



Comparison of original and weighted singularity index in separation of Pb- Zn mineralized zone in the Haft Savaran district, Central Iran

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Abstract

The Haft Saravan area is located in 90 km SE Arak in the central Iran. This area is a part of Malyer-Isfahan metallogenic belt. The article's objective is determining the most suitable method for identifying mineralized zones using original singularity and weighted singularity methods in three-dimensional space. In weighted singularity method, mineralized zones have a greater volume relative to original singularity method. Coefficient of areal association showed that percentage of overlapping of two methods is less than 50% for Pb, Fe and Mn and less than 55% for Zn. Two methods were compared by modified singularity method in order to select the best method. Percentage of overlapping is more than 98% both original and modified singularity methods for Pb, Zn and Mn and is less than 45% for weighted singularity method. Therefore, it can be said that original singularity method is suitable relative to weighted singularity in identifying mineralized zones due to the high overlapping of original and modified singularity method.

Keywords: Original singularity, *Weighted singularity, Mineralized zones, Haft Savaran*

1. Introduction

Mineralization as a nonlinear geo-process is accompanied by anomalous energy release and material accumulation in a narrow spatial-temporal interval. Simultaneously, it is a cascade process associated with various geological activities (e.g., magmatism, tectonism, etc.). Knowledge of these associated geo-activities is consequently beneficial to the exploration of mineralization. The singularity index mapping method in the context of fractal/multifractal efficient in separating geo-anomalies from both strong and weak background is applied to characterize variations of geological signatures. Geo-information extraction so-called geo-anomaly identification or separation from background is a primary task in geological exploration (Cheng et al. 1994; Afzal and Karami 2015; Afzal et al. 2017). After the introduction of multifractals by Mandelbrot (1972), various fractal/multifractal models were developed and commonly used to investigate geological issues (Carranza 2008; Carranza et al. 2009; Carranza and Sadeghi 2010; Afzal et al. 2016; Adib et al. 2017; Daneshvar 2017; Parsa et al. 2017a,b,c). Singularity, a multifractal concept proposed by Cheng (1999), is a quantitative and qualitative characteristic of particular natural phenomena that has been considered today, because of exposing hidden anomalies. Several researchers have tried to demonstrate geochemical anomalies by combination of singularity index along with other methods (Cheng 2007; Cheng and Agterberg 2009; Zuo et al. 2009; Cheng and Zhao 2011; Zuo et al. 2013; Liu et al. 2014; Zuo et al. 2014; Parsa et al. 2017

b,c; Zhao et al. 2017; Chen et al. 2018; Xiao et al. 2018).

Several varieties of the window-sliding based method, which is mainly dressed on procedures to improve its accuracy of computation and application to different measurements, has been developed in the literature including original algorithm singularity (Cheng et al. 2007), modified singularity analysis applied to negative values (Zuo et al. 2015) and weighted singularity index for detecting features with anisotropic patterns (Xiao et al. 2018). The original singularity index measures the gradient of relative changes within small neighbourhoods and can effectively detect the weak anomalies. This approach is also a kind of local neighbourhood statistical method that can reduce the effects of regional background and provide useful statistical information through involving data within small neighbourhood around a specific spatial location (Zuo 2014). The singularity index estimated using this method is influenced by the background value. In addition to the effects of background value, the original algorithm for calculating the original singularity index requires logarithmic transformation, which demands a positive density of area. Specific spatial pattern, frequently contain negative values, resulting in a negative density area. The modified method can process the data containing negative values and mitigate the effects of background values (Zuo et al. 2014). In addition to singularity, many other studies have suggested that the spatial anisotropy is another crucial property implicit in the nonlinear natural processes including hydrothermal mineralization (Zhang et al.

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2016). In mineral exploration, the mineralization associated geochemical anomalies are usually found to be characterized by anisotropic patterns that are controlled by the ore-controlling geological features such as strata, faults, folds, and magma intrusions, because they could supply the spaces, heats, fluids for the mineralization systems. As a result, incorporating these two significant properties that are both singularity and anisotropy inherent from mineralization systems could probably make the delineated anomaly targets be more geologically reasonable than the utilization of either one or even none of them (Zhang et al. 2016). Weighted singularity index is a useful tool for detecting features with anisotropic patterns and avoiding utilization of irrelevant samples or variables in the calculation of certain measurements for geo information extraction (Zhao et al. 2016).

The purpose of this study is to use original and weighted singularity indices to determine Pb, Zn, Fe and Mn mineralized zones and compare two models with modified singularity method and finally to select the best method for identification of depleted and enriched areas and background.

2. Geological setting

Haft Savaran deposit is located in the Malayer- Esfahan Pb-Zn belt (Fig 1). There has been known more than 600 Pb-Zn deposits in Iran (Zandy Ilghani et al. 2018). There are about 220 deposits in the Malayer- Esfahan belt. All this mineralization occurred in the lower Cretaceous carbonate facies (about 90%) and Jurassic sandstone and shale (about 10%). Mineralization area is situated about 25 km SW of Khomein and SE of Arak city in Sannanaj-Sirjanbelt. Based on the geological map of Haft-Savaran area, sandstone and shale of the lower Jurassic and Cretaceous limestone are the main rocks in Pb-Zn deposit (Fig 2). The typical sulfide mineral association is galena, pyrite, sphalerite, chalcopyrite, iron oxides and iron carbonates and other gangue minerals include, calcite, dolomite and quartz. The Pb + Zn, Pb / Pb + Zn, Cu + Pb + Zn and Cu ratios in the shales and sandstone showed that mineralization is a SEDEX strata-form. We have stratiform syngenetic - syn sedimentary ores in Haft Savaran region (Zandy Ilghani et al. 2018). The Pb and Zn mineralization has been occurred in shales and sandstones.

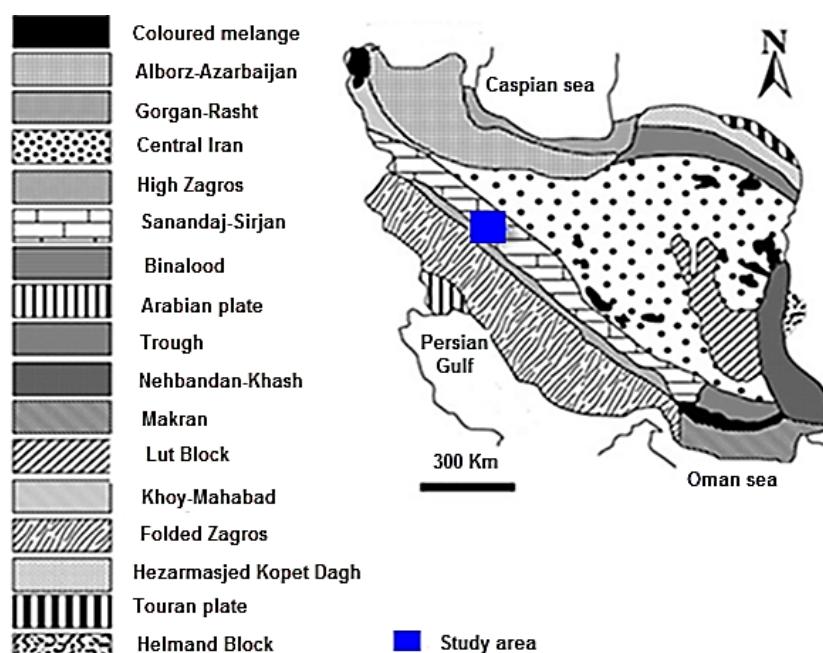


Fig 1. Location of HaftSavaran mineralization in Sanandaj-Sirjanbelt (Zandy Ilghani et al. 2018).

3. Material and methods

3.1. Sampling and analysis

About 170 samples were selected from 33 boreholes in Haft Savaran area (Fig 3). Samples were analyzed by inductively coupled plasma-mass spectrometry (ICP-MS) for 14 elements such as Pb, Zn, Fe, Mn and so on by Zarazma laboratory. The detection limits for Pb, Zn, Fe and Mn is 1, 1, 100 and 5 mg / kg, respectively.

3.2. Singularity index

According to multi-fractal theory, singularity index is related to the distribution of self-similarity (Cheng 2008). In modern nonlinear theory, the concept of multifractal is characterized by the law-power model. The window-based method is used to estimate singularity. Different types of windows such as square, circle, polygon and etc. could be applied to measure

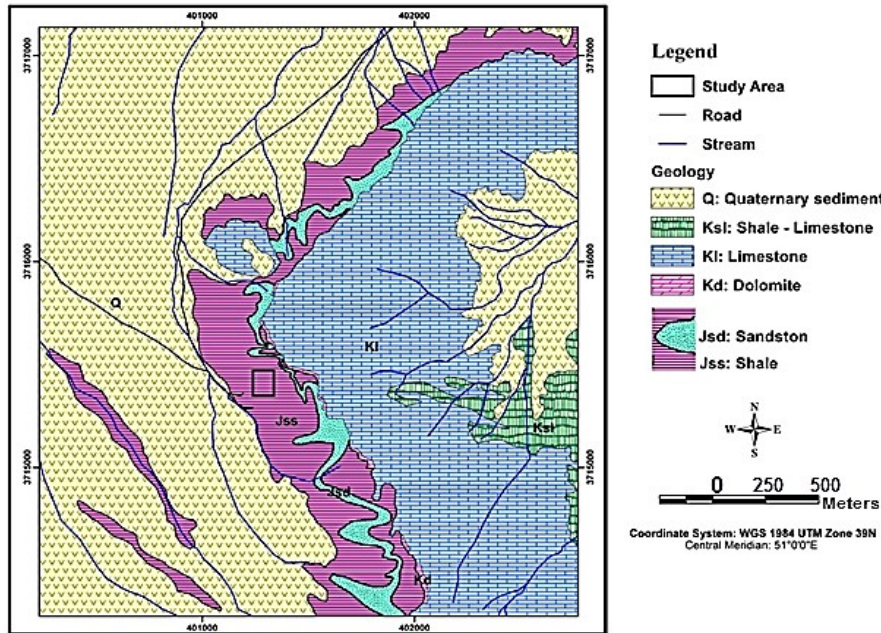


Fig 2. Geological map of the Haft Savaran mineralization area.

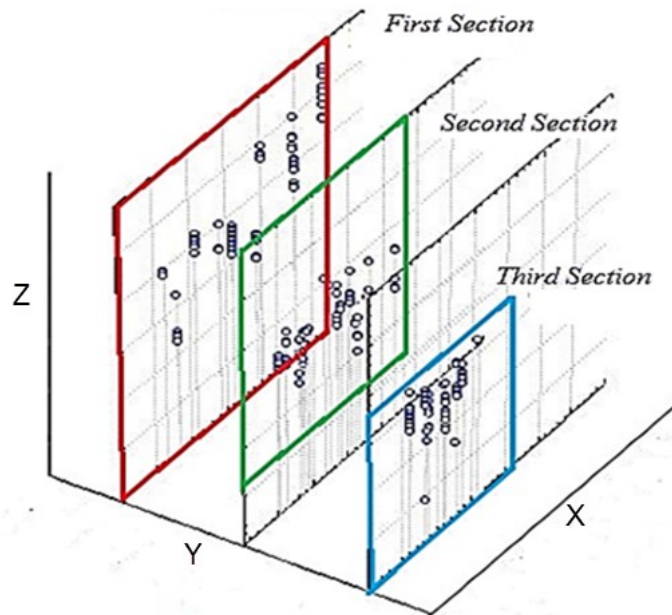


Fig 3. Location of borehole samples on three-dimensional space in Haft Savaran area.

concentration around a point. The singularity index is estimated in a small neighborhood according to Eq.(1).

$$C = c \cdot \varepsilon^{\alpha-E} \tag{1}$$

where C represents the mean concentration of the element, c the constant, α the index of singularity, ε the normalized distance and E the euclidean geometry. The singularity index is evaluated on a straight line with a

pair of data C and ε . Each point of the field is considered using the variables $A(r)$ (circle) with variable windows with sizes $r_{min} = r_1 < r_2 < \dots < r_n = r_{max}$. The mean value of the concentration is calculated for each window and shows linear relationship in the logarithmic diagram, Eq.(2).

$$\log C[A(r)] = c + (\alpha - 2)\log(r) \tag{2}$$

The estimated slope of the linear relation is introduced as $\alpha-2$. The values of the singularity index are typically 2 in the two-dimensional maps which is defined as non-singular points. The original singularity phenomenon in three-dimensional space can be described as a power-law relationship between volume V_i in a mineralization domain and total amount of metal $\mu (V_i)$ as Eq.(3)(Cheng 2007):

$$\mu(V) = cV^{\frac{\alpha}{3}} \quad (3)$$

where α denotes the singularity index. Because metal concentration $C(V_i)$ in V_i can be expressed as $C(V_i)=\mu(V_i)/V_i$, we also have Eq.(4):

$$C(V) = cV^{\left(\frac{\alpha}{3}\right)-1} \quad (4)$$

Using the Eqs.(3) and (4), a three-dimensional space-based method can be applied to estimate the singularity index α . It can be described as follows: given a location on the map, define a set of three-dimensional space $V(r)$ (with square) with variable network sizes, $r_{min}=r_1 < r_2 < \dots < r_n=r_{max}$, and calculate the average concentration value $C[V(r_i)]$ for each network size (Fig 4). There should be a linear relationship between $C[V(r_i)]$ ($i=1, \dots, n$) and r_i , Eq.(5):

$$\text{Log}[C[V(r)]] = c + (\alpha - 3)\text{Log}(r) \quad (5)$$

Value of $\alpha-3$ can be estimated as slope from this linear relationship. Applied to a three-dimensional map, mineralized zone positive singularity index values (with $\alpha < 3$) usually correspond to enrichments of element concentrations while mineralized zone negative singularity index values (with $\alpha > 3$) indicate depletion of element concentrations. At locations where the singularity index $\alpha \approx 3$, there is no positive or negative geochemical singularity. Then singularity distribution map can be created by a method similar to the method of three-dimensional space applied at all locations on the three-dimensional geochemical map. The standard error and squared correlation coefficient involved in the estimation can be calculated from the least squares fitting and these indices can be used for evaluating whether or not the power-law relationships of Eq.(5) exist. Thus, singularities on a three-dimensional space can provide significant information for identifying mineralized zone associated with mineralization (Cheng 2008; Cheng and Agterberg 2009; Zuo et al. 2009).

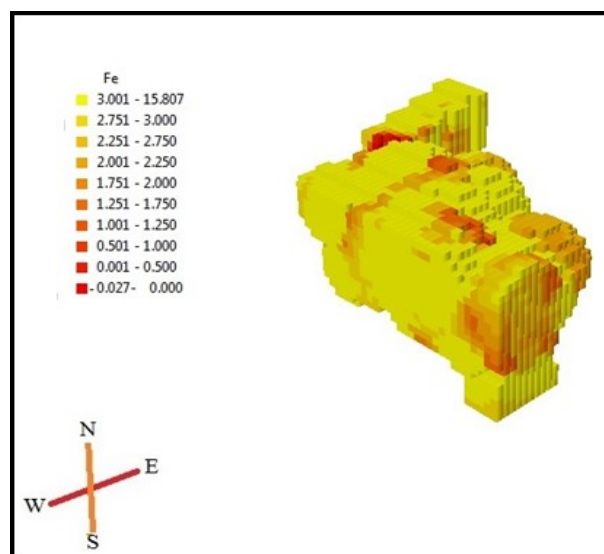


Fig 4. Distribution of Fe singularity index in three-dimensional space in Haft Savaran area.

The weighted singularity index is almost the same as original (simple) singularity index, only the average concentration value is used in calculations for each window. Mean value is obtained based on the squared distance inverse. An inverse distance method is an average weighting method in which data are weighed using the standard deviation of one point from other points using the networked nodes (Khavari 2016). In this method, weights are determined only with respect to the distance (d) of each point relative to the unknown point (Eq.6).

$$C = \frac{\frac{c_1}{d_1^n} + \frac{c_2}{d_2^n} + \dots + \frac{c_m}{d_m^n}}{\frac{1}{d_1^n} + \frac{1}{d_2^n} + \dots + \frac{1}{d_m^n}} \quad (6)$$

4. Results and discussion

4.1. Comparison of methods

Descriptive statistics for four metals used in this study are shown in Table 1. Most of the metals have a wide range of variations of several magnitudes. The mean Pb, Zn, Fe and Mn concentration is equal to 5381, 7268, 93443 and 7432 mg / kg, respectively. The objective of mineralized zones classification is separation of different geochemical population in the enriched areas.

Table 1. Statistical parameters of metals in shale and sandstone (mg/kg).

	Mean	Median	Minimum	Maximum	SD	Skewness	Kurtosis
Pb	5381	560	20	133930	16886	5.51	32.96
Zn	7268	1635	103	79180	13511	3.01	9.87
Fe	93443	71385	331	424700	70450	1.23	2.23
Mn	7432	3595	103	60820	9162	2.18	6.79

Therefore, we were deleted $\alpha > 3$, which represented the depleted areas. The singularity method is similar to the concentration-volume in fractal method, where the concentration is replaced by α -singularity index and volume by the volume containing any α . After plotting α - volume diagram, the best straight line was fitted to the data. Intersection is border between different populations of enrichment. Low α represent rich regions, which has low volume in three-dimensional mineralized zone map. In this classification, regions with a singularity index close to 3 are referred to as areas with very weak mineralized zones. The main enriched areas are divided into four levels populations for Pb, Fe and Mn, and five levels populations for Zn in original singularity index.

A part of the graph with the highest singularity index is introduced as areas with very weak mineralized zones. Therefore, we removed areas with very weak mineralized zones from rich regions (Fig 5) (Khavari 2016). But, Pb and Zn are classified into six, Fe into four populations and Mn to five populations in the weighted singularity index (Fig 6). Dimension and number of singularity indices populations are larger and more in weighted singularity than original singularity method (Tables 2 and 3). For example, Pb index dimension varies from 2.59 to 2.99 for original singularity and varies from -11 to 2.99 for weighted singularity method. Also, the number of Pb intersections is 3 in original singularity, but 5 for weighted singularity method.

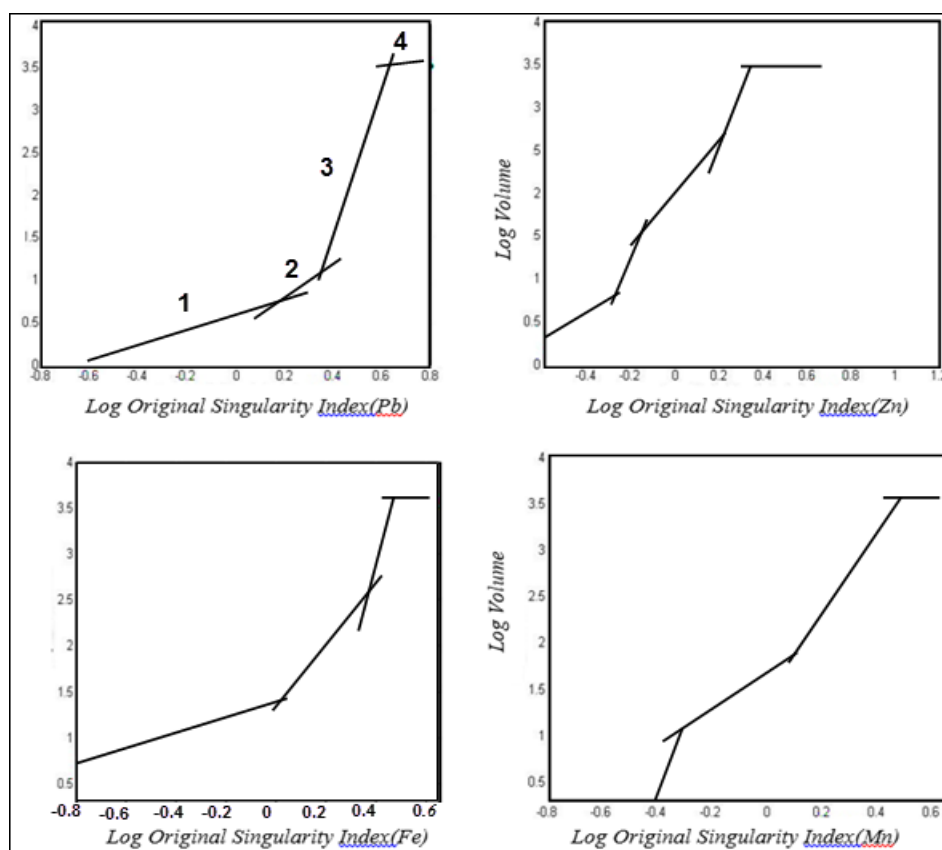


Fig 5. Original singularity index versus volume of Pb, Zn, Fe and Mn.

Comparison of Pb, Zn, Fe, and Mn mineralized zones is shown in three-dimensional space in Figs 7 and 8 in both original and weighted singularity method. In Figs 7 and 8, have been deleted areas with a singularity index more than 3 (background area). For all elements in Fig 7a and Fig 8a mineralized zones are expressed very poorly and degree of mineralized zones increases from b to d, but decreases its extent. It is obvious that the extent of the strong mineralized zones in weighted singularity is far greater than that original singularity. The extent of

weak to moderate mineralized zones is almost similar for Zn, Fe and Mn in original singularity (Fig 7a and b), but varies in strong mineralized zones (Fig 7c and d). However, the extent of weakness to strong mineralized zones show a relatively similar for Pb, Zn, Fe and Mn in weighted singularity method (Fig 8). On the other hand, the extent of each mineralized zones degree is far more in weighted singularity method than original singularity for above elements.

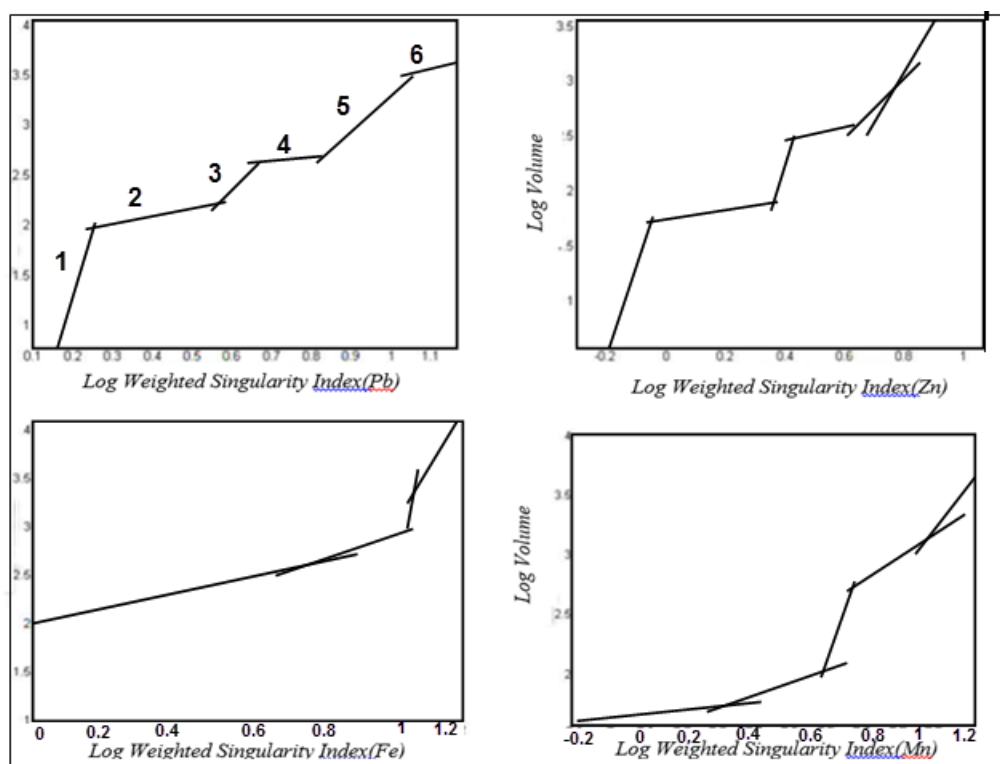


Fig 6. Weighted singularity index versus volume of Pb, Zn, Fe and Mn.

Table 2. Separation boundaries of enriched population in original singularity method.

Elements	Min. α	Max. α	First	Second	Third	Forth
Pb	-2.59	2.99	2.71	-0.34	-1.39	-
Zn	-0.85	2.99	2.86	1.32	-0.11	-0.35
Fe	-0.03	2.99	2.79	1.77	1.11	-
Mn	-0.54	2.99	2.79	0.87	-0.02	-

Table 3. Separation boundaries of enriched population in weighted singularity method.

Elements	Min. α	Max. α	First	Second	Third	Forth	Fifth
Pb	-11	2.99	2.18	-0.89	-4.41	-5.66	-6.77
Zn	-7.76	2.99	2.24	-1.37	-4.32	-5.16	-5.64
Fe	-12.19	2.99	1.93	0.55	-1.29	-	-
Mn	-7.1	2.99	2.44	0.24	-2.53	-3.56	-5.47

In addition, trend of mineralized zones is clear in original singularity method. But, there is no clear trend in mineralization in weighted singularity method. In Fig 9 first quarter indicates depletion and third quarter indicates the enrichment of the elements in both original and weighted singularity methods. The second quarter indicates depletion in original singularity method, but enrichment in weighted singularity method. The fourth quarter also indicates enrichment in original singularity method, but depletion in weighted singularity method. Two vertical axes in the amount of 3 indicate limit of depletion and enrichment. The points on the $Y=X$ line indicate similarity of two methods in estimating

mineralized zones of elements. A large dispersion of points in quadrants 1 and 3 of Pb relative to $Y=X$ line suggests incompatibility of two singularity methods. Distribution of points is almost similar in first quadrant in two methods relative to $Y=X$ line, but in third quarter, distribution of more mineralized points is close to original singularity index and is in weak mineralized zones range. A similar trend follow by Zn, Pb and Mn. Dimension of weighted singularity index is more than original singularity index in third quarter. High dispersion of points in second and fourth quarters is also indicates discrepancy between two methods in detecting mineralized zones especially for Pb and Zn.

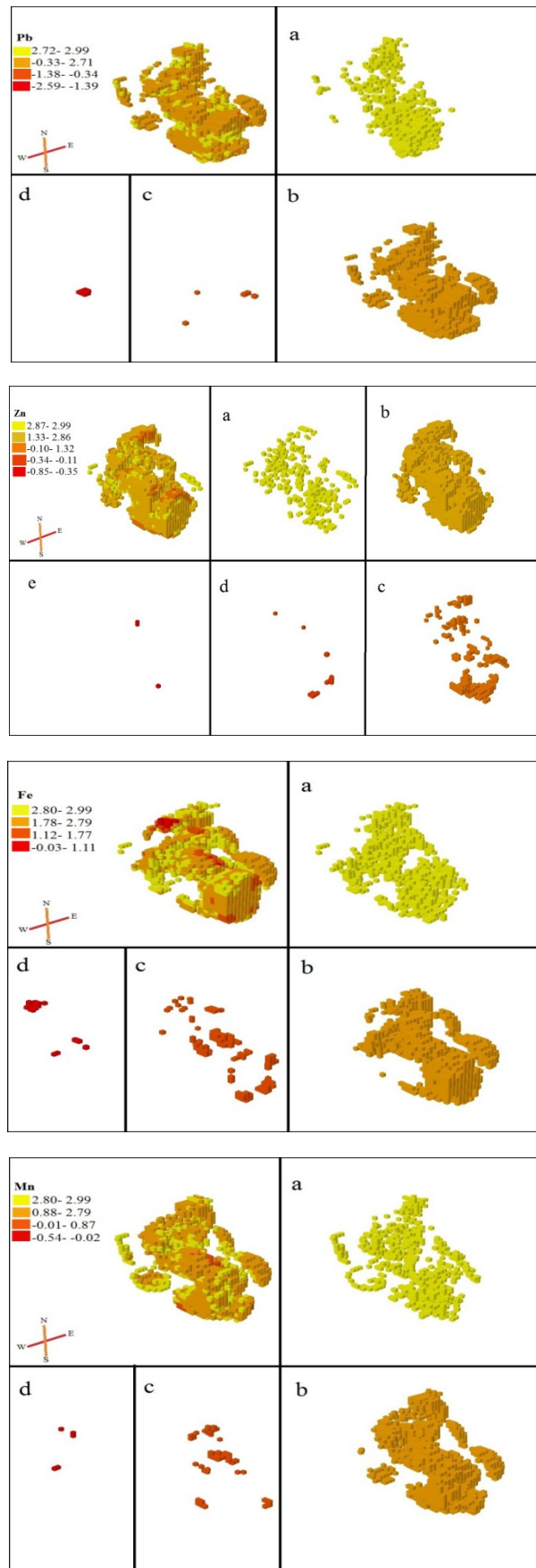


Fig 7. Distribution of original singularity index in three-dimensional mapping of Pb, Zn, Fe and Mn.

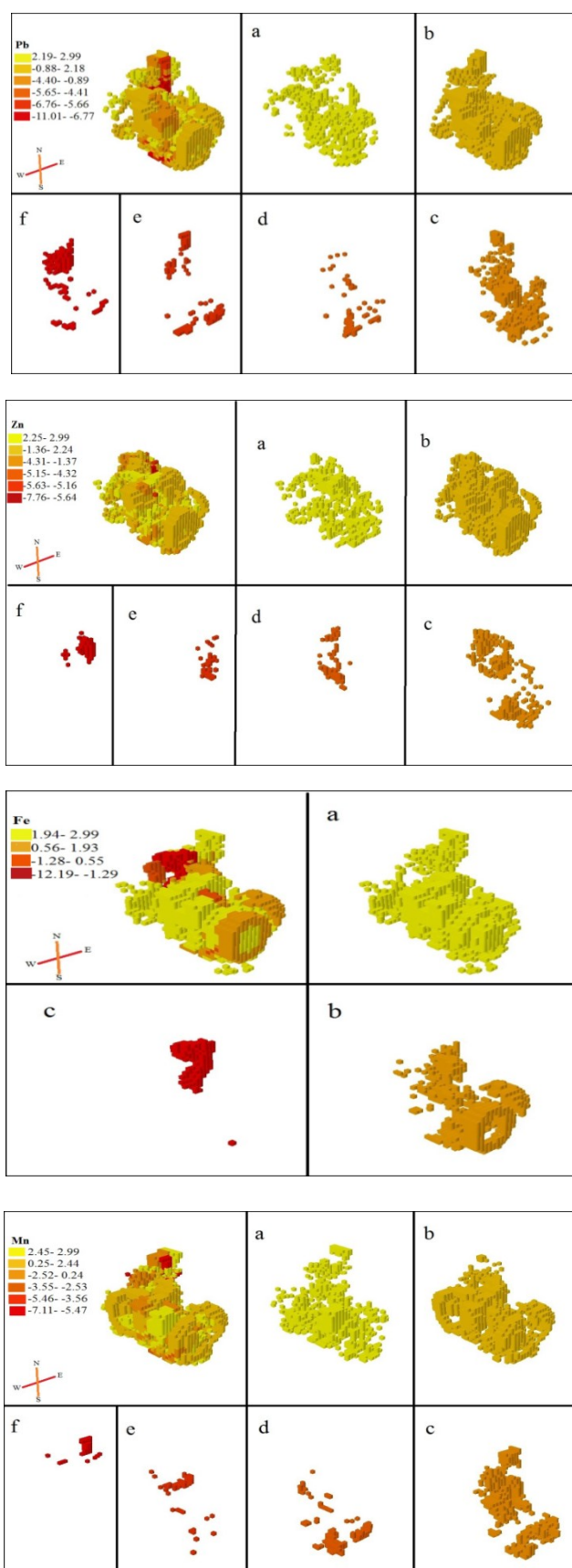


Fig 8. Distribution of weighted singularity index in three-dimensional mapping of Pb, Zn, Fe and Mn.

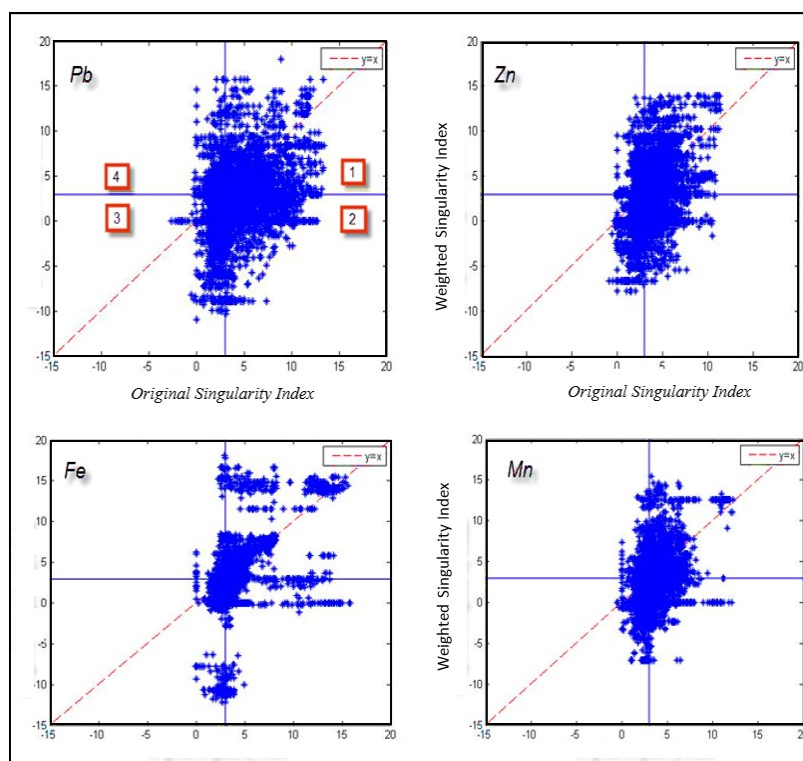


Fig 9. Scatter plot of original singularity index versus weighted singularity index.

4.2. Choice of method

To select the best method, we were used a comparison of original singularity method with modified singularity method (Fig 10). In Fig 10, there is a good adaptation between original and modified singularity method in Pb, Zn and Mn. But, this adaptation is not appropriate for Fe. Coefficient of areal association (CAA) is used to determine similarity of results of different methods. In this study, Taylor overlap coefficient (CAA) was used based on the Eq.(7).

$$CAA = \frac{S_{AA}+S_{BB}}{S_{AB}+S_{BA}+S_{AA}+S_{BB}} \quad (7)$$

where S denotes area, indices are the geochemical populations, A is mineralized zone, and B indicates background. The S_{AA} is area in first and second methods and S_{BA} is area in first method as background and in second method is mineralized zone (Shahrestani

and Mokhtari 2016). Coefficient of areal association is 40%, 54%, 32% and 38% for Pb, Zn, Fe and Mn, respectively in two original and weighted singularity methods (Table 4). This indicated a weak to moderate overlapping of original and weighted singularity method. We compared coefficient of areal association of original and weighted singularity methods with modified singularity method in order to select the best method for identifying mineralized zone elements. Comparison of weighted singularitymethod with modified singularity method shows that overlap is 46%, 54%, 29% and 41% for Pb, Zn, Fe and Mn, respectively. Coefficient of areal association is 98%, 99%, 32% and 98% for Pb, Zn, Fe and Mn between original singularity and modified singularitymethod.The high coefficient of areal association in original singularity and modified singularity method has shown original singularity is suitable for identifying the mineralized zones in the region.

Table 4. Coefficient of areal association of different methods for Pb, Zn, Fe and Mn.

	Pb		Zn		Fe		Mn	
	OS*	MS	OS	MS	OS	MS	OS	MS
OS	1.00	0.98	1	0.99	1	0.32	1	0.98
WS	0.40	0.46	WS	0.54	WS	0.29	WS	0.41
MS		1	MS	1	MS	1	MS	1

*OS: Original Singularity index; WS: Weighted singularity index; MS: Modified singularity index

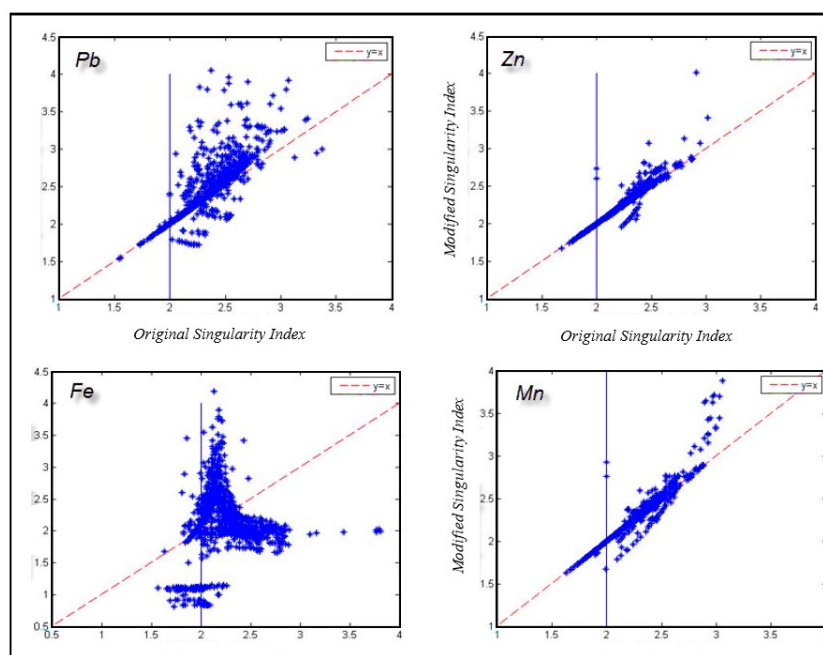


Fig 10. Scatter plot of original singularity index versus modified singularity index.

5. Conclusions

The singularity index is a multifractal concept used to identify hidden anomalies. This index is divided into various types of original singularity, weighted singularity, and modified singularity. In this paper, original singularities and weighted singularities were investigated in the identification of Pb, Zn, Fe and Mn mineralized zones. Separation of Pb, Zn, Fe and Mn mineralized zones by both original and weighted singularity methods showed that original singularity method is suitable for region in a three-dimensional space. Dimension, number, and volume of weighted singularity index are larger and more than original singularity index. The separation of the mineralized zone and its trend is clear in the original singularity method relative to the weighted singularity. Also, the range of mineralization changes in the original singularity is less than that lead to the mineralized zone. Comparison of original and weighted singularity with modified singularity showed that original singularity has well adaptation with modified singularity. Coefficient of areal association indicated original singularity index to be suitable for separation of mineralized zones.

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