Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 13, No. 3, 2021 Article ID IJIM-1044, 7 pages
DOR: http://dorl.net/dor/20.1001.1.20085621.2021.13.4.7.6
Research Article

An Interval Model in Interdiction Network Flow

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Received Date: 2017-03-08 Revised Date: 2019-09-03 Accepted Date: 2020-11-06

Abstract

One of the key issues raised in networks flow is its interdiction. Keshavarzi et al. scrutinized the issue with the multi-sources and multi-sinks conditions in mind while considering the specific conditions of flow being sent from sources to sinks [R. Keshavarzi et al., Multi-source-sinks network flow interdiction problem, International Journal of Academic Research, 2015]. Moreover, the matter was also examined in the state of multi-interdictors and a practical solution was presented [Keshavarzi and, Salehi, Multi commodity multi source-sinks network flow interdiction problem with several interdictors, Journal of Engineering and Applied Sciences, 2015]. In this paper, the networks flow interdiction in multi-source and multi-sink conditions was addressed; meanwhile, bearing in mind the uncertain data (from a specified beginning to an ending interval), an optimal interval was presented. Finally, a numerical example for this issue was provided and then solved by the program "Lingo".

Keywords : Interdiction problem; Network flow; Simplex method; Duality; Bi-level programming; Decomposition; Interval data.

1 Introduction

GEnerally speaking, network flow interdiction is an issue with two distinct decision makers (defender and attacker). Examples such as drug delivery, antidote delivery, narcotics systems and issues like these in which two decision makers with different goals use the network flow simultaneously are a kind of network flow interdiction. This matter was first taken under investigation by Wood [9].

Almost all studies prior to Wood [9] are specific to the applications stated above and are not extendable to more general contexts. Wood was the first to adopt a mathematical programming model to solve the problem. He developed a min-max formulation of Maximum Flow Network Interdiction problem and then converted it to an integer-programming model.
Cormican et al. [1] formulated and solved a stochastic version of the interdictor's problem. They minimized the expected maximum flow going through the network with interdiction variables being binary and random. Extensions were also made to handle uncertain arc capacities. Such stochastic integer programming problems can be used to interdict illegal drugs and to reduce the effectiveness of moving materials,

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troops, information, etc. in a military force through a network in wartime.
Wollmer [8] presented two algorithms for targeting strikes in a lines-of-communication (LOC) network. The LOCs were represented by a network of nodes and directed arcs. It was assumed that the user of the LOCs is attempting to achieve a circulation flow at a minimum cost. A very general goal was as special cases, maximizing the flow between two points, meeting the required flows between a source and a sink at a minimum cost, and combinations of these two.
Another category of the network-interdiction problem is that of maximizing the length of the shortest path where a set of network arcs are disabled in order to maximize the length of the shortest path between s and t through the usable portion of the network. Fulkerson and Harding [3] and Israeli [4] have notably contributed to resolving this problem.
In the real world, the network flow data may be expressed indecisively. For example, Network flow data such as capacity of arcs, the costs of sending the flow or flow interdiction on each arc are expressed in an interval or fuzzy form.
Nam [7] suggested a network in which its variables are located in a time period with a specified beginning and ending. They demonstrate their framework on vision tasks such as recognition and temporal segmentation of action sequence, or parsing and making future predictions online when running in streaming mode while observing an assembly task.
The present study sought to investigate the multi-source-sink network flow interdiction problem with interval data. This study dealt with two adverse elements: a network user/defender and multi interdictors/attackers. The network user attempts to maximize the out flow coming from a number of sources which are a subset of a node set V, in an undirected network. The interdictor tries to reduce the network user's maximum flow through the limited interdiction resource in order to block the arcs of the network.

2 Multi Source-Sinks Network Flow Interdiction problem

The topic under discussion in this paper was presented within an undirected, capacitated network $G(V, L)$. Also, it is supposed that V indicates the node set of the network and L represents arc set of the network. The arc set L includes ordered pairs (i, j) in which i and j are the vertices of network; additionally, the sending flow of each arc (i, j) is restricted by positive integral capacity u_{ij} from the top and by zero from the bottom.
In the proposed issue, the two subsets of the network's nodes are considered as the sets of sources and sinks which is shown with $N = \{n_1, n_2, \ldots, n_n\}$ and $D = \{d_1, d_2, \ldots, d_m\}$. Without excluding any generalities, it is supposed that these two sets have no common features. (i.e.$D \cap N = \varnothing$)
This problem is written in such a way that each source can send flows just to certain predetermined destinations. For the n_i source, the set of sinks is considered as the following:

$$D_i = \{d_j | d_j \in D \text{ and } d_j \text{ is a sink for } n_i\}$$

It is supposed that the sets D_i may have several joint nodes and each sink is the only possible receiver of flow. The issue under discussion has two separate decision makers: the target of the first decision maker is to send the max flow from the certain sources to the predetermined sinks for each source and the target of the second decision maker is to reduce this amount with destructing (eliminating) the network's arcs. The first and second decision makers will be orderly called attacker and interdictor. Also, we suppose that the interdictor uses a specified interdiction resource with a total amount of R units. Interdicting an arc (i, j) and eliminating it from the network requires $c_{ij} \geq 0$ units of the source.

3 Mathematical structure of the problem

In this problem, x_{ijk} be the amounts of flow out of the source node n_k on the (i,j) arc and the interdictor's variable for (i,j) arc is shown by w_{ij} .In this condition we suppose that the interdictor utilizes two motifs in confronting each arc: either eliminating the arc from the network $w_{ij}=1$ and considering the capacity of the arc as 0 or not eliminating the arc and with the capacity arc parameter (u_{ij}) remaining in the network. This decision of the interdictor is shown by placing the amounts $u_{ij}(1-w_{ij})$ instead of the arc capacity in the network. $u_{ij}(1-w_{ij})$. The set of intermediate nodes (which are neither a source nor a sink) with NP are shown. Also, it is presumed that the middle nodes have no supply and demand (problem of maximum flow) and their role is only going to be sending and receiving the flow.

Model 1

$min_w max \sum_{k=1}^{n} \sum_{(n_k,j)\in L} x_{n_k jk}$

s.t : $\sum_{k=1}^{n}(x_{ijk}+x_{jik}) \leq u_{ij}(1-w_{ij})\ \forall(i,j)\in L$ (1)

$\sum_{j:(i,j)\in L} x_{ijk} - \sum_{j:(j,i)\in L} x_{jik} = 0 \quad \forall i \in NP\ \forall k=1,\ldots,n$ (2)

$\sum_{(i,j)\in L, j\in D_k} x_{ijk} = \sum_{(n_k,j)\in L} x_{n_k jk} \quad \forall k = 1,\ldots,n$ (3)

$\sum_{(i,j)\in L} c_{ij}w_{ij} \leq R$ (4)

$w_{ij}\in\{0,1\} \quad \forall(i,j)\in L$ (5)

$x_{ijk}\geq 0 \quad \forall(i,j)\in L, k=1,\ldots,n$

The objective function in the above model calculates the true value of the output flow from sources with two purposes of Max and Min orderly for the attacker and the interdictor. The first constraint indicates the capacity of sending flow on each arc of the network. The second constraint (flow balance constraint) guarantees that the middle nodes (the nodes which are member of NP) have the supply and demand value of zero. The third constraint shows that the sent flow from the sources should be equal to the received flow in the predetermined destinations and the fourth constraint limits the expenditure of interdiction resources.

4 Interval Model:

In the suggested model, a number of the determined data in the problem (in this paper u_{ij}) is considered as an interval form, to this end, u_{ij}^l and u_{ij}^u are respectively considered as the lower and upper bounds for the capacity of (i,j) arc; in other words, for each (i,j) arc, we will have the relation of $u_{ij}^l \leq u_{ij} \leq u_{ij}^u$. This is called the Interval model since the arcs network capacity instead of being fixed is located in certain intervals. We note that the relation of $u_{ij}^l \leq u_{ij}^u$ and $u_{ij}^l, u_{ij}^u \geq 0$ are confirmed for each arc. If for a special arc, the relation is expressed equally $(u_{ij}^l = u_{ij}^u)$, this data will be defined by a determined amount. For writing the Interval Model: Model 1, we shall consider the interval capacity of the arcs as reformulated below:

interval model

$min_w max \sum_{k=1}^{n} \sum_{(n_k,j)\in L} x_{n_k jk}$

s.t : $\sum_{k=1}^{n}(x_{ijk}+x_{jik}) \leq u_{ij}(1-w_{ij})\ \forall(i,j)\in L$

$\sum_{j:(i,j)\in L} x_{ijk} - \sum_{j:(j,i)\in L} x_{jik} = 0 \quad \forall i \in NP\ ,\forall k=1,\ldots,n$

$\sum_{(i,j)\in L, j\in D_k} x_{ijk} = \sum_{(n_k,j)\in L} x_{n_k jk} \quad \forall k = 1,\ldots,n$

$\sum_{(i,j)\in L} c_{ij}w_{ij} \leq R$

$w_{ij}\in\{0,1\} \quad \forall(i,j)\in L$

$x_{ijk}\geq 0 \quad \forall(i,j)\in L, k=1,\ldots,n$

$u_{ij}^l \leq u_{ij} \leq u_{ij}^u \quad \forall(i,j)\in L$

In the interval model, arc capacities are located in the predetermined intervals instead of the real non-negative value. Since solving the model is nonlinear; henceforth, a strategy for linearizing and easily solving the model will be expressed. For this purpose, firstly we substitute the capacity of each arc which is an interval data with the upper bound corresponding to the interval and introduce the following model with the definite capacity for u_{ij}^u the (i,j) arc.

model U

$Z^u = min_w max \sum_{k=1}^{n} \sum_{(n_k,j)\in L} x_{n_kjk}$

s.t : $\sum_{k=1}^{n} x_{ijk} \leq u_{ij}^u(1-w_{ij}) \quad \forall(i,j)\in L$

$\sum_{j:(i,j)\in L} x_{ijk} - \sum_{j:(j,i)\in L} x_{jik} = 0 \quad \forall i \in NP \ , k=1,\dots,n$

$\sum_{(i,j)\in L, j\in D_k} x_{ijk} - \sum_{(n_k,j)\in L} x_{n_kjk} = 0 \quad k=1,\dots,n$

$\sum_{(i,j)\in L} c_{ij}w_{ij} \leq R$

$w_{ij}\in\{0,1\} \quad \forall(i,j)\in L$

$x_{ijk}\geq 0 \quad \forall(i,j)\in L, k=1,\dots,n$

In the following, a method to solve the model U is presented and its optimal answer Z^u will be considered as the bound of the optimization interval. To determine the lower bound of the optimization interval, model L is expressed as follows:

model L

$Z^l = min_w max \sum_{k=1}^{n} \sum_{(n_k,j)\in L} x_{n_kjk}$

s.t : $\sum_{k=1}^{n} x_{ijk} \leq u_{ij}^l(1-w_{ij}) \quad \forall(i,j)\in L$

$\sum_{j:(i,j)\in L} x_{ijk} - \sum_{j:(j,i)\in L} x_{jik} = 0 \quad \forall i \in NP \ , k=1,\dots,n$

$\sum_{(i,j)\in L, j\in D_k} x_{ijk} - \sum_{(n_k,j)\in L} x_{n_kjk} = 0 \quad k=1,\dots,n$

$\sum_{(i,j)\in L} c_{ij}w_{ij} \leq R$

$w_{ij}\in\{0,1\} \quad \forall(i,j)\in L$

$x_{ijk}\geq 0 \quad \forall(i,j)\in L, k=1,\dots,n$

Lemma 4.1. *Increasing (or decreasing) the capacity amount of arcs u_{ij} in the models U and L causes an increase (or decrease) in the amount of the flow sent in the interdicted network.*

Proof. In the model, L, the capacity of each arc u_{ij} with the least possible amount (the lower interval bound u_{ij}^l) is substituted. This number with the first bound has the most flow sent on each arc until the possible minimization and as a result, the maximum flow sent in the network is going to be lowered. In the situation, which the capacity of each arc with its upper bound is substituted, the similar results are obtained. Then by increasing the lower bound amount of arcs u_{ij}^l the flow sent under the interdicted network will not be decreased furthermore and vice versa. □

5 Solving Method:

To solve models 1, U and L, regarding to the existence of two decision makers with different purposes, the following strategy is presented. This strategy is presented for solving model 1, the rest of models are solvable similarly. Our method consists of 1. Taking the dual of the inner maximization by fixing w temporality and then releasing w to obtain a mix integer nonlinear "min-min" model which is simply a transferring the nonlinear model to a linear mix integer programming problem. The obtained result of the first step is:

Dual Model 1:

$min_{\alpha,\beta,w} \sum_{(i,j)\in L} u_{ij}(1-w_{ij})\beta_{ij}$

s.t : $-\alpha_{ik}+\alpha_{jk}+\beta_{ij}\geq 0 \quad k=1,\dots,n \ (i,j)\in L \ ; \forall i,j\in NP \quad (6)$

$-\alpha_{jk}+\alpha_{ik}+\beta_{ij}\geq 0 \quad k=1,\dots,n \ ; (i,j)\in L \ ; \forall i,j\in NP \quad (7)$

$\beta_{ij}-\gamma_k \geq 1 \quad i\in N, \ (i,j)\in L, \ k=1,\dots,n \quad (8)$

$\beta_{ij}-\gamma_k \geq 0 \quad (i,j)\in L, \ j\in D, \ k=1,\dots,n \quad (9)$

$\sum_{(i,j)\in L} c_{ij}w_{ij}\leq R$

$w_{ij}\in\{0,1\} \quad \forall(i,j)\in L$

α_{ik} free $\quad k=1,\dots,n, i\in NP$

$\beta_{ij}\geq 0 \quad (i,j)\in L$

γ_k free $\quad k=1,\dots,n$

The dual variables β_{ij}, α_{ik} and γ_k in the model above correspond to constraints (1) to (3), respectively.

Lemma 5.1. *There is an optimal solution to D-USRP such that: $-1\leq\alpha_{ik}\leq 0$, $\forall i\in NP$, $k=1,\dots,n$, $0\leq\beta_{ij}\leq 1$, $\forall(i,j)\in L$, $-1\leq\gamma_k\leq 0$, $\forall k=1,\dots,n$*

Table 1: Network parameters.

(i,j)	$(u^l_{ij}, u^u_{ij}, c_{ij})$	(i,j)	$(u^l_{ij}, u^u_{ij}, c_{ij})$
$(1,3)$	$(1,5,2)$	$(3,6)$	$(1,6,3)$
$(1,4)$	$(2,4,1)$	$(4,5)$	$(4,7,3)$
$(2,4)$	$(1,5,4)$	$(4,6)$	$(1,5,2)$
$(2,6)$	$(3,7,1)$	$(3,4)$	$(2,5,2)$
$(3,5)$	$(3,7,3)$		$R=6$

Table 2: Lingo software results.

Model	Optimal value	Iteration	Interdicted arc
Model L	15	5	$(1,4),(2,6)$
Model U	44	5	$(1,4),(2,6),(3,4)$
Interval Model	10/28654	25	$(1,3),(1,4),(2,6),(3,4)$

Proof. Note that (1) the coefficients of β_{ij} in the objective function are positive so that making each β_{ij}'s as small as permitted by the constraints, decreases the objective function value. (2) No two variables α_{ik} and $\alpha_{i\acute{k}}$ with the same node index i and different source indices k and $\acute{k}$ appear in the same constraint. Accordingly, the restriction of a variable α_{ik} to the interval $[-1,0]$ does not affect any other $\alpha_{i\acute{k}}$ for $k \neq \acute{k}$. (3) Constraints (6) and (7) imply that, for each arc $(i,j) \in L$, $i,j \in NP$, the variable β_{ij} is bounded below by $max_k = \{-\alpha_{jk} + \alpha_{ik}, -\alpha_{ik} + \alpha_{jk}\}$. The restriction of the variables α_{ik} and α_{jk} to the interval $[-1,0]$ implies that the lower bound on β_{ij}, enforced by constraints (6) and (7), is at most 1. Accordingly, restricting β_{ij} to the interval $[0,1]$ for such arcs, maintains feasibility without loss of optimality. Similarly, constraint (8) and (9) imply that β_{ij} is bounded below by $1+\gamma_k$ for $(i,j) \in L$, $i \in N$, $k = 1,\dots,n$ and by γ_k for $(i,j) \in L$, $j \in D$, $k = 1,\dots,n$. Restriction of γ_k's to the interval $[-1,0]$ for the related arcs implies that the maximum of these lower bounds is again at most 1. Therefore, we can conclude that $\beta_{ij} = 1$ (for the corresponding arcs) in optimal solution. This completes the proof. □

Since the objective function of this model is nonlinear, in order to make it linear, the new variable of p_{ij} is introduced in the following lemma:

Lemma 5.2. *If* $p_{ij} = (1-w_{ij})\beta_{ij}$, $\forall (i,j) \in L$ *then* $0 \leq p_{ij} \leq 1$ *and* $p_{ij} \geq \beta_{ij} - w_{ij}$, $\forall (i,j) \in L$.

Proof. Since $w_{ij} \in \{0,1\}$, $\forall (i,j) \in L$ then $0 \leq 1 - w_{ij} \leq 1$, $\forall (i,j) \in L$. According to $p_{ij} = \beta_{ij} - w_{ij}\beta_{ij}$ and $0 \leq \beta_{ij} \leq 1$, we have $0 \leq p_{ij} \leq 1$ and $p_{ij} \geq \beta_{ij} - w_{ij}$. □

Now using p_{ij}, Dual model 1 is written as:
$min_w min_{\alpha,\beta,p} \sum_{(i,j)\in L} u_{ij} p_{ij}$

s.t : $-\alpha_{ik} + \alpha_{jk} + \beta_{ij} \geq 0$ $\quad k = 1,\dots,n,\ (i,j) \in L,\ \forall i,j \in NP$

$-\alpha_{jk} + \alpha_{ik} + \beta_{ij} \geq 0$ $\quad k = 1,\dots,n,\ (i,j) \in L,\ \forall i,j \in NP$

$\beta_{ij} - \gamma_k \geq 1$ $\quad i \in N,\ (i,j) \in L,\ k = 1,\dots,n$

$\beta_{ij} - \gamma_k \geq 0$ $\quad \forall (i,j) \in L,\ j \in D,\ k = 1,\dots,n$

$\sum_{(i,j)\in L} c_{ij} w_{ij} \leq R$

$p_{ij} \geq \beta_{ij} - w_{ij}$ $\quad \forall (i,j) \in L$

$0 \leq p_{ij} \leq 1$ $\quad \forall (i,j) \in L$

$w_{ij} \in \{0,1\}$ $\quad \forall (i,j) \in L$

α_{ik} free $\quad k = 1,\dots,n\ \forall i \in NP$

$\beta_{ij} \geq 0$ $\quad \forall (i,j) \in L$

γ_k free $\qquad k = 1, \ldots, n$

Lemma 5.3. *The optimal value of p_{ij} is β_{ij}*

Proof. If $w_{ij} = 0$ is optimal in Dual Model 1, the corresponding term in the objective function is equal to $u_{ij}\beta_{ij}$. If $w_{ij} = 1$ is optimal then the corresponding term in objective function is 0. Thus, to linearize the model, it must be true that $p_{ij} = 0$ when $w_{ij} = 1$ and $p_{ij} = \beta_{ij}$ when $w_{ij} = 0$. When $w_{ij} = 1$, constraints $p_{ij} \geq \beta_{ij} - w_{ij}$, $\forall (i, j) \in L$ are satisfied for $0 \leq \beta_{ij} \leq 1$. However, because setting p_{ij} to any value greater than 0 increases the objective function, the value of p_{ij} must be zero. When $w_{ij} = 0$, constraints $p_{ij} \geq \beta_{ij} - w_{ij}$, $\forall (i, j) \in L$ are satisfied for $p_{ij} \geq \beta_{ij}$. However, due to the minimization goal ($\min_{\alpha,\beta,w,p} \sum_{(i,j)\in L} u_{ij} p_{ij}$), it must be true that $p_{ij} = \beta_{ij}$. This justifies the correctness of the linear objective function with binary and real variables of binary or real. □

6 Complexity:

Wood [9] indicated that the issue of network flow interdiction problem in the situation of having one source and one sink with the condition of certain data is a NP-Hard problem; here it is shown that the discussed problem in this paper is also a NP-Hard problem.
For the discussed problem (model 1), the network was partitioned into three sub-networks. This network partitioning is named as following:

$V_1 = N \cup \{(i, j) : i, j \in N\}$
$V_2 = NP \cup \{(i, j) : (i, j) \in L, (i, j) \notin V_1 \cup V_3\}$
$V_3 = D \cup \{(i, j) : i, j \in D\}$

The arcs and nodes of the network are divided into three groups including: (i) V_1 includes all sources and arcs that connect sources together, (ii) V_2 all middle nodes (no source and no sink) and edges which connect middle nodes together and edges connect middle nodes to the sources and the sinks, (iii) V_3 includes all sinks and edges that connect sinks together. Now we can assume V_1 as a pseudo-source and V_3 as a pseudo-sink. The user tries to maximize the network flow value from V_1 to V_3 and the interdictor tries to decrease this value as much as his accessible resource allows.
By doing the above, the model is rewritten as follows which has a virtual source and destination and is a NP-Hard problem.

$min_w min_{\alpha,\beta,p} \sum_{(i,j)\in L} u_{ij} p_{ij}$

s.t : $-\alpha_{ik} + \alpha_{jk} + \beta_{ij} \geq 0$
$k = 1, \ldots, n, \ (i, j) \in L, \ \forall i, j \in V_2$

$-\alpha_{jk} + \alpha_{ik} + \beta_{ij} \geq 0 \qquad k = 1, \ldots, n, \ (i, j) \in L, \ \forall i, j \in V_2$

$\beta_{ij} - \gamma_k \geq 1 \qquad i \in V_1 \ (i, j) \in L, \ k = 1, \ldots, n$

$\beta_{ij} - \gamma_k \geq 0 \qquad \forall (i, j) \in L, \ j \in V_3, \ k = 1, \ldots, n$

$\sum_{(i,j)\in L} c_{ij} w_{ij} \leq R$

$p_{ij} \geq \beta_{ij} - w_{ij} \qquad \forall (i, j) \in L$

$0 \leq p_{ij} \leq 1 \qquad \forall (i, j) \in L$

$w_{ij} \in \{0, 1\} \qquad \forall (i, j) \in L$

α_{ik} free $\qquad k = 1, \ldots, n, \ \forall i \in V_2$

$\beta_{ij} \geq 0 \qquad \forall (i, j) \in L$

γ_k free $\qquad k = 1, \ldots, n$

7 Numerical Example

Consider the undirected, in this network, the set of sources, destinations, and the possible destinations for receiving the flow from a particular source is considered as follows:
$V = \{1, 2, 3, 4, 5, 6\}$
$N = \{1, 2\} \qquad D = \{5, 6\}$
$D_1 = \{5\} \qquad D_2 = \{6\}$
In the table below,table 1,the triplet function vectors $(u^l_{ij}, u^u_{ij}, c_{ij})$ are similar to the arcs on the network. The authors of these vectors in order from the lower bound to upper bound of the capacity of arcs u_{ij} and the others show the cost of interdicting each arc. $R = 6$ shows the interdictor's source in the directional flow of the network. In the models, L and U, after having taken the prescribed steps in the solutions section with the predetermined conditions and boundaries linearly with the free, zero, one and non-negative variables reformulated. These two models with the

numerical example are solved by the program "Lingo". Repeated results are shown in the following table, Table 2. The interval model in this network is used for changing the non-linear function into a linear function, resulting in a model designed by Lingo and shown in the table below.

8 Conclusion

The Final Model is translated to linear programming model with two types of variables; binary and real variables. It should be noted that all constraints and objective functions are linear. Therefore, it may be solved by the mix Integer Programming Algorithms. Once more, it should be noted that the interdictor destroys arcs in order to minimize the attacker's optimal reward, subject to a budget R. The complete interdiction of each arc, (i, j) , incurs a cost of c_{ij}. We denote the interdictor's and attacker's decision variables by w_{ij} and x_{ijk} , respectively. The capacity of edge (i, j) is reduced to, where $w_{ij} = 1$ represents the complete interdiction of (i, j). The attacker variable x_{ijk} represents the amount of flow from the source $n_k \in N$ passing the arc $(i, j) \in L$ after interdiction. Now, due to the existence of capacity constraints, both the Final Model and Model 1 problems are feasible (one of the solutions is $w_{ij} = 0$, zero flow). Moreover, after presenting the given solutions, both primal optimal solutions of problem and optimal solutions of Final Model are equal (duality theorem).

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