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Portfolio Optimization Problem Considering Cardinality and Bounding Constraints Using a Metaheuristic Algorithm

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Abstract

The optimal portfolio selection problem is one of the most important problems in finance investigated by many researchers and professors over the last few decades. Of the exact methods and effective approximate solution algorithms, metaheuristic methods have also been successfully proposed to solve some practical and large-scale problems with large numbers of assets and constraints. Hence, in this study, it is tried to optimize the portfolio selection problem by the metaheuristic cuckoo search algorithm (CSA) considering cardinality constraints and show that the mentioned algorithm is capable of achieving suitable solutions. Once the algorithm is designed and run in MATLAB software, the efficient frontier diagram obtained from CSA is close and similar to the efficient frontier diagram obtained from the basic Markowitz model confirming the accuracy and validity of the results obtained from CSA. However, it should be noted that the convergence of the solutions obtained according to CSA is better. Finally, a general comparison between the results obtained from the use of CSA in this study and bee and genetic algorithms in other studies is shown. Based on the results, the average risk return according to CSA is higher than the other two algorithms. Moreover, the portfolio risk according to CSA is lower compared to the other algorithms.

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Introduction

A financial portfolio refers to a collection of financial assets such as stocks, bonds, and cash, the management of which is one of the most challenging issues in investment management and analysis. The main objective of forming a portfolio is to divide the investment risk into multiple assets; thus, the profit of one asset can compensate for the loss of another. One of the problems with the Markowitz portfolio optimization model is that the resulting portfolio usually encompasses small investments in a large number of assets in order to reduce risk. It should be noted that the management of portfolios including a large number of assets is difficult and also creates high transaction costs for the investor (Ma et al., 2012).

One of the methods to solve this problem is to limit the number of assets in the portfolio and to set a bound on the capital allocated to each asset, in which case the resulting model is called cardinality-constrained portfolio optimization. The portfolio selection and optimization problem in the case of minimizing risk while keeping the return constant can be solved using mathematical formulas and a quadratic equation. However, in practice and in the real world, due to the large number of choices, the mathematical approach used to solve this model requires extensive calculations and programming (Fabozzi et al., 2007).

The wide use of new tools enhances the motivation of individuals to enter the capital market and attract more resources and make financial markets more efficient. Despite the increasing use of portfolios and its rich literature, there still exist many unanswered questions in this regard. Additionally, Iranian stock markets, as growing markets, require local research so as to answer these questions. Efforts are being made to resolve optimization problems in order to support the general public in striking the balance between the factors affecting their choices and selecting the most desirable assets in the stock portfolio (Ong et al., 2005). Although minimizing risk and maximizing investment returns appears to be simple, in practice, several methods have been used to form an optimal portfolio. The closer the modeling of the optimal portfolio selection problem is to reality, the more complicated the mathematical problem, making the use of traditional methods impossible; therefore, we are forced to use metaheuristic algorithms. Metaheuristic algorithms also provide a suitable solution.

The research method is also based on the fact that there is a dataset with a certain number of stocks in the literature solved by various metaheuristic algorithms. In this research, we try to solve the aforementioned problem by the metaheuristic cuckoo search algorithm (CSA) and show that it is possible to achieve better solutions.

The second section investigates the theoretical foundations and background of the research. The third section describes the research methodology. The fourth section analyzes the numerical results, and finally, the fifth section presents conclusions and suggestions.

2. Theoretical Foundations and Background

Risk and return are the most important concepts in investment decision-making. Each stock or each portfolio of stocks, if purchased, held and sold within a certain period of time, would also provide its holder with a certain return. This return includes the change in price and the benefits derived from ownership. The term rate of return is used to describe the rate of increase or decrease in investment during the asset holding period. Whenever future returns are predicted and the probability of each of the predictions is multiplied and each of them is added together, the result will be the expected rate of return. Given the level of risk aversion or risk tolerance, the motivation to accept risk varies in individuals. The objective of measuring risk is to increase the ability to make better decisions. Risk taking can be defined as the probability of tolerating a loss. Usually, risk is the possibility of an undesirable event occurring (Ehrgott et al., 2004).

The stock selection optimization problem can be considered as a bi-objective optimization problem. Streichert et al. (2004) and Armañanzas and Lozano (2005) were among the researchers studying this problem with a bi-objective approach, one of which is to minimize risk and the other is to maximize the return on the stock portfolio. Moreover, this problem can be optimized by multiple objectives (more than two objectives). Efforts made by Ehrgott et al. (2004), Subbu et al. (2005) and Ong et al. (2005) can be mentioned as examples. However, it is worth noting that the multi-objective approach to solving the optimal portfolio problem has several disadvantages, which are fully described in Tolo and Rowley (2008).

Chang et al. (2000), Xia et al. (2000), and Kellerer and Maringer (2003) also suggested that the minimum budget constraint can be transferred to the objective function using the Lagrangian liberalization method. One of the advantages of this method is that the optimal stock portfolio is determined by considering the investor's risk tolerance or risk aversion.

The problem of determining the optimal stock portfolio is a quadratic model whose complexity increases with the increase in the number of assets and model constraints. In these circumstances, the use of heuristic methods, in particular metaheuristics, is inevitable and these methods will lead to proper solutions in a reasonable time. Several metaheuristic methods such as genetic, particle swarm optimization, ant colony optimization, simulated annealing, and Tabu search algorithms have been used in the field of stock portfolio model optimization. Among the efforts made in this field, the following papers can be mentioned:

Chen (2015) proposed a new probabilistic semi-absolute mean-standard deviation model considering transaction cost, cardinality, and quantity constraints. Then, he used the modified bee algorithm to solve the problem. Numerical results showed that the algorithm outperformed the standard ybee algorithm and other heuristic algorithms such as genetic, simulated annealing, particle swarm optimization, and evolutionary differential.

Lee and Yoo (2018) compared three types of neural networks for stock return prediction. The experimental results of the neural network showed that the long-term short-term memory (LSTM) neural network estimated the best model. They also built a prediction-based investment portfolio based on the prediction results of the LSTM neural network. This model was more data-driven in the design of the investment portfolio than the existing models. The experimental results showed that this investment portfolio had promising returns.

Fischer and Krauss (2018) used LSTM neural networks to predict directional movements. They found that LSTM-based neural networks outperformed portfolios based on memoryless classification. A common flaw in the reviewed models is that these models only use simple methods to construct their portfolios, such as the equal-weighted and threshold methods. The aforementioned methods for constructing portfolios do not analyze the risk of each stock; therefore, they are unable to balance the expected return and risk of the portfolio.

Wang et al. (2020) developed an investment portfolio optimization model using LSTM neural network for stock selection and mean-variance (MV) model for investment portfolio optimization. In this model, the LSTM neural network first selects k stocks from the whole stock pool, then the selected k stocks are used to build an MV-based investment portfolio model. They compared the LSTM neural network with support vector machines (SVM) and autoregressive integrated moving average (ARIMA) model in the stock selection process, then used the MV model for investment portfolio optimization. The experimental results showed that their proposed model outperformed the other models.

Ta et al. (2020) compared investment portfolios using the LSTM neural network and three portfolio optimization techniques such as equal-weighted method, Monte Carlo simulation, and MV model. Also, they used linear regression and SVM for comparison in the stock selection process. The experimental results showed that the LSTM neural network had higher prediction accuracy than linear regression and SVM and its constructed investment portfolio outperformed the other models. These investigated models apply different methods for stock selection, then build portfolio optimization models with the selected stocks for investment. These methods show promising results for building portfolio optimization models in practice. However, classical portfolio optimization models are often unsuitable for short-term practical investment. Therefore, it is important to explore a more efficient approach to combine prediction results with portfolio optimization models.

Yu et al. (2020) combined ARIMA model forecasts in advancing six portfolio optimization models, such as MV, market abuse directive (MAD), debt service ratio (DSR), liquidity adjusted value at risk (LVaR), conditional value at risk (CvaR), and Omega models. They first used the ARIMA model to forecast future stock returns, then used the forecast results to extend these portfolio optimization models. The experimental results showed that the advanced portfolio optimization models with the ARIMA forecast outperformed the conventional models, and the extended model with MV and Omega models performed best among these models.

Ma et al. (2021) combined models such as stochastic prediction and support vector regression, LSTM neural network, and deep multilayer perceptron to predict returns in portfolio formation. Specifically, this study used these prediction models to select stocks before portfolio formation. Then, their predicted results were followed in advancing the optimization MV and omega models.

In their study, Zhi et al. (2021) used copula and portfolio optimization models to investigate how inventory financing providers use market information to optimize the portfolio of market collateral to reduce default risks. By comparing the forecasting performance of copula strategies with a multivariate normal distribution strategy, it was shown that the conventional copula could characterize the dependence structure among collateral return series and had superior forecasting performance to the normal distribution strategy.

Doaei et al. (2021) proposed a framework for predicting the daily dividend price index of Tehran Stock Exchange through multilayer perceptron hybrid neural networks and metaheuristic algorithms consisting of genetic, particle swarm optimization, black hole and grey wolf algorithms. For this purpose, 18 technical indicators were extracted based on the daily dividend performance of Tehran Stock Exchange as input parameters. Experimental results showed that the grey wolf optimization algorithm had superior performance for training the hybrid neural network for stock market forecasting based on metaheuristics.

Gaspars-Wieloch (2022) presented a new decision rule for mixed uncertainty problems based on the goal programming method. This method can be used by pessimistic, optimistic, and balanced decision makers. One of the significant advantages of the new approach is the possibility of analyzing criteria that are not directly considered in the existing classical methods developed.

Cervellera (2023) introduced a data-driven scenario generation method based on the statistical concept of copula models, through which input parameters could be freely chosen without changing the structure of the joint multivariate dependence of the inputs. This approach is particularly suitable for implementing what-if simulation scenarios, in which the marginal distributions of the inputs change, while maintaining the joint dependence scheme. The proposed method can capture complex multivariate distributions of the simulation results and obtain reliable inferences in what-if analyses, significantly better than those where the joint dependence is ignored.

Stoilov et al. (2024) paid attention to minimizing economic risk in resource allocation planning as factors allowing for reducing potential losses for livestock management.

3. Methodology

In this research, the method is developmental-applied from in terms of purpose because it seeks to demonstrate a specific application of a problem in the real world. Also, in terms of nature, it uses a mathematical modeling approach. During modeling, a forward movement is always used, and we go back rarely and only to check the validity of the model. Various steps are envisaged to implement the modeling process, and these research stages are as follows: first, to present a model with innovation, the subject literature and papers related to the selection of investment portfolios should be reviewed and the characteristics of each paper, including research innovations, encompassing the consideration of the conditions of the units in terms of being classical (black box), dynamic, or non-dynamic, are extracted. The research gap indicates the methods and innovations used in different dimensions of the problem and reveals aspects of the subject that are not considered in the papers. In this research, the units with details and other subunits and the dynamism and the existence of a planning horizon in the model are considered. Then, the different aspects of the subject found in the research gap form the basis of our research problem, based on which the problem is stated and the hypotheses of the problem are determined. Once the mathematical model is designed and prepared, using the studies conducted in the literature, the proposed solution method is determined, then the data is collected and the desired problem is solved. Finally, the results are analyzed and a general conclusion is drawn from the research and suggestions for future research are presented. A summary of the stages of research implementation is described as follows.

Step 1: Review of existing domestic and foreign theoretical sources:

- Completing further studies and research and reviewing the literature on the subject;
- Identifying the criteria for selecting a stock portfolio using a review of financial research literature
- A detailed review of the most recent studies conducted in this field;

Step 2: Collecting information:

- Collecting proper information about important and effective characteristics and parameters in the problem environment;

Step 3: Problem modeling:

- Building and developing a suitable mathematical program model congruous with the characteristics and main assumptions of the problem;
- Coding the model in a proper software environment such as MATLAB software;
- Determining the solution set of the problem using a conventional deterministic method such as Simplex,
- Validating the developed model;
- Sensitivity analysis;

Step 4: Conclusion:

- Overall summary of the research results and suggestions for future research.

There are approximately 601 companies listed on the Tehran Stock Exchange. Since the main objective of this research is to investigate the efficiency of the CSA in optimizing the stock portfolio considering cardinality constraints, selecting the sample is not of great importance. Nevertheless, it was tried to select the companies from those more active on the Tehran Stock Exchange from various industries that have always been considered by active market participants and have had adequate liquidity. These stocks include:

Table 1. Selected stocks in the research

No.	Company
1	Iran Transfo Co.
2	Eghtesad Novin Bank
3	Tejart Bank
4	Sina Bank
5	Mellat Bank
6	SAIPA
7	Ghadir Investment Co.
8	Mobarakeh Steel Co.
9	Iranian Telecommunication Co.

10	National Iranian Copper Industry Co.

The abovementioned companies constitute the sample of the present study.

3.1. Proposed Portfolio Optimization Problem Without Cardinality Constraint

In this section, the portfolio optimization problem without cardinality constraints is investigated more thoroughly. The model of this problem is expressed as follows:

$$minimize \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \qquad \text{Relation (1)}$$

$$\text{Subjected to: } \sum_{i=1}^{n} w_i \mu_j = R^* \qquad \text{Relation (2)}$$

$$\sum_{i=1}^{n} w_i = 1 \qquad \text{Relation (3)}$$

$$\sum_{i=1}^{n} z_i = K \qquad \text{Relation (4)}$$

$$\varepsilon_i z_i \le w_i \le \delta_i z_i, \quad i = 1, \dots, n \qquad \text{Relation (5)}$$

$$z_i \in \{0,1\}, \quad i = 1, \dots, n \qquad \text{Relation (6)}$$

The objective function (1) minimizes the total variance (risk) of the portfolio, while constraint (2) sets the expected return of the portfolio equal to R^*. Constraint (3) guarantees the investment of the total available budget. Constraint (4) shows that exactly K items (stocks) must be placed in the portfolio. Constraint (5) states that if items (stocks) i are selected and placed in the portfolio ($z_i = 1$), then the stock allocated to this item must be greater than ε_i and less than δ_i ($0 \le \varepsilon_i \le \delta_i \le 1$). Given this constraint, it is clear that $w_i \ge 0$. If asset i is not selected and placed outside the portfolio ($z_i = 0$), then the stock allocated to the item, i.e. w_i, is equal to zero.

3.2. Cardinality and Bounding Constraints:

One of the problems with unconstrained portfolio optimization models is that, in order to reduce investment risk, the resulting portfolio usually includes small investments in a significant number of items. However, this investment policy creates problems in practice; it is difficult to manage portfolios including a large number of items and it also creates high transaction costs for the investor. One way to solve this problem is to set a limit on the number of items in the portfolio and also to set a limit on the allocation of capital to each item. For this purpose, suppose:

- ✓ K is the number of items in the portfolio,
- ✓ ϵ_i is the minimum possible stock to be assigned to item i if this item is selected (i=1,…,N)
- ✓ δ_i is the maximum possible stock to be assigned to item i if this item is selected (i=1,…,N)

Similarly, $0 \leq \epsilon_i \leq \delta_i \leq 1$, then the variable z_i is defined as follows:

$$z_i = \begin{cases} 1 & \text{If item i is in the portfolio} \\ 0 & otherwise \end{cases}$$ Relation (7)

Now, the cardinality and bounding constraints can be expressed as follows:

$$\sum_{i=1}^{n} z_i \leq K$$
$$\varepsilon_i z_i \leq w_i \leq \delta_i z_i, \quad i = 1, \dots, n$$ Relation (8)

Some papers studying the portfolio optimization problem with cardinality constraints consider the cardinality constraint as unequal and some as equal. In the following, we will refer to the problem involving these two constraints as the cardinality-constrained portfolio optimization problem. In the Markowitz mean-variance portfolio optimization with cardinality constraints, the goal is to create portfolios that, in addition to balancing risk and return, allow control over the number of assets held in the portfolio. By introducing a cardinality constraint, the number of assets is limited. In other words, we only invest in assets reducing risk for a given level of return. One practical reason for imposing a cardinality constraint is that it is easier for an investor to find a portfolio with only a certain number of assets, and in this case, by choosing from the assets available in the portfolio, the investor can control the overall shape of the portfolio.

3.3. Problem Solving Method: Cuckoo Search Algorithm

Cuckoo search algorithm (CSA) is a metaheuristic optimization method with an evolutionary approach in searching for the optimal solution and was proposed by Yang and Deb (2009). This method is inspired by the interesting behavior of some species of cuckoo with their eggs. Studies conducted on this algorithm have shown that this method has good efficiency for many problems. In experiments, this algorithm has achieved higher accuracy and a higher success rate than other evolutionary algorithms.

Most bird species have a similar lifestyle. The female lays eggs. Since bird eggs contain a lot of protein and nutrients, most birds have to keep their eggs in safe places to protect them from various types of predators. Finding a suitable place to care for the eggs and raise them until they become independent is one of the most challenging instinctive tasks of various birds. Most birds build their nests among the leaves and branches of trees and take care of their eggs in them.

In this algorithm, adult cuckoos and their eggs form the initial population of CSA. Adult cuckoos lay eggs in the nests of other birds. If the cuckoo eggs are not detected and destroyed by the adult host birds, they will grow and develop into adult cuckoos.

Adult cuckoos migrate in groups under the influence of environmental characteristics and in the hope of finding an optimal environment for life and reproduction. In this algorithm, the optimal environment will be the “global optimum” in the objective function of the optimization problem. Rajabion (2011) introduced CSA for continuous nonlinear problems and showed that it is one of the most powerful evolutionary optimization methods introduced so far.

4. Numerical Results

In this subsection, we want to investigate and validate the performance of the metaheuristic algorithm in optimizing the optimal portfolio selection problem. First, the proposed model is examined for a standard set introduced in the literature (with five assets or stocks), and then the efficiency frontier diagram obtained from the metaheuristic CSA is compared with the efficiency frontier diagram obtained from the basic Markowitz model. Sefiane and Benbouziane (2012) and Pakanin and Pelevich (2013) used the metaheuristic "genetic" and "artificial bee" algorithms to optimize a standard five-stock portfolio introduced in the literature, respectively. The average values of the return of each stock and the covariance between the stocks are shown in Tables 2 and 3.

Table 2. Average return on assets per stock

Name of stock	Average return
Stock 1	0.116
Stock 2	0.226
Stock 3	0.252
Stock 4	0.204
Stock 5	0.110

Table 3. Covariance between stocks

	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5
Stock 1	0.2173	0.0034-	0.0535-	0.0093-	0.0106

Stock 2	0.0034-	0.0025	0.0085	0.0024	0.0046-
Stock 3	0.0535-	0.0085	0.2225	0.0311-	0.0239-
Stock 4	0.0093-	0.0024	0.0311-	0.0407	0.0028
Stock 5	0.0106	0.0046-	0.0239-	0.0028	0.0168

The results obtained from Sefiane and Benbouziane (2012) and Pakanin and Pelevich (2013) are as shown in Table 4.

Table 4. Best stock portfolio obtained from different algorithms

Portfolio values	Bee algorithm	Genetic algorithm with different hybrid operators		
		One-point	Two-point	Arithmetic
X_1	0.066	0.0511	0.1167	0.0535
X_2	0.393	0.2050	0.0791	0.4074
X_3	0.387	0.3284	0.6360	0.3912
X_4	0.058	0.2550	0.1167	0.0950
X_5	0.096	0.1605	0.0515	0.0529
Average portfolio return	**0.2164**	**0.2049**	**0.2213**	**0.2222**
Average portfolio risk	**0.0313**	**0.0194**	**0.0801**	**0.0325**

As can be seen, none of the results obtained from the genetic algorithm had a higher return and lower risk compared to the solution obtained from the bee algorithm. In the following, in order to implement CSA for the abovementioned five-stock set, the components of the problem and the algorithm are considered as shown in Table 5.

Table 5. Values of the components of selecting the optimal portfolio for a standard set of stocks

Problem parameters	**CSA parameters**
Number of variables: 5	Number of initial cuckoos: 20
Lower bound of each variable: 0	Maximum number of cuckoos in the population: 50
Upper bound of each variable: 1	Number of groups (classes for migration): 4
Minimum number of assets in the optimal portfolio: 0	Movement coefficient (F): 0.1

Maximum number of assets in the optimal portfolio: 5	Egg-laying radius coefficient (α): 0.1
Risk coefficient (λ): 0.0; 0.2; 0.4; 0.6; 0.8 and 1.0	Minimum and maximum number of eggs per cuckoo: 2 and 4
Risk-free return (RF): 0.08	Maximum number of iterations: 100

In order to optimize the aforementioned five-stock dataset, CSA was run five times independently for each risk coefficient, and the best results obtained are as shown in the following graphs:

a) Running CSA for λ = 0.0

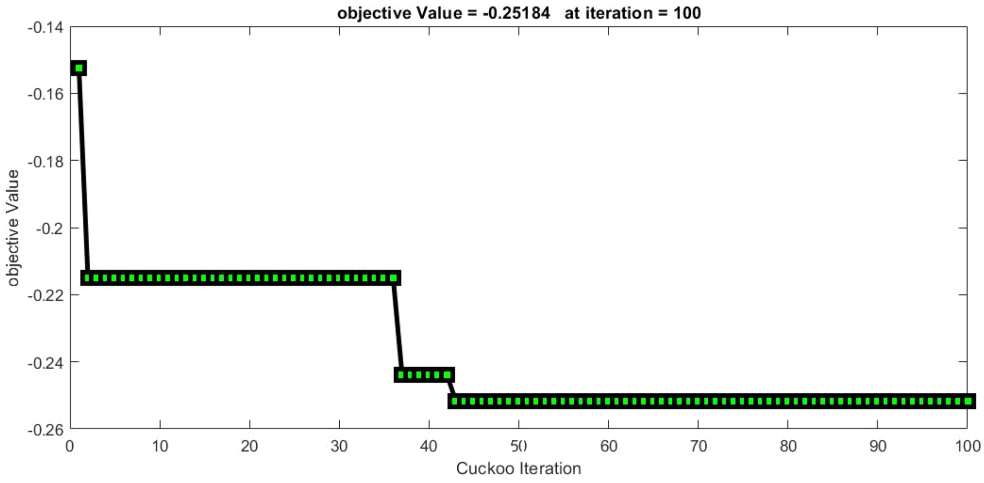

Figure 1. Steps of running CSA for a balance factor of 0.0

According to the run SCA in this case, the average return of the portfolio is 0.2144 and the average risk of the portfolio is 0.46.

B) Running CSA for λ = 0.2

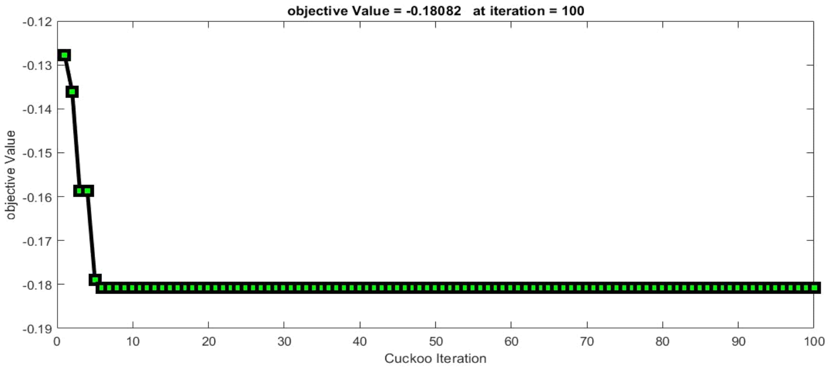

Figure 2. Steps of running CSA for a balance factor of 0.2

According to the run SCA in this case, the average return of the portfolio is 0.2144 and the average risk of the portfolio is 0.46.

C) Running CSA for λ = 0.4

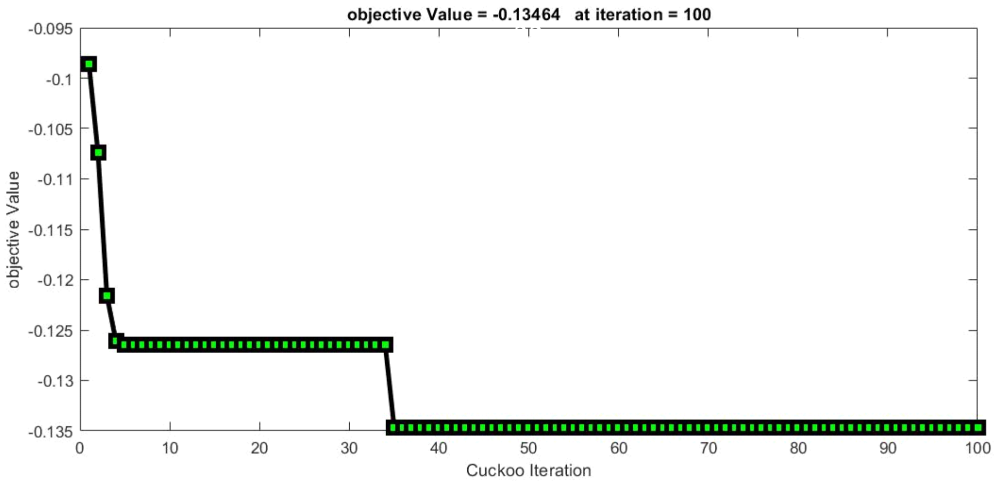

Figure 3. Steps of running CSA for a balance factor of 0.4

According to the run SCA in this case, the average return of the portfolio is 0.2144 and the average risk of the portfolio is 0.46.

D) Running CSA for λ = 0.6

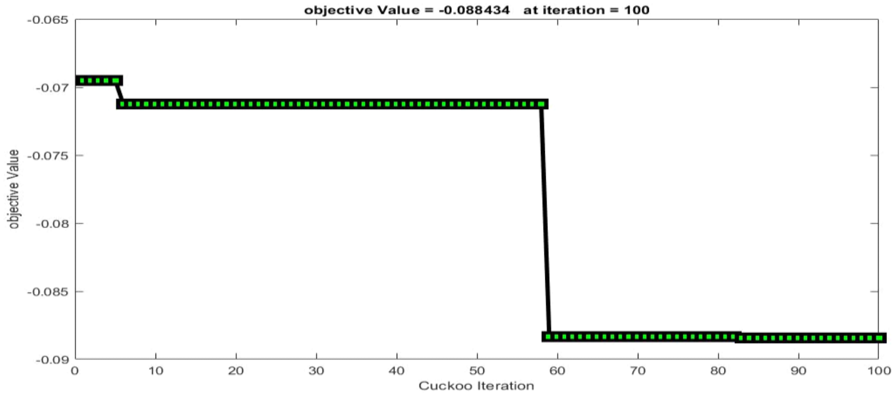

Figure 4. Steps of running CSA for a balance factor of 0.6

According to the run SCA in this case, the average return of the portfolio is 0.2144 and the average risk of the portfolio is 0.46.

E) Running CSA for λ = 0.8

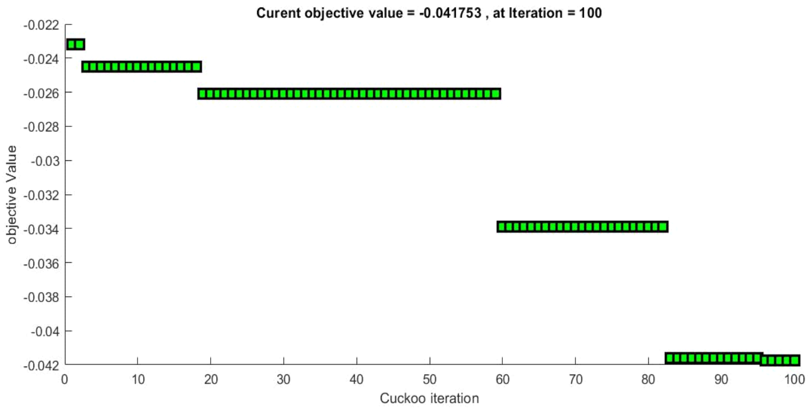

Figure 5. Steps of running CSA for a balance factor of 0.8

According to the run SCA in this case, the average return of the portfolio is 0.2144 and the average risk of the portfolio is 0.46.

F) Running CSA for λ = 1.0

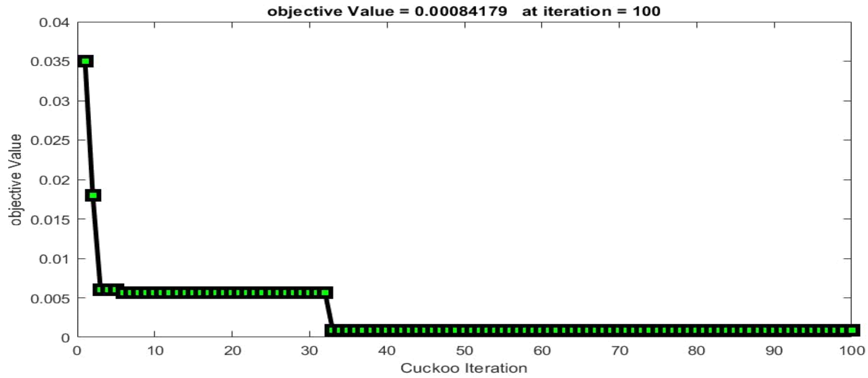

Figure 6. Steps of running CSA for a balance factor of 1.0

According to the run SCA in this case, the average return of the portfolio is 0.2144 and the average risk of the portfolio is 0.46.

Finally, the summary of the results is listed in Table 6.

Table 6. Results of running CSA for the standard five-stock set

	0.0 = λ	0.2 = λ	0.4 = λ	0.6 = λ	0.8 = λ	1.0 = λ
X_1	0/0000	0/0002	0/0000	0/0000	0/0000	0/0213
X_2	0/0008	0/5877	0/9009	0/9868	0/8842	0/7235
X_3	0/9964	0/4094	0/0955	0/0013	0/0000	0/0000
X_4	0/0028	0/0027	0/0000	0/0000	0/0241	0/0000
X_5	0/0000	0/0000	0/0036	0/0119	0/0917	0/2551
Average portfolio return	0/2518	0/2366	0/2281	0/2247	0/2148	0/1941
Average portfolio risk	0/4698	0/2053	0/0741	0/0488	0/0389	0/0290

By comparing the information in Tables 5 and 6, it is seen that some results obtained from CSA are superior to the results of the genetic algorithm with a two-point operator and that, in general, compared to other solutions obtained from running various metaheuristic algorithms, it has achieved valid and acceptable results, indicating the correct performance of the proposed CSA.

4.1. Examination of the Efficient Frontier obtained from CSA and Markowitz Model

In this subsection of the report, the basic Markowitz model is implemented for a standard five-stock set for different expected returns (between the minimum return and the maximum return of stocks) and the obtained results are given in Table 7.

Table 7. Results from running the basic Markowitz model for a set of standard stocks

Minimum expected returns	0.2520	0.2236	0.1952	0.1668	0.1384	0.1100
X_1	0/0000	0/0000	0/0000	0/0000	0/0005	0/0035
X_2	0/0000	0/4250	0/8501	0/9383	0/8431	0/7443
X_3	1/0000	0/5750	0/1499	0/0000	0/0000	0/0027
X_4	0/0000	0/0000	0/0000	0/0000	0/0000	0/0000
X_5	0/0000	0/0000	0/0000	0/0617	0/1565	0/2495
Average portfolio return	0/2520	0/2409	0/2299	0/2188	0/2078	0/1967
Average portfolio risk	0/4717	0/2795	0/0948	0/0420	0/0317	0/0275

According to the values obtained for average return and risk for different values of expected return, the corresponding efficient frontier diagram is as follows.

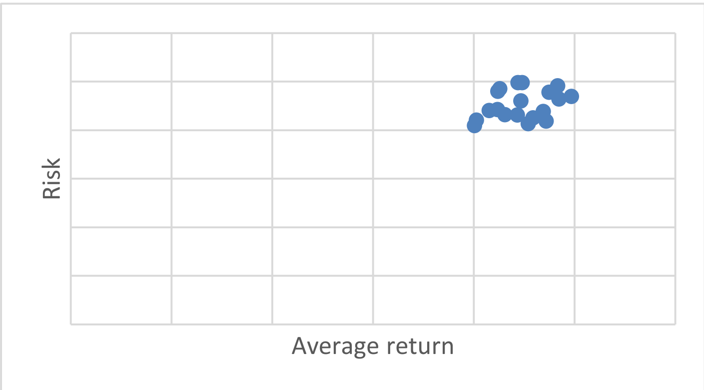

Figure 7. Efficient frontier of the standard stock set using the Markowitz model

In the following, in order to compare the results, the efficient frontier diagram according to the results obtained from CSA is demonstrated in Figure 8.

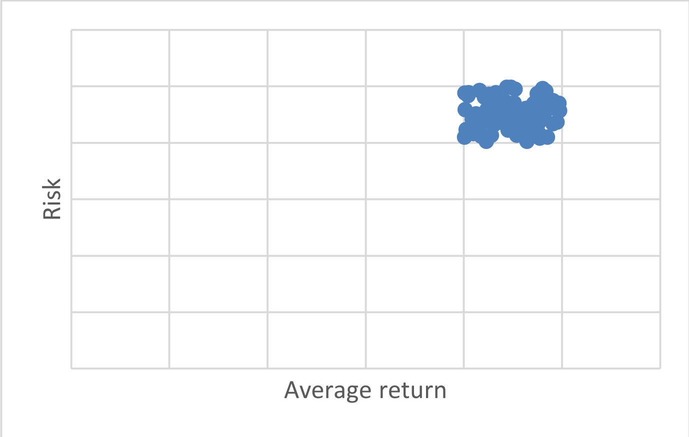

Figure 8. Efficient frontier of standard stock portfolio using CSA

As can be seen, the efficient frontier diagram obtained from CSA is close and similar to the efficient frontier diagram obtained from the basic Markowitz model, confirming the accuracy and validity of the results obtained from CSA. However, it should be noted that the convergence of the solutions obtained according to CSA is better. Finally, a general comparison is shown in Table 8 between the results obtained from the CSA used in this study and bee and genetic algorithms in other studies. Based on the obtained results, the average risk return according to CSA is higher than that in the other two algorithms. Also, the portfolio risk according to CSA is lower than that in other algorithms.

Table 8. Comparison of CSA and bee and genetic algorithms

Portfolio values	Bee algorithm	Genetic algorithm with different hybrid operators			CSA
		One-point	Two-point	Arithmetic	
X_1	0.066	0.0511	0.1167	0.0535	0.2050
X_2	0.393	0.2050	0.0791	0.4074	0.3284
X_3	0.387	0.3284	0.6360	0.3912	0.2550
X_4	0.058	0.2550	0.1167	0.0950	0.387
X_5	0.096	0.1605	0.0515	0.0529	0.058
Average portfolio return	**0.2164**	**0.2049**	**0.2213**	**0.2222**	**0.235**
Average portfolio risk	**0.0313**	**0.0194**	**0.0801**	**0.0325**	**0.0011**

5. Conclusion and Suggestions

The way stocks are selected in the stock exchange has been one of the major concerns of investors in recent years. Selecting a stock or portfolio with the highest return and the lowest risk in terms of profitability, increase price and profitability is of great importance. Therefore, optimizing the selection of a stock or portfolio has been considered as a critical matter by all analysts, and extensive efforts have been made in this field so as to control and make predictable the irregular and nonlinear models affected by many factors.

In this study, an attempt was made to propose a heuristic method for determining the optimal portfolio formation method while investigating the performance of this method. The main objective of this study was to answer the question of whether the use of the metaheuristic CSA in solving the optimization problem considering cardinality constraints leads to a better solution than other metaheuristic algorithms. After designing and implementing the algorithm in MATLAB software, the efficient frontier diagram obtained from CSA was close and similar to the efficient frontier diagram obtained from the basic Markowitz model, confirming the accuracy and validity of the results obtained from CSA. However, it should be noted that the convergence of the solutions obtained according to CSA is better. Finally, a general comparison was made between the results obtained from the CSA used in this study and bee and genetic algorithms in other studies. According to the results obtained, the average risk return according to CSA is higher than that in the other two algorithms. Also, the portfolio risk according to CSA is lower compared to other algorithms.

5.1. Suggestions for Future Research

In order to complete and further this research, the following suggestions are presented for future research:

a) Using other metaheuristic algorithms such as sequential minimal optimization, bacterial search optimization, etc. and evaluating their efficiency in comparison with the algorithm used in this research.

b) Optimizing the selected stock portfolio by the algorithm proposed in this research assuming the existence of short selling in the market.

c) The hypothetical investor considered in this research is an investor who has allocated their entire stock portfolio to stocks available in the capital market. Mixed assets such as gold, currency, bonds, etc. could also be considered in the set of assets of this investor.

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