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A New Method for Optimization of Inefficient Cost Units In The Presence of Undesirable Outputs

S. Sadri *, M. Rostamy- Malkhalifeh †‡, N. Shoja §

Abstract

Undesirable Output such as pollution and waste may occasionally occur in the production process, which should be reduced to improve efficiency. In the present study, cost efficiency model is presented in the presence of undesirable output by Inverse Linear Programming, and the desirable cost is calculated in order to achieve cost efficiency for units that are technically efficient but not cost-efficient.

Keywords : Data envelopment analysis; Undesirable output; Cost efficiency; Inverse linear programming.

1 Introduction

Since the past few years, the issue of constraints on resources and production facilities has always been discussed, and it will also affect economic conditions to a greater extent in the future. Hence, it has become an important issue to make optimal use of available resources and enhance efficiency to achieve prosperity and respond to growing needs.

Undesirable data have been investigated in Data Envelopment Analysis. Methods, presented in the face of undesirable output, can be divided into direct and indirect categories. In the direct method, the data are used without changes in the introduced models. This is while in indirect methods, data are changed or transmitted so that they can be used as desirable data in the Data Envelopment Analysis Models. Koopmans [12] has presented the first indirect method in which, the undesirable output values are multiplied by (-1) in order to convert the desirable output. Accordingly, Seiford and Zhu [19] presented a similar method in which, a suitable transmission vector was also combined with undesirable outputs in addition to multiplying in (1). The inverse multiplicative method is another indirect method introduced by Golany and Roll [8] and used by Lovell and Pastor [17], In this method, the inverse outputs were used to convert undesirable outputs to desirable outputs. Also, data envelopment analysis with fixed inputs, undesirable out puts and negative data has been presented by Esmaeili and Rostamy-Malkhalifeh [20]. Sadri, Rostamy-Malkhalifeh and Shoja used Inverse Linear Programming in Cost Efficiency [18]. Khalili-Damghani, Tavana, and Haji-Saami has been developed DEA model in presence of interval data and undesirable outputs [10]. Khoshgova, and Rostamy-Malkhalifeh proposed a model which is capable of calculating

*Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran.
†Corresponding author. mohsen˙rostamy@yahoo.com, Tel:+98(912)2992645.
‡Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.
§Department of Mathematics, Firuozkooh Branch, Islamic Azad University, Firuozkooh, Iran

cost efficiency with integer data in the absence of convexity [11]. Ghiyasi extended the inverse DEA models when price information is available. His proposed techniques are based on the cost efficiency problem. These methods preserve not only technical efficiency but the cost efficiency score of DMUs for an output perturbation [7]. Dash investigated the cost efficiency of Indian life insurance service providers. He applied DEA to assess it and probe the relationship between efficiency and market power in the Indian life insurance industry [4]. Kordrostami, et al. Offered an alternative technique for measuring the efficiency and ranking DMUs which undesirable and fuzzy factors present [13]. No research has yet been conducted in the field of cost efficiency with undesirable data by the Inverse Linear Programming method. There is a feasible solution that is not optimal under the objective function in Inverse linear programming method, and the objective function coefficients must be calculated so that the solution is optimized. Inverse Problems were raised in order to investigate the optimality of the feasible solution. These problems have been widely studied by researchers with geophysical data and have been used in various cases such as traffic control, airplane design, and healthcare. For the first time, Burton and Toint [3] introduced the inverse problem of hybrid optimization. They used the inverse problem for traffic and calculation of the shortest seismic wave path. The problem of Inverse linear programming was firstly introduced by Zhang and Liu [23]. They formulated the Inverse linear programming problem as a new linear problem and showed how to solve the new problem in order to solve the problem inversely. Zhang and Liu [23] also proposed a method for adjusting the cost coefficients of linear programming problem so that the feasible solution is optimized. A group of Inverse Optimization Problems was formulated by Zhang and Yang [22] as a similar linear programming model with two methods of calculation. One of the methods was a general method that generates the columns needed for the Simplex method by solving the original optimization problem; another method was the application of the ellipse method, which can solve a series of inverse problems. Ahuja and Orlin [1] proved that when the optimization problem can be solved with the polynomials linear cost function, then the inverse problem can be solved under the L_1 and L_∞ polynomials norm. Hurkmans [9] used Inverse optimization in radiation therapy. Emrouznejad and Amin [2] used the Inverse Linear Programming Problem in Data Envelopment Analysis and provided an effective method for Data Envelopment Analysis and collective model. However, there are no applications of Inverse Linear Programming in the field of undesirable output. The present paper is structured as follows:

In the second section, the Inverse Linear Programming Problem is expressed and the undesirable output model is expressed the third part. The fourth part discusses about the cost model with Inverse Linear Programming in the presence of undesirable output. Section 5 explains the numerical example for the proposed model. The conclusion is presented in the sixth section.

2 Inverse Linear Programming

Suppose a set of feasible solutions of S, x^o as a given feasible solution and c as the vector of the objective function. Consider the following problem:

$$\begin{array}{ll} \min & cx \\ & Ax = b \\ & x \geq 0 \end{array} \tag{2.1}$$

The Inverse Linear Programming disturbs the objective function vector to $\tilde{c}$ such that the feasible point $x^o$is the optimal solution of the problem relative to the vector $\tilde{c}$ and has the smallest distance with the vector c. Therefore, the goal is to minimize $\|\tilde{c} - c\|$. The Inverse Problem is presented by Zhang and Liu [23] using the optimality conditions as follows:

$$\begin{array}{lll} \min & \|\theta\| & \\ & \pi P_j - \theta_j = C_j, & j \in \bar{J} \\ & \pi P_j - \theta_j = C_j, & j \in J \\ & \theta_j \geq 0, & j \in \bar{J} \end{array} \tag{2.2}$$

Where sets J and $\bar{J}$ are defined as follows:

$$\bar{J} = \left\{ j | x_j^o > 0 \right\}, \ J = \left\{ j | x_j^o = 0 \right\} \tag{2.3}$$

3 Undesirable outputs:

One of the most common methods, introduced for undesirable outputs, is to use the principle of

Table 1: Data for 4 DMU

DMU	Input 1	Input 2	desirable outputs 1	desirable outputs 2	undesirable outputs 1	undesirable outputs 2
1	10	11	21	26	20	15
2	32	23	13	19	20	15
3	12	22	24	20	20	23
4	31	19	21	26	20	15

Table 2: Computational results for 4 DMU

DMU	Model (3.4)	Efficiency Cost (4.6)	Inverse model (4.8)	V_2	V_1
1	1	1	0	17	34
2	0.4781	0.4103	51	0	0
3	1	0.4211	22.96104	17	11.03896
4	0.5788	0.4637	51	0	0

Weak Disposability for outputs in the production possibilities set. The Weak Disposability to outputs from Shepherd's [21] perspective is expressed as follows: "Outputs are Weak Disposability if any proportional contraction is feasible from outputs." The principle is as follows in the presence of undesirable outputs:

$$\forall(x, y, w) \in T \& 0 \leq \theta \leq 1 \Rightarrow (x, \theta y, \theta w) \in T$$

There are two views on the outputs contraction in the Weak Disposability Principle. Fare and Grosskopf [5] provided the same contraction factor for all units, while Kuosmanen [15] claimed that there was no reason to use the same factors and considered separate contraction factors θ_j for each unit, and defined the set of production possibilities as follows:

$$T_k = \Big\{ (x, y, w) | \sum_{j=1}^{n} \lambda_j X_j \leq X \& \sum_{j=1}^{n} \theta_j \lambda_j y_j \geq Y \\ \& \sum_{j=1}^{J} \theta_j \lambda_j w_j = w \& \sum_{j=1}^{n} \lambda_j = 1 \\ \& \lambda_j \geq 0,\ 0 \leq \theta_j \leq 1,\ j = 1, \ldots, n \Big\}$$

Clearly, by setting $\theta_1 = \theta_2 = \cdots = \theta_n$, the Fare and Grosskopf production possibilities sets are equal. The Kuosmanen possibilities set constraints are non-linear. Kuosmanen [15] converted the constraints to linear form by placing $\mu_j = (1 - \theta_j)\lambda_j$ and $\lambda_j = \mu_j + \eta_j$.

Suppose that n input decision unit I are used to generate M desirable and N undesirable output, and the input vectors, the desirable output and the undesirable output are defined as $(i = 1, \ldots, I) x_{ij}$, $(m = 1, \ldots, M) v_{mj}$ and $(d = 1, \ldots, N) w_{dj}$, respectively for the decision unit $(j = 1, \ldots, n)$. Suppose we show the unit under consideration with $_o$, $(j \neq 0)$. The efficiency of the $_o$ in the input-oriented modle is calculated as follows:

$$\begin{array}{lll} \min & \theta & \\ & \sum_{j=1}^{n} (\mu_j + \eta_j) x_{ij} \leq \theta x_{io} & i = 1, \ldots, I \\ & \sum_{j=1}^{n} \eta_j v_{mj} \geq v_{mo} & m = 1, \cdots, M \\ & \sum_{j=1}^{n} \eta_j w_{dj} = w_{do} & d = 1, \ldots, N \\ & \sum_{j=1}^{n} (\mu_j + \eta_j) = 1 & \\ & \eta_j, \mu_j \geq 0 & j = 1, \ldots, J \end{array} \tag{3.4}$$

Kuosmanen and Podinovski [16] also showed with one example that the use of the same contraction factors creates the possibility of non-convex production possibilities set and proved that the set of production possibilities that would be created by taking into account the individual contraction factors, would be the smallest set that is true in the contextual axioms, the Strong Disposability of inputs and outputs and Weak Disposability to all outputs.

4 Inverse programming with undesirable output

The lowest cost with undesirable output is calculated using the following model.

$$\begin{array}{lll} \min & \sum_{i=1}^{I} C_i x_i & \\ & \sum_{j=1}^{n} (\mu_j+\eta_j)x_{ij} \le x_i & i=1,\ldots,I \quad (a) \\ & \sum_{j=1}^{n} \eta_j v_{mj} \ge v_{mo} & m=1,\ldots,M \quad (b) \\ & \sum_{j=1}^{n} \eta_j w_{dj} = w_{do} & d=1,\ldots,N \quad (c) \\ & \sum_{j=1}^{n} (\mu_j+\eta_j) = 1 & \quad (d) \\ & \eta_j,\mu_j \ge 0 & j=1,\ldots,n \end{array} \tag{4.5}$$

Definition 4.1 *Suppose (x^*,η^*,μ^*) is the optimal solution of the model; the cost efficiency with undesirable output is obtained using equation (4.6).*

$$E_C = \frac{\sum_{i=1}^{I} C_{io}x_i^*}{\sum_{i=1}^{I} C_i x_{io}} \tag{4.6}$$

$$0 < E_{Cj} \le 1$$

The unit under evaluation is called cost-effective if and only if E_C= 1 according to Farrell's definition [6].

Suppose (x^0,η^0,μ^0) is an answer to the model (4.5), and e_i is the dual variable of the ith constraint (a), u_m is the duel variable of the mth constraint (b), f_n is the duel variable of the nth constraint (c) and g is the duel variable of the constraint (d) in model (4.5); the model (4.5) dual is obtained as follows:

$$\begin{array}{ll} \max & \sum_{m=1}^{M} u_m v_{mo} + \sum_{d=1}^{N} f_d w_{do} + g \\ & \sum_{m=1}^{M} u_m v_{mj} + \sum_{d=1}^{N} f_d w_{dj} - \sum_{i=1}^{I} e_i x_{ij} + g \le 0 \\ & \qquad j=1,\ldots,n \\ & g - \sum_{i=1}^{I} e_i x_{ij} \le 0 \\ & e_i = C_i \ i=1,\ldots,I \\ & e_i \ge 0 \ i=1,\ldots,I \\ & u_m \ge 0 \ m=1,\ldots,M \end{array} \tag{4.7}$$

The objective function vector C to the vector e is disturbed so that the feasible point is optimal relative to the vector e. So the goal is to minimize $\|C-e\|$. We use the norm one to linearize $\|C-e\|$. Using the Karush-Kuhn-Tucker [14] optimality conditions:

1) (x^0,η^0,μ^0) is the initial feasible solution.

2) (e,u,f,g) is the dual feasible solution.

3) There is a redundant supplementary condition.

By applying the inverse linear programming problem for cost efficiency with an undesirable output, the Inverse programming model with undesirable output is obtained as follows:

$$\begin{array}{ll} \min & \|C-e\| = \min \sum_{i=1}^{I} |C_i - e_i| \\ & \sum_{j=1}^{n} (\mu_j+\eta_j)x_{ij} \le x_{io} \ i=1,\ldots,I \\ & \sum_{j=1}^{n} \eta_j v_{mj} \ge v_{mo} \ m=1,\ldots,M \\ & \sum_{j=1}^{n} \eta_j w_{dj} = w_{do} \ d=1,\ldots,N \\ & \sum_{j=1}^{J} (\mu_j+\eta_j) = 1 \\ & \sum_{m=1}^{M} u_m v_{mj} + \sum_{d=1}^{N} f_d w_{dj} - \sum_{i=1}^{I} e_i x_{ij} + g \le 0 \\ & \qquad j=1,\ldots,n \\ & g - \sum_{i=1}^{I} e_i x_{ij} \le 0 \\ & e_i(\sum_{j=1}^{n} (\mu_j+\eta_j)x_{ij} - x_{io}) = 0, \ i=1,\ldots,I \\ & u_m(\sum_{j=1}^{n} \eta_j v_{mj} - v_{mo}) = 0 \ m=1,\ldots,M \\ & \eta_j(\sum_{m=1}^{M} u_m v_{mj} + \sum_{d=1}^{N} f_d w_{dj} - \sum_{i=1}^{I} e_i x_{ij} + g) = 0 \\ & \qquad j=1,\ldots,n \\ & \mu_j(g - \sum_{i=1}^{I} e_i x_{ij}) = 0 \ j=1,\ldots,n \\ & e_i \ge 0 \ i=1,\ldots,I \\ & u_m \ge 0 \ m=1,\ldots,M \\ & \eta_j,\mu_j \ge 0 \ j=1,\ldots,n \end{array} \tag{4.8}$$

Model (4.8) is a nonlinear model. Using the change of the following variable, it can be transformed into a linear model.

Linear form of the model (4.8) is obtained as follows:

$$\begin{array}{l} t_i = C_i - e_i,\ t_i = \alpha_i - \beta_i,\ i = 1, \dots, I \\ U_{mj} = u_m \eta_j,\ m = 1, \dots, M,\ j = 1, \dots, n \\ M_{ij} = e_i \mu_j,\ i = 1, \dots, I,\ j = 1, \dots, n \\ F_{dj} = f_d \eta_j,\ n = 1, \dots, N,\ j = 1, \dots, n \\ E_{ij} = e_i \eta_j,\ i = 1, \dots, I,\ j = 1, \dots, n \end{array} \tag{4.9}$$

As a result we have:

$$\begin{array}{ll} \min & \sum_{i=1}^{I} \alpha_i + \beta_i \\ & \sum_{j=1}^{n} (\mu_j + \eta_j) x_{ij} \leq x_{io}\ i = 1, \dots, I \\ & \sum_{j=1}^{n} \eta_j v_{mj} \geq v_{mo}\ m = 1, \dots, M \\ & \sum_{j=1}^{n} \eta_j w_{dj} = w_{do}\ d = 1, \dots, N \\ & \sum_{j=1}^{n} (\mu_j + \eta_j) = 1 \\ & \sum_{m=1}^{M} u_m v_{mj} + \sum_{d=1}^{N} f_d w_{dj} - \sum_{i=1}^{I} e_i x_{ij} + g \leq 0 \\ & \qquad j = 1, \dots, n \\ & g - \sum_{i=1}^{I} e_i x_{ij} \leq 0\ j = 1, \dots, J \\ & \sum_{j=1}^{n} (M_{ij} + E_{ij}) x_{ij} - e_i x_{io} = 0,\ i = 1, \dots, I \\ & (\sum_{j=1}^{n} U_{mj} v_{mj} - u_m v_{mo}) = 0\ m = 1, \dots, M \\ & \sum_{m=1}^{M} U_{mj} v_{mj} + \sum_{d=1}^{N} F_{dj} w_{dj} - \sum_{i=1}^{I} E_{ij} x_{ij} + \eta_j g \\ & = 0 \\ & \qquad j = 1, \dots, n \\ & \mu_j g - \sum_{i=1}^{I} M_{ij} x_{ij} = 0\ j = 1, \dots, n \\ & e_i = C_i - \alpha_i + \beta_i\ i = 1, \dots, I \\ & \alpha_i \geq 0\ i = 1, \dots, I\ \beta_i \geq 0\ i = 1, \dots, I \\ & e_i \geq 0\ i = 1, \dots, I \\ & u_m \geq 0\ m = 1, \dots, M \\ & \eta_j,\ \mu_j \geq 0\ j = 1, \dots, J \\ & U_{mj} \geq 0\ m = 1, \dots, M,\ j = 1, \dots, J \\ & M_{ij} \geq 0\ i = 1, \dots, I,\ j = 1, \dots, J \\ & f_d \geq 0\ d = 1, \dots, N \\ & F_{dj} \geq 0\ d = 1, \dots, N,\ j = 1, \dots, J \\ & E_{ij} \geq 0\ i = 1, \dots, I,\ j = 1, \dots, J \end{array} \tag{4.10}$$

Theorem 4.1 *If $_o$ happens to be inefficient, no cost vector will exist for which $_o$ can become cost-efficient.*

Following the principle of reduction ad absurdum, assume that $\bar{C} > 0$ for which $_o$ renders cost-efficient; i.e., for any arbitrary $\begin{pmatrix} X \\ V_o \\ W_o \end{pmatrix} \in PPS$, we have $\bar{C}X \geq \bar{C}X_o$. On the other hand, we know $\begin{pmatrix} \theta^* X_o \\ V_o \\ W_o \end{pmatrix} \in PPS$ and $\theta^* < 1$, hence one can write$\bar{C}\theta^* X_o < \bar{C}X_o$, representing a contradiction.

Theorem 4.2 *If the DMU_o is the strong efficient of CCR, then there is a cost vector such as C, so that the DMU_o is cost effective.*

Consider the following model:

$$\begin{array}{ll} \max & \sum_{m=1}^{M} u_m v_{mo} + \sum_{d=1}^{N} f_d w_{do} + g \\ & \sum_{m=1}^{M} u_m v_{mj} + \sum_{d=1}^{N} f_d w_{dj} - \sum_{i=1}^{I} e_i x_{ij} \\ & +g \leq 0 \qquad j = 1, \dots, n \\ & g - \sum_{i=1}^{I} e_i x_{ij} \leq 0 \\ & e_i = C_i\ i = 1, \dots, I \\ & e_i \geq 0\ i = 1, \dots, I \\ & u_m \geq 0\ m = 1, \dots, M \end{array}$$

Taking (U^*, f^*, e^*, g^*) as the optimum solution of the problem

$$U^* v_o + f^* w_o - e^* x_o + g^* = 0$$

And knowing that $C = e^*$ and $\forall (X, v_o, w_o) \in T$ gives:

$$U^* v_o + f^* w_o - e^* X + g^* \leq 0$$

$$U^* v_o + f^* w_o + g^* = 1,\ e^* x_o = 1$$

As such, we have

$$U^* v_o + f^* w_o - e^* x_o + g^* = 0$$

$$\rightarrow U^* v_o + f^* w_o + g^* = e^* x_o$$

Therefore, we have

$$e^* x_o - e^* X \leq 0 \rightarrow e^* x_o \leq e^* X$$

$$\forall \begin{pmatrix} X \\ Y \end{pmatrix} \in T \rightarrow e^* x_o \leq e^* X$$

5 Numerical Example

Suppose the data of the four hypothetical units is as Table 1. Each one consists of two inputs and desirable outputs and two undesirable outputs and the input cost is the same for all units. Consider the first input cost as 17 and the second input cost as 34.

In the second column of Table 2, the efficiency of the units was checked in accordance with the Kuosmanen efficiency model (3.4), and in the third column, the cost efficiency was calculated according to equation (4.6), and in the fourth column, the value of the objective function of the inverse model (4.8) is calculated for the data of the four units of Table 1. According to the inverse model for undesirable outputs, the costs obtained for the first and second inputs are shown in columns 5 and 6. As shown in Table 2, Units 1 and 3 are technically efficient and, according to the cost-effectiveness model, unit 1 is also cost-effective, and the mentioned costs were calculated by solving the inverse model. According to the Theorem 4.1, unit 3 is technically efficient, but it is not cost-efficient, and by solving the inverse model, a different amount of costs was obtained, that the unit 3 becomes cost-efficient with these costs. While, because the units 2 and 4 are not technically efficient, we cannot find the unit which can be cost effective by solving the inverse model for these units and a zero value is obtained.

According to Case 1, we were able to find a cost for a unit that is not cost-effective so that the evaluated unit becomes cost-effective.

6 Conclusions

The inverse linear programming method in Data Envelopment Analysis in the field of undesirable output has not been used so far. For this reason, the present study provided cost efficiency model using Inverse linear programming in the presence of undesirable outputs, and the proposed model has been used for a numerical example. There are three cases according to the theorems 4.1, 4.2 and the presented example: the first case is the units that are not technically efficient. In these cases, it is not possible to obtain the cost for the purpose of cost efficiency. The second case is related to the units having technical efficiency but they are not cost-effective. In this case, the appropriate cost can be calculated for the purpose of cost-effectiveness using the provided Inverse model. The third mode involves units with technical and cost efficiency. In these cases, the same previous cost is obtained by implementing the inverse model. If the new costs obtained in the second mode are replaced for units in the third mode, the cost efficiency will be achieved again. In the current research, an appropriate cost was obtained for the purpose of cost efficiency due to the provided Inverse Model.

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Soroor Sadri is a faculty member of Islamic Azad University, Abhar Branch, Abhar, Iran. She received her Ph.D. in applied mathematics (Operation Research) in January 2018 from Islamic Azad University, Karaj Branch. She got her Ms.C from Islamic Azad University, Zahedan Branch. Her research areas in mainly Data Envelopment Analysis (DEA). She has published papers on (DEA) in Advances and Applications in Statistics.

Mohsen Rostamy- Malkhalifeh is an Associate professor at the Applied Mathematics Operations Research group in Islamic Azad University in Science and Research Branch in Tehran, Iran. His research interests include performance management, efficiency, Fuzzy and especially those based on the broad set of methods known as Data Envelopment Analysis (DEA). Dr. Rostamy-Malkhalifeh research studies are published in such journals as Applied Mathematics and Computation, International Journal of Fuzzy Systems, International Journal of Applied Decision Sciences, European Journal of Operational Research, Mathematics Scientific Journal, The Scientific World Journal and etc.

Naghi Shoja is an Associate professor of the Applied Mathematics Operations Research group in Islamic Azad University in Firuozkooh, Iran. His research interests include performance management, efficiency and especially those based on the broad set of methods known as Data Envelopment Analysis (DEA). Dr. Shoja research studies are published in such journals as Applied Mathematics and Computation, International Journal of Industrial Mathematics, Optimization letters, Mathematical and Computational Applications, The Scientific World Journal and etc.