model in this study will be implemented for performance assessment of an investment company in Section 4. Finally, the conclusions of this research are given in Section 5.

2 Two-Stage Network Data Envelopment Analysis

Network DMUs have various types of structure such as basic two-stage, general two-stage, series, parallel, mixed, hierarchical, and dynamic. It should be noted that among the structures that have been mentioned, the two-stage structure has been widely discussed in the NDEA literature [13]. Accordingly, in this study, window network DEA model will proposed based on extended two-stage structure presented in Fig. 1. As can be seen in Fig. 1, the network structure

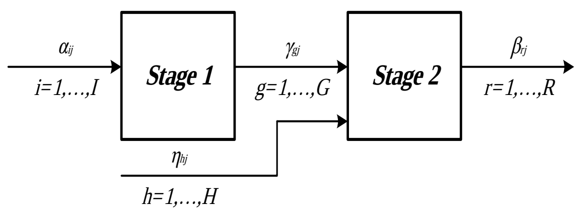

Figure 1: Extended Two-Stage Process

is a two-stage process with added inputs to the second stage, where there is a set of r_i homogenous DMU$_j (j = 1, \cdots, n)$ that each DMU has I inputs $\alpha_{ij} (i = 1, \cdots, I)$ in the first stage, G intermediate variables $\gamma_{gj} (g = 1, \cdots, G)$ that linking first stage and second stage, H additional inputs $\eta_{hj} (h = 1, \cdots, H)$ in the second stage and finally R outputs $\beta_{rj} (r = 1, \cdots, R)$ in the second stage.

Note that the NDEA models, like the classic DEA models could be proposed based on different return to scale (RTS) including constant return to scale (CRS) and variable return to scale (VRS) assumptions that presented by Charnes et al. [6] and Banker et al. [3], respectively. Since, the VRS is a realistic assumption in different applciation and real word problems, the modeling of NDEA in this section and subsequently modeling of WNDEA in next section are presented under VRS assumption.

Table 1: The Value of Parameters in WNDEA Model

Parameters	Value
Number of DMUs	5
Number of Periods	5
Width of Window	2
Number of Windows	4

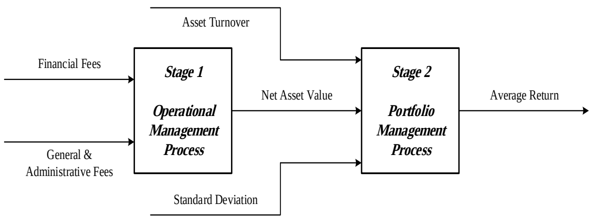

Figure 2: Extended Two-Stage Process

Now, after providing the necessary preliminaries, modeling the network data envelopment analysis based on additive efficiency decomposition approach that presented by Chen et al. [8], will be discussed. Acording to the BCC model that intruduced by Banker et al. [3], the efficiency scores based on varibele return to sclae assumtion for DMU under investigation in the stages 1 and 2 can be estimated by the following Models (2.1) and (2.2), respectively:

$$\begin{aligned} \Theta_p^1 = \text{Max } & \frac{\sum_{g=1}^{G} w_g \gamma_{gp} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ip}}, \\ \text{S. t. } & \frac{\sum_{g=1}^{G} w_g \gamma_{gj} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ij}} \leq 1, \quad \forall j, \\ & v_i, w_g \geq 0, \quad \forall i, g. \end{aligned} \tag{2.1}$$

and

$$\begin{aligned} \Theta_p^2 = \text{Max } & \frac{\sum_{r=1}^{R} u_r \beta_{rp} + \Psi_p^2}{\sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp}}, \\ \text{S. t. } & \frac{\sum_{r=1}^{R} u_r \beta_{rj} + \Psi_p^2}{\sum_{g=1}^{G} w_g \gamma_{gj} + \sum_{h=1}^{H} k_h \eta_{hj}} \leq 1, \quad \forall j, \\ & w_i, k_h, u_r \geq 0, \quad \forall g, h, r. \end{aligned} \tag{2.2}$$

Based upon the idea of Chen et al. [8], the overall efficiency of the two-stage process with added inputs to the second stage will be defined as Eq. (2.3):

$$\begin{aligned} \Theta_p &= \lambda_1 \Theta_p^1 + \lambda_2 \Theta_p^2 \\ &= \lambda_1 \left(\frac{\sum_{g=1}^{G} w_g \gamma_{gp} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ip}} \right) \\ &\quad + \lambda_2 \left(\frac{\sum_{r=1}^{R} u_r \beta_{rp} + \Psi_p^2}{\sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp}} \right) \end{aligned} \tag{2.3}$$

Note that in Eq. (2.3), λ_1 and λ_2 are user-specified weights such that $\lambda_1 + \lambda_2 = 1$. In other words, λ_1 and λ_2 are the relative importance of the performances of first stage and second stage, respectively, to the overall performance of the decision-making unit. Accordingly, the overall efficiency of the process is calculated by solving the Model (2.4) as follows:

$$\begin{aligned} \Theta_p = & \text{Max } \lambda_1 \left(\frac{\sum_{g=1}^{G} w_g \gamma_{gp} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ip}} \right) \\ & + \lambda_2 \left(\frac{\sum_{r=1}^{R} u_r \beta_{rp} + \Psi_p^2}{\sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp}} \right), \\ \text{S.t. } & \frac{\sum_{g=1}^{G} w_g \gamma_{gj} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ij}} \leq 1, \quad \forall j, \\ & \frac{\sum_{r=1}^{R} u_r \beta_{rj} + \Psi_p^2}{\sum_{g=1}^{G} w_g \gamma_{gj} + \sum_{h=1}^{H} k_h \eta_{hj}} \quad \forall j, \\ & v_i, w_g, k_h, u_r \geq 0, \quad \forall i, g, h, r. \end{aligned} \tag{2.4}$$

Table 2: The Results of Window Network DEA Model - Overall

ICs	Windows	2013	2014	2015	2016	2017	Average
5*IC 01	Window 1	1	1				1
	Window 2		1	1			1
	Window 3			1	1		1
	Window 4				1	1	1
	Average	1	1	1	1	1	1
5*IC 02	Window 1	1	1				1
	Window 2		1	1			1
	Window 3			1	1		1
	Window 4				1	0.99290	0.99645
	Average	1	1	1	1	0.99290	0.99858
5*IC 03	Window 1	0.99690	0.83923				0.91807
	Window 2		0.82803	1			0.91402
	Window 3			1	0.59596		0.79798
	Window 4				0.68208	0.75637	0.71923
	Average	0.99690	0.83363	1	0.63902	0.75637	0.84518
5*IC 04	Window 1	1	1				1
	Window 2		1	0.71294			0.85647
	Window 3			0.64944	0.50507		0.57725
	Window 4				0.59205	0.59205	0.59205
	Average	1	1	0.68119	0.54856	0.59205	0.76436
5*IC 05	Window 1	1	1				1
	Window 2		1	0.84627			0.92314
	Window 3			0.87239	0.87877		0.87558
	Window 4				0.83742	1	0.91871
	Average	1	1	0.85933	0.85809	1	0.94348

As it can be seen in Model (2.4), this model cannot be turned into a linear program (LP) by applying the usual Charnes and Cooper [4]. For eliminating this issue, Chen et al. [8] suggested and as Eq. (2.5) Eq. (2.6), respectively:

$$\lambda_1 = \frac{\sum_{i=1}^{I} v_i \alpha_{ip}}{\sum_{i=1}^{I} v_i \alpha_{ip} + \sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp}} \tag{2.5}$$

$$\lambda_1 = \frac{\sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp}}{\sum_{i=1}^{I} v_i \alpha_{ip} + \sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp}} \tag{2.6}$$

Thus, by utilizing the above equations, Model (2.4) will be converted to Model (2.7) as follows:

$$\begin{aligned}
\Theta_p = &\text{Max} \ \frac{\sum_{g=1}^{G} w_g \gamma_{gp} + \Psi_p^1 + \sum_{r=1}^{R} u_r \beta_{rp} + \Psi_p^2}{\sum_{i=1}^{R} v_i \alpha_{ip} + \sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp}} \\
\text{S.t.} \ & \frac{\sum_{g=1}^{G} w_g \gamma_{gj} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ij}} \le 1, \quad \forall j, \\
& \frac{\sum_{r=1}^{R} u_r \beta_{rj} + \Psi_p^2}{\sum_{g=1}^{G} w_g \gamma_{gj} + \sum_{h=1}^{H} k_h \eta_{hj}} \quad \forall j, \\
& v_i, w_g, k_h, u_r \ge 0, \quad \forall i, g, h, r.
\end{aligned} \tag{2.7}$$

Now, by applying Charnes and Cooper [4] transformation, Model (2.7) is equivalent to Model (2.8):

$$\begin{aligned}
\Theta_p = &\text{Max} \sum_{g=1}^{G} w_g \gamma_{gp} + \Psi_p^1 + \sum_{r=1}^{R} u_r \beta_{rp} + \Psi_p^2, \\
\text{S.t.} \ & \sum_{i=1}^{R} v_i \alpha_{ip} + \sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{hp} = 1, \\
& \sum_{g=1}^{G} w_g \gamma_{gj} - \sum_{i=1}^{R} v_i \alpha_{ij} + \Psi_p^1 \le 0 \quad \forall j, \\
& \sum_{r=1}^{R} u_r \beta_{rj} - \sum_{g=1}^{G} w_g \gamma_{gj}
\end{aligned} \tag{2.8}$$

Table 3: The Results of Window Network DEA Model - Stage 1

ICs	Windows	2013	2014	2015	2016	2017	Average
5*IC 01	Window 1	1	1				1
	Window 2		1	1			1
	Window 3			1	1		1
	Window 4				1	1	1
	Average	1	1	1	1	1	1
5*IC 02	Window 1	1	1				1
	Window 2		1	1			1
	Window 3			1	1		1
	Window 4				1	0.99290	0.99645
	Average	1	1	1	1	0.99290	0.99858
5*IC 03	Window 1	0.99690	0.98337				0.99014
	Window 2		1	1			1
	Window 3			1	0.59596		0.79798
	Window 4				0.94198	0.75637	0.84917
	Average	0.99690	0.99168	1.00000	0.76897	0.75637	0.90278
5*IC 04	Window 1	1	1				1
	Window 2		1	0.71294			0.85647
	Window 3			0.74897	0.50507		0.62702
	Window 4				0.86928	0.86928	0.86928
	Average	1	1	0.73096	0.68717	0.86928	0.85748
5*IC 05	Window 1	1	1				1
	Window 2		1	1			1
	Window 3			0.87239	0.87877		0.87558
	Window 4				1.00000	1	1.00000
	Average	1	1	0.93619	0.93938	1	0.97512

$$-\sum_{h=1}^{H} k_h \eta_{hj} + \Psi_p^2 \leq 0 \quad \forall j,$$
$$v_i, w_g, k_h, u_r \geq 0 \quad \forall i, g, h, r.$$

It should be noted that the optimal multipliers that are solved from Model (2.8) may not be unique. As a result, the decomposition of the overall efficiency defined in Eq. (2.3) would not be unique. Kao and Hwang [14] suggested an approach to find a set of multipliers which produces the maximum efficiency score for stage 1 (or stage 2) while maintaining the overall efficiency score. By assuming that the efficiency of the stage 1 is more important for the decision maker (DM), Θ_p^1 will be estimated by solving Model (2.9) while calculating the Θ_p by Model (2.8).

$$\Theta_p^1 = \text{Max} \; \frac{\sum_{g=1}^{G} w_g \gamma_{gp} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ip}} \tag{2.9}$$

$$\text{S.t.} \; \frac{\sum_{g=1}^{G} w_g \gamma_{gj} + \Psi_p^1}{\sum_{i=1}^{I} v_i \alpha_{ij}} \leq 1, \quad \forall j,$$
$$\frac{\sum_{r=1}^{R} u_r \beta_{rj} + \Psi_p^2}{\sum_{g=1}^{G} w_g \gamma_{gj} + \sum_{h=1}^{H} k_h \eta_{kj}} \leq 1, \quad \forall j,$$
$$\frac{\sum_{g=1}^{G} w_g \gamma_{gj} + \Psi_p^1 + \sum_{r=1}^{R} u_r \beta_{rp} + \Psi_p^2}{\sum_{i=1}^{I} v_i \alpha_{ip} + \sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{h=1}^{H} k_h \eta_{kp}} = \Theta_p^*,$$
$$v_i, w_g, k_h, u_r \geq 0 \quad \forall i, g, h, r.$$

Since Model (2.9) is a linear fractional program, using the transformation of Charnes and Cooper [4], this model will be equivalent to Model (2.10):

$$\Theta_p^1 = \text{Max} \sum_{g=1}^{G} w_g \gamma_{gp} + \Psi_p^1,$$
$$\text{S.t.} \sum_{i=1}^{I} v_i \alpha_{ip} = 1, \tag{2.10}$$

Table 4: The Results of Window Network DEA Model - Stage 2

ICs	Windows	2013	2014	2015	2016	2017	Average
5*IC 01	Window 1	1	1				1
	Window 2		1	1			1
	Window 3			1	1		1
	Window 4				1	1	1
	Average	1	1	1	1	1	1
5*IC 02	Window 1	1	1				1
	Window 2		1	1			1
	Window 3			1	0.64790		0.82395
	Window 4				0.45024	0.50000	0.47512
	Average	1	1	1	0.54907	0.50000	0.80981
5*IC 03	Window 1	0.99690	0.75693				0.87692
	Window 2		0.78612	1			0.89306
	Window 3			1	0.59596		0.79798
	Window 4				0.14049	0.75637	0.44843
	Average	0.99690	0.77153	1	0.36822	0.75637	0.77861
5*IC 04	Window 1	1	1				1
	Window 2		1	0.71294			0.85647
	Window 3			0.47368	0.50507		0.48937
	Window 4				0.08573	0.08573	0.08573
	Average	1	1	0.59331	0.29540	0.08573	0.59489
5*IC 05	Window 1	1	1				1
	Window 2		1	0.75042			0.87521
	Window 3			0.87239	0.87877		0.87558
	Window 4				0.83405	1	0.91702
	Average	1	1	0.81140	0.85641	1	0.93356

Table 5: The Average Efficiency Score and Ranking of Investment Companies

2*ICs	Overall		Stage 1		Stage 2	
	Average Efficiency	Rank	Average Efficiency	Rank	Average Efficiency	Rank
IC 01	1	1	1	1	1	1
IC 02	0.99858	2	0.99858	2	0.80981	3
IC 03	0.84518	4	0.90278	4	0.77861	4
IC 04	0.76436	5	0.85748	5	0.59489	5
IC 05	0.94348	3	0.97512	3	0.93356	2

$$\sum_{g=1}^{G} w_g \gamma_{gj} - \sum_{i=1}^{I} v_i \alpha_{ij} + \Psi_p^1 \leq 0 \quad \forall j,$$

$$\sum_{r=1}^{R} u_r \beta_{rj} - \sum_{g=1}^{G} w_g \gamma_{gj}$$

$$- \sum_{h=1}^{H} k_h \eta_{hj} + \Psi_p^2 \leq 0, \quad \forall j,$$

$$\sum_{g=1}^{G} w_g \gamma_{gp} + \sum_{r=1}^{R} u_r \beta_{rp} - \Theta_p^* \Big(\sum_{g=1}^{G} w_g \gamma_{gp}$$

$$+ \sum_{h=1}^{H} k_h \eta_{hp} \Big) + \Psi_p^1 + \Psi_p^2 = \Theta_p^*,$$

$$v_i, w_g, k_h, u_r \geq 0 \quad \forall i, g, h, r.$$

After calculating Θ_p^{1*} using the Model (2.10), the efficiency score of the stage 2 is obtained from Eq. (2.11):

$$\Theta_p^{2*} = \frac{\theta_p^* - \lambda_1^* \Theta_p^{1*}}{\lambda_2^*} \tag{2.11}$$

Alternatively, if the efficiency of the stage 2 is more important for the DM, Θ_p^2 will be estimated by solving the Model (2.12) while calculating the Θ_p^* by Model (2.8).

$$\begin{aligned}
\Theta_p^2 =&\text{Max } \frac{\sum_{r=1}^{R} u_r\beta_{rp} + \Psi_p^2}{\sum_{g=1}^{G} w_g\gamma_{gp} + \sum_{h=1}^{H} k_h\eta_{hp}} \\
\text{S.t. } & \frac{\sum_{g=1}^{G} w_g\gamma_{gj} + \Psi_p^1}{\sum_{i=1}^{I} v_i\alpha_{ij}} \le 1, \quad \forall j, \\
& \frac{\sum_{r=1}^{R} u_r\beta_{rj} + \Psi_p^2}{\sum_{g=1}^{G} w_g\gamma_{gj} + \sum_{h=1}^{H} k_h\eta_{kj}} \le 1, \quad \forall j, \\
& \frac{\sum_{g=1}^{G} w_g\gamma_{gp} + \Psi_p^1 + \sum_{r=1}^{R} u_r\beta_{rp} + \Psi_p^2}{\sum_{i=1}^{I} v_i\alpha_{ip} + \sum_{g=1}^{G} w_g\gamma_{gp} + \sum_{h=1}^{H} k_h\eta_{kp}} = \Theta_p^*, \\
& v_i, w_g, k_h, u_r \ge 0 \quad \forall i, g, h, r.
\end{aligned} \tag{2.12}$$

Like the previous fractional models in this section, by employing the Charnes and Cooper [4] transformation, Model (2.12) will be equivalent to Model (2.13):

$$\begin{aligned}
\Theta_p^2 =&\text{Max } \sum_{r=1}^{R} u_r\beta_{rp} + \Psi_p^2, \\
\text{S.t.} & \sum_{g=1}^{G} w_g\gamma_{gp} + \sum_{h=1}^{H} k_h\eta_{hp} = 1, \\
& \sum_{g=1}^{G} w_g\gamma_{gj} - \sum_{i=1}^{I} v_i\alpha_{ij} + \Psi_p^1 \le 0 \quad \forall j, \\
& \sum_{r=1}^{R} u_r\beta_{rj} - \sum_{g=1}^{G} w_g\gamma_{gj} \\
& \quad - \sum_{h=1}^{H} k_h\eta_{hj} + \Psi_p^2 \le 0, \quad \forall j, \\
& \sum_{g=1}^{G} w_g\gamma_{gp} + \sum_{r=1}^{R} u_r\beta_{rp} - \Theta_p^* \left(\sum_{i=1}^{I} v_i\alpha_{ip} \right) \\
& \quad + \Psi_p^1 + \Psi_p^2 = \Theta_p^*, \\
& v_i, w_g, k_h, u_r \ge 0 \quad \forall i, g, h, r.
\end{aligned} \tag{2.13}$$

Finally, after Θ_p^{2*} is calculated from the Model (2.13), the efficiency score of the stage 1 is obtained from Eq. (2.14):

$$\Theta_p^{1*} = \frac{\theta_p^* - \lambda_2^*\Theta_p^{2*}}{\lambda_1^*} \tag{2.14}$$

It should be noted that the two-stage data envelopment analysis models presented in this section are input-oriented. The window network data envelopment analysis model for a two-stage process with added inputs to the second stage under VRS assumption will be proposed in the next section.

3 Window Network Data Envelopment Analysis

The combination of window analysis approach and DEA models is a very applicable and useful methodology to investigate the dynamic changes of the efficiency of each DMU comprehensively, both horizontally and vertically. The goal of this section is to propose window network DEA model for dynamic performance measurement of network DMUs that is capable to be used in the presence of panel data for performance appraisal of DMUs with network structure. In order to propose WNDEA model, consider an extended two-stage process with added inputs to the second stage as depicted in Fig. 1, as well as the indices, parameters and variables that introduced in previous section. Note that, in window analysis methodology, the same DMU in different period of time are considered as entirely different DMUs and moving average approach is used to choose different reference sets in order to measure the relative efficiency of each DMU.

Accordingly, consider a set of r_i DMUs with two-stage structure in $T(t = 1, \cdots, T)$ period of time. Let indices of qz denote the window start at the time point of q and the width of window is $z(1 \le z \le T - q)$, Λ_{qz} is the set of DMUs that exists in window with characteristics of qz. The window two-stage DEA model for measuring the overall efficiency of the process is proposed as Model (3.15):

$$\begin{aligned}
\Theta_{pqz} =&\text{Max } \sum_{g=1}^{G} w_g\gamma_{gpqz} + \Psi_{pqz}^1 + \sum_{r=1}^{R} u_r\beta_{rpqz} + \Psi_{pqz}^2, \\
\text{S.t. } & \sum_{i=1}^{R} v_i\alpha_{ipqz} + \sum_{g=1}^{G} w_g\gamma_{gpqz} + \sum_{h=1}^{H} k_h\eta_{hpqz} = 1, \\
& \sum_{g=1}^{G} w_g\gamma_{gjt} - \sum_{i=1}^{I} v_i\alpha_{ijt} + \Psi_{pqz}^1 \le 0 \quad \forall j, t \in \Lambda_{qz} \\
& \sum_{r=1}^{R} u_r\beta_{rjt} - \sum_{g=1}^{G} w_g\gamma_{gjt}
\end{aligned} \tag{3.15}$$

$$- \sum_{h=1}^{H} k_h \eta_{hjt} + \Psi_p^2 \leq 0 \quad \forall j, t \in \Lambda_{qz}$$
$$v_i, w_g, k_h, u_r \geq 0 \quad \forall i, g, h, r.$$

As the previous section, if first sub process is assumed to be more important, $\Theta_1(pqz)$ will be calculated by solving the Model (3.16) while measuring the $\Theta^*(pqz)$ by Model (3.15).

$$\begin{aligned}
\Theta^1_{pqz} =& \text{Max} \sum_{g=1}^{G} w_g \gamma_{gpqz} + \Psi^1_{pqz}, \\
& \text{S.t.} \sum_{i=1}^{I} v_i \alpha_{ipqz} = 1, \\
& \sum_{g=1}^{G} w_g \gamma_{gjt} - \sum_{i=1}^{I} v_i \alpha_{ijt} \\
& + \Psi^1_{pqz} \leq 0 \quad \forall j, t \in \Lambda_{qz}, \\
& \sum_{r=1}^{R} u_r \beta_{rjt} - \sum_{g=1}^{G} w_g \gamma_{gjt} \\
& - \sum_{h=1}^{H} k_h \eta_{hjt} + \Psi^2_{pqz} \leq 0, \forall j, t \in \Lambda_{qz}, \\
& \sum_{g=1}^{G} w_g \gamma_{gpqz} + \sum_{r=1}^{R} u_r \beta_{rpqz} \\
& - \Theta^*_{pqz} \left(\sum_{g=1}^{G} w_g \gamma_{gpqz} + \sum_{h=1}^{H} k_h \eta_{hpqz} \right) \\
& + \Psi^1_{pqz} + \Psi^2_{pqz} = \Theta^*_{pqz}, \\
& v_i, w_g, k_h, u_r \geq 0 \quad \forall i, g, h, r.
\end{aligned} \tag{3.16}$$

And the efficiency of the second sub process is calculated by Eq. (3.17):

$$\Theta^{2*}_{pqz} = \frac{\theta^*_{pqz} - \lambda^*_1 \Theta^{1*}_{pqz}}{\lambda^*_2} \tag{3.17}$$

In a similar manner, if second sub process is assumed to be more important, Θ^2_{pqz} will be estimated by solving the Model (3.18) while measuring the Θ^*_{pqz} by Model (3.15).

$$\begin{aligned}
\Theta^2_{pqz} =& \text{Max} \sum_{r=1}^{R} u_r \beta_{rpqz} + \Psi^2_{pqz}, \\
& \text{S.t.} \sum_{g=1}^{G} w_g \gamma_{gpqz} + \sum_{h=1}^{H} k_h \eta_{hpqz} = 1, \\
& \sum_{g=1}^{G} w_g \gamma_{gjt} - \sum_{i=1}^{I} v_i \alpha_{ijt} \\
& + \Psi^1_{pqz} \leq 0 \quad \forall j, t \in \Lambda_{qz}, \\
& \sum_{r=1}^{R} u_r \beta_{rjt} - \sum_{g=1}^{G} w_g \gamma_{gjt} - \sum_{h=1}^{H} k_h \eta_{hjt} \\
& + \Psi^2_{pqz} \leq 0, \quad \forall j, t \in \Lambda_{qz}, \\
& \sum_{g=1}^{G} w_g \gamma_{gpqz} + \sum_{r=1}^{R} u_r \beta_{rpqz} \\
& - \Theta^*_{pqz} \left(\sum_{i=1}^{I} v_i \alpha_{ipqz} \right) \\
& + \Psi^1_{pqz} + \Psi^2_{pqz} = \Theta^*_{pqz}, \\
& v_i, w_g, k_h, u_r \geq 0 \quad \forall i, g, h, r.
\end{aligned} \tag{3.18}$$

And the efficiency of the first sub process is then calculated as follows:

$$\Theta^{1*}_{pqz} = \frac{\theta^*_{pqz} - \lambda^*_2 \Theta^{2*}_{pqz}}{\lambda^*_1} \tag{3.19}$$

It should be noted that applying the WNDEA model required to choose the window and the number of windows depends on the time span considered. A real-life case study from financial market is applied to demonstrate the applicability, efficacy and effectiveness of the proposed WNDEA model.

4 Application: Investment Company

In this section the presented WNDEA model of this research, will be implemented for 5 investment companies (ICs) from Tehran stock exchange. ICs are one of the most important financial institutions in the capital market that invest the money received from investors on a specific investment plan, and each investor will be shared in the investment incomes and risks in proportion to his/her interest in the ICs [16]. As a result, the activities of investment companies can be viewed as a two-stage process that the management of ICs seeks to attract funds from investors in stage 1, and focuses on the optimal portfolio construction in stage 2. Fig. 2 depicts the empirical framework of the activities of ICs.

As shown in Fig. 2, the overall efficiency of the ICs is decomposed into two stages that the first stage indicates the operational management process and the second stage indicates the portfolio management process. In the first stage, two input variables including financial fees and general and administrative fees are considered. Net asset value (NAV) is the intermediate measure that is linking first and second stage. In the second stage, asset turnover and standard deviation of the returns are the input variables and average return is the output variable.

Now, for employing the WNDEA model in order to performance assessment of 5 ICs from Tehran stock exchange during the period 2013–2017, a 2 year window width was chosen and therefore four overlapping windows will be analyzed over the 5 year study period. The value of parameters that used in WNDEA model are given in Table 1.

Now, by assuming that the first stage is more important for DM, the results of window network DEA model for overall, stage 1 and stage 2, will be calculated using Model (3.15), Model (3.16) and Equation (3.17), respectively. It should be noted that LINGO software was used for solving all models. Accordingly, Tables 2 to 4, present the overall, first stage and second stage efficiency based on WNDEA approach, respectively.

As can be seen in Tables 2 to 4, in addition to calculating the efficiency of each IC per window, three types of average efficiency including the average efficiency scores of ICs for all years, the average efficiency scores of ICs for all windows and the average of all efficiency scores for each IC are measured. Accordingly, in order to evaluate and rank all ICs comprehensively, the average of all efficiency scores for each IC are extracted from Tables 2 to 4 and summarized in Table 5.

Numerical assessment of the proposed window network DEA model during period from 2013 to 2017 reveals that the presented model is able to highlight the investment companies that may have managed their portfolios well. Also, which of the two-stage including operational management process and portfolio management process, may have been the contributory factor to their good or bad performance.

With respect to the results from WNDEA model, the IC 01 is the best investment company in comparison to other ICs in period 2013–2017. It should be noted that the introduced information of Table 5 can help investors to make informed decisions and enables administrators of investment companies to judge how well their portfolio managers have performed relative to their competitors over the period 2013–2017.

5 Conclusion

This study presents a window network data envelopment analysis model based on additive efficiency decomposition and VRS assumptions for assessing the relative performance of investment companies. It should be noted that this WNDEA model is presented for extended two-stage structure with added inputs to the second stage. The applicability of the WNDEA model is demonstrated by performance measurement of 5 investment companies from Tehran Stock Exchange across the period 2013–2017. For future research, the WNDEA method can be extended based on uncertainty programming approaches for dealing with uncertain panel data (for more details see [17, 18, 19]). Moreover, the window network DEA model can also be applied to other financial institutions with network structure, such as banks and insurance companies.

References

[1] A. Al-Refaie, R. Najdawi, E. Sy, Using DEA window analysis to measure the efficiencies of blowing machines in plastics industry, *Jordan Journal of Mechanical and Industrial Engineering* 10 (2016) 1-11.

[2] M. Arefrad, H. Alipoor, Performance evaluation of guilan refah bank branches using data

envelopment analysis-DEA (window model), *Indian Journal of Fundamental and Applied Life Sciences* 5 (2015) 1291-1297.

[3] R. D. Banker, A. Charnes, W. W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management Science* 30 (1984) 1078-1092.

[4] A. Charnes, W. W. Cooper, Programming with linear fractional functionals, *Naval Research Logistics Quarterly* 9 (1962) 181-186.

[5] A. Charnes, W. W. Cooper, B. Golany, A development study of DEA in measuring the effect of maintenance units in the US Air Force, *Annals of Operations Research* 2 (1985) 95-112.

[6] A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficiency of decision making units, *European Journal of Operational Research* 2 (1978) 429-444.

[7] X. Chen, Y. Gao, Q. An, Z. Wang, L. Nerali,). Energy efficiency measurement of Chinese Yangtze River Deltas cities transportation: a DEA window analysis approach, *Energy Efficiency* (2018) 1-13.

[8] Y. Chen, W. D. Cook, N. Li, J. Zhu, Additive efficiency decomposition in two-stage DEA, *European Journal of Operational Research* 196 (2009) 1170-1176.

[9] A. Flokou, V. Aletras, D. Niakas, A window-DEA based efficiency evaluation of the public hospital sector in Greece during the 5-year economic crisis, *PloS One* 12 (2017) 177-194.

[10] G. E. Halkos, M. L. Polemis, The impact of economic growth on environmental efficiency of the electricity sector: A hybrid window DEA methodology for the USA, *Journal of Environmental Management* 211 (2018) 334-346.

[11] A. Hemmasi, M. Talaeipour, H. Khademi-Eslam, S. H. Pourmousa, Using DEA window analysis for performance evaluation of Iranian wood panels industry, *African Journal of Agricultural Research* 6 (2011) 1802-1806.

[12] T. Jia, H. Yuan, The application of DEA (Data Envelopment Analysis) window analysis in the assessment of influence on operational efficiencies after the establishment of branched hospitals, *BMC Health Services Research* 17 (2017) 265-275.

[13] C. Kao, Network data envelopment analysis: A review, *European Journal of Operational Research* 239 (2014) 1-16.

[14] C. Kao, S. N. Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan, *European Journal of Operational Research* 185 (2008) 418-429.

[15] Y. Ohe, N. Peypoch, Efficiency analysis of Japanese Ryokans: A window DEA approach, *Tourism Economics* 22 (2016) 1261-1273.

[16] P. Peykani, E. Mohammadi, Interval network data envelopment analysis model for classification of investment companies in the presence of uncertain data, *Journal of Industrial and Systems Engineering* 11 (2018) 63-72.

[17] P. Peykani, E. Mohammadi, A. Jabbarzadeh, A. Jandaghian, Utilizing robust data envelopment analysis model for measuring efficiency of stock, a case study: Tehran stock exchange, *Journal of New Research in Mathematics* 1 (2016) 15-24.

[18] P. Peykani, E. Mohammadi, M. S. Pishvaee, M. Rostamy-Malkhalifeh, A. Jabbarzadeh, A novel fuzzy data envelopment analysis based on robust possibilistic programming: possibility, necessity and credibility-based approaches, *RAIRO-Operations Research* 52 (2018) 1445-1463.

[19] P. Peykani, E. Mohammadi, M. Rostamy-Malkhalifeh, F. Hosseinzadeh Lotfi, Fuzzy data envelopment analysis approach for ranking of stocks with an application to Tehran stock exchange, *Advances in Mathematical Finance and Applications* 4 (2019) 31-43.

[20] D. Pjevevi, A. Radonji, Z. Hrle, V. oli, DEA window analysis for measuring port efficiencies in Serbia, *Promet-Traffic & Transportation* 24 (2012) 63-72.

[21] M. Pulina, C. Detotto, A. Paba, An investigation into the relationship between size and efficiency of the Italian hospitality sector: A window DEA approach, *European Journal of Operational Research* 204 (2010) 613-620.

[22] A. Salem Al-Eraqi, A. Mustafa, A. Tajudin Khader, An extended DEA windows analysis: Middle East and East African seaports, *Journal of Economic Studies* 37 (2010) 208-218.

[23] K. Wang, S. Yu, W. Zhang, Chinas regional energy and environmental efficiency: A DEA window analysis based dynamic evaluation, *Mathematical and Computer Modelling* 58 (2013) 1117-1127.

[24] R. Webb, Levels of efficiency in UK retail banks: a DEA window analysis, *International Journal of the Economics of Business* 10 (2003) 305-322.

[25] Q. D. Wu, Y. Shi, Q. Xia, W. D. Zhu, Effectiveness of the policy of circular economy in China: A DEA-based analysis for the period of 11th five-year-plan, *Resources, Conservation and Recycling* 83 (2014) 163-175.

[26] H. H. Yang, C. Y. Chang, Using DEA window analysis to measure efficiencies of Taiwan's integrated telecommunication firms, *Telecommunications Policy* 33 (2009) 98-108.

Pejman Peykani is a PhD Candidate at the School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran. He received his MSc in Financial Engineering in 2014 from the K. N. Toosi University of Technology, Tehran, Iran. His areas of research interests include Data Envelopment Analysis, Financial Modeling, Portfolio Management, Robust Optimization, and Fuzzy Mathematical Programing.

Emran Mohammadi is an Assistant Professor of Industrial Engineering at the Iran University of Science and Technology, since 2011. He received his PhD. in Industrial Engineering in 2010 from Iran University of Science and Technology, Tehran, Iran. His areas of research interests include Mathematical Modeling, Performance Measurement and Management, Efficiency and Productivity Analysis, Uncertainty Programming.