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New Arithmetic Operations on Developed Parabolic Fuzzy Numbers and Its Application

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Abstract

In this paper, we have studied the basic arithmetic operations for developed parabolic fuzzy numbers by using the concept of the transmission average, which was already implied in [F. Abbasi et al., A new attitude coupled with fuzzy thinking to fuzzy rings and fields, Journal of Intelligent and Fuzzy Systems, 2015] in its rudimentary form and was finally presented in its fully-fledged form in [F. Abbasi et al., A new and efficient method for elementary fuzzy arithmetic operations on pseudo-geometric fuzzy numbers, Journal of Fuzzy Set Valued Analysis, 2016]. The major advantage of these operations is that they findings are closer to reality than extension principle-based fuzzy arithmetic operations (in the domain of the membership function) or interval arithmetic (in the domain of α-cuts). A technical example is given to illustrate applying the method. The proposed method can model and analyze the fuzzy system reliability in a more flexible and intelligent manner in comparison with the other methods.

Keywords : Fuzzy arithmetic operations; Developed parabolic fuzzy numbers; Extension principle; Fuzzy system reliability; Transmission average.

1 Introduction

IN order to use fuzzy numbers and relations in any intelligent system one must be able to perform arithmetic operations, addition, subtraction, multiplication and division, employing these fuzzy quantities, the process of which is called fuzzy arithmetic. The usual arithmetic operations on real numbers can be extended to the ones defined on fuzzy numbers by means of Zadeh's EP [12].

It is well known that we have some problem

in subtraction operator, division operator and obtaining the membership functions using of of the fuzzy arithmetic operations based on the extension principle (in the domain of the membership function) or the interval arithmetics (in the domain of the α- cuts) [10]. Although with the revised definitions on subtraction and division [8, 9], usage of an interval arithmetic for fuzzy operators have been permitted, because it always exists, but its not efficient, it means that results support is major agent(dependence effect)and also complex calculations of interval arithmetic in determining the membership function of operators based on the extension principle, are not yet resolved. Hence, most researchers apply interval operators on triangular or trapezoidal

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fuzzy numbers; otherwise the results could not be easily obtained, as they would become dependent upon the max-min of non-linear functions. Therefore, we eliminated such deficiency with the fuzzy arithmetic operations based on TA [1, 2].

In this paper we propose the new fuzzy arithmetic operations based on TA on developed parabolic fuzzy numbers. Then, we analyze the reliability of fuzzy system (particularly, series and parallel system) with independent and nonidentically distributed components, and using the new operations of TA. Finally, An electric network model of a "Dark room" is considered in fuzzy environment. The reliability of components of the proposed model is considered as developed parabolic fuzzy numbers. The paper is organized as follows. In section 2, we present the basic definitions and concepts related to the subject. The new fuzzy arithmetic operators based on TA proposed on developed parabolic fuzzy numbers and its basic properties are investigated in section 3. A technical example to illustrate applying the method and comparing the results of the new method with the previous methods is given in section 4. Finally, conclusions and future research are drawn in section 5.

2   Preliminaries and notations

In this subsection, some notations and background about the concept are brought.

Definition 2.1 *[13] Let, $X = X_1 \times \cdots \times X_n$, be the Cartesian product of universes, and $A = A_1 \times \cdots \times A_n$ be fuzzy sets in each universe respectively. Also let Y be another universe and $B \in Y$ be a fuzzy set such that $B = f(A_1 \times \cdots \times A_n)$, where $f : X \to Y$ is a monotonic mapping. Then Zadeh's extension principle is defined as follows:*

$$\mu_B(y) = \sup \min(\mu_{A_1}(x_1), ..., \mu_{A_n}(x_1)) \qquad (2.1)$$
$$(x_1 \times \cdots \times x_n) \in f^{-1}(y)$$

where $f^{-1}(y)$ is the inverse function of $y = f(x_1 \times \cdots \times x_n)$).

Definition 2.2 *[13] (α -cut Representation) A fuzzy set, A can be represented (decomposed) as,*

$$A = \bigcup_{\alpha \in (0,1]} \alpha.A_\alpha, \qquad A_\alpha = [\underline{A}_\alpha, \overline{A}_\alpha]. \qquad (2.2)$$

where,

$$A_\alpha = \{x | \mu_A(x) \geq \alpha\}, \qquad (2.3)$$
$$\alpha.A_\alpha = \{(x, \alpha) | x \in A_\alpha\}.$$

It is easy to check that the following holds:

$$\mu_A(x) = sup_{x \in A_\alpha} \alpha. \qquad (2.4)$$

Definition 2.3 *[7] (Fuzzy number) A fuzzy set A in $\mathbb{R}$ is called a fuzzy number if it satisfies the following conditions:*

(i) A is normal,
(ii) A_α is a closed interval for every $\alpha \in (0, 1]$,
(iii) the support of A is bounded.

According to definition of fuzzy number mentioned above and the our emphasis on non-triangular and trapezoidal fuzzy numbers, we use the definition of a developed parabolic fuzzy number as follows:

Definition 2.4 *(developed parabolic fuzzy number) A fuzzy number $\widetilde{A}$ is called a developed parabolic fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} (\frac{x-\underline{a}}{a_1-\underline{a}})^2, & \underline{a} \leq x \leq a_1, \\ 1, & a_1 \leq x \leq a_2, \\ (\frac{\overline{a}-x}{\overline{a}-a_2})^2, & a_2 \leq x \leq \overline{a}, \\ 0, & \text{otherwise.} \end{cases} \qquad (2.5)$$

The developed parabolic fuzzy number $\widetilde{A}$ is denoted by

$$\widetilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)),$$

where $l_{\widetilde{A}}(x)$ and $r_{\widetilde{A}}(x)$ are parabolic functions. A parabolic fuzzy number is a particular developed parabolic fuzzy number, when the $a_1 = a_2$. The parabolic fuzzy number $\widetilde{A}$ is denoted by

$$\widetilde{A} = (\underline{a}, a, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)).$$

Fig.. 1 Graphical representation of a developed parabolic fuzzy number
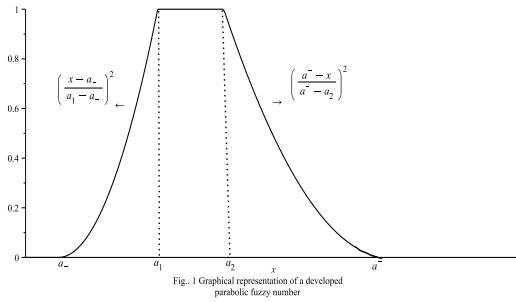
Figure 1

3 The new fuzzy arithmetic operations based on TA on developed parabolic fuzzy numbers

As regards fuzzy arithmetic operations using of the extension principle (in the domain of the membership function) or the interval arithmetics (in the domain of the α- cuts), we have some problem in subtraction operator, division operator and obtaining the membership functions of operators. Although with the revised definitions on subtraction and division, usage of an interval arithmetic for fuzzy operators have been permitted, because it always exists, but its not efficient, it means that results support is major agent(dependence effect)and also complex calculations of interval arithmetic in determining the membership function of operators based on the extension principle, are not yet resolved. Therefore, we eliminated such deficiency with the fuzzy arithmetic operations based on TA [1, 2].
We define fuzzy arithmetic operations based on TA for addition, subtraction, multiplication and division on developed parabolic fuzzy numbers as follows:

Consider two developed parabolic fuzzy numbers,

$$\widetilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)),$$

$$\widetilde{B} = (\underline{b}, b_1, b_2, \overline{b}, l_{\widetilde{B}}(x), r_{\widetilde{B}}(x)),$$

with the following α-cut forms:

$$A = \bigcup_{\alpha \in (0,1]} \alpha . A_\alpha, \qquad A_\alpha = [\underline{A}_\alpha, \overline{A}_\alpha],$$

$$\underline{A}_\alpha = \underline{a} + (a_1 - \underline{a})\sqrt{\alpha}, \quad \overline{A}_\alpha = \overline{a} - (\overline{a} - a_2)\sqrt{\alpha},$$
$$B = \bigcup_{\alpha \in (0,1]} \alpha . B_\alpha, \qquad B_\alpha = [\underline{B}_\alpha, \overline{B}_\alpha],$$

$$\underline{B}_\alpha = \underline{b} + (b_1 - \underline{b})\sqrt{\alpha}, \quad \overline{B}_\alpha = \overline{b} - (\overline{b} - b_2)\sqrt{\alpha}.$$
Let

$$\phi = \frac{a_1 + a_2}{2}, \qquad \psi = \frac{b_1 + b_2}{2},$$

then,

i) addition,

$$A + B = \bigcup_{\alpha \in (0,1]} \alpha . (A + B)_\alpha,$$
$$(A + B)_\alpha = [\underline{(A + B)}_\alpha, \overline{(A + B)}_\alpha],$$
(3.6)

where,

$$\underline{(A + B)}_\alpha = \frac{\phi + \psi}{2} + (\frac{\underline{A}_\alpha + \underline{B}_\alpha}{2}),$$
$$\overline{(A + B)}_\alpha = \frac{\phi + \psi}{2} + (\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2}).$$
(3.7)

ii) subtraction,

Firstly,

$$-B = \bigcup_{\alpha \in (0,1]} \alpha . (-B)_\alpha, (-B)_\alpha = [\underline{(-B)}_\alpha, \overline{(-B)}_\alpha],$$
(3.8)

where,

$$\underline{(-B)}_\alpha = -2\psi + \underline{B}_\alpha,$$
$$\overline{(-B)}_\alpha = -2\psi + \overline{A}_\alpha.$$
(3.9)

finally,

$$A - B = A + (-B),$$

$$A - B = \bigcup_{\alpha \in (0,1]} \alpha . (A - B)_\alpha,$$
$$(A - B)_\alpha = [\underline{(A - B)}_\alpha, \overline{(A - B)}_\alpha],$$
(3.10)

where,

$$\underline{(A - B)}_\alpha = \frac{\phi - 3\psi}{2} + (\frac{\underline{A}_\alpha + \underline{B}_\alpha}{2}),$$
$$\overline{(A - B)}_\alpha = \frac{\phi - 3\psi}{2} + (\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2}).$$
(3.11)

iii) multiplication,

$$A.B = \bigcup_{\alpha \in (0,1]} \alpha.(A.B)_\alpha, \qquad (3.12)$$

$$(A.B)_\alpha = [\underline{(A.B)}_\alpha, \overline{(A.B)}_\alpha],$$

where,

iv) division,

Firstly,

$$B^{-1} = \bigcup_{\alpha \in (0,1]} \alpha.(B^{-1})_\alpha,$$

$$(B^{-1})_\alpha = [\underline{(B^{-1})}_\alpha, \overline{(B^{-1})}_\alpha], \qquad (3.14)$$

where,

$$\underline{(B^{-1})}_\alpha = (\frac{1}{\psi^2})\underline{B}_\alpha,$$
$$\overline{(B^{-1})}_\alpha = (\frac{1}{\psi^2})\overline{B}_\alpha. \qquad (3.15)$$

finally,

$$A.B^{-1} = \bigcup_{\alpha \in (0,1]} \alpha.(A.B^{-1})_\alpha,$$

$$(A.B^{-1})_\alpha = [\underline{(A.B^{-1})}_\alpha, \overline{(A.B^{-1})}_\alpha], \qquad (3.16)$$

where,

Remark 3.1 *Division on developed parabolic fuzzy number*

$\tilde{0} = (a_1, -a, a, a_2, \underline{0}(x), \overline{0}(x))$ is not able to define.

4 Theorems and Properties

In this section, some lemma and theorem about the concept are brought. Since the developed parabolic fuzzy numbers are special case of pseudo-geometric fuzzy numbers, so we have the following lemma and theorems from the [2] for the developed parabolic fuzzy numbers.

Theorem 4.1 *(Lack dependence effect) The results support of fuzzy arithmetic operations based on TA (in the domain of the transmission average of support) are smaller than fuzzy arithmetic operations based on the EP (in the domain of the membership function) and the interval arithmetic (in the domain of the α-cuts).*

$$[\underline{(A.B)}_\alpha, \overline{(A.B)}_\alpha] = \begin{cases} [(\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha, (\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha], & \phi \geq 0, \psi \geq 0, \\ [(\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha, (\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha], & \phi \geq 0, \psi \leq 0, \\ [(\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha, (\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha], & \phi \leq 0, \psi \geq 0, \\ [(\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha, (\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha], & \phi \leq 0, \psi \leq 0, \end{cases}$$

$$(3.13)$$

$$[\underline{(A.B^{-1})}_\alpha, \overline{(A.B^{-1})}_\alpha] = \begin{cases} [(\tfrac{1}{2\psi})\underline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\underline{B}_\alpha, (\tfrac{1}{2\psi})\overline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\overline{B}_\alpha], \phi \geq 0, \psi > 0, \\ [(\tfrac{1}{2\psi})\overline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\underline{B}_\alpha, (\tfrac{1}{2\psi})\underline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\overline{B}_\alpha], \phi \geq 0, \psi < 0, \\ [(\tfrac{1}{2\psi})\overline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\overline{B}_\alpha, (\tfrac{1}{2\psi})\underline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\underline{B}_\alpha], \phi \leq 0, \psi < 0, \\ [(\tfrac{1}{2\psi})\underline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\overline{B}_\alpha, (\tfrac{1}{2\psi})\overline{A}_\alpha + (\tfrac{\phi}{2\psi^2})\underline{B}_\alpha], \phi \leq 0, \psi > 0. \end{cases} \tag{3.17}$$

Theorem 4.2 *(Fuzzy neutral elements) Let $F_C p(\mathbb{R})$ is a set of developed parabolic fuzzy numbers defined on set of real numbers, then*

$\forall \tilde{A} \; \exists \tilde{0}_{\tilde{A}}$ *such that* $\widetilde{A + 0}_{\tilde{A}} = \widetilde{0_{\tilde{A}} + A} = \tilde{A}$ *and* $\widetilde{A - A} = \tilde{0}_{\tilde{A}}$,
$\forall \tilde{A} \exists \tilde{1}_{\tilde{A}}$ *such that* $\widetilde{A.1}_{\tilde{A}} = \widetilde{1_{\tilde{A}}.A} = \tilde{A}$ *and* $\widetilde{A.A^{-1}} = \tilde{1}_{\tilde{A}}$.

Remark 4.1 *Let* $\widetilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x))$, $a = \frac{a_1 + a_2}{2}$, *then,*

$$\tilde{0}_{\tilde{A}} =$$

$$(\underline{a} - a, a_1 - a, a_2 - a, \overline{a} - a, l_{\widetilde{A}}(x + a), r_{\widetilde{A}}(x + a)), \tag{4.18}$$

and,

$$\tilde{1}_{\tilde{A}} = \{ \; (\tfrac{\underline{a}}{a}, \tfrac{a_1}{a}, \tfrac{a_2}{a}, \tfrac{\overline{a}}{a}, l_{\widetilde{A}}(a.x), r_{\widetilde{A}}(a.x)), \tag{4.19}$$

Lemma 4.1 *Let* $\tilde{A}, \tilde{B}$ *are the developed parabolic fuzzy numbers, then*
(i) $\widetilde{A + B} = \widetilde{B + A}$,
(ii) $\widetilde{A.B} = \widetilde{B.A}$,
(iii) $\widetilde{-(-A)} = \tilde{A}$,
(iv) $\widetilde{-(A + B)} = \widetilde{-A - B}$,
(v) $\widetilde{A + A} = 2.\tilde{A}$.

Definition 4.1 *(Fuzzy approximation) Let,*
$\widetilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x))$, $a = \frac{a_1 + a_2}{2}$,
$\widetilde{B} = (\underline{b}, b_1, b_2, \overline{a}, l_{\widetilde{B}}(x), r_{\widetilde{B}}(x))$, $b = \frac{b_1 + b_2}{2}$,

then
$$\tilde{A} \cong \tilde{B} \; if \; and \; only \; if \; a = b.$$

Lemma 4.2 *Let* $\tilde{A}, \tilde{B}, \tilde{C}$ *are the developed parabolic fuzzy numbers, then*

(i) $\widetilde{0 + A} \cong \tilde{A}$,
(ii) $\widetilde{1.A} \cong \tilde{A}$,
(iii) $\widetilde{A + A} + A \cong 3.\tilde{A}$,
(iv) $\widetilde{A + (B + C)} \cong \widetilde{(A + B)} + C$,
(v) $\widetilde{A.(B.C)} \cong \widetilde{(A.B).C}$,

(vi) $A.\widetilde{(B + C)} \cong \widetilde{(A.B) + (A.C)}$,

(vii)$\widetilde{(A + B)}.C \cong \widetilde{(A.C) + (B.C)}$.

In the following, we provided several numerical samples to illustrate the application of the proposed method on developed parabolic fuzzy numbers. We also compared the results of the new method with the previous methods.

Example 4.1 *In this example, we compare the results TA method with EP (α-cut) method.*

Let $A = (0, 3, 5, 7, (\frac{x-0}{3-0})^2, (\frac{7-x}{7-5})^2)$, $B = (3, 4, 6, 7, (\frac{x-3}{4-3})^2, (\frac{7-x}{7-6})^2)$ with the following α-cut forms:

$$A = \bigcup_{\alpha \in (0,1]} \alpha.A_\alpha, A_\alpha = [3\sqrt{\alpha}, 7 - 2\sqrt{\alpha}],$$

$$B = \bigcup_{\alpha \in (0,1]} \alpha.B_\alpha, B_\alpha = [3 + \sqrt{\alpha}, 7 - \sqrt{\alpha}],$$

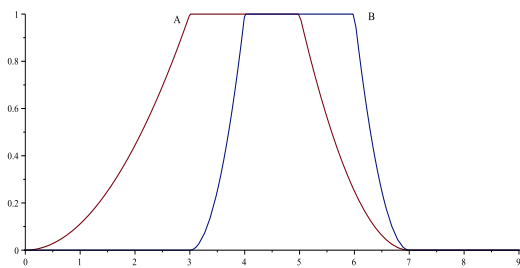Then using of the elementary fuzzy arithmetic

Fig. 2. The developed parabolic fuzzy numbers in the example 5.1

Figure 2

operations based on the EP (α -cut) and TA, we get:

Based on the EP (α -cut):

$A + B = \bigcup_{\alpha \in (0,1]} \alpha.(A + B)_\alpha$, $\quad (A + B)_\alpha = [3 + 4\sqrt{\alpha}, 14 - 3\sqrt{\alpha}]$,

$- B = \bigcup_{\alpha \in (0,1]} \alpha.(-B)_\alpha$, $\quad (-B)_\alpha = [-7 + \sqrt{\alpha}, -3 - \sqrt{\alpha}]$,

$A - B = \bigcup_{\alpha \in (0,1]} \alpha.(A - B)_\alpha$, $\quad (A - B)_\alpha = [-7 + 4\sqrt{\alpha}, 4 - 3\sqrt{\alpha}]$,

$A.B = \bigcup_{\alpha \in (0,1]} \alpha.(A.B)_\alpha$, $\quad (A.B)_\alpha = [12\sqrt{\alpha}, 49 - 19\sqrt{\alpha}]$,

$B^{-1} = \bigcup_{\alpha \in (0,1]} \alpha.(B^{-1})_\alpha$, $\quad (B^{-1})_\alpha = [\frac{1}{7} + \frac{1}{42}\sqrt{\alpha}, \frac{1}{3} - \frac{1}{12}\sqrt{\alpha}]$,

$A.B^{-1} = \bigcup_{\alpha \in (0,1]} \alpha.(A.B^{-1})_\alpha$, $\quad (A.B^{-1})_\alpha = [\frac{1}{2}\sqrt{\alpha}, \frac{7}{3} - \frac{13}{12}\sqrt{\alpha}]$.

Based on the TA:

$A + B = \bigcup_{\alpha \in (0,1]} \alpha.(A + B)_\alpha$, $\quad (A + B)_\alpha = [6 + 2\sqrt{\alpha}, \frac{23 - 3\sqrt{\alpha}}{2}]$,

$- B = \bigcup_{\alpha \in (0,1]} \alpha.(-B)_\alpha$, $\quad (-B)_\alpha = [-7 + \sqrt{\alpha}, -3 - \sqrt{\alpha}]$,

$A - B = \bigcup_{\alpha \in (0,1]} \alpha.(A - B)_\alpha$, $\quad (A - B)_\alpha = [-4 + 2\sqrt{\alpha}, \frac{3 - 3\sqrt{\alpha}}{2}]$,

$A.B = \bigcup_{\alpha \in (0,1]} \alpha.(A.B)_\alpha$, $\quad (A.B)_\alpha = [\frac{15}{2}\sqrt{\alpha} + 2(3 + \sqrt{\alpha}), \frac{5}{2}(7 - 2\sqrt{\alpha}) + 2(7 - \sqrt{\alpha})]$,

$B^{-1} = \bigcup_{\alpha \in (0,1]} \alpha.(B^{-1})_\alpha$, $\quad (B^{-1})_\alpha = [\frac{1}{25}(3 + \sqrt{\alpha}), \frac{1}{25}(7 - \sqrt{\alpha})]$,

$A.B^{-1} = \bigcup_{\alpha \in (0,1]} \alpha.(A.B^{-1})_\alpha$, $(A.B^{-1})_\alpha = [\frac{3}{10}\sqrt{\alpha} + \frac{2}{25}(3 + \sqrt{\alpha}), \frac{1}{10}(7 - 2\sqrt{\alpha}) + \frac{2}{25}(7 - \sqrt{\alpha})]$. The graphical comparison is shown in figures 3, 4, 5 and 6.

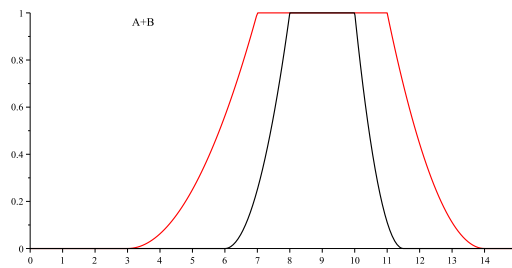

Fig. 3. The red graph is based on the extension principle (α-cut). The black graph is based on the transmission average.

Figure 3

Example 4.2 *(Ambiguity of the area and perimeter) Let the length and breadth of a rectangle are two developed parabolic fuzzy numbers $A = (1, 2, 6, 8, (\frac{x-1}{2-1})^2, (\frac{8-x}{8-6})^2)$, and $B = (0, 2, 4, 7, (\frac{x-0}{2-0})^2, (\frac{7-x}{7-4})^2)$, then the area and perimeter of the rectangle are given by $A.B$, $2(A + B)$ respectively.*

Fig. 4. The red graph is based on the extension principle (α-cut). The black graph is based on the transmission average.

Figure 4

Fig. 5. The red graph is based on the extension principle (α-cut). The black graph is based on the transmission average.

Figure 5

Fig. 6. The red graph is based on the extension principle (α-cut). The black graph is based on the transmission average.
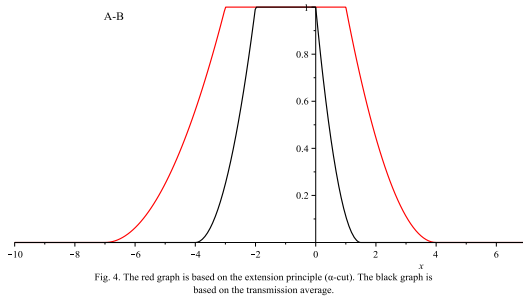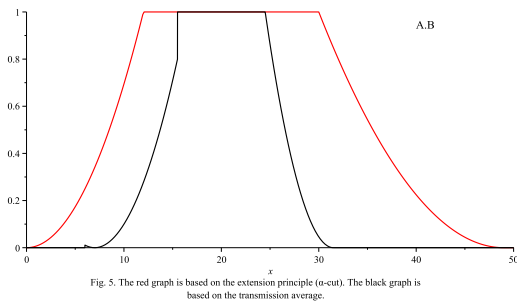
Figure 6

Now based on the TA, we obtain the ambiguity of the area and perimeter, as follows:

Area=$(\frac{3}{2}, 7, 17, 26, (\frac{x-\frac{3}{2}}{7-\frac{3}{2}})^2, (\frac{26-x}{26-17})^2)$,

Perimeter=$(11, \frac{25}{2}, \frac{31}{2}, 18, (\frac{x-11}{\frac{25}{2}-11})^2, (\frac{18-x}{18-\frac{31}{2}})^2)$.

5 A Technical Example

Assume that a windowless room contains one switch and four light bulbs and the switch can only fail to close. A fault tree for the top event Dark room (i.e., no light in the room) is shown in figure 7. In this example, we show an electric network model of a "Dark room" in fuzzy environment. The reliability of components of the proposed model is considered as developed parabolic fuzzy numbers. At first, some notations and background about the concept are brought.

5.1 Fault tree analysis

The concept of fault tree analysis (FTA) was developed in 1962 at Bell telephone laboratories. FTA is a logical and diagrammatic method which has been extensively used to evaluate the probability of an accident resulting from sequences and
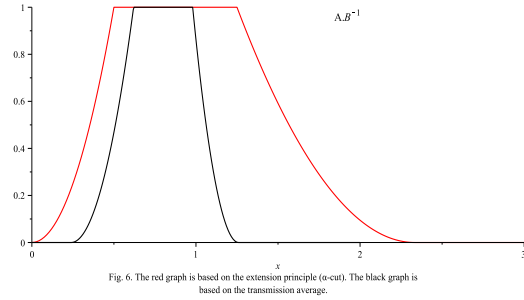
combinations of faults and failure events. A typical fault tree consists of the top event, the basic events, and the logic gates. Gates reflect relationships between events. During the design process of system, the logic diagram (fault tree) is drawn to analyze the potential factors (hard-ware, software, environment, human factor) leading to a system failure. The probability of system failure is calculated based on the known combinations and probabilities of basic events. Several investigators pay attention to applying the fuzzy sets theory to reliability analysis [4, 5, 6, 11].

5.2 Fuzzy operators based on TA of fault tree analysis

During the fuzzy fault tree analysis, the probabilities of basic events are described as fuzzy numbers and the traditional logic gate operators are replaced by fuzzy logic gate operators to obtain the fuzzy probability of the top event.

In this subsection, we present a new method for analyzing fuzzy system reliability based on TA, where the reliability of the components of a system is represented by developed parabolic fuzzy number.

Lemma 5.1 *Let* $A_1, A_2, ..., A_n$ *be developed parabolic fuzzy numbers as follows:*
$A_i = (\underline{a_i}, a_{1i}, a_{2i}, \overline{a_i}, l_{A_i}(x), r_{A_i}(x))$, $a_i = \frac{a_{1i}+a_{2i}}{2}$, $(a_i > 0)$,
with the following α-cut forms:
$A_i = \bigcup_{\alpha \in (0,1]} \alpha.A_{i\alpha}$,
$A_{i\alpha} = [l_{Ai}^{-1}(\alpha), r_{Ai}^{-1}(\alpha))]$,
$l_{Ai}^{-1}(\alpha) = \underline{a_i} + (a_{1i} - \underline{a_i})\sqrt{\alpha}$,
$r_{Ai}^{-1}(\alpha) = \overline{a_i} - (\overline{a_i} - a_{2i})\sqrt{\alpha}$,
then,
1)
$\prod_{i=1}^{n} A_i = \bigcup_{\alpha \in (0,1]} \alpha. \prod_{i=1}^{n} A_{i\alpha}$,

$\prod_{i=1}^n A_{i\alpha} = [l_{\prod_{i=1}^n Ai}^{-1}(\alpha), r_{\prod_{i=1}^n Ai}^{-1}(\alpha)],$
where,
$l_{\prod_{i=1}^n Ai}^{-1}(\alpha) =$
$\sum_{k=0}^{n-2}(\frac{\prod_{i=1, i \neq n-k}^n a_i}{2^{k+1}})l_{A(n-k)}^{-1}(\alpha) + (\frac{\prod_{i=2}^n a_i}{2^{n-1}})l_{A1}^{-1}(\alpha),$
$r_{\prod_{i=1}^n Ai}^{-1}(\alpha)) =$
$\sum_{k=0}^{n-2}(\frac{\prod_{i=1, i \neq n-k}^n a_i}{2^{k+1}})r_{A(n-k)}^{-1}(\alpha) + (\frac{\prod_{i=2}^n a_i}{2^{n-1}})r_{A1}^{-1}(\alpha),$

2)
$1 - A_i = \bigcup_{\alpha \in (0,1]} \alpha.(1-A)_{i\alpha},$
$(1-A)_{i\alpha} = [l_{(1-A)i}^{-1}(\alpha), r_{(1-A)i}^{-1}(\alpha)],$
where,
$l_{(1-A)i}^{-1}(\alpha) = \frac{1}{2}(2 - 3a_i + l_{Ai}^{-1}(\alpha)),$
$r_{(1-A)i}^{-1}(\alpha) = \frac{1}{2}(2 - 3a_i + r_{Ai}^{-1}(\alpha)).$

Proof. *We have the above cases, by mathematical induction and according to the fuzzy arithmetic operations of TA on developed parabolic fuzzy numbers.*

Consider a serial system shown in Fig. 8, where the reliability R_i of component x_i is represented by a developed parabolic fuzzy number defined in the universe of discourse [0, 1]: $R_i =$
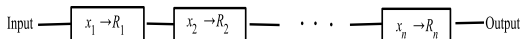

Fig. 8. Configuration of a serial system.

Figure 7

$\bigcup_{\alpha \in (0,1]} \alpha.R_{i\alpha}, \qquad R_{i\alpha} = [l_{Ri}^{-1}(\alpha), r_{Ri}^{-1}(\alpha)],$
$l_{Ri}^{-1}(\alpha) = \underline{r_i} + (r_{1i} - \underline{r_i})\sqrt{\alpha}, \qquad r_{Ri}^{-1}(\alpha) = \overline{r_i} - (\overline{r_i} - r_{2i})\sqrt{\alpha},$
$r_i = \frac{r_{1i}+r_{2i}}{2}.$

Then, the reliability R of the serial system can be evaluated by the (5.1) lemma as follows:
$R = R_1.R_2. \ldots .R_n$
$= \prod_{i=1}^n R_i$
$= \bigcup_{\alpha \in (0,1]} \alpha. \prod_{i=1}^n R_{i\alpha}, \qquad \prod_{i=1}^n R_{i\alpha} =$
$[l_{\prod_{i=1}^n R_i}^{-1}(\alpha), r_{\prod_{i=1}^n R_i}^{-1}(\alpha)],$

where,
$l_{\prod_{i=1}^n Ri}^{-1}(\alpha) =$
$\sum_{k=0}^{n-2}(\frac{\prod_{i=1, i \neq n-k}^n r_i}{2^{k+1}})l_{R(n-k)}^{-1}(\alpha) + (\frac{\prod_{i=2}^n r_i}{2^{n-1}})l_{R1}^{-1}(\alpha),$
$r_{\prod_{i=1}^n Ri}^{-1}(\alpha)) =$
$\sum_{k=0}^{n-2}(\frac{\prod_{i=1, i \neq n-k}^n r_i}{2^{k+1}})r_{R(n-k)}^{-1}(\alpha) + (\frac{\prod_{i=2}^n r_i}{2^{n-1}})r_{R1}^{-1}(\alpha),$
Furthermore, consider the parallel system shown in Fig. 9, where the reliability R_i of component x_i is represented by a developed parabolic fuzzy number defined in the universe of discourse [0, 1]: $R_i = \bigcup_{\alpha \in (0,1]} \alpha.R_{i\alpha}, R_{i\alpha} = [l_{Ri}^{-1}(\alpha), r_{Ri}^{-1}(\alpha)],$

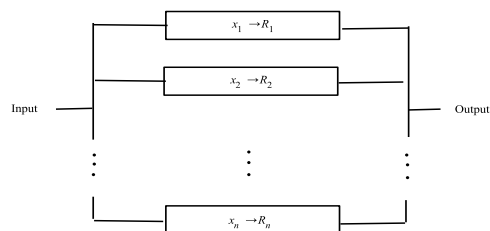Fig. 9. Configuration of a parallel system.

Figure 8

$l_{Ri}^{-1}(\alpha) = \underline{r_i} + (r_{1i} - \underline{r_i})\sqrt{\alpha}, \qquad r_{Ri}^{-1}(\alpha) = \overline{r_i} - (\overline{r_i} - r_{2i})\sqrt{\alpha},$
$r_i = \frac{r_{1i}+r_{2i}}{2}$
Then, the reliability R of the parallel system can be evaluated as follows:
$R = 1 - \prod_{i=1}^n (1-R_i) = \bigcup_{\alpha \in (0,1]} \alpha.R_\alpha,$
$R_\alpha = [\frac{1}{2}\{2 - 3(1 - \prod_{i=1}^n r_i) + l_{\prod_{i=1}^n (1-R)i}^{-1}(\alpha)\}, \frac{1}{2}\{2 - 3(1 - \prod_{i=1}^n r_i) + r_{\prod_{i=1}^n (1-R)i}^{-1}(\alpha)\}],$

where,
$l_{\prod_{i=1}^n (1-R)i}^{-1}(\alpha) =$
$\sum_{k=0}^{n-2}(\frac{\prod_{i=1, i \neq n-k}^n (1-r_i)}{2^{k+1}})l_{(1-R)(n-k)}^{-1}(\alpha) \qquad +$
$(\frac{\prod_{i=2}^n (1-r_i)}{2^{n-1}})l_{(1-R)1}^{-1}(\alpha),$
$r_{\prod_{i=1}^n (1-R)i}^{-1}(\alpha)) =$
$\sum_{k=0}^{n-2}(\frac{\prod_{i=1, i \neq n-k}^n (1-r_i)}{2^{k+1}})r_{(1-R)(n-k)}^{-1}(\alpha) \qquad +$
$(\frac{\prod_{i=2}^n (1-r_i)}{2^{n-1}})r_{(1-R)1}^{-1}(\alpha),$

$l_{(1-R)i}^{-1}(\alpha) = \frac{1}{2}(2 - 3r_i + l_{Ri}^{-1}(\alpha)),$
$r_{(1-R)i}^{-1}(\alpha) = \frac{1}{2}(2 - 3r_i + r_{Ri}^{-1}(\alpha)).$

Finally, we have an electric network model of a "Dark room" by the components of developed parabolic fuzzy numbers as follows:
where the reliability of the components

$X_3, X_4, ..., X_{10}$ are $R_3, R_4, ..., R_{10}$, respectively.

Based on the previous discussion, we can see that the reliability R of the system can be evaluated as follows:

$R = 1 - [\prod_{i=1}^{3}(1 - R_i)]$,

where,

$R_1 = 1 - [\prod_{i=4}^{6}(1 - R_i)]$,

$R_2 = \prod_{i=7}^{10} R_i$.

As regards,

$\prod_{i=4}^{6}(1 - R_i) = \bigcup_{\alpha \in (0,1]} \alpha . \prod_{i=4}^{6}(1 - R)_{i\alpha}$,

$\prod_{i=4}^{6}(1-R)_{i\alpha} = [l^{-1}_{\prod_{i=4}^{6}(1-R)_i}(\alpha), r^{-1}_{\prod_{i=4}^{6}(1-R)_i}(\alpha))]$,

$l^{-1}_{\prod_{i=4}^{6}(1-R)_i}(\alpha) =$

$\sum_{k=0}^{1}(\frac{\prod_{i=4, i\neq 6-k}^{6}(1-r_i)}{2^{k+1}})l^{-1}_{(1-R)(6-k)}(\alpha)$ $+$

$(\frac{\prod_{i=5}^{6}(1-r_i)}{2^2})l^{-1}_{(1-R)4}(\alpha)$,

$r^{-1}_{\prod_{i=4}^{6}(1-R)_i}(\alpha)) =$

$\sum_{k=0}^{1}(\frac{\prod_{i=4, i\neq 6-k}^{6}(1-r_i)}{2^{k+1}})r^{-1}_{(1-R)(6-k)}(\alpha)$ $+$

$(\frac{\prod_{i=5}^{6}(1-r_i)}{2^2})r^{-1}_{(1-R)4}(\alpha)$,

$l^{-1}_{(1-R)i}(\alpha) = \frac{1}{2}(2 - 3r_i + l^{-1}_{Ri}(\alpha))$,

$r^{-1}_{(1-R)i}(\alpha) = \frac{1}{2}(2 - 3r_i + r^{-1}_{Ri}(\alpha))$,

and,

$\prod_{i=7}^{10} R_i = \bigcup_{\alpha \in (0,1]} \alpha . \prod_{i=7}^{10} R_{i\alpha}$,

$\prod_{i=7}^{10} R_{i\alpha} = [l^{-1}_{\prod_{i=7}^{10} R_i}(\alpha), r^{-1}_{\prod_{i=7}^{10} R_i}(\alpha))]$,

where,

$l^{-1}_{\prod_{i=7}^{10} Ri}(\alpha) =$

$\sum_{k=0}^{2}(\frac{\prod_{i=7, i\neq 10-k}^{10} r_i}{2^{k+1}})l^{-1}_{R(10-k)}(\alpha) + (\frac{\prod_{i=8}^{10} r_i}{2^3})l^{-1}_{R7}(\alpha)$,

$r^{-1}_{\prod_{i=7}^{10} Ri}(\alpha)) =$

$\sum_{k=0}^{2}(\frac{\prod_{i=7, i\neq 10-k}^{10} r_i}{2^{k+1}})r^{-1}_{R(10-k)}(\alpha) + (\frac{\prod_{i=8}^{10} r_i}{2^3})r^{-1}_{R7}(\alpha)$,

finally, we can get the system reliability R of a "Dark room". If we required the system to have a fault probability of x_0 as a limit, then, $\alpha \geq \alpha_0$ is necessary, where

$$\alpha_0 = inf\{\alpha | x_0 \notin R_\alpha\}.$$

In this case, we allow the system to be uncertain and flexible to an extend that, the fault probabilities be in the R_{α_0}.

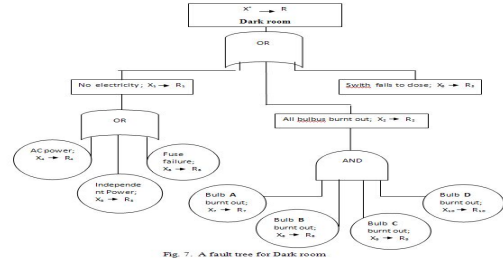It is worth mentioning, the proposed model is applicable for the every "Dark room" with having the numerical data.

Figure 9

6 Conclusion and future research

The arithmetic operations on fuzzy numbers are usually approached either through the use of EP (in the domain of membership function [12]), or interval arithmetic (in the domain of α-cuts) as outlined by Dubois and Prade [3]. The interval arithmetic operations have certain shortcomings in terms of subtraction and division. Although with the revised definitions on subtraction and division, one has become able to employ interval arithmetic for fuzzy operations, because it always exists, it is not efficient, meaning that, the results support is major agent (dependence effect) and we have not yet calculated interval arithmetic in determining the membership function of EP-based operations. Hence, in this paper, we did not consider the fuzzy attitude on the fuzzy arithmetic based on interval arithmetic. Accordingly, with a completely fuzzy thought, we used the new fuzzy arithmetic operators based on TA from [2] for the fuzzy arithmetic operators on developed parabolic fuzzy numbers.

Based on the results of the study, the proposed operations findings are closer to reality than EP-based fuzzy arithmetic operations (in the domain of the membership function) or interval arithmetic (in the domain of α-cuts). The scheme was tested with several numerical examples, all of which vouch for the reliability and efficiency of the proposed method.

We have also discussed the system reliability of a dark room with components having failure distribution as developed parabolic fuzzy numbers. Further, the proposed approach can be ap-

plied to the uncertainty analysis and engineering and mathematical science problems which can be taken for further research.

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