



Resources classification using fractal modelling in Eastern Kahang Cu-Mo porphyry deposit, Central Iran

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Abstract

Resources/reserves classification is crucial for block model creation utilised in mine planning and feasibility study. Selection of estimation methods is an essential part of mineral exploration and mining activities. In other word, resources classification is an issue for mining companies, investors, financial institutions and authorities, but it remains subject to some confusion. The aim of this paper is to determine a resources classification for a Cu block model generated by an Ordinary Kriging (OK) and a Concentration-Volume (C-V) fractal modelling based on estimated variance in Eastern Kahang Cu-Mo porphyry deposit, Central Iran. Variography, block modelling and cell declustering for dataset with respect to Cu concentrations as the main target in this deposit were conducted firstly. Then, Cu distribution model was carried out by the OK and estimated variances were calculated for all voxels. According to a C-V log-log plot, three populations for estimated variances were detected. "Measured" resources contain voxels with estimated variances lower than 0.08 and more than 7 samples. Estimated variances varied between 0.08 and 0.24 in which more than 3 samples were engaged for estimation of "Indicated" resources. "Inferred" resources include estimated variances over 0.24 which are located in marginal parts of this deposit. Results derived via this study reveal that the C-V fractal modelling can be used for resources classification in different ore deposits.

Keywords: Ordinary Kriging (OK); Concentration-Volume (C-V) fractal model, Resources classification, Estimated variance, Eastern Kahang Cu-Mo porphyry deposit.

1. Introduction

The classification of geochemical populations and elemental distribution is important in mineral exploration, mineral resource classification and mine planning (Vural and Erdogan, 2014; Yasrebi 2014; Sadeghi et al. 2015). Mineral Resources as well as occurrences of intrinsic economic interest based on quality and quantity are reasonable prospects for eventual economic extraction (Armstrong and Boufassa 1988; Clark 1999; Afzal et al. 2016; Yasrebi et al. 2017). Mineral exploration in a mining district is a challenging operation because it is followed by a correspondingly rise in cost and risk of targeting deposits at increasingly great depths which requires further detailed data and more expensive computational methods (Houlding 2007; De Kemp et al. 2011; Wang et al. 2011a; Wang et al. 2011b). Combing geostatistical methods, 3D and mathematical analysis (e.g., fractal modeling in this scenario) helps in improving and consequent understanding of the distribution of mineral resources at depth and their relationships among geostatistical parameters for mineral exploration (Wang et al. 2013). 3D modelling is an important technology in quantitative assessment and prediction of mineral resources on a district scale. As a result, conventional geostatistics has been developed to use 3D reverse/

resource estimation (Wilson et al. 2011). Although various factors of mineral deposit formation control the variability in geochemical data such as grades or concentrations of ore elements have been considered a normal (Gaussian) or log-normal distribution in traditional statistical methods for data analysis (Armstrong and Boufassa 1988; Clark 1999; Limpert et al. 2001). However, many scientists and researchers have suggested and developed the frequency distributions of elemental concentrations which are commonly not normally distributed (Reimann and Filzmoser 2000; Li et al. 2003; Bai et al. 2010; He et al. 2013; Luz et al. 2014). Mineral resources are further sub-divided into three classes of "Inferred", "Indicated" and "Measured" classes, as depicted in Fig 1 (Wang et al. 2013; Sadeghi et al. 2015). "Measured" and "Indicated" categories have high level exploratory and geological data and modelling with a proper level for operation a mining project. "Inferred" resources are estimated with a low level of confidence based on tonnage, grade and mineral content (JORC 2012). Several international standards were proposed for resources classification such as Council of Mining and Metallurgical Institutions (CMMI) which was established in 1994 comprising representatives from mining and metallurgical institutions from the United States (SME), Australia (AusIMM), Canada (CIM), the United Kingdom (IMM) and South Africa (SAIMM)

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(CRIRSCO 2013). Resources and reserves classification is utilised commonly for feasibility study, mine planning and equipment selection. Resources classes are determined based on level of geological knowledge and confidence (David 1970; Mwasinga 2001; Asghari and Madani Esfahani 2013; Maleki et al. 2013; Shahbeik et al. 2014). Most of these methods have been established on geostatistical modelling and simulation (Emery et al. 2004; Emery 2005). Separation of “Measured” and “Indicated” from “Inferred” resources is essential because summation of “Measured” and “Indicated” levels are considered as positive significance for feasibility study and a Net Present Value (NPV) calculation (Yasrebi 2014).

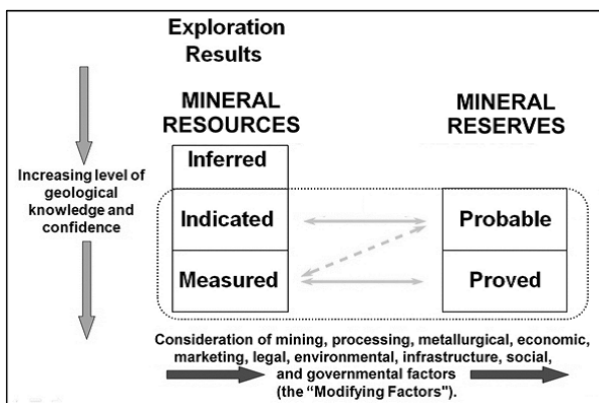


Fig 1. Relationship between exploration results, mineral resources and reserves (CRIRSCO 2013).

Fractal and multifractal methodology was established by Mandelbrot (1983) and has been utilised in mining engineering (Mandelbrot 1983; Agterberg et al 1993; Li et al. 2003; Carranza 2009; Carranza 2011; Afzal et al. 2013; Yasrebi et al. 2013). Several fractal models have been used for delineation of different attributes in various ore deposits based on geochemical, geophysical, geomechanical and economical parameters. A Concentration-Volume (C-V) fractal model was proposed by Afzal et al. (2011) for delineation of various mineralised zones in porphyry deposits (Afzal et al. 2011). This method is carried out for detection of different zones according to various regionalised variables.

Estimated variance for interpolation methods such as kriging is utilised as a classification approach (Sinclair and Blackwell, 2002). David (1988) used a relative kriging standard deviation (SD) which is applied as the ratio between kriging standard deviation and resource classification for an estimated block model. Arik (1999) proposed a classification for combination of the ordinary kriging variance and weighted average of squared difference between estimated grade of a voxel and raw data values. Yamamoto (2000) suggested a classification method based on estimated variance for an ordinary kriging technique. Emery et al. (2004) introduced interpolation variance as a criterion for

mineral resource classification. Silva and Boisvert (2014) proposed a resource classification method based on kriging variance. In this study, the C-V fractal model is used for recognition of different resources classification in Eastern Kahang Cu-Mo porphyry deposit (Central Iran) based on geostatistical parameters especially estimated variances.

2. Methodology

2.1. C-V fractal modelling

The C-V fractal model was proposed by Afzal et al. (2011) for detection of various populations from background (known as the first population) in various ore deposits is addressed as:

$$V(\rho \leq v) \propto \rho^{-a_1}; \quad V(\rho \geq v) \propto \rho^{-a_2} \quad \text{Equation 1}$$

where $V(\rho \leq v)$ and $V(\rho \geq v)$ indicate two volumes with concentration values (e.g., ore grade, density of rocks, estimated variance in this scenario) less than/equal to and greater than/equal to the value ρ ; v reveals the threshold value of a zone (or volume); and a_1 and a_2 are characteristic exponents. To calculate $V(\rho \leq v)$ and $V(\rho \geq v)$ enclosed by an estimated variance contour in a 3D model was used for this study. Volumes $V(\rho \leq v)$ and $V(\rho \geq v)$ are equal to the unit volume of a voxel (or volume cell) multiplied by the number of voxels with estimated variances (ρ) that are, respectively, smaller and greater than a certain estimated variance threshold value (v). A log-log plot of the estimated variances contours versus the corresponding volumes [$V(\rho \leq v)$ and $V(\rho \geq v)$] follows a power-law relationship. Depicted arrows in the log-log plot show threshold values.

2.2. Ordinary Kriging (OK)

Kriging is introduced as a geostatistical methods' group for the interpolation of different regionalised variable values which includes OK, universal kriging, indicator kriging, co-kriging and others (Emery 2005; Bayraktar and Turalioglu 2005; Hormozi et al. 2012; Vural 2018). The select of which kriging method to be utilised depends on the data characteristics and the type of spatial model. The most commonly geostatistical method is the OK which was employed for this study. The necessities of this method include high values of data, boreholes and a dense grid drilling for generation of proper variogram. In this study, 48 boreholes with 7146 core samples were used for variography and estimation. The OK plays a special role because of its compatibility with a stationary model which involves a variogram. The OK estimates based on a moving average of the variable of interest satisfy various dispersion forms of data e.g. sparse sampling points. Moreover, it is a linear model based on local neighbourhood structure. Variograms and anisotropic ellipsoids are a set of widely used statistical tools for spatial estimation and interpolation, which are the fundamental components for geostatistical modelling, especially the OK (Ver Hoef and Cressie 1993).

2.2.1 Kriging variance

The kriging variance is dependent only on the estimation location, the position of samples and the variogram. The most common classification approaches require the definition of thresholds to differentiate categories. The advantage of using kriging variance as an index for classification is related to spatial structure of the variable and the redundancy between samples. Closest sample location as the kriging variance is very low, resulting in patches of "Measured" resources (Silva and Boisvert, 2014).

3. Case study

The Kahang deposit is located about 73 km NE of Isfahan, Central Iran. This deposit contains more than 100 million tonnes of sulphide ore with an average grade of 0.2 wt.% and 50 ppm for Cu and Mo,

respectively. The deposit is situated on the Cenozoic Urumieh-Dokhtar magmatic belt, depicted in Fig 2 (Berberian and King 1981; Alavi 2004; Yasrebi 2014). This deposit is mainly composed of Eocene volcanic-pyroclastic rocks, which were intruded by porphyritic quartz diorite, dacite, andesite as the major lithological units. These intrusions are roots of acidic to intermediate domes within the Kahang porphyry deposit. Studies of the pattern of zonation in the eastern part of Kahang deposit demonstrate that the most significant mineralisation (in terms of ore zone size) is hypogene containing a high percentage of chalcopyrite accompanied by pyrite. The major alteration zones of potassic, phyllic, argillic and propylitic types have been accompanied by the vein to veinlets fillings of quartz, quartz-magnetite and Fe-hydroxides (Yasrebi 2014; Afzal et al. 2011; Afzal et al. 2016).

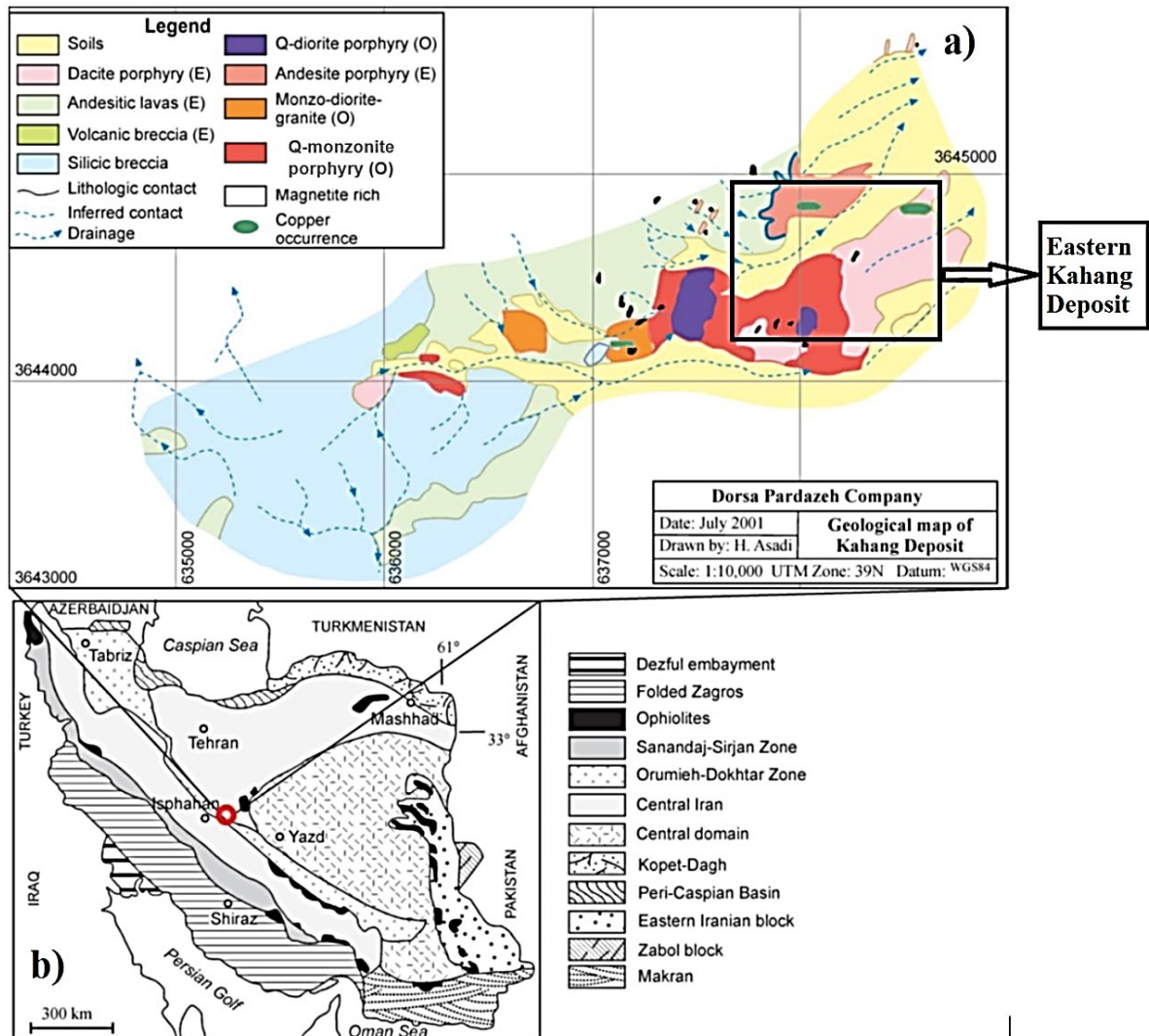


Fig 2. a) Geological map of the Kahang area, scale: 1: 10,000 (Tabatabaei and Asadi Haroni 2006; Yasrebi et al. 2013), b) structural map of Iran, showing the Urumieh-Dokhtar volcanic belt (Alavi 1994).

4. Assay Quality Assurance and Quality Control

Sampling is the fundamental part in a geochemical investigation for different stages of mineral exploration and environmental purposes. The optimum sampling strategy should be based on geochemical methods followed by the field observations, variety of sampling, sample preparation and analytical approaches. The estimate of reproducibility (precision) allows us to quantify variation of sampling and laboratory analysis which is an integral part of the geochemical data interpretation. As a result, any mistake in sampling and sample preparation may influence the results of the survey (Thompson and Howarth 1978; Fletcher 1981; Demetriades 2014).

From 48 drill holes in the Kahang deposit, 7146 lithochemical samples have been collected at 2 m intervals. These samples were analysed using ICP-MS for 48 elements by ALS Chemex (ALS Canada Ltd) and Zarazma Mineral Studies Company certified by Geostats Pty Ltd (Australia). Detection limits for Cu and Mo are 0.2 ppm and 0.05 ppm, respectively. Moreover, 399 randomised samples for Cu determination were selected and analysed for quality assurance and quality control purposes, assessed using Thompson-Howarth

error analysis. The following procedure is suggested for estimation of precision from a minimum of 50 pairs of duplicate samples (Thompson and Howarth 1978):

- (1) From the duplicate analyses, obtain a list of the means and absolute difference.
- (2) Arrange a list (in Excel software) in increasing order of concentration means.
- (3) From the first 11 results obtain the mean concentration and absolute difference of the two results (controlling samples) from that group (each group contains 11 duplicated/reanalysed samples).
- (4) Repeat step 3 for each successive group of 11 results, ignoring any remainder less than 11.
- (5) The mean of each replicate pair is plotted against the absolute difference between the two analyses.

The highest value up the % scale on the right axis gives the precision. A precision around 5% is normal. If the precision is around 1%, the Y axis has not been properly calculated with respect to the procedure mentioned above. The precision greater than 5% may have cause for concern and reconsideration. However, the precision for Cu is around 2% in the Kahang deposit with respect to 399 duplicated sample for Cu (Fig 3).

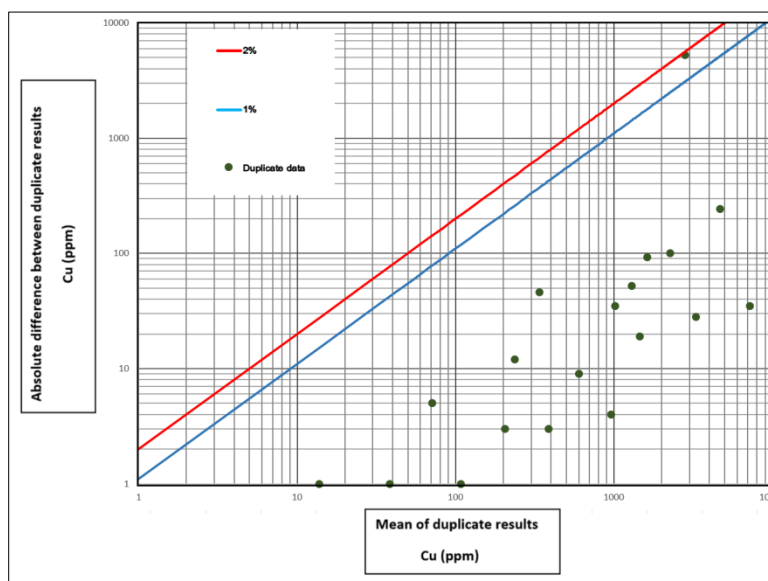


Fig 3. Precision estimation of Cu analyses using diagram of Thompson and Howarth (1978). The mean of the replicate pairs is plotted along the X-axis, the absolute difference of the two results along the Y-axis.

4.1. Comparison of Geochemical Data Variances via F-Distribution

F-distribution test is used to identify variances equality of duplicated samples (e.g., geochemical data). This is the theoretical distribution of values which are expected by randomly sampling from a normal population and calculating, for all possible pairs of sample variances, the ratios as follow (Davis 1987; Deutsch and Journal 1998; Emery 2012):

$$F = \frac{S_1^2}{S_2^2} \quad S_1 \geq S_2 \quad \text{Equation 2}$$

Where F, S1 and S2 represent F-distribution or continuous probability distribution and variances for pair of samples ($S_1 = 0.222$ and $S_2 = 0.219$). The variances of double samples vary if the number of observations used in their calculation is small. Therefore, the shape of the F-Distribution is expected to change with changes in terms of samples amounts.

The F-Distribution has two degrees of freedom equal to n_1-1 and n_2-1 in which n_1 and n_2 represent the number of observations equal to 398. Fisher showed that significance level, $1-\alpha$ (α : probability value) is calculated in the cases of one-tailed and two-tailed distributions depending on the defining alternative hypothesis. The hypotheses are as follows (Fisher and Tippett 1928; Emery 2012):

$$\text{Null hypothesis: } H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{Equation 3}$$

$$\text{Alternative hypothesis: } H_1 : \sigma_1^2 \neq \sigma_2^2 \quad \text{Equation 4}$$

Where σ_1 and σ_2 denote variances of populations. Based on the F-test, $F(398,398) = 1.015$ which is less than 1.2175 (Emery 2012). With respect to the confidence level of 97.5% ($\alpha = 0.025$). As a result, the Null hypothesis is acceptable representing that two variances obtained from the paired samples are almost equal to each other.

4.2. Comparison of Geochemical Data Means via Paired T-Test

A paired T-test is utilised to compare between means of two populations. The paired sample T-tests typically include a sample of matched pairs of similar units (e.g., Cu wt.% in this scenario), or one group of units that has been tested twice (Davis 1987; Emery 2012).

The correct rejection of the null hypothesis (no difference between mean values) can become much more likely. Because half of the sample now depends on the other half, the paired version of Student's T-test has only " $n/2-1$ " degrees of freedom (n is the total number of observations). Pairs are individual test units and the sample has to be doubled to achieve the same number of degrees of freedom.

To achieve the null hypothesis which the true mean difference is zero, the procedure is as follows:

1. Calculate the difference between the two observations on each pair as follow:

$$d_i = y_i - x_i \quad \text{Equation 5}$$

2. Calculate the mean difference of the pair samples in terms of their grades (\bar{d}). The grades' means for the paired samples are 0.194% and 0.196% so \bar{d} is 0.002%.

3. Calculate the differences of standard deviation ($Sd = Sd_1^2 - Sd_2^2$) for the pair of samples. To do this, the standard deviation of each sample (Sd_1 and Sd_2) was calculated and they are equal to 0.468 and 0.472, respectively. Subsequently, standard error of the mean difference was calculated (Equation 6) which is 0.47

$$SE(\bar{d}) = \frac{Sd}{\sqrt{n}} \quad \text{Equation 6}$$

Where n is equal to 2 because there is a pair of samples. 4. Calculate the T-test statistic under the null hypothesis, this statistic follows a T-distribution with $n - 1$ degrees of freedom.

$$T = \frac{\bar{d}}{SE(\bar{d})\sqrt{\frac{2}{n}}} \quad \text{Equation 7}$$

Where n is the number of paired samples which is 399. 5. Use table of the T-distribution to compare value for T to the T_{n-1} distribution. This will give a T critical (p-value), defined as the smallest level of significance at which the null hypothesis would be rejected for a specific test, for the paired T-test (Davis 1987). The calculated T from paired samples is -0.06 according to the Equation 7 and the T critical for "two-tailed test" with respect to confidence level (probability value for $\alpha = 0.025$) of 97.5% is equal to ± 1.9629 which indicates that the Null hypothesis is again acceptable. Therefore, the mean values of the paired samples are equal (Emery 2012). Consequently, results derived from T- and Fisher tests show that there is no significant difference between results obtained via raw and controlling samples giving an analytical accuracy in this deposit.

5. Discussion

5.1. Statistical Characteristics

In the studied deposit, 7146 core samples were collected from 48 boreholes at 2 m intervals, and analysed by ICP-MS for Cu and Mo. The Cu and Mo distribution functions are not normal, with Cu and Mo averages of 0.166 wt.% and 28 ppm, respectively, derived via RockWorksTM v. 15 (Fig 4a and b). The elemental distributions show an L shape with most of the volume of the deposit containing low grades for Cu and Mo. Most values of Cu and Mo are lower than 1 wt.% and 200 ppm, respectively. Variation between maximum and minimum of these data shows a wide range among elemental concentrations (Table 1). Based on the abnormal elemental distributions, Cu and Mo medians are assumed to be equal to threshold values for separation of 'barren' host rocks and mineralisation which are 0.087 wt.% for Cu and 9.9 ppm for Mo (Davis 1987). Figure for the original data sets used for Cu (as the main target in this deposit) values has been generated using MATLAB software, as depicted in Fig 4c.

Table 1. Statistical characteristics for Cu and Mo

Elements Grades	Minimum Value	Maximum Value	Range	Mean	Standard Deviation	Median	Variance	Skewness	Kurtosis
Cu (wt.%)	0.0003	4.92	4.91	0.16	0.271	0.087	0.073	6.6	74.5
Mo (ppm)	0.5	1,479	1,478.5	28.27	76.178	9.9	5,803.132	8.4	96.1

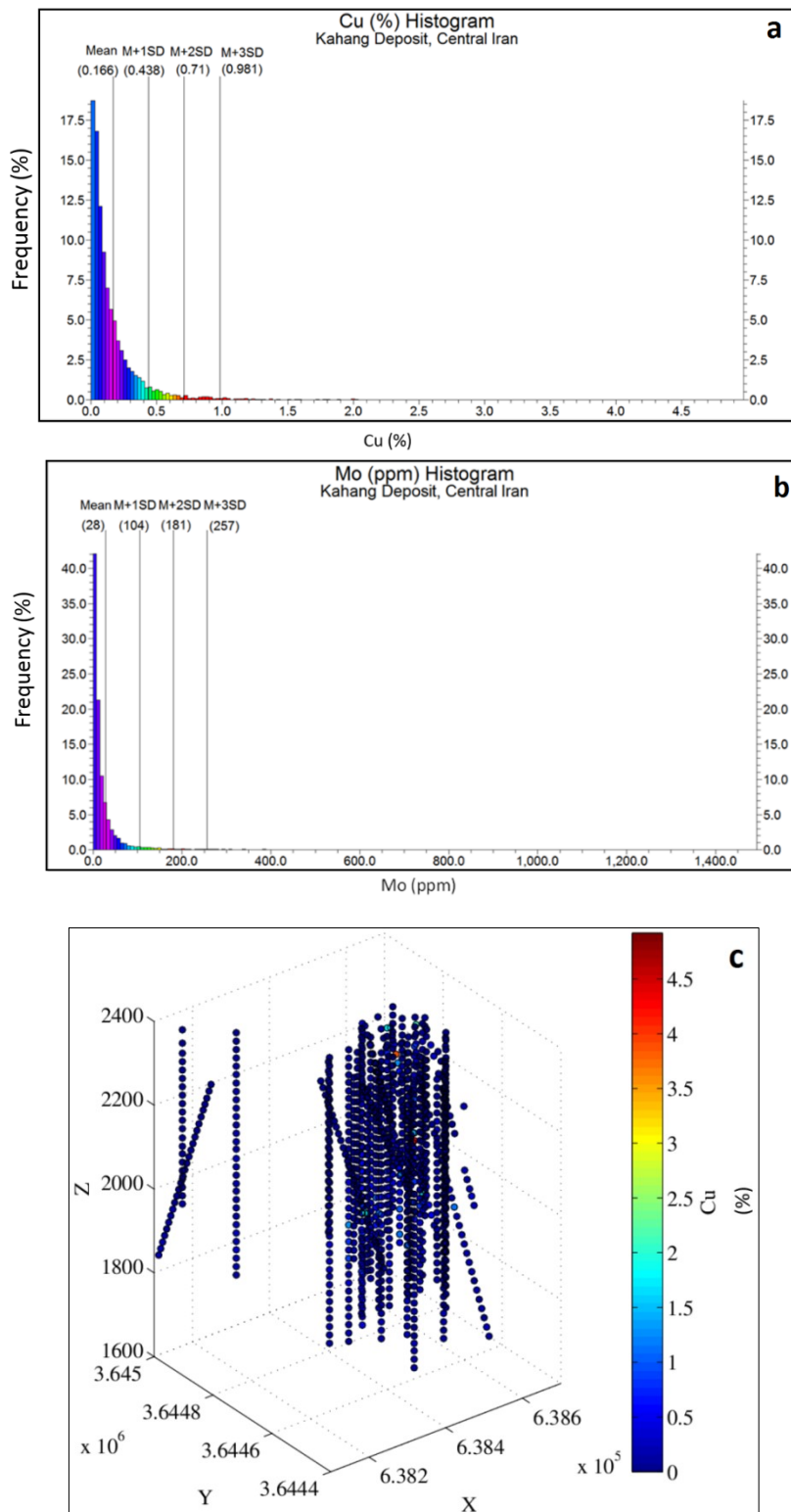


Fig 4. Histograms for data from the Kahang deposit: a) Cu wt.%, b) Mo ppm and c) 3D map of original datasets for Cu wt.%

5.2. Block modelling and cell declustering

It is necessary to select an optimal voxel size with respect to the deposit geometry and drilling pattern because most of the geostatistical software, e.g. Datamine Studio which was employed in this study, estimates an ultimate block model based on the closest points considering parameters such as ore element concentrations (Yasrebi et al. 2015). In other words, choosing a suitable voxel size for evaluation of a reserve/resource is crucial for minimising errors (Asghari and Madani Esfahani 2013; Shahbeik et al. 2014). This problem has been assessed for estimated block models using ordinary kriging (OK). Results obtained by the estimation methods relate to the determination of voxel size in block modelling (Cressie 1993; Soltani Mohammadi et al. 2012). Utilising a larger voxel size will increase the averaging effect in the estimated block model in terms of concentrations (Emery and González 2007; Emery and Ortiz 2011). Additionally, a smaller voxel size will show more details, but potentially more error in an anisotropic environment. As a result, reducing the voxel size results in an increase in estimated errors (variance and standard deviation) for a final block model. Moreover, increasing the voxel size in the block model changes the higher or lower grades of mineralised zones by smoothing of these points with high or low values within a large voxel (Yasrebi et al. 2015). Therefore, it is necessary to select an optimal voxel size because estimation of an ultimate block model is based on the closest points considering particular parameters such as ore element concentrations. David (1970) proposed an applicable method for an operation based on geometrical particulars of the different types of ore deposits and grid

drilling. Based on the method, voxel dimensions are calculated as follows:

a) Length and width of each voxel is equal to between half and quarter of the distance between the drill cores according to along the least variability deposit.

b) Height of each voxel is delineated due to the type of the deposit. In 'massive' deposits such as magmatic deposits (e.g., porphyry deposits), the parameter is equal to the height of excavating benches in the open pit mines (Hustrulid and Kuchta 2006). The 3D map which indicated the location of 48 boreholes drilled in the Kahang deposit was constructed by MATLAB software package (Fig 4c). As can be seen, the grid drilling pattern within this deposit is not uniform and systematic so the geochemical data need to be declustered. The Kahang deposit was modelled with 489,927 voxels and each voxel has a dimension of $4 \times 4 \times 10$ m, corresponding the project dimensions of 600, 660 and 780 m, in the X, Y and Z directions, respectively.

Data are often spatially clustered because of preferential sampling which makes it difficult to determine whether they are representative of the entire area of interest. To obtain a representative distribution, one approach is to assign declustering weights whereby values in cells with more data receive less weight than those in sparsely sampled areas. As mentioned above, data need to be declustered. This operation was carried out using the Declus program which incorporates the GSLIB library (Fig 5). In addition, Cu mean of above-mentioned voxel size should be close to Cu mean value obtained from the declustered data (Deutsch and Journel 1998; Richmond 2002; Olea 2007; Sadeghi et al. 2015). The Cu mean and standard deviation of the declustered data are 0.145 wt.% and 0.22077% (Fig 5).

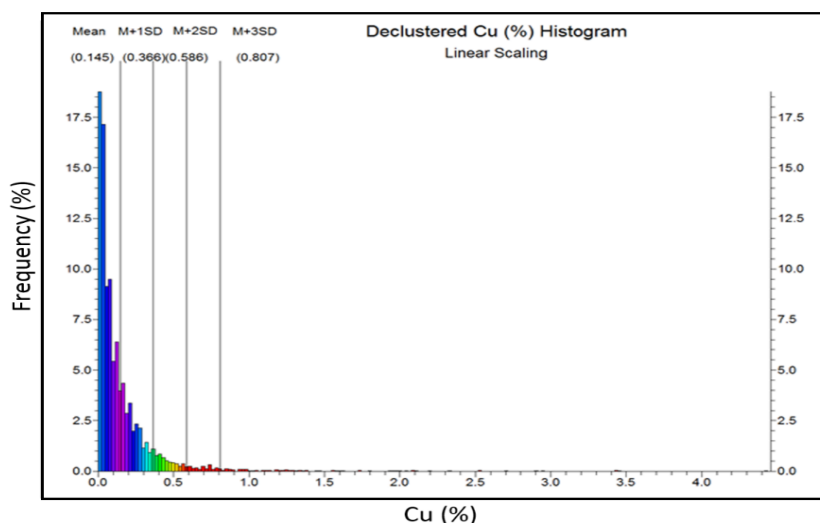


Fig 5. Cu histogram based on declustered data.

5.3. Variography and anisotropic ellipsoid

The experimental variograms in horizontal (Azimuth: 0 and Dip: 0) and vertical (Azimuth: 0 and Dip: -90) directions were generated using MATLAB software with respect to log transformations of Cu grades (Fig 6). The horizontal and vertical ranges for Cu are 56 m and 270 m, respectively. The spherical model was fitted to the experimental variograms. Accordingly, the theoretical variogram for Cu grade values is as:

$$\gamma_{Cu}(h) = 0.41 + 0.36 sph(10.10.25) + 0.85 sph(56.56.270) \quad \text{Equation 8}$$

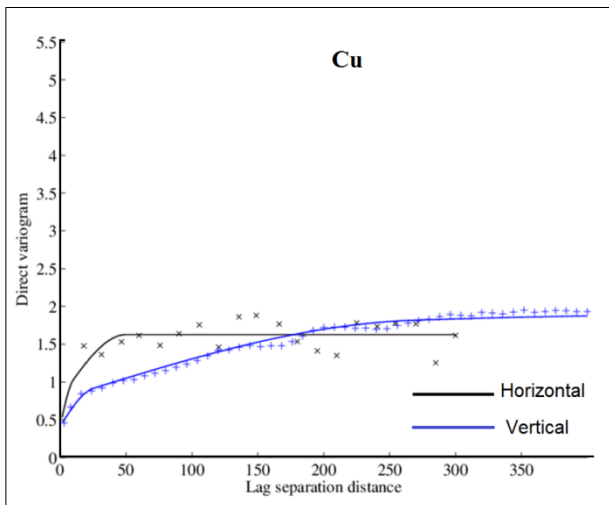


Fig 6. Experimental and theoretical variograms for Cu.

Anisotropic ellipsoid was calculated based on variograms. The horizontal and vertical ranges were recognised based on the combined variograms with lags' spacing of 15 m and 8 m for horizontal and vertical directions with respect to the theoretical variograms.

Subsequently, a Cu block model with voxel size of 4 m × 4 m × 10 m, according to David (1970) method as mentioned above, was generated by OK utilising Datamine Studio software. For validation of the chosen voxel dimension, standard deviation (SD) and an average Cu value have been calculated for the block model with voxel dimension of 4 m × 4 m × 10 for X, Y and Z, respectively.

The Cu mean and standard deviation values for this block model are 0.15823 wt.% and 0.20134% which are relatively close to the Cu average and SD value obtained from the declustered data (e.g., 0.145 wt.% and 0.22077%; Table. 2).

5.4. OK application

After variography, determination of estimation parameters and providing block model, the Cu values in the deposit was estimated by the OK technique. Maximum and minimum samples are 10 and 2 which used for the OK estimation. There are 489,927 estimated voxels in the Kahang deposit. Cu values, estimated

variances and engaged samples' number were calculated for total voxels in the studied deposit (Fig 7). A 3D model for estimated variances revealed that this parameter is increased in marginal part of this deposit, as depicted in Fig 7. Estimated variances are important for resources classification in this research.

Table 2. Comparison between standard deviation and average values for pre-estimated Cu and declustered data.

	Standard Deviation (%)	Cu Average (wt.%)
Block Model Dimensions 4 × 4 × 10 (m ³)	0.20134	0.15823
Declustered Data	0.22077	0.145

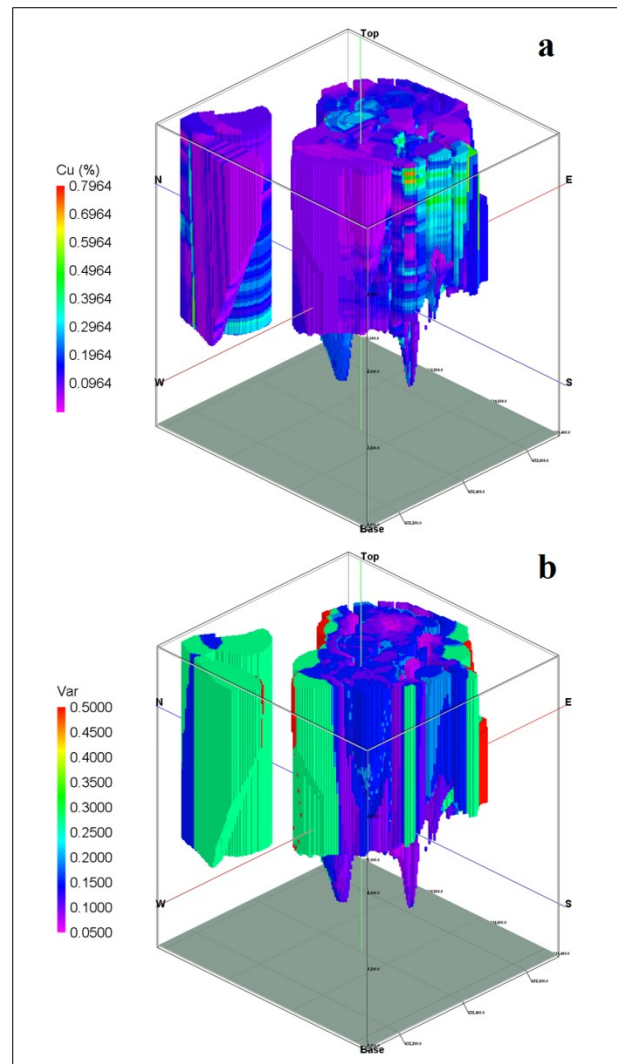


Fig 7. 3D models for Cu values (a) and estimated variances (b) in this studied deposit.

5.5. Application of the C-V fractal modelling

A C-V log-log plot was generated based on estimated variances block model, as depicted in Fig 8. There are three populations with two threshold values for estimated variances corresponding to 0.08 and 0.24, respectively. “Measured” resources contain low values of estimated variances based on high density of boreholes and data. Voxels with estimated variances lower than 0.083 (first population) are named as “Measured” resources (Fig 9a). However, engaged samples in estimation process for the “Measured” resources are more than 7 and 3 boreholes. This category is situated in the central part of this deposit

with high density of boreholes. In addition, “Indicated” resources is determined as the second population with estimated variances between 0.08 and 0.24. These voxels are located within the marginal and NE parts of the deposit, as depicted in Fig. 9b. Voxels of this class consist of more than 3 samples which were engaged for estimation. Moreover, “Inferred” category which includes voxels with estimated variances higher than 0.24 is located in the marginal, NE and NW parts of this deposit, as shown in Fig 9c. Summation of the “Measured” and “Indicated” classes contain Cu values between 0.075% and 0.686% (Fig 10).

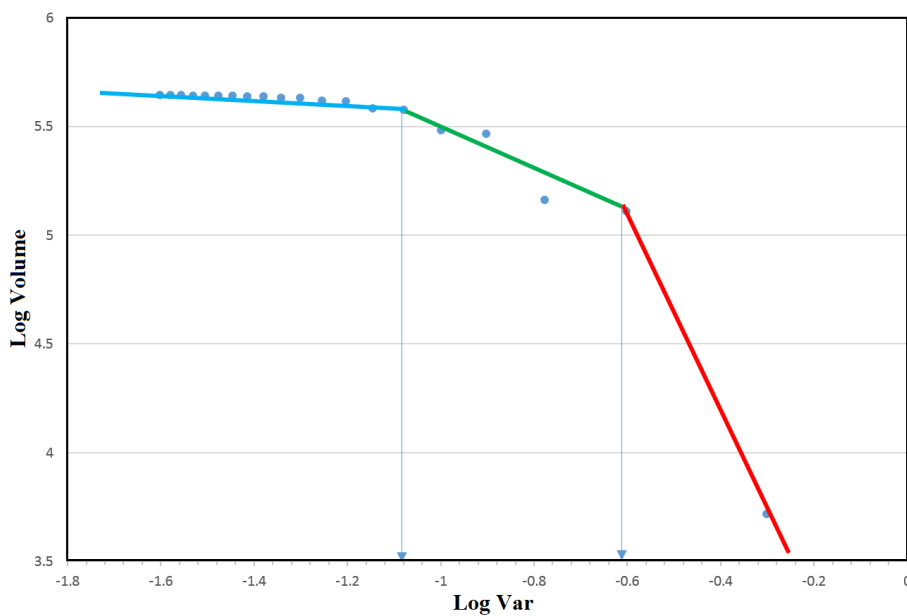


Fig 8. C-V log-log plot for estimated variances.

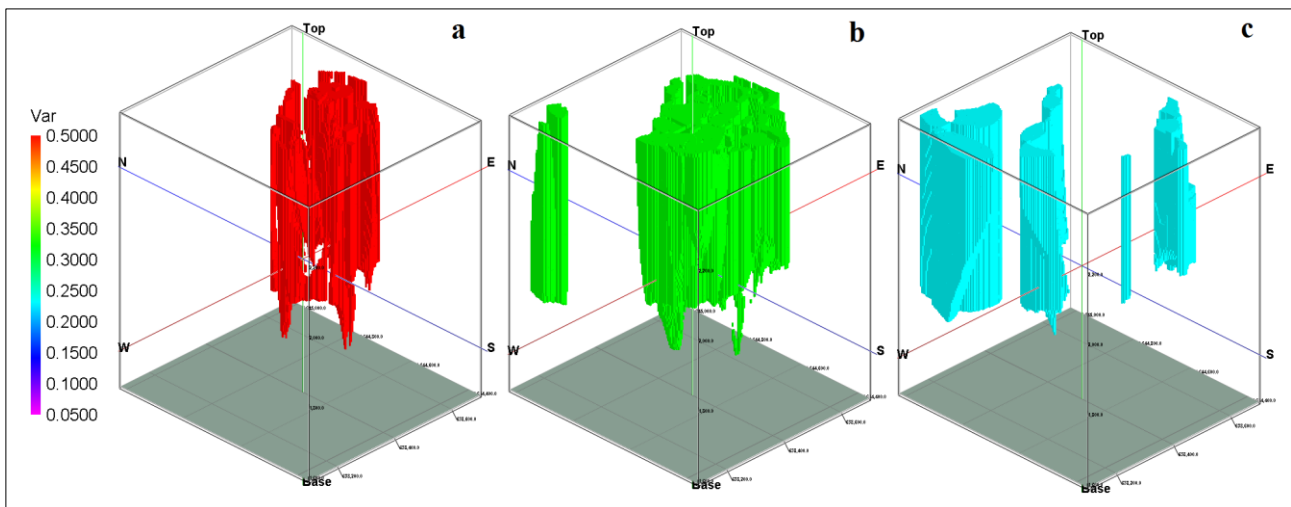


Fig 9. 3D models for a) Measured, b) Indicated and c) Inferred resources.

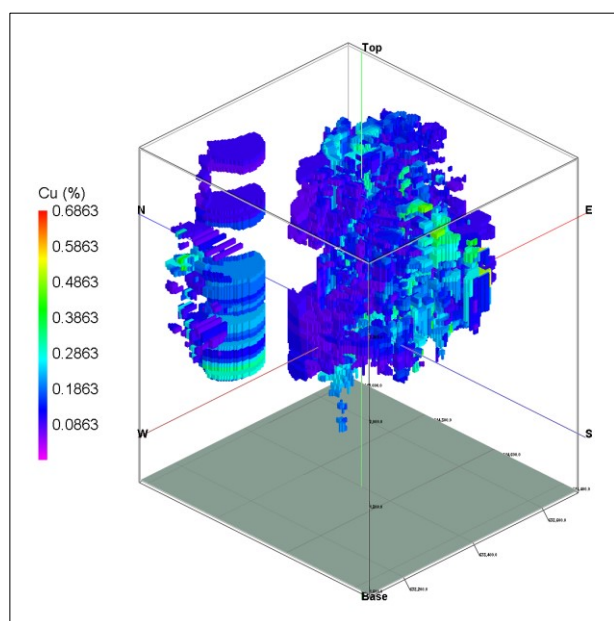


Fig 10. Cu values in the Measured and Indicated resources of the Kahang deposit.

6. Conclusions

Results obtained by this study represent that the C-V fractal modelling is a proper method for classification of different resources based on estimated variances. Estimated variance is a fundamental parameter for resources classification because it is related to density and frequency of samples and boreholes in a studied deposit. The “Measured”, “Indicated” and “Inferred” categories were detected using a C-V fractal modelling based on an OK estimated block model for Cu in the Eastern part of Kahang deposit. Threshold values for estimated variances, with respect to the C-V log-log plot, are 0.08 and 0.24. The resulted classes were correlated with engaged samples for estimation and boreholes. There is a direct correlation between increasing of levels with samples number and boreholes. Furthermore, “Measured”, “Indicated” and “Inferred” classes are located from central to marginal parts of this deposit, respectively. Finally, results obtained by this methodology indicate that this method can be suggested for other ore deposits in detailed exploration stage.

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