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Research Article

An Efficient Numerical Method for a Class of Boundary Value Problems Based on Shifted Jacobi-Gauss Collocation Scheme

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Abstract

This paper proposes a numerical method to deal with the two-dimensional hyperbolic equations with nonlocal integral conditions. The nonlocal integral equation usually is of major challenge in the frame work of the numerical solutions of partial differential equations. The method benefits from collocation radial basis function method, the generalized thin plate splines (GTPS) radial basis functions are used. Therefore, it does not require any struggle to determine shape parameter (In other RBFs, it is time-consuming step). The present technique is one of the truly meshless methods in where it does not require any background integration cells over local or global domains and it is in contrast to weak form methods in where all integrations are carried out locally or globally over quadrature domains of regular shapes, such as lines in one dimensions, circles or squares in two dimensions and spheres or cubes in three dimensions. The obtained results for some numerical examples reveal that the proposed technique is very effective, convenient and quite accurate to such considered problems.

Keywords : Collocation method; Shifted Jacobi polynomial; Singular boundary value problem; Exponential nonlinearity; Product nonlinearity.

1 Introduction

Spectral methods are one of the principal methods of discretization for the numerical solution of boundary value problems, initial value problems and so on [1, 2, 3, 4, 5]. The most widely used spectral versions are the Galerkin, collocation, and Tau methods [6]. Collocation methods are very popular for solving such problems, also they are very applicable in providing highly accurate solutions to these problems. In this paper, we extended the application of Jacobi polynomials from Galerkin method for solving nonlinear second-order initial value problems (see [7, 8]) to collocation method to solve a class of boundary value problems on the unit interval which feature a type of exponential and product nonlinearities.

A well-known advantage of a spectral method is that it achieves high accuracy with relatively fewer spatial grid points in comparison by a finite-difference method. Also, in using spectral methods, we meet to full matrices, partially negating the gain in efficiency due to the fewer number of grid points (see [9, 10]). The use of Jacobi polynomials has the advantage of obtaining the solutions of nonlinear differential equations, [8, 11].

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The fundamental goal of this paper is comparison of the result of the numerical solution of boundary value problem (BVP) by collocation methods with Adomian Decomposition Method (ADM) and Reproducing Kernel Method (RKM) and we will observe that present method is both efficient and accurate.

The BVP is collocated only at nodes of the shifted Jacobi-Gauss interpolation as collocation points. The main equation together with initial conditions generate a system of algebraic equations which can be solved using Newton's iterative method.

This paper is organized as follows. In Section 2, an overview of shifted Jacobi polynomials and their relevant properties needed hereafter is presented. In Section 3, we construct the collocation method by using the shifted Jacobi polynomials in two cases, for a two-point boundary value problem for the fourth-order nonlinear differential equation with an exponential nonlinearity, for a two-point boundary value problem for the fourth-order nonlinear differential equation with a product nonlinearity, respectively. In Section 4, we present some numerical results exhibiting the accuracy and efficiency of our numerical algorithms, and a brief conclusion in Section 5.

2 Preliminaries

Let $\alpha > -1,\ \beta > -1$, and $P_n^{(\alpha,\beta)}(r)$ be the standard Jacobi polynomial of degree n. Obviously, we have

$$\begin{aligned} P_n^{(\alpha,\beta)}(-r) &= (-1)^n P_n^{(\alpha,\beta)}(r), \\ P_n^{(\alpha,\beta)}(-1) &= \frac{(-1)^n \Gamma(n+\beta+1)}{n!\,\Gamma(\beta+1)}, \\ P_n^{(\alpha,\beta)}(1) &= \frac{\Gamma(n+\alpha+1)}{n!\,\Gamma(\alpha+1)}. \end{aligned} \tag{2.1}$$

The m-th derivative of $P_n^{(\alpha,\beta)}(r)$ is defined as

$$D^m P_n^{(\alpha,\beta)}(r) = 2^{-m}\frac{\Gamma(m+n+\alpha+\beta+1)}{\Gamma(n+\alpha+\beta+1)} P_{n-m}^{(\alpha+m,\beta+m)}(r). \tag{2.2}$$

The set of Jacobi polynomials with the weight function $\omega^{(\alpha,\beta)}(r) = (1-r)^\alpha(1+r)^\beta$ forms a weighted Hilbert space $L^2_{\omega^{(\alpha,\beta)}}[-1,1]$, which is also a complete system by standard inner product. The shifted Jacobi polynomial of degree n is defined by $J_n^{(\alpha,\beta)}(r) = P_n^{(\alpha,\beta)}(2r-1)$, and by using (2.1) and (2.2), it can be shown that

$$D^m J_n^{(\alpha,\beta)}(r) = \frac{\Gamma(m+n+\alpha+\beta+1)}{\Gamma(n+\alpha+\beta+1)} J_{n-m}^{(\alpha+m,\beta+m)}(r). \tag{2.3}$$

The set of shifted Jacobi polynomials with the weight function $\chi^{(\alpha,\beta)}(r) = (1-r)^\alpha r^\beta$ forms a weighted Hilbert space $L^2_{\chi^{(\alpha,\beta)}}[0,1]$, which is also a complete system by standard inner product. Moreover, we have

$$\| J_n^{(\alpha,\beta)} \|^2_{\chi^{(\alpha,\beta)}} = \left(\frac{1}{2}\right)^{\alpha+\beta+1} \| P_n^{(\alpha,\beta)} \|^2_{\omega^{(\alpha,\beta)}}.$$

The symmetric Jacobi polynomials, the shifted Chebyshev of the first kind, the shifted Chebyshev of the second kind and the shifted Legendre polynomials are recovered by $\alpha=\beta$, $\alpha=\beta=-0.5$, $\alpha=\beta=+0.5$, $\alpha=\beta=0$, respectively.

The nodes of the standard Jacobi-Gauss interpolation on the interval $[-1,1]$ and their corresponding Christoffel numbers are denoted by $r_{N,j}^{(\alpha,\beta)}$ and $\varpi_{N,j}^{(\alpha,\beta)}$, $0 \le j \le N$, respectively. We denote by $\theta_{N,j}^{(\alpha,\beta)}$, $0 \le j \le N$ the nodes of the shifted Jacobi-Gauss interpolation on $[0,1]$, which are the zeros of $J_{N+1}^{(\alpha,\beta)}$. It can be shown $\theta_{N,j}^{(\alpha,\beta)} = (r_{N,j}^{(\alpha,\beta)}+1)/2$ and their corresponding Christoffel numbers are $\vartheta_{N,j}^{(\alpha,\beta)} = (\frac{1}{2})^{\alpha+\beta+1}\varpi_{N,j}^{(\alpha,\beta)}$ for $j=0,1,\ldots,N$. By the properties of the standard Jacobi-Gauss quadrature, if ϕ be a polynomial of degree at most $2N+1$, we have [7]

$$\int_0^1 (1-r)^\alpha r^\beta \phi(r)\mathrm{d}r = \sum_{j=0}^{N} \vartheta_{N,j}^{(\alpha,\beta)} \phi(\theta_{N,j}^{(\alpha,\beta)}).$$

In practice, only the first $(N+1)$ terms shifted Jacobi polynomials are considered.

3 Shifted Jacobi-Gauss Collocation Method

In this section, we consider shifted Jacobi-Gauss collocation method approach to numerically solve a class of boundary value problems on the unit interval which feature a type of exponential and product nonlinearities.

3.1 Exponential nonlinearity

Consider the two-point boundary value problem for the fourth-order nonlinear differential equation with an exponential nonlinearity [13]

$$u''''(r) + 6e^{-4u(r)} = 0, \quad r \in [0, 4-e], \tag{3.4}$$

subject to

$$u(0) = 1,\ u''(0) = -\frac{1}{e^2}, \\ u(4-e) = \ln(4), u''(4-e) = -\frac{1}{16}. \tag{3.5}$$

One of the important step in the collocation method is the choice of collocation points which effect on efficiency. Let us first introduce some basic notation. We set

$$S_N(0,1) = span\{J_0^{(\alpha,\beta)}(r), J_1^{(\alpha,\beta)}(r), ..., J_N^{(\alpha,\beta)}(r)\} \tag{3.6}$$

and we define the discrete inner product and norm as follows:

$$(u,v)_{\chi^{(\alpha,\beta)},N} = \sum_{j=0}^{N} u(\theta_{N,j}^{(\alpha,\beta)}) v(\theta_{N,j}^{(\alpha,\beta)}) \vartheta_{N,j}^{(\alpha,\beta)}, \tag{3.7}$$

$$\|u\|_{\chi^{(\alpha,\beta)},N} = \sqrt{(u,v)_{\chi^{(\alpha,\beta)},N}}.$$

Here $\theta_{N,j}^{(\alpha,\beta)}$ and $\vartheta_{N,j}^{(\alpha,\beta)}$ are the nodes and the corresponding weights of the shifted Jacobi-Gauss quadrature formula on the interval $(0,1)$, respectively. Obviously,

$$(u,v)_{\chi^{(\alpha,\beta)},N} = (u,v)_{\chi^{(\alpha,\beta)}}, \quad \forall u,v \in S_{2N-1}. \tag{3.8}$$

Thus, for any $u \in S_N(0,1)$, the norms $\|u\|_{\chi^{(\alpha,\beta)},N}$ and $\|u\|_{\chi^{(\alpha,\beta)}}$ are equal.

Associating with this quadrature rule, we denote by $I_N^{J(\alpha,\beta)}$ the shifted Jacobi-Gauss interpolation,

$$I_N^{J(\alpha,\beta)} u(\theta_{N,j}^{(\alpha,\beta)}) = u(\theta_{N,j}^{(\alpha,\beta)}), \quad 0 \le j \le N. \tag{3.9}$$

The shifted Jacobi-Gauss collocation method for solving (3.4) and (3.5) is to seek $v_N(x) \in S_N(0,1)$, such that

$$u''''(\theta_{N,j}^{(\alpha,\beta)}) + 6e^{-4u(\theta_{N,j}^{(\alpha,\beta)})} = 0, \tag{3.10}$$
$$j = 0, 1, ..., N,$$
$$u_N(0) = 1,\ u_N''(0) = -\frac{1}{e^2},$$
$$u_N(4-e) = \ln(4),\ u_N''(4-e) = -\frac{1}{16}.$$

We now derive an algorithm for solving (3.5) and (3.4). To do this, let

$$u_N(r) = \sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(r), \\ a = (a_0, a_1, ..., a_N)^T. \tag{3.11}$$

We first approximate $u(r), u'(r), u''(r)$, as (3.11). By substituting these approximation in (3.4), we get

$$\sum_{j=0}^{N} a_j D^4 J_j^{(\alpha,\beta)}(r) \\ + 6e^{-4\sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(r)} = 0. \tag{3.12}$$

Then, by virtue, we deduce that

$$\sum_{j=0}^{N} a_j c_{j1} c_{j2} c_{j3} c_{j4} J_{j-4}^{(\alpha+4,\beta+4)}(r) \tag{3.13}$$
$$+ 6e^{-4\sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(r)} = 0,$$

where $c_{ji} = \alpha + \beta + j + i$.

Also, by substituting (3.11) in (3.5) we obtain

$$\sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(0) = 1,$$
$$\sum_{j=0}^{N} a_j D^2 J_j^{(\alpha,\beta)}(0) = -\frac{1}{e^2},$$
$$\sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(4-e) = \ln(4),$$
$$\sum_{j=0}^{N} a_j D^2 J_j^{(\alpha,\beta)}(4-e) = -\frac{1}{16}. \tag{3.14}$$

To find the solution $u_N(r)$, we first collocate (3.13) at the Jacobi rational roots, yielding

$$\sum_{j=0}^{N} a_j c_{j1} c_{j2} c_{j3} c_{j4} J_{j-4}^{(\alpha+4,\beta+4)}(r) + 6e^{-4\sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(r)} = 0. \quad (3.15)$$

Equation (3.14), after using of virtue of Shifted Jacobi-Gauss polynomial, can be written as

$$\sum_{j=0}^{N} \frac{(-1)^j \Gamma(j+\beta+1)}{\Gamma(j+1)\Gamma(\beta+1)} a_j - 1 = 0,$$

$$\sum_{j=0}^{N} \frac{(-1)^{j-2} c_{j1} c_{j2} \Gamma(j+\beta+1)}{\Gamma(j-1)\Gamma(\beta+3)} a_j + \frac{1}{e^2} = 0,$$

$$\sum_{j=0}^{N} \frac{\Gamma(j+\alpha+1)}{\Gamma(j+1)\Gamma(c_{j1})} \times \sum_{s=0}^{j} \frac{\Gamma(c_{j1}+s)}{\Gamma(\alpha+s+1)} (3-e)^s a_j - \ln(4) = 0,$$

$$\sum_{j=0}^{N} \frac{\Gamma(j+\alpha+1)}{\Gamma(j-1)\Gamma(c_{j3})} \times \sum_{s=0}^{j} \frac{\Gamma(c_{j3}+s)}{\Gamma(\alpha+s+3)} (3-e)^s a_j + \frac{1}{16} = 0. \quad (3.16)$$

Finally, from (3.15) and (3.16), we get a system of nonlinear algebraic equations which can be solved for the unknown coefficients a_j by using any standard iteration technique, like Newton's iteration method. Consequently, $u_N(r)$ given in (3.11) can be evaluated.

3.2 Product nonlinearity

Consider the two-point boundary value problem for the fourth-order nonlinear differential equation with a product nonlinearity [12, 13]

$$u''''(r) + u(r)u'(r) - 4r^7 - 24 = 0, \ r \in [0,1], \quad (3.17)$$

subject to

$$u(0) = 0, \ u'''(0.25) = 6, \ u'(0.5) = 3, \ u(1) = 1. \quad (3.18)$$

The shifted Jacobi-Gauss collocation method for solving (3.17) and (3.18) is to seek $v_N(x) \in S_N(0,1)$, such that

$$u''''(\theta_{N,j}^{(\alpha,\beta)}) + u(\theta_{N,j}^{(\alpha,\beta)})u'(\theta_{N,j}^{(\alpha,\beta)}) - 4(\theta_{N,j}^{(\alpha,\beta)})^7 - 24 = 0, \ j = 0,1,...,N,$$
$$u_N(0) = 0, \ u_N'''(0.25) = 6, \ u_N'(0.5) = 3, \ u_N(1) = 1. \quad (3.19)$$

We now derive an algorithm for solving (3.17) and (3.18). To do this, let

$$u_N(r) = \sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(r), \ a = (a_0, a_1, ..., a_N)^T. \quad (3.20)$$

We first approximate $u(r), u'(r), u''(r)$, as (3.20). By substituting these approximation in (3.17), we get

$$\sum_{j=0}^{N} a_j D^4 J^{(\alpha,\beta)} J_j(r) + \sum_{j=0}^{N} a_j J^{(\alpha,\beta)} J_j(r) \sum_{j=0}^{N} a_j D J^{(\alpha,\beta)} J_j(r) - 4r^7 - 24 = 0. \quad (3.21)$$

Then, by virtue, we deduce that

$$\sum_{j=0}^{N} a_j c_{j1} c_{j2} c_{j3} c_{j4} J^{(\alpha,\beta)} J_{j-4}^{(\alpha+4,\beta+4)}(r) - \sum_{j=0}^{N} a_j J^{(\alpha,\beta)} J_j^{(\alpha,\beta)}(r) \times \sum_{j=0}^{N} a_j c_{j1} a_j J^{(\alpha,\beta)} J_{j-1}^{(\alpha+1,\beta+1)}(r) - 4r^7 - 24 = 0. \quad (3.22)$$

Also, by substituting (3.20) in (3.18) we obtain

$$\sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(0) = 0,$$
$$\sum_{j=0}^{N} a_j D^3 J_j^{(\alpha,\beta)}(0.25) = 6,$$
$$\sum_{j=0}^{N} a_j D^2 J_j^{(\alpha,\beta)}(0.5) = 3,$$
$$\sum_{j=0}^{N} a_j J_j^{(\alpha,\beta)}(1) = 1. \quad (3.23)$$

To find the solution $u_N(r)$, we first collocate (3.22) at the Jacobi rational roots, yielding

$$\sum_{j=0}^{N} a_j c_{j1} c_{j2} c_{j3} c_{j4} J^{(\alpha,\beta)} J_{j-4}^{(\alpha+4,\beta+4)}(r) - \sum_{j=0}^{N} a_j J^{(\alpha,\beta)} J_j^{(\alpha,\beta)}(r) \times \sum_{j=0}^{N} a_j c_{j1} J^{(\alpha,\beta)} J_{j-1}^{(\alpha+1,\beta+1)}(r) - 4r^7 - 24 = 0. \quad (3.24)$$

Equation (3.23), after using of virtue of Shifted Jacobi-Gauss polynomial, can be written as

$$\begin{aligned}
&\sum_{j=0}^{N} \frac{(-1)^j \Gamma(j+\beta+1)}{\Gamma(j+1)\Gamma(\beta+1)} a_j = 0, \\
&\sum_{j=0}^{N} \frac{c_{j1} c_{j2} \Gamma(j+\alpha+1)}{\Gamma(j-1)\Gamma(c_{j3})} \times \sum_{s=0}^{j} \frac{\Gamma(c_{j3}+s)}{\Gamma(\alpha+s+3)} (-0.75)^s a_j - 6 = 0, \\
&\sum_{j=0}^{N} \frac{c_{j1} c_{j2} \Gamma(j+\alpha+1)}{\Gamma(j-1)\Gamma(c_{j3})} \times \sum_{s=0}^{j} \frac{\Gamma(c_{j3}+s)}{\Gamma(\alpha+s+3)} (-0.5)^s a_j - 3 = 0, \\
&\sum_{j=0}^{N} \frac{\Gamma(j+\alpha+1)}{\Gamma(j+1)\Gamma(\alpha+1)} a_j = 0. \qquad (3.25)
\end{aligned}$$

Finally, from (3.24) and (3.25), we get a system of nonlinear algebraic equations which can be solved for the unknown coefficients a_j by using any standard iteration technique, like Newton's iteration method. Consequently, $u_N(r)$ given in (3.20) can be evaluated.

4 Numerical Examples

In this section, we have a comparison between the proposed method explained in pervious sections and some other numerical methods in solving linear and nonlinear differential equations. This comparison shows the validity and applicability of our proposed method.

Example 4.1 *Consider the two-point boundary value problem for the fourth-order nonlinear differential equation with an exponential nonlinearity (3.4) and (3.5), [13, 14]. The exact solution of this problem is* $u(x) = \ln(e + x)$.

The best error obtained in [13] by ADM is 2.3×10^{-5}, approximately. Also, the best error obtained in [14] by RKM is 5.8×10^{-8}. The absolute errors $|u(r) - u_N(r)|$, reported in Table 1, show the accuracy of the present method.

Example 4.2 *Consider the two-point boundary value problem for the fourth-order nonlinear differential equation with a product nonlinearity (3.17) and (3.18), [12, 13, 14]. The exact solution of this problem is* $u(r) = r^4$.

This problem is consider in [12] by Adomian decomposition method (ADM), and in [13] by modified Adomian decomposition method (MADM). The best error obtained in [12] is 1.0×10^{-10}, and in [13] is 5.4×10^{-9}, approximately. Also, the best error obtained in [14] by RKM is 2.2×10^{-14}. The absolute errors $|u(r) - u_N(r)|$, reported in Table 2, show the accuracy of the present method.

Example 4.3 *Consider the following singular fourth order four-point boundary value problem [14, 15]*

$$\begin{aligned}
&\sin(r)(e^r - 1)^2 u''''(r) + 300 e^{\frac{r}{2}} u'(r) \\
&\quad + 200 \sin(\sqrt{r}) u''(r) + r \sinh(r) u'(r) \\
&\quad + r \sin(u(r)) = f(r), \quad r \in [0, 1], \\
&u(0) = 0, \ u(\tfrac{1}{3}) = \sin(\tfrac{1}{3}), \\
&u(\tfrac{2}{3}) = \sin(\tfrac{2}{3}), \ u(1) = \sin(1),
\end{aligned}$$

where

$$f(r) = (-1 + e^r)^2 \sin^2(r) - 2 \sin(\sqrt{r}) \sin(r) - e^{\frac{r}{2}} \cos(r) + r \sin(\sin(r)) + \cos(r) \sinh(r).$$

The exact solution of this problem is $u(r) = \sin(r)$.

The best error obtained in [14] by RKM is 3.5×10^{-8}. Also, the best error obtained in [15] by combining of the homotopy perturbed method (HPM) and RKM is 2.3×10^{-8}. The absolute errors $|u(r) - u_N(r)|$, reported in Table 3, show the accuracy of the present method.

Table 1: Absolute errors with $N = 31$ for Example 4.1.

$\alpha = \beta = -\frac{1}{2}$	$\alpha = \beta = 0$	$\alpha = \beta = \frac{1}{2}$	r
$1. \times 10^{-20}$	$3. \times 10^{-20}$	$1. \times 10^{-19}$	0.0
$7.9516053250797606810 \times 10^{-23}$	$3.3434710028806590559 \times 10^{-22}$	$3.9392630304801456252 \times 10^{-22}$	0.1
$2.0000006108250784288 \times 10^{-19}$	$1.0000215024548864886 \times 10^{-19}$	$2.0000149618669976087 \times 10^{-19}$	0.2
$2.2751751705451664619 \times 10^{-22}$	$9.5176590214182709569 times 10^{-22}$	$2.0000316084519611054 \times 10^{-19}$	0.3
$1.0000042129860206793 \times 10^{-19}$	$1.2097647203036968936 \times 10^{-21}$	$3.0000341664421918505 \times 10^{-19}$	0.4
$1.0000058389525492761 \times 10^{-19}$	$1.4172826887112441484 \times 10^{-21}$	$1.0001412930681577235 \times 10^{-19}$	0.5
$1.0000071832906491791 \times 10^{-19}$	$1.0001219615783720258 \times 10^{-19}$	$2.0000862855539263074 \times 10^{-19}$	0.6
$1.0000079738265055987 \times 10^{-19}$	$1.0001330048377446228 \times 10^{-19}$	$2.0000948144228097547 \times 10^{-19}$	0.7
$3.9983394948964359366 \times 10^{-22}$	$1.6124273746517058476 \times 10^{-21}$	$1.9355975958912418494 \times 10^{-21}$	0.8
$1.0000071268506422337 \times 10^{-19}$	$1.4938250446988961272 \times 10^{-21}$	$5.0000326781265528444 \times 10^{-19}$	0.9
$1.0000054021297995555 \times 10^{-19}$	$1.0000799115193710960 \times 10^{-19}$	$2.0000599932646300490 \times 10^{-19}$	1.0

Table 2: Absolute errors with $N = 7$ for Example 4.2.

$\alpha = \beta = -\frac{1}{2}$	$\alpha = \beta = 0$	$\alpha = \beta = \frac{1}{2}$	r
0.0	0.0	0.0	0.0
1.97×10^{-21}	2.95×10^{-21}	3.8×10^{-22}	0.1
4.00×10^{-21}	7.20×10^{-21}	2.3×10^{-21}	0.2
6.10×10^{-21}	1.23×10^{-20}	6.2×10^{-21}	0.3
9.00×10^{-21}	1.80×10^{-20}	1.1×10^{-20}	0.4
1.30×10^{-20}	2.30×10^{-20}	1.9×10^{-20}	0.5
1.00×10^{-20}	3.00×10^{-20}	3.0×10^{-20}	0.6
1.00×10^{-20}	3.00×10^{-20}	4.0×10^{-20}	0.7
2.00×10^{-20}	3.00×10^{-20}	4.0×10^{-20}	0.8
3.00×10^{-20}	2.00×10^{-20}	6.0×10^{-20}	0.9
3.00×10^{-20}	0.0	2.0×10^{-19}	1.0

Table 3: Absolute errors with $N = 11$ for Example 4.3.

$\alpha = \beta = -\frac{1}{2}$	$\alpha = \beta = 0$	$\alpha = \beta = \frac{1}{2}$	r
$1. \times 10^{-20}$	$1. \times 10^{-20}$	$1. \times 10^{-20}$	0.0
3.5807×10^{-17}	2.12286×10^{-16}	6.59712×10^{-16}	0.1
4.509×10^{-17}	1.0000×10^{-16}	3.4955×10^{-16}	0.2
6.495×10^{-17}	3.786×10^{-17}	1.02×10^{-18}	0.3
4.104×10^{-17}	5.371×10^{-17}	1.1417×10^{-16}	0.4
2.3349×10^{-16}	2.7981×10^{-16}	5.0367×10^{-16}	0.5
3.423×10^{-17}	2.527×10^{-17}	1.6451×10^{-16}	0.6
1.4038×10^{-16}	1.3110×10^{-16}	1.7176×10^{-16}	0.7
4.9256×10^{-16}	7.7813×10^{-16}	1.63389×10^{-15}	0.8
6.9288×10^{-16}	1.24280×10^{-15}	2.63556×10^{-15}	0.9
1.0×10^{-19}	0.0	6.0×10^{-20}	1.0

5 Conclusions

In this article, we have proposed a numerical algorithm to solve a class of boundary value problems. The Shifted Jacobi-Gauss collocation method was developed to solve these problems. We used nodes of the shifted Jacobi-Gauss interpolation on $[0, 1]$. These equations together with initial conditions generate a system of algebraic equations which can be solved using Newtons iterative method. Numerical results were given to show the accuracy and applicability of the presented method. The method is rather robust, hence it may be applied to other type of singular nonlinear boundary value problems with more complicated forms of nonlinearity.

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