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Decision Support System for Dynamic Pricing of Parallel Flights

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Abstract

In the recent years, traditional revenue management (RM) models are shifting from them from quantity-based to price-based techniques and incorporating individuals' decisions within optimization models. In this paper, we have replaced, quantity-based with price-based techniques and proposed the MNL to capture more choice probabilities Computation results indicate the obtained revenue by using proposed model for deciding about the most appropriate product for offering to the customers.

***Keywords**:* price-based techniques, multinomial logit model (MNL), dynamic pricing, revenue management (RM)

1.Introduction

Recently, revenue management (RM) has played a very significant role in a wide range of industries. This technique was initiated in the United States airlines industries and is now extending to such other domains as railways, cruises, hotels, manufacturing, and so on. RM is the application of disciplined tactics which predict consumer behavior at micro market levels and optimize product availability and price in order to maximize the revenue growth (Cross, 1997).

Traditional RM models are based on the assumption that demands for each fare class is independent of fare availability controls; this assumption was concluded, a decade ago, to have serious limitations. New RM models insert the customer's choice in the traditional models to overcome this limitation. Multinomial logit (MNL) model is the most popular tool for the customer's choice modeling and incorporating it in the optimization model.

Recently, considerable attention has been paid to the modeling of consumers choice among a set of multiple products and applying the realistic discrete choice model of the consumer behavior in normative revenue management models while simultaneously keeping problem complexity at a reasonable level (Schon, 2010).

Another reason for the evolution in RM models is the expansion of low-cost businesses; they have caused these models to focus on quantity-based instead of on price-based techniques. In recent years, an increasing number of firms have successfully implemented low-cost business strategies that operate without complicated tariffs (Meissner & Strauss, 2010). Specific characteristics of such low-cost carriers in the airline industry as the simplified fare structure, point to point non-stop flights, and multiple same-day parallel flights between specific origins and destinations, have forced them to apply pricing techniques based on the flight schedule, price, capacity and remaining time horizon, to optimize the revenue.

The organization of this paper is as follows: Related researches and discrete choice models and their characteristics are reviewed and presented in sections 2 and 3 respectively. In section 4, the proposed model is described, two specific choice models are incorporated in the optimization module, and their solutions are analyzed. Computations for the parallel flights network and comparison of the results associated with different conditions are given in section 5. And, finally, a brief summary of the paper and conclusions are presented in section 6.

2. Literature Review

In this section, we have reviewed the most relevant literature on choice-based quantity models and price-based revenue management models. Traditional revenue management models are based on independent demand assumption. A comprehensive survey on traditional revenue management models can be found in (Talluri & van Ryzin, 2004). Various price-based revenue management models are available in (Bitran and Caldentey, 2003), (Talluri & Van Ryzin, 2004b), and (Elmaghraby & Keskinocak, 2003). In this literature review, price-based RM models that are most related to our context are reviewed, then a survey of choice-based RM models is conducted, and finally, the outstanding discrete choice models are reviewed.

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Gallego and van Ryzin (1994) analyzed the dynamic pricing problem with price sensitive demands and found an upper bound on the expected revenue for general demand functions. Gallego and van Ryzin (1997) studied the multiproduct dynamic pricing problem and, assuming the demand to be a function of the price vector, offered two heuristic solutions for stochastic problems. Zhao and Zhang (2000) considered a dynamic pricing model for perishable products over a finite time horizon, followed a non-homogeneous Poisson process for customers' arrivals, and analyzed price changes under pre-specified conditions.

Suh and Aydin (2011) studied the dynamic pricing problem of two substitutable products over a finite selling horizon and applied multinomial logit to model the customer's choice. They showed that under the optimal pricing policy, the marginal value of a resource increases in the remaining time and decreases in its own (and other products') stock level, and that the optimal price is not monotonic in the remaining time or the stock level.

Dong and et al. (2009) considered dynamic pricing of substitutable products when a consumer's choice is based on the multinomial logit model. They studied the effects of time and inventory depletion on the optimal pricing and found out that dynamic pricing is of great value in the presence of inventory scarcity, and that initial inventory decisions are quite robust in the pricing scheme. Maglaras and Meissner (2006), applying a combination of dynamic pricing and capacity allocation controls, considered a model to maximize the firm's revenue.

Zhang and Cooper (2009) offered a heuristic solution for Markov's decision process formulation of the dynamic pricing of parallel substitutable products. Schon (2010) presented a dynamic pricing model for a single resource finite horizon when the firm is to select a price from pre-specified points, and analyzed the structural properties of specific choice models in this problem.

The importance of considering a customer's behavior of choice decision was shown by Belobaba and Hopperstad (1999). They studied, using simulation, passengers' purchase behavior to analyze their preference sensitivities toward an airline's time and date of departure, path, and ticket price.

Andersson (1998) , Algers and Baser (2001) reported the results of a project in the Scandinavian Airlines System (SAS) regarding the estimation of the recapture and buy up using the stated and revealed preferences data.

Zhang and Cooper (2005) used the Markovian decision process for simultaneous seat-inventory control of the set of parallel flights from common origins to common destinations considering customers' choices among the flights. Their model assumed that the customer chooses within the same fare class among different flights, but not among different fare classes. They proposed heuristics and simulation-based techniques to solve this problem. They also applied the general choice model to consider the customer's behavior.

Van Ryzin and Vulcano (2008) considered the network capacity control problem where customers choose from the various products offered by a firm. They modeled customers' choices assuming that they individually have an ordered list of preferences. They assumed that the firm controls the availability of products using a virtual nesting control strategy.

Chen and Homem-de-Mello (2010) considered a network airline revenue management model in which the customer's choice model was based on the concept of preference of orders. They proposed a new model using mathematical programming techniques to determine the seat allocation.

Talluri and van Ryzin (2004) provided a complete characterization of an optimal policy under a general discrete choice model of customers' behavior in a single legged revenue management model. They reminded that an optimal policy is made up of a selected set of efficient offer sets that are a sequence of no dominated sets which provide the highest positive exchange among expected capacity assumptions and revenues.

Gallego et al. (2004) provided a customer choice-based LP model for the network revenue management. They supposed that the firm has the ability to provide customers' alternative products to serve the same market's demands with a flexible product offer. One limitation of their market demand model was that it did not allow any segmentation to happen.

Liu and Van Ryzin (2008) used the analysis of the model provided by Gallego et al. (Gallego et al., 2004) to extend the concept of efficient sets. They proved that when the capacity and the demand are scaled up proportionately, the revenue obtained by the choice-based deterministic linear programming converges to the optimal revenue under the exact formulation. They presented a market segmentation model to describe the choice behavior. The segments were defined by disjoint consideration sets (i.e. subsets) of products that customers consider as options provided by the firm.

Bront et al. (Bront et al., 2009) extended the work of Liu and Van Ryzin (2008) by allowing the customers to consider products belonging to an overlapping segment and proved that column generation sub-problem is Np-hard, and proposed a greedy heuristic to solve it. Etebari and Aghaei (2012) used CDLP formulation for the dynamic pricing of parallel flights by the multinomial logit choice model.

Kunnukal and Topaloglu (2008) proposed a new deterministic linear program for the network revenue management problem with customers' choice behavior. They generated bid prices that depended on the time left until departure. Their model's main drawback was that the number of constraints was significantly larger than that used in Liu and Van Ryzin's linear programming formulation (Liu and Van Ryzin, 2008).

Vulcano et al. (2010) developed the most likely estimation algorithm in discrete choice models for the airline revenue management. Their simulation results showed an improvement of 1-5% in the average revenue with the help of choice-based revenue management.

Etebari et al. (2013) proposed a nested logit model for incorporating a correlation alternatives in different nests. The column generation algorithm and a hybrid heuristic algorithm is proposed for solving this problem. Etebari and Najafi (2016) developed a knowledge acquisition subsystem for choosing the most suitable choice model in the choice-based network revenue management. They incorporated the artificial neural network for predicting revenue improvement obtained by using the more realistic choice model.

Hosseinalifam and et al. (2016) developed a new model for estimating time-dependent bid prices. Column generation algorithm is proposed for solving this problem.

Ben-Akiva and Lerman (1985) analyzed different discrete choice models and provided the most advanced elements of the estimation and usage of discrete choice models that required simulation. Garrow (2010) provided a comprehensive overview of discrete choice models and their application in the airline industry. Potoglou (2008), Nurul Habib (2012) believe there is extensive literature on the application of these models for the estimation of shares of different alternatives in real life.

3. Multinomial Logit Model

In order to model the customer-choice behavior, we can assume that each customer wants to maximize his/her utility while his utility of alternatives is a random variable. The firm offers a set of alternatives for customer n who has a consideration set of C_n with the utility U_{in} for each alternative$i \in C_n$. This can be decomposed into a deterministic (or expected) utility denoted by v_{in} and a mean-zero random component ε_{in} without losing generality. Hence, we can have a utility function as follows:

$$U_{in} = v_{in} + \varepsilon_{in} \tag{1}$$

In many cases, the representative component v_{in} is modeled as a linear combination of several attributes,

$$v_{in} = \beta^T x_{in} \tag{2}$$

where β is an unknown weight vector that should be estimated by the data, and x_{in} is the vector of such observable attributes as time and date of departure, price, airline brand, and so on, associated with the alternative i available to the customer n.

Multinomial logit (MNL) is a well-known widely used model to study how customers make their choices (Train, 2009). It is assumed, here, that ε_{in}s in the utility functions are independent and identically-distributed random variables with a Gumbel distribution. The probability that customer n will choose the alternative $i \in C_n$ in an MNL model is given by:

$$P_n(i) = \frac{e^{\beta^T x_{in}}}{\sum_{j \epsilon C_n} e^{\beta^T x_{in}} + 1} \tag{3}$$

4. Model

To understand our model, consider a network with m substitutable resources (legs) every one of which has N_i pre-defined price points. It is assumed that every price point forms a virtual product, and only one virtual product of a resource can be offered in each period. There will be $N = \sum_{i=1}^{m} N_i$ virtual products (prices) that can be offered to the customers; N_i is the number of virtual products belonging to resource i, $N = \{1,2, \dots, \sum_{i=1}^{m} N_i\}$ is all the products and r_j denotes the price of the jth virtual product. Capacity usages can be studied by defining vector $c = (c_1, c_2, \dots, c_m)$ which denotes the initial capacities of resources. The resource usage, according to the corresponding product, is presented by defining an incidence matrix $A = [a_{ij}] \in B^{m \times n}$ the entries of which are defined by $a_{ij} = 1$ if resource i is used by product j, and by $a_{ij} = 0$ if otherwise. Time has discrete periods and runs forward until the finite number T; $t = 1,2, \dots, T$, and it is undertaken that we have at most one arrival for each period of time, and each customer can buy only one single product. Customers are divided into L different segments and a Poisson process is considered for their arriving streams from segment l with a rate of λ and the total arriving rate of $\lambda = \sum_{l=1}^{L} \lambda_l$. If there is only one arrival, then p_l will represent the probability that an arriving customer belongs to segment l with $\sum_{l=1}^{L} p_l = 1$. In each time period t, the firm should decide about choosing an appropriate price point for each resource. These price points constitute an offer set for a leg (i.e. a subset $S \subset N$ of price points that the firm makes available for customers). If set S is offered, the deterministic quantity $P_j(S)$ will indicate the probability of choosing product$j \in S$; otherwise, $P_j(S) = 0$. By total probability low, we have $\sum_{j \in S} P_j(S) + P_0(S) = 1$; where $P_0(S)$ indicates the no-purchase probability.

In order to describe Markov's decision process, it is assumed that $x = (x_1, x_2, \dots, x_m)$ denotes the unsold remaining capacity and $V_t(x)$ is the maximum expected revenue from period t to the end of the horizon. Then, the optimality equation under Markov's decision process, according to (Talluri and Van Ryzin, 2004b), is given by:

$$
\begin{aligned}
V_t(x) = \max_{\substack{S \subset N \\ n(S \cap N_i)=1}} \sum_{j \epsilon S} \lambda P_j(S)[r_j + V_{t+1}(x - A_j) + [1 - \lambda \\
+ \lambda P_0(S)]V_{t+1}(x)] \\
= \max_{\substack{S \subset N \\ n(S \cap N_i)=1}} \{\sum_{j \epsilon S} \lambda P_j(S)[r_j \\
- \left(V_{t+1}(x) - V_{t+1}(x - A_j)\right)]\} \\
+ V_{t+1}(x); \quad \forall i, x, t
\end{aligned} \tag{4}
$$

where $\Delta V_{t+1}(x) = V_{t+1}(x) - V_{t+1}(x - A_j)$ is the marginal value of the resources. The boundary conditions are given by $V_{T+1}(x) = 0$ and $V_0(x) = 0$ for all x_s. $n(S \cap N_i) = 1$ states that in each offer set, there is only one virtual product of each resource that can be offered. This problem's state space dimension makes it intractable and we should find a substitutable solution for it.

In order to solve the above mentioned problem, we can estimate the marginal value of the capacity and use it to determine the offer set during each period. In this paper, use has been made of the choice-based deterministic linear programming model[2] assuming that there are many candidate price points for each resource and we should select appropriate price points based on such different factors as the initial capacity, time horizon, virtual products utilities, and no-purchase utility. These selected price points will be used during the booking horizon to choose the optimal offer set. The CDLP solution represents the candidate price points for each resource. Simultaneous with selecting appropriate price points (virtual products) the resources' optimal dual values can be used to approximate the marginal value (of each resource) which can itself be used for selecting appropriate price points in each period.

Offering set $S \subset N$, $n(S \cap N_i) = 1$ for an arriving customer, the expected revenue is given by:

$R(S) = \sum_{j \in S} r_j P_j(S)$. Given that set S is offered, let $P(S) = (P_1(S), \dots, P_n(S))^T$ be the purchase probability vector and A the incidence matrix of the resources used by products. Then $Q(S)$ (the capacity consumption probability vector) is given by $Q(S) = A.P(S)$

where $Q(S) = (Q_1(S), \dots, Q_m(S))^T$, and $Q_i(S)$ indicates the probability of using one unit of capacity on leg $i, i = 1,2, \dots, m$. $t(S)$ represents the number of periods during which set S is going to be offered. However, as choice probabilities are time-homogeneous and demand is deterministic, it only matters how many times a set S is offered; knowing during exactly which period is not important. Based on these assumptions, the CDLP formulation is:

$$
\begin{aligned}
& V^{CDLP} = max \sum_{\substack{S \subset N \\ n(S \cap N_i)=1}} \lambda R(S) t(S) \\
& S.t. \sum_{\substack{S \subset N \\ n(S \cap N_i)=1}} \lambda Q(S) t(S) \le c; \qquad \forall i \\
& \sum_{\substack{S \subset N \\ n(S \cap N_i)=1}} t(S) \le T; \qquad \forall i \\
& t(S) \ge 0, \forall S \subset N, n(S \cap N_i) = 1, \forall i
\end{aligned} \tag{5}
$$

The main difference between this and the well-known CDLP model is that if we can suppose the network has specific legs, unlike the previous model, the number of variables ($\prod_{i=1}^{m} N_i$) is not exponential, and any operation research software package can be used to solve it.

Solving this problem will lead to the optimal values of the primal and dual variables. The first can be adopted to specify the suitable price points at the beginning of the products' offering period, and can be used during the booking horizon to determine the specific price point for each resource in each period. Choosing specific pre-specified price points for our analyses during products offering has two advantages. First, there are different potential price points in the beginning and the use of a limited number of them (during the booking horizon) is preferred due to system restrictions or firm preferences. Second, with a large number of potential points, selecting optimal ones for each leg during each period is quite time consuming, and since this process is repeated several times (during the booking horizon), the firm will save time through using specific pre-specified price points.

Optimal values of dual variables are used in estimating the marginal values of different resources. Liu et al.(2008), Bront et al.(Bront et al., 2009), Zhang and Adelman (2009) and Meissner and Strauss (2012) tried to use the equation that maximizes the difference between the products fare and their used resources' marginal value considering the probability of these products choices, to determine the optimal offer set. Based on this policy, the following problem should be solved at the beginning of each period in order to choose the more suitable products to offer:

$$
\max_{\substack{S \subset N \\ S \cap N_i = 1}} \sum_{j} \lambda P_j(S)[r_j - \pi . A_j] \tag{6}
$$

Binary variable ($y \epsilon B^n$) used in equation (10) represents the candidate price point for offering. If product j is selected to be offered to the customers, it will be equal to one; otherwise, it is zero.

In the next section, we will incorporate multinomial logit within problem (9) and present the proposed solutions.

4.1.Multinomial Logit Model

Suppose customers choose their products based on the MNL model. This assumption will lead to the following problem which specifies optimal price points:

$$\begin{aligned}&max\{\sum_{l=1}^{L}\lambda_l \frac{\sum_{j\epsilon C_l}(r_j-A_j^T\pi)e^{V_{lj}}y_j}{\sum_{i\epsilon C_l}e^{V_{li}}y_i+e^{V_{lo}}}\}\\&\sum_{j\epsilon C_{leg}} y_j \leq 1\,; \quad \forall leg\\&y_j = 0,1\end{aligned} \tag{7}$$

where V_{lj} represents the observed utility of product j belonging to segment l, and V_{l0} is the observed utility of the customer departure without purchase belonging to segment l. The objective function can be transformed to a mixed integer programming problem (Prokopyev et al., 2005). In order to achieve this goal, a new variable will be defined as follows:

$$x_l = \frac{1}{\sum_{i\epsilon C_l} e^{V_{li}}y_i + e^{V_{lo}}}, \quad l = 1,2,\dots,L \tag{8}$$

Replacement of this variable in the equation 7 will lead to:

$$\begin{aligned}&max \sum_{l=1}^{L}\sum_{j\epsilon C_l}\lambda_l(\,r_j - A_j^T\pi)e^{V_{lj}}y_j x_l\\&x_l e^{V_{lo}} + \sum_{i\epsilon C_l} e^{V_{li}}y_i x_l = 1; \quad l = 1,2,\dots,L\\&\sum_{j\epsilon C_{leg}} y_j \leq 1\,; \quad \forall leg\\&y_j = 0,1; \quad x_l \geq 0\end{aligned} \tag{9}$$

The nonlinear terms $z_{li} = x_l y_i$ can be linearized (Wu, 1997) by replacing the nonlinear term with three linear constraints as follows:

$$\begin{aligned}&x_l - z_{li} \leq K - Ky_i\\&z_{li} \leq x_l\\&z_{li} \leq Ky_i\end{aligned} \tag{10}$$

where K is a large number. Replacing these terms, the problem will become:

$$\begin{aligned}&max\ \sum_{l=1}^{L}\sum_{j\epsilon C_l}\lambda_l(\,r_j - A_j^T\pi)e^{V_{lj}}z_{li}\\&x_l e^{V_{lo}} + \sum_{i\epsilon C_l} e^{V_{li}}z_{li} = 1; \quad \forall l\\&x_l - z_{li} \leq K - Ky_i; \quad \forall l, i\epsilon C_l\\&z_{li} \leq x_l; \quad \forall l, i\epsilon C_l\\&z_{li} \leq Ky_i; \quad \forall l, i\epsilon C_l\\&\sum_{j\epsilon N_i} y_j \leq 1\,; \quad \forall i\\&y_j = 0,1; \quad x_l \geq 0; \quad z_{li} \geq 0\end{aligned} \tag{11}$$

This is an MIP problem that may be solved with any optimization software.

5. Computational Results

In this section, a network of parallel flights is considered for the dynamic pricing of products segmented in the nested logit way and chosen by customers based on a nested logit model.
To simulate a customer's choice behavior, use has been made of Mont Carlo's simulation method, and to analyze correlation impacts among the products in a nest, a simulation has been done according to two distinct scenarios assuming that customers choose products based on the nested logit model while the firm can apply either a multinomial or a nested logit model to determine the price points.

To determine products' offering prices, we will first solve the CDLP formulation and determine the resources' optimal dual values which are used to specify the particular price point for each resource. Dual values are updated during specific time periods and price points are selected at the beginning of each time period. To better evaluate the algorithms, different product offering capacities are considered by assuming different booking periods of 600, 1300 and 2000.

5.1. Parallel Flights Network

Consider a network with a specific origin and destination that has four parallel legs of 4 pre-specified price points each and a leg capacity
$C = (150,120,180,150)$. A firm should decide about offering 4 among 16 virtual products during each period, and customers can choose from among them or decide to leave without purchasing. The problem consists of finding a policy which should lead to choosing the most suitable price points to be offered to the customers at any period of time during the booking horizon while the revenue of the firm should be maximized. Table (1) describes the available products in this network.

Table1
Product definition for parallel flight

Products	Legs	Fare	Products	Legs	Fare
1	1	500	9	3	300
2	1	600	10	3	400
3	1	700	11	3	500
4	1	750	12	3	600
5	2	300	13	4	500
6	2	400	14	4	600
7	2	500	15	4	700
8	2	600	16	4	750

Table 2
Revenue simulation results when a firm offers its products based on the MNL Model

		Correlation				
Time Periods		**0**	**0.2**	**0.4**	**0.6**	**0.8**
600	Revenue	151071.67	150843.33	152236.67	153328.33	153328.33
	Load factor	47.72	48.77	48.81	49.07	49.07
1300	Revenue	329221.67	327820.00	3332900.00	339788.33	342820.00
	Load factor	91.29	91.28	92.19	93.58	94.22
2000	Revenue	404976.67	404925.00	404966.67	404925.00	404915.00
	Load factor	100.00	100.00	100.00	100.00	100.00

Table (2) presents similar results when the firm uses multinomial logit model (to specify offering price points) and customers choose products based on the nested logit model.

According to these tables, the results can be interpreted under three states: 1) resource is abundant and nearly fifty percent will remain unused, 2) capacity is strictly scarce and it is certain that all will be used, and 3) a state between these two extremes. The first rows in the tables of results are related to the first.

The last rows in the abovementioned tables are related to the second extreme where capacity is scarce. Here, the firm will offer the highest possible prices; the whole capacity will be used with this price and, therefore, application of all choice models will lead to the same result.

The second row is related to the moderate state; results show that when correlation (within the nests) increases, the nested logit outperforms in comparison with the multinomial model.

6. Conclusions

In this article, we tried to analyze the price-based revenue management of substitutable products with two dominant customer choice models when there are pre-specified price points to be chosen by the firm. Effort was made to use quantity-choice-based revenue management techniques for dynamic pricing of products supposing that at the beginning of the booking horizon there are pre-specified price points (assumed to be virtual products) among which a firm should select at the beginning of each period. Most researches that focus on choice-based quantity and price-based revenue management models usually apply the multinomial logit choice model. Choice-based deterministic linear programming model, used to compute the resources' marginal values, is one of the most applicable revenue management models. These values are used in a 0-1 fractional programming to select the most appropriate price points at the beginning of each period. Fractional programming, obtained by applying the multinomial logit model and sensitive to specific choice models, can be transformed to a linear 0-1 programming and then solved with ordinary software programs.
Results have shown that during the two extreme conditions (capacity abundance and scarcity), the selected prices change to lower and higher, choice models do not disturb these results considerably, and dynamic pricing approaches towards the fixed pricing policy. During the moderate state (enough capacity), selecting a suitable choice model is important and will influence the firm's revenue. Applying more accurate choice models in this condition will increase the load factor of capacities and then decrease the total amount of the required flight time to move passengers. This approach will moderate their negative impact on the environment.

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