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Research Article

Revenue -Profit Measurement in Data Envelopment Analysis with Dynamic Network Structures: A Relational Model

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Abstract

The correlated models are introduced in this article regarding revenue efficiency and profit efficiency in dynamic network production systems. The proposed models are not only applicable in measuring efficiency of divisional, periodical and overall efficiencies, but recognizing the exact sources of inefficiency with respect to revenue and profit efficiencies. Two numerical examples, consisting of twoperiods and , are presented to illustrate these proposed models.

Keywords : DEA; Static network revenue efficiency (SNRE); Dynamic Network revenue Efficiency (DNRE); Static network profit efficiency (SNPE); Dynamic Network profit Efficiency (DNPE).

1 Introduction

DAta envelopment analysis (DEA) is a non parametric analytical method to evaluate the decision making units (DMUs) that consume multiple inputs to produce multiple outputs. This method was introduced by Charnes, Cooper and Rhodes and named (CCR) [1] and then extended by Banker, Charnes and Cooper [2]. The DEA model is commonly applied for technical efficiencyanalysis. Although analysis of Technical efficiency is important, management seeks forinformation regarding revenue and profit aspects of the performance.The procedures for empirical implementations of revenue efficiency measure in DEA are initially introduced by Fare et al. [5]. The authors in [6] developed an overall output price efficiency measure in the context of revenue maximization. Fre et al. [7] provided a decomposition of profit inefficiency inorder to identifythe inefficiency sources,where the profit efficiency is decomposed into technical and allocative inefficiency. Portela and Thanassoulis [8] highlight some drawbacks embodied in the existing approaches and proposed a new measure of profit efficiency system based on a geometric mean of input/output adjustments needed for maximizing profits. Fukuyama [9] developed a new indicator of profit inefficiency, based on decision-makers by choosing the amount of money to be spent on each input and the amount to be earned on each output, rather than choosing the physical quantities of inputs and outputs.Park and Cho [10] developed a simple and practical linear programming model for measuring the highest possible measure of profit efficiency without any price-cost data. Aparicio et al. [11] reveal how DEA can be applied in measuring and decomposing revenue inefficiency, where all sources of technical waste

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in an application context is a concern to assess the Designation of Origin (DO). The reviewed literature reveals that in the majority of studies the revenue and profit efficiency of a DMU is calculated as a black-box. There existfew studies where the internal activities' impact on DMU regarding the revenue and profit efficiency measurement is assessed.In classic DEA models, the internal activities of DMUs are ignored,therefore, the obtained efficiency measurement cannot reflect the underlying performance in an accurate manner.To overcome this drawback, the network DEA (NDEA) model is introduced by Fare and Grosskopf [3] . Modeling network structures has been and is a critical issue of debate addressed by Fare and Grosskopf [3], Lewis and sexton [12, 13], Golany [14], Prieto and Zofio [15], Kao [16, 17, 18], Tone and Tsutsui [19], and Cook et al. [20]. Banihashemi and Tohidi [21] introduced models of cost, revenue and profit efficiency in network DEA. Avkiran [22] applied network slack-based measure (NSBM) for banks operating in the United Arab Emirates. It is argued that NDEAoutperforms the standard DEA technique since it provides adequate detail information for management to identify the specific determinants of inefficiency. In practice, activities between two consecutive time periods usually influence the whole systems overall performance. The dynamic systems, where the operations of a DMU are evaluated based on different periods and two consecutive periods connected by carryovers, areproposed by Fare and Grosskopf [4]. This issue is of a major concern among researches to name a few:Nemoto and Goto [23, 24], Tone and Tsutsui [25, 26]. The importance of calculating the profit and revenue efficiencies of dynamic network systems' issue is assessed in this study. The revenue and profit efficiency scores of dynamic network systems and the correlational model to calculate efficiency scores of its divisions are decomposed in this study. These proposed models are named static network revenue efficiency (SNRE) and static profit efficiency models (SNPE) and dynamic network revenue efficiency (DNRE) and dynamic network profit model (DNPE) to be applied in calculating revenue and profit efficiencies in network structures. By applying these proposed models, the period, divisional and dynamic efficiencies can be determined in a simultaneous manner.

This article organized as follow: the revenue and profit efficiencies are reviewed in Section 22; the Network-structure production systems are presented in Section 3; the network revenue efficiency is discussed in Section 4; The network profit efficiency is discussed in Section 5; the numerical examples are presented in Section 6, and the article is included in Section 7.

2 Revenue and Profit Efficiency

According to Farrell [27] efficiency consists of two elements: technical efficiency (TE) and allocative efficiency (AE). The TE refers to production where the best available technologies are applied and AE refers to allocation of inputs and products to different producers. Together, these efficiencies are named the economic efficiency, (EE). The (EE) is expressed in different manners, depending on how the best available production technology is defined. The (EE) is expressed in terms of cost minimization, revenue maximization or profit maximization. If revenue maximization is assumed, the (EE) is expressed as (RE).In this case, RE constitutes a combination of outputs that generates the maximum possible revenue. In a similar manner, if maximization of profit is of concern, the (EE) is expressed as profit efficiency (PE), that is, the amount of output that maximizes profit. Assume $j = 1, \dots, n$ are the number of DMU and $\{(X_j, Y_j)|j = 1, \dots, n\}\}$, where $X_j = (x1j, \dots, x_{mj})^T\}$ and $Y_j = (y_{1j}, \dots, y_{rj})^T$ are the vectors of input and output values forDMUj , respectively. The problem of maximizing both the revenue and profit is solved through the DEA models, both of which are output-oriented programs. Let p_{ro} be the price of the under evaluated unit (DMUo) output r,then the DEA model of revenue maximization is:

$$
\begin{aligned}
R_0^* = \max & \sum_{r=1}^{s} p_{ro}\bar{y}_{ro} \\
S.T. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{io} \qquad i = 1, \dots, m; \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant \bar{y}_{ro} \qquad r = 1, \dots, s; \\
& \lambda_j, \bar{y}_{ro} \geqslant 0.
\end{aligned}
\tag{2.1}
$$

Model (2.1) is a constant return to scale (CRS). The revenue obtained through the DMU_0 is presented as $\sum_{r=1}^{s} p_{ro}y_{ro}$. The (RE) of DMU_0 (RE_0) is measured through:

$$RE_0 = \frac{\sum_{r=1}^{s} p_{ro}y_{ro}}{\sum_{r=1}^{s} p_{ro}\bar{y}^*_{ro}} \tag{2.2}$$

where, λ^* and $\bar{y}^*_{ro}$ are the optimal solutions of model (2.1) The profit maximization problem is solved as follows:

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} p_{ro}\bar{y}_{ro} - \sum_{i=1}^{m} c_{i0}\bar{x}_{io} \\ S.T. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant \bar{x}_{io} \qquad i = 1,\dots,m; \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant \bar{y}_{ro} \qquad r = 1,\dots,s; \\ & \bar{x}_{io} \leqslant x_{io}, \bar{y}_{ro} \geqslant y_{ro} \\ & \lambda_j \geqslant 0. \end{aligned} \tag{2.3}$$

where, c_{io} is the price of input i and p_{ro} is the price of output r of DMU_0. The profit obtainedby the DMU_0 is $\sum_{r=1}^{s} p_{ro}y_{ro} - \sum_{i=1}^{m} c_{io}x_{io}$ and (PE) of DMU_0 (PE_o) is measured as follows:

$$PE_o = \frac{\sum_{r=1}^{s} p_{ro}y_{ro} - \sum_{i=1}^{m} c_{io}x_{io}}{\sum_{r=1}^{s} p_{ro}\bar{y}^*_{ro} - \sum_{i=1}^{m} c_{io}\bar{x}^*_{io}} \tag{2.4}$$

where, $\bar{x}^*_{io}$ and $\bar{y}^*_{ro}$ are the the optimal solutions of the model (2.3).

3 Network-Structure Production Systems

Classical DEA assumes a production technology as a blackbox where a set of inputs feed a process, which in turn generates the outputs. It computes efficiency scores by disregarding the correlation among divisions within the system. Ignoring intermediate activities in network structures may lead to inaccurate results. In many cases, DMUs consist of a network structure available in hospitals, universities, court houses etc. To assess network model, consider two static and dynamic structure for internal parts of the DMU.
The static network models assess production systems that contain a set of network-structured subsystems.Consider n $DMUs = (j = 1,\dots,n)$ consisting of K divisions $(k = 1,\dots,K)$; Let mk, rkand Lkh be the number of inputs and outputs to division k and the set of links leading from k to divisionh, respectively. The term $x_{jkj} \in R^+(i = 1,\dots,mk; k = 1,\dots,K; j = 1,\dots,n)$) is applied for the input resource i to DMU_j to produce output $y_{rkj} \in R^+(r = 1,\dots,r_k; k = 1,\dots,K; j = 1,\dots,n)$,), that is, the output r from DMU_j. Following this, the term $z_l(kh)_j \in R^+(j = 1,\dots,n; l = 1,\dots,L_{kh})$) is applied to link the intermediate products from division k to division h, Figure 1. As a rule, in practice, time

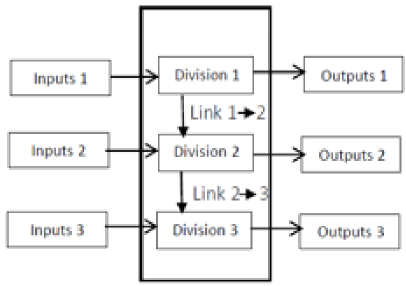

Figure 1: Static structure

is an important factor in improving performance. Therefore, the dynamic effects are incorporated into the SNRE model, Figure 2 For DMU_j, the following structure should be defined:

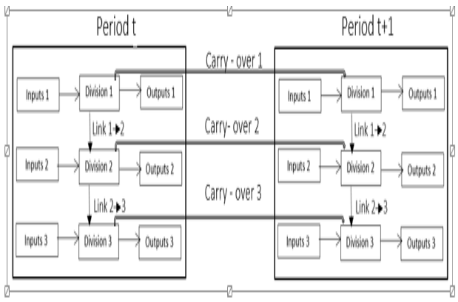

Figure 2: Dynamic structure

(a) $x^t_{jkj} \in R^+(i = 1,\dots,mk; k = 1,\dots,K; j = 1,\dots,n; t = 1,\dots,T)$ is the input resource i to DMU_j for division k in period t.

(b) $y^t_{rkj} \in R^+(r = 1,\dots,rk; k = 1,\dots,K; j = 1,\dots,n; t = 1,\dots,T)$ is the output resource r to DMU_j for division k in period t.

(c) $z_l(kh)^t_j \in R^+(j = 1,\dots,n; l = 1,\dots,Lkh; t = 1,\dots,T)$ is the linking intermediate products of DMU_j from division k to division h in period t.

(d) $z^{(t,t+1)}_{lk_j} \in R^+(j = 1,\dots,n; l = 1,\dots,L_k; k = 1,\dots,K; t = 1,\dots,T-1)$

is the carry-over product l produced by division k in period t and consumed in period $t+1$.
In the next sextions, the above definitions are used to expand revenue and profit efficiencies in network structures.

4 Network Revenue Efficiency

In traditional revenue efficiency model, $DMUs$ are treated as black-boxes, and the revenue efficiency score is computed without considering the interrelation among divisions within the system. Ignoring the intermediate activities in network structures may **lead to inequitable** measures.The revenue efficiency measurement is enhanced by introducing static network revenue efficiency and it is adopted as a dynamic network revenue efficiency.

4.1 Static Network Revenue Efficiency(SNRE)

Let the unit prices of all exogenous outputs be known and let $p_{rkj} \in R^+$ be the price of output r from DMU_j related to division k. Given these assumptions, the SNRE model can be written as follows:

$$\begin{aligned}
\max \quad & \sum_{r=1}^{r_k} p_{rko}\bar{y}_{rko} \\
S.T. \quad & \sum_{j=1}^{n} \lambda_{kj} x_{ikj} \leqslant x_{iko} \qquad i=1,\dots,m_k; k=1,\dots,K \\
& \sum_{j=1}^{n} \lambda_{kj} y_{rkj} \geqslant \bar{y}_{rko} \qquad r=1,\dots,r_k; k=1,\dots,K \\
& \sum_{j=1}^{n} \lambda_{kj} z_{l(kh)_j} \geqslant \tilde{z}_{l(kh)_o} \qquad l=1,\dots,L_{kh}; h,k=1,\dots,K \\
& \sum_{j=1}^{n} \lambda_{kj} z_{l(hk)_j} \leqslant \tilde{z}_{l(kh)_o} \qquad l=1,\dots,L_{hk}; h,k=1,\dots,K \\
& \lambda_{jk}, \bar{y}_{rko} \geqslant 0. \qquad r=1,\dots,r_k; k=1,\dots,K; j=1,\dots,n
\end{aligned} \tag{4.5}$$

where, the $\tilde{z}_{l(kh)_o}$ is the unknown decision variables for the intermediate products. Tone and Tsutsui [19] name this variable the "free link" case.After the optimal solution is obtained through SNRE model, the network revenue efficiency score of division k related to $DMUo(NRE_ko)$ is obtained as:

$$NRE_{ko} = \frac{\sum_{r=1}^{r_k} p_{rko}y_{rko}}{\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}} \tag{4.6}$$

Here, it is claimed that the overall revenue efficiency score is the weighted arithmetic mean of the divisional revenue efficiency scores and presented as:

$$SNRE_o = \sum_{k=1}^{K} w_{ko}.NRE_{ko} \tag{4.7}$$

where, w_ko is the weight of division k and w_ko is the proportion of the maximum revenue from division k of $DMUo$ to maximum revenue of the total $DMUo$, where both meet the following condition:

$$\sum_{k=1}^{K} w_{ko} = 1, \quad w_{ko} \geqslant 0 \tag{4.8}$$

consequently the weight related to division k of $DMUo$ is defined by:

$$w_{ko} = \frac{\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}} \tag{4.9}$$

According to the above definition of w_ko, the overall revenue efficiency is

$$\begin{aligned}
SNRE_o &= \sum_{k=1}^{K} w_{ko}.NRE_{ko} \\
&= \sum_{k=1}^{K} \left(\frac{\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}} \right) \left(\frac{\sum_{r=1}^{r_k} p_{rko}y_{rko}}{\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}} \right) \\
&= \sum_{k=1}^{K} \left(\frac{\sum_{r=1}^{r_k} p_{rko}y_{rko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}} \right) = \\
&\frac{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}y_{rko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko}}
\end{aligned} \tag{4.10}$$

4.2 Dynamic Network Revenue Efficiency (DNRE)

The optimal revenue of division k of DMU_o in period t can be obtained by solving the following linear programming problem:

$$\begin{aligned}
&\max \sum_{r=1}^{r_k} p^t_{rko}\bar{y}^t_{rko} \\
&s.t. \sum_{j=1}^{n} \lambda^t_{kj} x^t_{ikj} \leqslant x^t_{iko} \qquad i=1,\ldots,m_k; k=1,\ldots,K; t=1,\ldots,T \\
&\sum_{j=1}^{n} \lambda^t_{kj} y^t_{rkj} \geqslant \bar{y}^t_{rko} \qquad r=1,\ldots,r_k; k=1,\ldots,K; t=1,\ldots,T \\
&\sum_{j=1}^{n} \lambda^t_{kj} z^t_{l(kh)_j} \geqslant \tilde{z}^t_{l(kh)_o}, l=1toL_{kh}; h,k=1,\ldots,K; t=1,\ldots,T \\
&\sum_{j=1}^{n} \lambda^t_{kj} z^t_{l(hk)_j} \leqslant \tilde{z}^t_{l(kh)_o}, l=1toL_{hk}; h,k=1toK; t=1toT \\
&\sum_{j=1}^{n} \lambda^t_{kj} z^{(t,t+1)}_{lk_j} \geqslant z^{(t,t+1)}_{lk_o}, l=1toL_k; k=1toK; t=1toT-1 \\
&\sum_{j=1}^{n} \lambda^{t+1}_{kj} z^{(t,t+1)}_{lk_j} \leqslant z^{(t,t+1)}_{lk_o}, l=1toL_k; k=1toK; t=1toT-1 \\
&\lambda^t_{jk}, \bar{y}^t_{rko} \geqslant 0, r=1tor_k; k=1toK; j=1ton; t=1toT.
\end{aligned} \tag{4.11}$$

The above linear programming problem is solved for $t=1\ldots,T$ followed by calculating the dynamic divisional efficiency as follows:

$$NRE^t_{ko} = \frac{\sum_{r=1}^{r_k} p^t_{rko} y^t_{rko}}{\sum_{r=1}^{r_k} p^t_{rko} \bar{y}^*_{rko}} \tag{4.12}$$

The SNRE is adopted as follows:

$$DNRE_o = \sum_{t=1}^{T} W^t_o \left(\sum_{k=1}^{K} w^t_{ko}.NRE^t_{ko} \right) \tag{4.13}$$

where w^t_{ko} and W^t_o are the weights of division k in period t and the aggregated weight related to DMU subject to assessment in period t,respectively and they meet the condition: $\sum_{k=1}^{K} w^t_{ko} = 1, \sum_{t=1}^{T} W^t_o = 1, w_{ko}, W^t_o \geqslant 0$. These weights are defined through:

$$\begin{aligned}
w^t_{ko} &= \frac{\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}} \\
W^t_O &= \frac{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}}{\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}}
\end{aligned} \tag{4.14}$$

Thus, the following equations are yielded

$$\begin{aligned}
DNRE_o &= \sum_{t=1}^{T} W^t_O \left(\sum_{ko}^{t} .NRE^t_{ko} \right) \\
&= \sum_{t=1}^{T} \left(\frac{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}}{\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}} \right) \\
&\left(\sum_{k=1}^{K} \frac{\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}} . \right. \\
&\left. \left(\frac{\sum_{r=1}^{r_k} p^t_{rko} y^t_{rko}}{\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}} \right) \right) \\
&\sum_{t=1}^{T} \left(\frac{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}}{\sum_{t=1}^{T}\sum_{r=1}^{s} p^t_{rko}\bar{y}^{t^*}_{rko}} \right) \\
&\left(\frac{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko} y^t_{rko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}} \right) \\
&= \frac{\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko} y^t_{rko}}{\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t^*}_{rko}}
\end{aligned} \tag{4.15}$$

5 Network Profit Efficiency

5.1 Static Network Profit Efficiency (SNPE)

By applying model (4.5), the SNPE model is introduced as follows:

$$\begin{aligned}
&\max \sum_{r=1}^{r_k} p_{rko}\bar{y}_{rko} - \sum_{i=1}^{m_k} c_{ko}\bar{x}_{iko} \\
&S.T. \sum_{j=1}^{n} \lambda_{kj} x_{ikj} \leqslant \bar{x}_{iko} \qquad i=1,\ldots,m_k; k=1,\ldots,K \\
&\sum_{j=1}^{n} \lambda_{kj} y_{rkj} \geqslant \bar{y}_{rko} \qquad r=1,\ldots,r_k; k=1,\ldots,K \\
&\sum_{j=1}^{n} \lambda_{hj} z_{l(kh)_j} \geqslant \tilde{z}_{l(kh)_o} \qquad l=1,\ldots,L_{kh}; h,k=1,\ldots,K \\
&\sum_{j=1}^{n} \lambda_{kj} z_{l(hk)_j} \leqslant \tilde{z}_{l(kh)_o} \qquad l=1,\ldots,L_{hk}; h,k=1,\ldots,K \\
&\bar{x}_{io} \leqslant x_{io}, \bar{y}_{ro} \geqslant y_{ro}; \\
&\lambda_j \geqslant 0.
\end{aligned} \tag{5.16}$$

where the NPE score related to division k of DMU_o is defined as:

$$NPE_{ko} = \frac{\sum_{r=1}^{r_k} p_{rko} y_{rko} - \sum_{i=1}^{m_k} c_{iko} x_{iko}}{\sum_{r=1}^{r_k} p_{rko} \bar{y}^*_{rko} - \sum_{i=1}^{m_k} c_{iko} \bar{x}^*_{iko}} \tag{5.17}$$

Note that $SNPE_{ko}$ is bounded between 0 and 1 except when the observed profit related to division k of DMU_o is negative and its maximum profit is positive. If the maximum profit is negative the $SNPEk0$ exceeds unity. The $SNPE$ of DMU_o is defined as before:

$$SNPE_o = \sum_{k=1}^{K} w_{ko}.NPE_{ko} \tag{5.18}$$

where w_{ko} is the weight of division k of DMU_o intended to show the proportion of the maximum profit of the kth division in the maximum profit and it is defined as follows:

$$w_{ko} = \frac{\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko} - \sum_{i=1}^{m_k} c_{iko}\bar{x}^*_{iko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko} - \sum_{k=1}^{K}\sum_{i=1}^{m_k} c_{iko}\bar{x}^*_{iko}} \tag{5.19}$$

Note that w_{ko}, by definition, can be negative, thus:

$$SNPE_0 = \sum_{k=1}^{K} w_{ko}.SNPE_{ko} = \frac{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}y_{rko} - \sum_{k=1}^{K}\sum_{i=1}^{m_k} c_{iko}x_{iko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko} - \sum_{k=1}^{K}\sum_{i=1}^{m_k} c_{iko}\bar{x}^*_{iko}} \tag{5.20}$$

5.2 Dynamic Network Profit Efficiency (DNPE)

The dynamic network profit efficiency is calculated similar to that of dynamic network revenue efficiency where the result is applied in $SNPE$ model to solve the following linear problem for every time period.

$$\begin{aligned}
\max\ & \sum_{r=1}^{r_k} p^t_{rko}\bar{y}^t_{rko} - \sum_{i=1}^{m_k} c^t_{ko}\bar{x}^t_{iko} \\
S.T.\ & \sum_{j=1}^{n} \lambda^t_{kj}x^t_{ikj} \leqslant \bar{x}^t_{iko} \qquad i = 1tom_k; k = 1toK; t = 1toT \\
& \sum_{j=1}^{n} \lambda^t_{kj}y^t_{rkj} \geqslant \bar{y}^t_{rko} \qquad r = 1tor_k; k = 1,\dots,K; t = 1toT \\
& \sum_{j=1}^{n} \lambda^t_{hj}z^t_{l(kh)_j} \geqslant \tilde{z}^t_{l(kh)_o} \qquad l = 1toL_{kh}; h,k = 1toK; t = 1toT \\
& \sum_{j=1}^{n} \lambda^t_{kj}z^t_{l(hk)_j} \leqslant \tilde{z}^t_{l(kh)_o} \qquad l = 1toL_{hk}; h,k = 1toK; t = 1toT \\
& \sum_{j=1}^{n} \lambda^t_{kj}z^{(t,t+1)}_{lk_j} \geqslant z^{(t,t+1)}_{lk_o} \qquad l = 1toL_k; k = 1toK; t = 1toT-1 \\
& \sum_{j=1}^{n} \lambda^{t+1}_{kj}z^{(t,t+1)}_{lk_j} \leqslant z^{(t,t+1)}_{lk_o} \qquad l = 1toL_k; k = 1toK; t = 1toT-1 \\
& \bar{x}^t_{iko} \leqslant x^t_{iko}, \bar{y}^t_{rko} \geqslant y^t_{rkj}; \\
& \lambda^t_{kj} \geqslant 0; t = 1,\dots,T.
\end{aligned} \tag{5.21}$$

After the optimal result is obtained through $DNPE$, the network profit efficiency score of division k related to $DMU_o(NPE_{ko})$ in period t is defined as the observed profit to maximum profit ratio, as follows:

$$NPE^t_{ko} = \frac{\sum_{r=1}^{r_k} p^t_{rko}y^t_{rko} - \sum_{i=1}^{m_k} c^t_{iko}x^t_{iko}}{\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t*}_{rko} - \sum_{i=1}^{m_k} c^t_{iko}\bar{x}^{t*}_{iko}} \tag{5.22}$$

Here it is claimed that $DNPE$ score is the weighted arithmetic mean of the divisional profit efficiencies scores presented as:

$$DNPE_o = \sum_{t=1}^{T} W^t_o \left(\sum_{k=1}^{K} w^t_{ko}.NPE^t_{ko}\right) \tag{5.23}$$

where the weights w^t_{ko} and W^t_o s are defined as:

$$w^t_{ko} = \frac{\sum_{r=1}^{r_k} p^t_{rko}\bar{y}^{t*}_{rko} - \sum_{i=1}^{m_k} c^t_{iko}\bar{x}^{t*}_{iko}}{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}\bar{y}^*_{rko} - \sum_{i=1}^{m_k} c_{iko}\bar{x}^*_{iko}} \tag{5.24}$$

$$w^t_o = \frac{\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{iko}\bar{y}^*_{rko} - \sum_{k=1}^{K}\sum_{i=1}^{m_k} c_{iko}\bar{x}^*_{iko}}{\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{iko}\bar{y}^*_{rko} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{m_k} c_{iko}\bar{x}^*_{iko}} \tag{5.25}$$

Note that w^t_{ko} and W^t_o represent

$$\begin{aligned}
DNPE_o &= \sum_{t=1}^{T} W^t_o \left(\sum_{k=1}^{K} NPE_{ko}\right) \\
&= \frac{\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}y^t_{rko} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{m_k} c_{iko}x^t_{iko}}{\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{r=1}^{r_k} p_{rko}y^*_{rko} - \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i=1}^{m_k} c_{iko}x^*_{iko}}
\end{aligned} \tag{5.26}$$

6 Numerical examples

The proposed models in two numerical examples consisting of $10DMUs$ over two periods of t and $t+1$ are applied here. Each one of the $DMUs$ includes three divisions; where each division includes one exogenous input and one exogenous output. In Period t, the inputs of division 1, division 2 and division 3 are characterized by X^t_{11}, X^t_{12}, and X^t_{13} , respectively. The outputs are represented by Y^t_{11}, Y^t_{12}, and Y^t_{13}.There exist two intermediate measures, named $Z^t_{1(12)}$ (division 1 to division 2 links) and $Z^t_{1(23)}$ (division 2 to division 3 links). The data related to periods t and $t+1$ are tabulated in Tables 1 and 2, respectively. There are three carryovers between periods t and $t+1$, named, $Z^{(t,t+1)}_{11}$ (related to division 1) , $Z^{(t,t+1)}_{12}$ (related to division 2) and $Z^{(t,t+1)}_{13}$ (related to division 3), Table 3.

6.1 Numerical example 1

The $DNRE$ model with the output prices of $p^t_{11} = p^t_{12} = p^t_{13} = 4in$ period t and output prices $p^{(t+1)}_{11} = p^{(t+1)}_{12} = p^{(t+1)}_{13} = 5$ in period $t+1$ are of concern here. Both the divisional and period revenue efficiency scores and the divisional weights related to the two periods t and $t+1$ subject to $DNRE$ model are tabulated in Tables 4 and 5, respectively. The period weights and dynamic revenue efficiency scores are tabulated in Table 6. By applying model (4.5), the divisional revenue efficiency scores of three divisions associated with the two period of t and $t+1$ are highlighted in columns $2-4$ of Tables 4 and 5, respectively. The results of static revenue efficiency are obtained

Table 1: Inputs, outputs and links in three divisions in period t.

X_{11}^t	X_{12}^t	X_{13}^t	Y_{11}^t	Y_{12}^t	Y_{13}^t	$Z_{1(12)}^t$	$Z_{1(23)}^t$
0.962	0.221	0.879	0.337	0.894	0.362	0.948	
1.330	0.132	0.443	0.232	0.538	0.18	0.678	0.188
0.621	0.045	0.482	0.423	0.911	0.198	0.83	0.207
1.783	0.111	0.467	0.514	0.57	0.491	0.869	0.516
1.892	0.208	1.073	0.351	1.086	0.372	0.693	0.407
0.99	0.139	0.545	0.021	0.722	0.253	0.966	0.269
0.151	0.075	0.366	0.312	0.509	0.241	0.647	0.257
0.108	0.074	0.229	0.723	0.619	0.097	0.756	0.103
1.364	0.061	0.691	0.833	1.023	0.38	1.191	0.402
1.922	0.149	0.337	0.133	0.769	0.178	0.792	0.187

Table 2: Inputs, outputs and links in three divisions in period $t+1$.

X_{11}^{t+1}	X_{12}^{t+1}	X_{13}^{t+1}	Y_{11}^{t+1}	Y_{12}^{t+1}	Y_{13}^{t+1}	$Z_{1(12)}^{t+1}$	$Z_{1(23)}^{t+1}$
0.838	0.277	0.962	0.111	0.879	0.337	0.494	0.262
1.233	0.132	0.443	0.212	0.538	0.18	0.578	0.138
0.321	0.045	0.482	0.123	0.911	0.198	0.736	0.307
1.483	0.111	0.467	0.214	0.57	0.491	0.769	0.616
1.592	0.208	1.073	0.321	1.086	0.372	0.793	0.107
0.79	0.139	0.545	0.121	0.722	0.253	0.866	0.169
0.451	0.075	0.366	0.412	0.509	0.241	0.547	0.157
0.408	0.074	0.229	0.323	0.619	0.097	0.446	0.403
1.864	0.061	0.691	0.233	1.023	0.38	1.441	0.442
1.222	0.149	0.337	0.333	0.769	0.178	0.492	0.147

Table 3: Carry-overs related to three divisions.

$Z_{11}^{(t,t+1)}$	$Z_{12}^{(t,t+1)}$	$Z_{13}^{(t,t+1)}$
0.603	0.133	0.463
0.982	0.073	0.245
0.979	0.053	0.568
0.720	0.054	0.227
0.595	0.072	0.373
0.936	0.094	0.370
0.906	0.084	0.410
0.574	0.104	0.322
0.713	0.023	0.264
0.715	0.87	0.197

through model (4.5) and expressed in column 5 Tables 4 and 4. The revenue efficiency calculated by treating the DMU as a black-box and by applying model (2.1) is expressed in column 6 of tables 4 and 5.When the system is considered as a black-box, the $DMUs$ C, H and I are efficient in periodt. By applying model (4.5), DMU H is efficient. In period $t+1$, these proposed models identify all $DMUs$ as inefficient, whereas the black-box model identifies $DMUs$ C and H as efficient.It is found that these proposed models identify the first division of $DMUs$ C and Gas-efficient and that they outperform their counterparts.

The associated weights for the three divisions of period t and period $t+1$ are expressed in the last two columns of Tables 4 and 5, respectively. These weights indicate the importance of maximum revenue of each corresponding DMU division in relation to maximum revenue of the same

Table 4: Revenue resultsof Period t.

DMU	NRE_1^t	NRE_2^t	NRE_3^t	NRE^t	$Black-boxRE$	w_1^t	w_2^t	w_3^t
A	0.035	1	0.972	0.190	0.256	0.838	0.116	**0.046**
B	0.026		0.096	0.360	0. 897	0.085	**0.018**	
C	0.102	1	1	0.291	1	0.789	0.173	**0.038**
D	0.043	1	1	0.121	0.652	0.918	0.044	**0.038**
E	0.028	1	0.955	0.128	0.336	0.896	0.077	**0.028**
F	0.003	0.643	0.984	0.124	0.338	0.828	0.140	**0.032**
G	1	1	1	1	0.704	0.294	0.479	**0.227**
H	1	1	1	1	1	0.502	0.430	**0.067**
I	0.091	1	0.990	0.212	1	0.866	0.097	**0.036**
J	0.010	0.796	0.997	0.077	0.510	0.918	0.069	**0.013**

Table 5: Revenue resultsof Period t.

DMU	NRE_1^{t+1}	NRE_2^{t+1}	NRE_3^{t+1}	NRE^{t+1}	$Black-boxRE$	w_1^{t+1}	w_2^{t+1}	w_3^{t+1}
A	0.145	1	0.687	0.621	0.472	0.359	0.412	**0.230**
B	0.188	0.634	0.742	0.419	0.486	0.508	0.383	**0.109**
C	1	1	0.548	0.883	1	0.088	0.653	**0.259**
D	0.158	0.546	1	0.441	0.718	0.469	0.361	**0.170**
E	0.221	0.904	1	0.588	0.484	0.481	0.396	**0.123**
F	0.168	0.604	0.850	0.495	0.509	0.326	0.540	**0.134**
G	1	0.691	1	0.836	0.903	0.296	0.530	**0.173**
H	0.867	1	0.403	0.843	1	0.302	0.502	**0.195**
I	0.137	1	0.732	0.504	0.980	0.525	0.315	**0.160**
J	0.298	1	0.798	0.607	0.837	0.529	0.365	**0.106**

Table 6: Results of Dynamic revenue system.

DMU NRE^{t+1} w_3^{t+1}	NRE_1^{t+1} $Black-boxRE$	NRE_2^{t+1} w_1^{t+1}	NRE_3^{t+1} w_2^{t+1}
A	0.739	0.261	**0.302**
B	0.782	0.218	**0.166**
C	0.751	0.249	**0.438**
D	0.783	0.217	**0.191**
E	0.789	0.211	**0.225**
F	0.743	0.257	**0.220**
G	0.379	0.621	**0.898**
H	0.483	0.517	**0.919**
I	0.722	0.278	**0.293**
J	0.842	0.158	**0.161**

DMU. DMU H is applied to explain this phenomenon; the revenue efficiency of the three divisions is 1, whereas the weight related to division 3, w_3^t, is smaller than those of the other two divisions. This fact indicates that contribution of maximum revenue obtained by prices of outputs of division 3 to maximum revenue obtained by prices of outputs of DMUHis less than those of the two other divisions. The period weights and overall dynamic revenue efficiency are tabulated in Table 6. It is observed that the results of the dynamic revenue efficiencies obtained from model (4.5) are set between the network revenue efficiency in period t and network revenue efficiency in period $t+1$. These proposed models decompose $DNRE$ efficiency into NRE efficiencies of the two periods. It is worthwhile to inform the decision makers on which divisions lead to ineffi-

Table 7: Profit efficiency of period t.

DMU	NPE_1^t	NPE_2^t	NPE_3^t	NPE^t	$Black-boxPE$	w_1^t	w_2^t	w_3^t
A	−0.135	1	5.071	−0.128	−0.195	0.919	0.102	−0.021
B	−0.145	0.567	1	−0.119	−0.524	0.944	0.089	−0.033
C	−0.056	1	1	0.093	1	0.859	0.210	−0.069
D	−0.125	1	1	−0.074	1	0.995	0.043	0.002
E	−0.143	1	9.940	−0.118	−0.502	0.930	0.076	−0.006
F	−0.172	0.592	2.815	−0.104	−0.387	0.865	0.151	−0.016
G	1	1	1	1	1	0.343	0.923	−0.266
H	1	1	1	1	1	0.598	0.530	−0.128
I	−0.068	1	14.467	0.014	1	0.892	0.110	−0.002
J	−0.163	0.758	2.740	−0.113	−0.139	0.935	0.070	−0.005

Table 8: Profit efficiency of period $t+1$

DMU	NPE_1^{t+1}	NPE_2^{t+1}	NPE_3^{t+1}	NPE^{t+1}	$Black-boxPE$	w_1^{t+1}	w_2^{t+1}	w_3^{t+1}
A	20.641	0.545	0.687	0.136	−0.853	−0.023	0.708	0.315
B	24.775	0.368	0.742	−0.333	−1.454	−0.032	0.846	0.186
C	1	0.633	0.548	0.565	1	−0.129	0.893	0.236
D	23.145	0.161	1	−0.097	1	−0.017	0.868	0.149
E	22.480	0.307	1	−0.007	−0.949	−0.018	0.901	0.117
F	10.835	0.204	0.850	0.054	−0.591	−0.020	0.924	0.096
G	0.152	1	1	0.208	1	−0.013	0.934	0.079
H	2.673	0.191	0.403	0.182	1	−0.010	0.932	0.079
I	15.875	0.337	0.732	−0.088	1	−0.031	0.873	0.159
J	25.343	0.217	0.798	−0.030	−1.243	−0.012	0.938	0.073

ciency at which periods periodsand for doing this improved strategies must be provided for better system performance.

7 Numerical example 2

Here, consider the profit maximization for the output prices of $p_{11}^t = p_{12}^t = p_{13}^t = 4$ and in-

Table 9: Overall profit efficiency

DMU	w^t	w^{t+1}	$DNPE$
A	0.751	0.249	**-0.062**
B	0.831	0.169	**-0.155**
C	0.683	0.317	**0.243**
D	0.721	0.279	**-0.080**
E	0.745	0.255	**-0.089**
F	0.628	0.372	**-0.045**
G	0.109	0.891	**0.295**
H	0.211	0.789	**0.355**
I	0.680	0.320	**-0.019**
J	0.755	0.245	**-0.093**

put prices of $c^t_{11} = c^t_{12} = c^t_{13} = 4$ in period t and output prices of $p^{(t+1)}_{11} = p^{(t+1)}_{12} = p^{(t+1)}_{13} = 5$ and input prices of $c^{(t+1)}_{11} = c^{(t+1)}_{12} = c^{(t+1)}_{13} = 5$ in period $t+1$. The divisional and period profit efficiencies and the divisional weights related to the two periods t and $t+1$ subject to $DNPE$ model are tabulated in Tables 7 and reft8,respectively. The given period's weights and the overall dynamic profit efficiency are tabulated in Table 9.

The periods weights and the overall dynamic profit efficiencies are tabulated in Table 8. As observed the results of the dynamic profit efficiencies obtained through model (5.21) are between the network profit efficiency period t and network profit efficiency period $t+1$. These proposed models decompose $DNPE$ efficiency into NPE efficiencies of both the periods, therefore, they provide strategies to improve system performance. The results indicate that a DMU can be totally efficient if and only if it is efficient for all divisions.

8 Conclusions

The correlated models for calculating revenue, and profit efficiencies of dynamic vs. static systems with network structures are proposed in this study. The contributions of this study consist of an additive weighted model and an improved Farrell's cost model. Though attempts are made in several studies to improve the Farrell's revenue model, the dynamic effects are usually neglected. Furthermore, the importance of every division on overall revenue and profit efficiencies can be determined by its own weight. Two numerical examples, consisting of twotime periods t and $t+1$, are applied to describe these proposed models.

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