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Research Article

Applying MCDEA Models to Rank Decision Making Units with Stochastic Data

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Abstract

As a technique based on mathematical programming, Data Envelopment Analysis (DEA) is used for evaluating the efficiency of homogeneous Decision Making Units (DMUs). DEA models need accurate input and output data. In many situations, on the one hand, accurate measurement of inputs and outputs is difficult due to their volatility and complexity. This conflict results in uncertain DEA models. Its main problem is transformation of deterministic equivalent of stochastic model into quadratic programming, time-consuming and complexity and it requires presuppositions. By means of Bi-objective multiple criteria DEA (Bio-MCDEA) model that considers stochastic data, our proposed model reduces some of these problems and facilitates problem solving through presenting primary presupposition and final linear model. The efficiency score of DMUs is determined by applying stochastic Bio- MCDEA model. Eventually, we used the data of seventeen Iranian electricity distribution companies to illustrate the methods developed in the present paper.

Keywords : Data envelopment analysis (DEA); Multiple criteria DEA (MCDEA); Stochastic Data; Ranking; Probability.

1 Introduction

As an objective method, data envelopment analysis (DEA) is used for comparing the relative efficiency of decision- making units (DMUs) with similar multiple inputs and outputs. The originators of this method were Charnes, Cooper and Rhodes (CCR). Banker, Charnels and Cooper (BCC) presented a variable return to scale version of the CCR model, called the BCC model. As a result of CCR and BCC models, other DEA models were presented in the DEA literature. Conventional DEA has been criticized for not allowing stochastic information to be incorporated in input and output data, which in true, lead to the DEA efficiency measure to be sensitive to such information. Udhayakumar [20]. To evaluate DMU with different kinds of data such as deterministic, fuzzy and interval data, many DEA models have been in-

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troduced in various fields Khodabakhshi & Asgharian, [13]. Moreover, managers handle the units with imprecise data as random variable in many practical problems Liu & Wang, [1] .To incorporate possible uncertainty in inputs and outputs, stochastic formulation of original DEA models was introduced Cooper, Seiford, &Tone [11]; Sueyoshi [19] ; Khodabakhshi & Asgharian, [15]; Jingliang [16]. Morita, Hiroshi, and Seiford examined carefully the robustness of efficiency results when input and output data depend on stochastic measurement error, while Jess, Jongen, Neralic, and Stein presented a semi-infinite programming model in DEA to examine a chemical engineering problem.Recently, researchers such as Tavana and Kiani Mavi, Jingling and Zhou, Khodabakhshi and Asgharian [12, 13, 14, 15] have studied stochastic input and output variations in DEA. For discussions on linear programming programs, refer to Kall (1976). In this paper, we extend bi-objective MCDEA model to sto-chastic one. Then, we apply a deterministic equivalent to our stochastic model. Following a multiple objective decision- making framework, the multiple criteria (or Multi-objective) DEA model Li & Reeves [10] was proposed as a means to overcome the problems of discriminating power and weight dispersion. Although the original formulation of Li and Reeves does not promise complete ranking, it presupposes the decision maker to use its models of three objectives interactively. Therefore, the three objectives are analyzed separately in the MCDEA model and no preference order is set for those objectives. To simultaneously solve all three objectives of the MCDEA model Bal, Orkcu, & Celebioglu [5] have recently proposed the goal programming approach. Their GPDEA models, namely, constant returns to scale and variable returns to scale were claimed to improve the dispersion of weights and discriminating power in a MCDEA framework. This study introduces new stochastic bi-objective multiple criteria DEA (Bio-MCDEA) model, which can solve the disadvantages. We organize the rest of the paper as follows: in section 2, the proposed stochastic ranking method is described. stochastic version of bi-objective weighted model is developed in section 3 and its deterministic equivalent is obtained. In section 4, the model, as an empirical example, is applied to data of Iranian electricity distribution companies. Section 5 concludes the paper.

2 Preliminaries

It is claimed that the basic DEA model does not always provide good discriminatory characteristics among alternatives; particularly in situations where some alternatives may have scores equal to one (i.e., are efficient). To well discriminate among alternative scores, a number of techniques have been proposed .We can use MCDEA to improve the discriminating power of classical DEA method. Its solution is to find non-dominated solutions and to help select the most preferred one. The form of MCDEA model depends on the efficiency criteria used. In the present study, we use the model suggested by Li and Reeves [10] as follows:

$$\begin{aligned}
&\min d_0\\
&\min M\\
&\min \sum_{j=1}^{n} d_j\\
&s.t\\
&\sum_{i=1}^{m} v_i x_{i0} = 1\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad j = 1,2,...,n\\
&M - d_j \geq 0 \quad j = 1,2,...,n\\
&u_r \geq 1_s\varepsilon \quad r = 1,2,...,s\\
&v_i \geq 1_m\varepsilon \quad r = 1,2,...,m\\
&d_j \geq 0 \quad j = 1,2,...,n
\end{aligned} \tag{2.1}$$

The following is the detailed information about the model (2.1):
This model has three criteria, namely, minimizing d_0, minimizing the maximum deviation M, and minimizing the sum of the deviations. u_r, v_i and d_0 are the weight and deviation variable for DMU0 respectively, and the variable M in the second objective represents the maximum quantity among all deviation variables (d_j $j = 1, ..., n$). The third objective function is a straightforward representation of the deviation sum. In the above

three definitions, irrespective of DMU0 efficiency, its DEA efficiency score is $1-d_0$,however, the values of d_0 may vary under different criteria. Remembering Li and Reeves (1999) approach, the MCDEA models objective function include three parts: Min d_0 , Min M, and $\sum_{j=1}^{n} d_j$, as defined in model 2.1. We can rewrite a MCDEAs three objective function in a weighted method as follows: $w_i d_0 + w_2 M + w_3 \sum_{j=1}^{n} d_j$ for the single weighted objective equivalent. To obtain different efficient solutions $w_i(i = 1, 2, 3)$ can vary. Thus, given that the first objective w1 is the equivalent to a conventional CCR model, it can be eliminated from the MCDEA in the weighted objective sense. Moreover, Li and Reeves had suggested that the first objective yields lower discrimination power in compared to the other two objectives. Thus, we solved the bi-objective weighted problem for Bio-MCDEA model [3], using both the second and third objectives. The value of w_1 is set equal to zero because whenever $\sum_{j=1}^{n} d_j$ is minimized, d_0 will be minimized as well. Thus, we have the following model. Bio-objective MCDEA model under CCR

$$
\begin{aligned}
& \min w_2 M + w_3 \sum_{j=1}^{n} d_j \\
& s.t \\
& \sum_{i=1}^{m} v_i x_{i0} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad j = 1, 2, ..., n \\
& M - d_j \geq 0 \quad j = 1, 2, ..., n \\
& u_r \geq 1_s \varepsilon \quad r = 1, 2, ..., s \\
& v_i \geq 1_m \varepsilon \quad r = 1, 2, ..., m \\
& d_j \geq 0 \quad j = 1, 2, ..., n
\end{aligned}
\tag{2.2}
$$

Stochastic Bio-MCDEA Model

A large number of parameters are required for Stochastic DEA. We can greatly facilitate the practical application of the technique by some assumptions simplifying the computational task in such estimation of parameters and while reducing the almost 100% confidence chance constraint programming problem 2.2 to an ordinal linear programming problem. In the rest of this paper, it is assumed that components of the inputs and outputs are related only through common relationships with some basic factor. The component of any input and output is solely determined by this single factor. Based on the analysis of randomness in the introduction, this study assumes that the stochastic variable $(\tilde{x}_{ij})$ of each input is expressed by $\tilde{x}_{ij} = \overline{x}_{ij} + a_{ij}\epsilon$ ($i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$), where $(\overline{x}_{ij})$ is an expected value of$\tilde{x}_{ij}$ and a_{ij} is standard deviation. The stochastic variable of output is represented as $\tilde{y}_{rj} = \overline{y}_{rj} + \overline{b}_{rj}\epsilon$ where $\overline{y}_{rj}$ is an expected value of $\tilde{y}_{rj}$ and b_j^- is its standard deviation.It is naturally supposed that the random variable (ϵ) follows normal distribution $(N(0, \sigma^2))$. After the preparation of the above assumptions, it is better to acquire the equivalent deterministic form in order to facilitate the mode1†s solution.

Methodology

In order to describe the following models, clearly, this study assumes that there are n DMU_j $j = 1, 2, ..., n$. naturally considering the environmental effect. At the same time, inputs and outputs depend upon external fac-tors such as social changes, economic conditions, and other socioeconomic factors affecting the magnitude of inputs and outputs. Thus, this study also considers the existence of data's randomness. Based on the above for DMU_j, let $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, ..., \tilde{x}_{mj})^T$ and $\tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, ..., \tilde{y}_{rj})^T$ represent stochastic variables for the input and output vectors. It is assumed that all DMUs have input and output vectors with positive components . Assume that $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, ..., \tilde{x}_{mj})^T$ and $\tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, ..., \tilde{y}_{rj})^T$ are the stochastic input and output vectors. By applying the chance constrained programming (CCP) on the model, the objective function is subject to uncertainty, the objective function is expressed as $\min \delta$ and the constraint
$w_2 M + w_3 (\sum_{i=1}^{m} v_i \tilde{x}_i - \sum_{r=1}^{s} u_r \tilde{y}_r)) \leq \delta$ is added to the constraints.
The probabilistic constraint form of (Bio-MCDEA) model with stochastic data is as fol-

lows:

$$\min \delta \tag{2.3}$$
$$s.t$$
$$E(\sum_{i=1}^{m} v_i \tilde{x}_{i0}) = 1 \quad (a)$$

$$P_r((w_2M + w_3(\sum_{i=1}^{m} v_i \tilde{x}_i - \sum_{r=1}^{s} u_r \tilde{y}_r)) \leq \delta)$$
$$\geq 1-\alpha \quad (b)$$
$$Pr(\sum_{r=1}^{s} u_r \tilde{y}_r - \sum_{i=1}^{m} v_i \tilde{x}_i \leq 0))1-\alpha$$
$$j = 1, 2, ..., n \quad (c)$$
$$P_r(\sum_{i=1}^{m} v_i \tilde{x}_i - \sum_{r=1}^{s} u_r \tilde{y}_r \leq M) \geq 1-\alpha$$
$$j = 1, 2, ..., n \quad (d)$$
$$u_r \geq 1_s \varepsilon \quad r = 1, 2, ..., s$$
$$v_i \geq 1_m \varepsilon \quad i = 1, 2, ..., m$$
$$M \geq 0$$
$$\delta \geq 0$$

Here in the above models, P means probability and α is a level of error between 0 and 1, which is a predetermined number. The above model can be converted into the deterministic model through the following procedures.

3 Deterministic equivalent for the Bio-MCDEA model

In order to have a better understanding of the performance of the proposed method, in this section an illustrative process is presented in this section.

The constraints of Eq. (2.3-a), including the stochastic process, can be rewritten respectively as follows:

$$E(\sum_{i=1}^{m} v_i \tilde{x}_{i0}) = \sum_{i=1}^{m} v_i \bar{x}_{i0} = 1 \tag{3.4}$$

The input constraints can be transformed into equality form by adding $\eta > 0$

$$P_r(\delta - w_2M - w_3(\sum_{i=1}^{m} v_i \tilde{X}_i - \sum_{i=1}^{s} u_r \tilde{Y}_r) \geq 0)$$
$$= 1-\alpha+\eta \quad \eta > 0 \tag{3.5}$$

Remark 3.1 *Let T be a random variable and a, b and c constant numbers, if $P(T < a) = c$ and$b < a$, then there exists $d < c$ such that $P(T < b) = d$. [7]*

By employing the above remark, there exist $\Delta \geq 0$ such that:

$$P_r(\delta - w_2M - w_3(\sum_{i=1}^{m} v_i \bar{X}_i - \sum_{i=1}^{s} u_r \tilde{Y}_r) \geq \Delta)$$
$$= 1-\alpha \quad \eta > 0 \tag{3.6}$$

Since the random variable $\sum_{i=1}^{m} v_i \bar{X}_i - \sum_{i=1}^{s} u_r \tilde{Y}_r$ has normal distribution, the above expression can be converted into the following deterministic expression through using the standard normal distribution:

$$E\Big(w_2M + w_3(\sum_{i=1}^{m} v_i \bar{x}_i - \sum_{i=1}^{s} u_r \tilde{y}_r)\Big)$$
$$= E\Big(w_2M + w_3 \sum_{j=1}^{n}(\sum_{i=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \bar{x}_{ij})\Big)$$
$$= w_2M + w_3\Big(\sum_{r=1}^{s} u_r \sum_{j=1}^{n} E(\bar{y}_{rj})$$
$$- \sum_{i=1}^{m} v_i \sum_{j=1}^{n} E(\bar{x}_{ij})\Big)$$
$$= w_2M + w_3\Big(\sum_{r=1}^{s} u_r \underbrace{\sum_{j=1}^{n} \bar{y}_{rj}}_{Y_r} - \sum_{i=1}^{m} v_i \underbrace{\sum_{i=1}^{m} \bar{X}_{ij}}_{X_i}\Big) \tag{3.7}$$

$$P_r(\delta - w_2M - w_3(\sum_{i=1}^{m} v_i \tilde{X}_i - \sum_{i=1}^{s} u_r \tilde{Y}_r) \geq \Delta)$$
$$= 1-\alpha \tag{3.8}$$

where

$$\bar{X}_i = \sum_{i=1}^{m} \bar{x}_{ij} \quad and \quad E(\bar{X}_i) = E(\sum_{i=1}^{m} \bar{x}_{ij}) = \sum_{i=1}^{m} \bar{x}_{ij} \tag{3.9}$$

$$\bar{Y}_r = \sum_{r=1}^{s} \bar{y}_{rj} \quad and \quad E(\bar{Y}_r) = E(\sum_{r=1}^{s} \bar{x}_{rj}) = \sum_{r=1}^{s} \bar{y}_{rj} \tag{3.10}$$

The normality assumption is used to introduce a deterministic equivalent to the model

$$P_r(\frac{\overbrace{\overbrace{\sum_{i=1}^{m} v_i \bar{X}_i - \sum_{r=1}^{s} u_r \tilde{Y}_r}^{x} - \overbrace{(\sum_{i=1}^{m} v_i \bar{X}_i - \sum_{r=1}^{s} u_r \tilde{Y}_r)}^{\mu}}^{Z}}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_{ij}) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}} \le \frac{\frac{1}{w_3}(\delta - w_2 M) - \Delta - (\overbrace{(\sum_{i=1}^{m} v_i \bar{X}_i - \sum_{r=1}^{s} u_r \tilde{Y}_r)}^{\mu})}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_i j) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}} \tag{3.11}$$

Where Z is normal standard variable, and we can have

$$\varphi(\frac{\frac{1}{w_3}(\delta - w_2 M) - \Delta - (\overbrace{(\sum_{i=1}^{m} v_i \bar{X}_i - \sum_{r=1}^{s} u_r \tilde{Y}_r)}^{\mu})}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_i j) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}}) = (1 - \alpha) \tag{3.12}$$

$$\frac{\frac{1}{w_3}(\delta - w_2 M) - \Delta - (\overbrace{(\sum_{i=1}^{m} v_i \bar{X}_i - \sum_{r=1}^{s} u_r \tilde{Y}_r)}^{\mu})}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_i j) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}} = \varphi^{-1}(1 - \alpha) \tag{3.13}$$

or

$$\sum_{i=1}^{m} v_i \bar{x}_i - \sum_{r=1}^{s} u_r \bar{y}_r = \frac{1}{w_3}(\delta - w_2 M) - \varphi^{-1}(1 - \alpha)(\sum_{i=1}^{m} v_i a_{ij} - \sum_{r=1}^{s} u_r b_{rj})\delta - \Delta \tag{3.14}$$

It should be noted that φ is the cumulative normal distribution function. Like what has been done to the constraints, the deterministic form for and constraints can be achieved as follows. Where (E) andb(Var) separately indicate the mean and variance of each random variable ($\sum_{i=1}^{m} v_i \tilde{x}_{ij} - \sum_{r=1}^{s} u_r \tilde{y}_{rj} \quad j = 1, 2, ..., n$) we will have

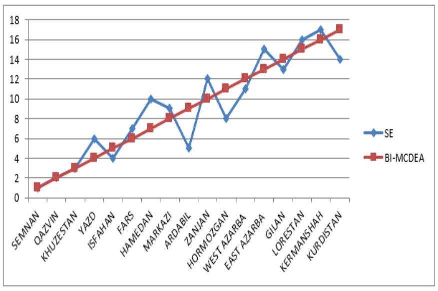

Figure 1: Comparison of ranks SE and Bio-MCDEA models

$$E(\sum_{i=1}^{m} v_i \tilde{x}_{ij} - \sum_{r=1}^{s} u_r \tilde{y}_{rj}) = \sum_{i=1}^{m} v_i E(\tilde{x}_{ij}) - \sum_{r=1}^{s} u_r E(\tilde{y}_{rj}) = \sum_{i=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj} \tag{3.15}$$

Table 1: Data for electricity distribution units

DMU	Company		Inputs		Outputs	
		Net. len. (km)	Tr. Cap (MVA)	Empl.	Un. del (MW h)	S.area
1	East Azarbaijan	15,151	990	867	1825	40,968
2	West Azarbaijan	19,610	1306	1047	5297	37,463
3	Isfahan	25,566	2713	1072	5835	97,923
4	Hamadan	12,340	1101	595	2369	19574
5	Khuzestan	19,380	3932	1471	8048	53,442
6	Zanjan	10,347	788	351	1526	21.841
7	Kurdistan	11.697	725	426	1174	28.817
8	Fars	24.624	1617	872	4015	83.575
9	Ardabil	10.129	507	441	936	17.881
10	Markazi	14.505	1489	600	3063	29.406
11	Lorestan	12.078	1006	594	1784	28.392
12	Qazvin	8766	946	302	2414	15.491
13	Semnan	8063	730	374	1418	96.816
14	Kermanshah	12.795	1147	538	18.6	24.641
15	Gilan	21,187	1534	938	2848	13.952
16	Hormozgan	16.185	1786	938	3411	71.193
17	Yazd	11,990	963	576	2400	73.467
	Mean Standard Deviation	14966.06 (5188.39)	1369.412 (815.3)	706 (309.2)	2951.118 (1841.3)	44402.47 (28239.3)

and

$$var(\sum_{i=1}^{m} v_i\tilde{x}_{ij} - \sum_{r=1}^{s} u_r\tilde{y}_{rj})$$
$$var(\sum_{i=1}^{m} v_i(\bar{x}_{ij} + \varepsilon a_{ij})$$
$$-\sum_{r=1}^{s} u_r(\bar{y}_{rj} + \varepsilon b_{ij})$$
$$= var(\sum_{i=1}^{m} v_i(\bar{x}_{ij}) - \sum_{r=1}^{s} u_r(\bar{y}_{rj})$$
$$+ \varepsilon_{N(0,\sigma^2)} \sum_{i=1}^{m} v_i(a_{ij}) - \sum_{r=1}^{s} u_r(b_{rj})$$
$$= (\sum_{i=1}^{m} v_i(a_{ij}) - \sum_{r=1}^{s} u_r(b_{rj}))^2\sigma^2 \qquad (3.16)$$

In a similar fashion, the following deterministic equivalent to the constraint 2.3-d can be obtained as follows:

$$Pr(M - (\sum_{i=1}^{m} v_i\bar{x}_{ij} - \sum_{r=1}^{s} u_r\bar{y}_{rj}) \geq \Delta_j)$$
$$= (1-\alpha) \qquad (3.17)$$

therefore,

$$P_r(\frac{\overbrace{\underbrace{\sum_{i=1}^{m} v_i\bar{x}_{ij} - \sum_{r=1}^{s} u_r\tilde{y}_{rj}}_{x} - \underbrace{(\sum_{i=1}^{m} v_i\bar{x}_{ij} - \sum_{r=1}^{s} u_r\bar{y}_{rj})}_{\mu}}^{Z}}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_{ij}) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}} \qquad (3.18)$$

Table 2: Stochastic bio-MCDEA model and supper efficiency model (from Khodabakshi 2010) results of the 17 units with ($\alpha = 0.4$)

DMU	Company	Stocha.score (SE)	Rank	Stocha.score(Bio-MCDEA)	Rank
1	East Azarbaijan	1.341	15	0.8525	13
2	West Azarbaijan	1.246	11	0.8736	12
3	Isfahan	0.802	4	1.159	5
4	Hamadan	1.159	10	1.021	7
5	Khuzestan	0.567	3	1.315	3
6	Zanjan	1.284	12	0.9313	10
7	Kurdistan	0.567	14	0.7414	17
8	Fars	0.912	7	1.116	6
9	Ardabil	0.850	5	0.979	9
10	Markazi	1.142	9	1.005	8
11	Lorestan	1.402	16	0.8204	15
12	Qazvin	0.529	2	1.443	2
13	Semnan	0.146	1	1.879	1
14	Kermanshah	1.564	17	0.7716	16
15	Gilan	1.316	13	0.8287	14
16	Hormozgan	1.141	8	0.9107	11
17	Yazd	0.911	9	1.203	4

$$\leq \frac{M - \Delta_j - (\overbrace{\sum_{i=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj})}^{\mu}}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_{ij}) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}}$$

Where Z is normal standard variable, and we can have

$$\varphi \frac{M - \Delta_j - ((\overbrace{\sum_{i=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj}))}^{\mu}}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_i j) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}} = (1 - \alpha) \tag{3.19}$$

In other word,

$$\frac{M - \Delta_j - ((\overbrace{\sum_{i=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj}))}^{\mu}}{\underbrace{\sigma(\sum_{i=1}^{m} v_i(a_i j) - \sum_{r=1}^{r} u_r(b_{ij}))}_{\sigma}} = \varphi^{-1}(1 - \alpha) \tag{3.20}$$

or

$$\sum_{i=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj} = M - \delta_j - \varphi^{-1}(1 - \alpha)(\sum_{i=1}^{m} v_i a_{ij} - \sum_{r=1}^{s} u_r b_{rj})) \tag{3.21}$$

Therefore, the deterministic form of the (BIO-MCDEA) model with stochastic data which has been derived from model 2.3 is as follows:

$$
\begin{aligned}
&\min \delta \\
&s.t \\
&\sum_{i=1}^{m} v_i \bar{x}_{i0} = 1 \\
&\sum_{i=1}^{m} v_i \bar{x}_i - \sum_{r=1}^{s} u_r \bar{y}_r = \frac{1}{w_3}(\delta - w_2 M) - \varphi^{-1}(1-\alpha) \\
&(\sum_{i=1}^{m} v_i \bar{a}_{ij} - \sum_{r=1}^{s} u_r \bar{b}_{rj})\delta - \Delta \\
&\sum_{r=1}^{s} u_r \bar{y}_{rj} - \sum_{i=1}^{m} v_i \bar{x}_{ij} = -\varphi^{-1}(1-\alpha) \\
&(\sum_{i=1}^{m} v_i \bar{a}_{ij} - \sum_{r=1}^{s} u_r \bar{b}_{rj})\delta - \gamma_j \;\; j = 1,2,...,n \\
&\sum_{i=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj} \\
&= M - \Delta_j - \varphi^{-1}(1-\alpha)
\end{aligned}
$$

$$
\begin{aligned}
&(\sum_{i=1}^{m} v_i \bar{a}_{ij} - \sum_{r=1}^{s} u_r \bar{b}_{rj})\delta - \gamma_j \;\; j = 1,2,...,n \\
&u_r \geq 1_s \varepsilon \quad r = 1,2,...,s \\
&v_i \geq 1_m \varepsilon \quad i = 1,2,...,m \\
&M \geq 0 \\
&\gamma_j \geq 0 \quad j = 1,2,...,n \\
&\Delta_j \geq 0 \quad j = 1,2,...,n \\
&\delta \geq 0
\end{aligned}
$$

Based on this assumption $\forall i \; (\delta v_i = P_i)$, $\forall r \; (\delta u_r = Q_r)$ model 3.5 is transformed into the following linear programming problem. Finally, following deterministic equivalent, our stochastic bio-MCDEA model is obtained

$$
\begin{aligned}
&\min \delta \\
&s.t \\
&\sum_{i=1}^{m} v_i \bar{x}_{i0} = 1 \\
&\sum_{i=1}^{m} v_i \bar{x}_i - \sum_{r=1}^{s} u_r \bar{y}_r = \frac{1}{w_3}(\delta - w_2 M) - \varphi^{-1}(1-\alpha) \\
&(\sum_{i=1}^{m} P_i \bar{a}_{ij} - \sum_{r=1}^{s} Q_r \bar{b}_{rj})\delta - \Delta \\
&\sum_{r=1}^{s} u_r \bar{y}_{rj} - \sum_{i=1}^{m} v_i \bar{x}_{ij} = -\varphi^{-1}(1-\alpha) \\
&(\sum_{i=1}^{m} P_i \bar{a}_{ij} - \sum_{r=1}^{s} Q_r \bar{b}_{rj}) - \gamma_j \;\; j = 1,2,...,n \\
&\sum_{i=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj} \\
&= M - \Delta_j - \varphi^{-1}(1-\alpha) \\
&(\sum_{i=1}^{m} P_i \bar{a}_{ij} - \sum_{r=1}^{s} Q_r \bar{b}_{rj}) \;\; j = 1,2,...,n \\
&M \geq 0 \quad i = 1,2,...,m \\
&\Delta \geq 0 \\
&\gamma_j \geq 0 \quad j = 1,2,...,n \\
&\Delta_j \geq 0 \quad j = 1,2,...,n \\
&P_j \geq 0 \quad i = 1,2,...,m \\
&Q_r \geq 0 \quad r = 1,2,...,s \\
&\delta \geq 0
\end{aligned} \tag{3.22}
$$

Suppose (U^*, V^*) to be the optimal solution of the model, the efficiency score of DMU_j is $\theta_j^* = \frac{\sum_{r=1}^{s} u_r^* \bar{y}_{rj}}{\sum v_i^* \bar{x}_{ij}}$ and $\bar{y}_{rj}, \bar{x}_{ij}$ are the mean vectors of the stochastic output and input, respectively. Hence, the more the efficiency score, the better DMU_j will be.

4 Application

As an empirical example, the proposed method is applied to Iranian electricity distribution units using some actual data of year 2000. Established in 1992, the Iranian electricity distribution units are public ones and they act under supervision of TAVANIR Co. (Iran's Power, Generation, Transmission and Distribution Management Company).Based on extensive review in Jamasband Pollitt (2001) [15], operating costs, number of employees, transformer capacity and the length of network are the most frequently used inputs. Furhermore, units of delivered energy, number of customers and the size of the service area are the most widely used outputs. The cost data is not available. We use network length, transformer capacity, and employee variables as inputs and units' delivery, and service area variables as outputs in this study.

The size of service area is considered as an environmental variable in our study. Considering that the electricity distribution units are public and act in provinces of Iran, the service area is out of control of the units. The computational results of the equivalent deterministic problem are presented in columns 3 and 4 of Table 1. To compute result for stochastic data, has been considered to be equal to 0.4 , Although all DMUs are assumed to have similar variance, they can have different means. Therefore the variances for outputs and Inputs can be estimated by:

$$var(\bar{y}_r) = \frac{1}{17}\sum_{j=1}^{17}(y_{rj} - \bar{y}_r)^2 \tag{4.23}$$

$$var(\bar{x}_i) = \frac{1}{17}\sum_{j=1}^{17}(x_{ij} - \bar{x}_i)^2 \tag{4.24}$$

Where

$$\bar{y}_r = \frac{1}{17}\sum_{j=1}^{17} y_{rj} \tag{4.25}$$

$$\bar{x}_i = \frac{1}{17}\sum_{j=1}^{17} x_{ij} \tag{4.26}$$

Also, x_{ij} and y_{rj} are the expected values of inputs and outputs for DMU_j used as an estimate for the expected value of the stochastic inputs and outputs. It is also assumed that outputs and inputs for different DMUs are independent. The stochastic bio-MCDEA scores obtained from software LINDO are presented in Table 2. Again, the best company is DMU 13, With the stochastic score 1.879. Semnan corresponds to =0.4.The worst company is DMU 17 with score 0.7414. Both models have set Semnan unit as superior rank and Qazvin unit in the second rank, the model presented by Yazd has been ranked fourth with score1.203while its SE model (Khodabakhshi 2010) is ranked sixth. In the comparison of computational complexity, computation rate of the presented model is more cost effective than the previous model and it has been influenced mostly by outputs and is computable with operation research ordinary software. Software LINDO has been used for computation.

Comparing numerical result presented in columns 3-4 and 5-6 of Table 2, DMU 14, Kermanshah and DMU 15, Gilan, DMU 10, Markazi, DMU 16, Hormozgan, DMU 4

, Hamedan, DMU 6, Zanjan, DMU 2, West Azarbaijan, DMU 1, East Azarbaijan, DMU 7, Kurdistan and DMU 11, Lorestan are inefficient with both Stochastic models. Therefore the computational results are quit identical.

5 Conclusion

In this paper, we presented Stochastic MCDEA model for ranking all Stochastic version of the proposed Bi-objective MCDEA model has been developed and a deterministic equivalent was obtained for the stochastic version using this model. The new model may provide practitioners with a quite robust measure to get more accurate picture of efficiency allowing for random fluctuation in the data used to compare DMUs. It should be noted that our method in this study is interested in controlling the quantity of inputs as our decision variable, whilst being unable to control outputs, since these quantities depend upon external factors such as economic condition, a demographic change, and other socio-economic factors that influenceing the magnitude of outputs. The Stochastic MCDEA model shows that DMU 13, Semnan is the best company in terms of technical efficiency. Finally, it is hoped that this study make a small contribution towards DEA-based management planning.

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