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A note on "Supplier selection by the pair of nondiscretionary factors-imprecise data envelopment analysis models"

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Abstract

Recently, Farzipoor Saen [Journal of the Operational Research Society, **60**(11), 1575–1582 (2009)] proposed a method based on data envelopment analysis to identify optimistic efficient suppliers in the presence of nondiscretionary factors-imprecise data. This short communication aims at showing a computational error in computing the value of preference intensity parameter in Farzipoor Saen's [1] article. Then, a ranking method is used to identify the suppliers with the best performance.

Keywords: Data envelopment analysis; supplier selection; imprecise data, ranking.

1. Introduction

The supplier selection models based on cardinal data do not emphasize much on the ordinal data; however, with the extensive use of production philosophies, such as just-in-time method, more emphasis was placed on considering both cardinal and ordinal data at the same time in supplier selection process [1]. The earlier studies showed how data envelopment analysis (DEA) could be used to evaluate suppliers on several criteria. They also specified benchmark rate. Farzipoor Saen [1] studied suppliers'

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selection using DEA while there are nondiscretionary factors and imprecise data. The present note shows a computational error in computing the value of preference intensity parameter in Farzipoor Saen's [1] article. Therefore, another verification test is conducted here to achieve the optimistic efficiency interval of suppliers.

2. The models proposed by Farzipoor Saen [1] to select a supplier

In DEA analysis, it is usually assumed that there are n production units that use m different inputs and produce s different outputs. Specially, j^{th} production unit consumes x_{ij} units of input i (i=1,…,m) and produces y_{rj} units of output r (r =1,...,s). In spite of the preliminary DEA model, it is assumed in the interval DEA that some definitive values of input x_{ij} and output y_{rj} are not known. We know that the inputs/outputs data are within the bounded intervals, i.e. $x_{ij} \in [x_{ij}^L, x_{ij}^U]$ and $y_{rj} \in [y_{rj}^L, y_{rj}^U]$ and the intervals of the upper and lower bounds are given as the fixed numbers. It is also assumed that $y_{rj}^L > 0$ and $x_{ij}^L > 0$.

In addition, assume that we can divide the input variables into two subsets including discretionary subset (I_D) and nondiscretionary subset (I_N). Therefore, we have:

$$I = \{1,\dots,m\} = I_D \cup I_N, \quad I_D \cap I_N = \varnothing.$$

To work with such an uncertain situation, Farzipoor Saen [1] presented the following linear programming models to create upper and lower bounds of optimistic efficiency interval for each decision-making unit (DMU):

$$\min\ c_o^U = \theta_o - \varepsilon\left(\sum_{i\in I_D} s_i^- + \sum_{r=1}^{s} s_r^+\right) \tag{1}$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{j=1}^{n} y_{rj}^U \lambda_j - s_r^+ = y_{ro}^U, \quad r = 1,\dots,s,\\
& x_{io}^L \theta_o - \sum_{j=1}^{n} x_{ij}^L \lambda_j - s_i^- = 0, \quad i \in I_D,\\
& x_{io}^L - \sum_{j=1}^{n} x_{ij}^L \lambda_j = 0, \quad i \in I_N,\\
& \theta_o, \text{ free},\\
& \lambda_j \ge 0, \quad j = 1,\dots,n,\\
& s_i^- \ge 0, \quad i \in I_D,\\
& s_r^+ \ge 0, \quad r = 1,\dots,s.
\end{aligned}$$

$$\min\ c_o^L = \theta_o - \varepsilon\left(\sum_{i\in I_D} s_i^- + \sum_{r=1}^{s} s_r^+\right) \tag{2}$$

$$\text{s.t.}\qquad \sum_{j=1}^{n} y_{rj}^U \lambda_j - s_r^+ = y_{ro}^L, \quad r = 1,\ldots,s,$$

$$x_{io}^U \theta_o - \sum_{j=1}^{n} x_{ij}^L \lambda_j - s_i^- = 0, \quad i \in I_D,$$

$$x_{io}^U - \sum_{j=1}^{n} x_{ij}^L \lambda_j = 0, \quad i \in I_N,$$

$$\theta_o, \text{free},$$

$$\lambda_j \geq 0, \quad j = 1,\ldots,n,$$

$$s_i^- \geq 0, \quad i \in I_D,$$

$$s_r^+ \geq 0, \quad r = 1,\ldots,s.$$

Where "o" indicates the DMU under evaluation. ε is non-Archimedes infinitesimal. θ_o, λ_j, s_i^- and s_r^+ are dual variables. θ_o is the radial input shrinkage factor and $\lambda = \{\lambda_j\}$ is the vector of DMU loads. Variable s_r^+ is shortfall amount of output r and s_i^- is surplus amount of input i. c_o^{U*} which indicates the upper bound of optimistic efficiency interval of DMU_o and c_o^{L*} denotes the interval lower bound of optimistic efficiency of DMU_o. These show the optimistic efficiency interval of $[c_o^{L*}, c_o^{U*}]$. If $c_o^{U*} = 1$ then DMU_o is called optimistic efficient; otherwise, that is called optimistic non-efficient. All DMUs of optimistic efficient form an efficient production frontier.

Models (1) and (2) were created to work with interval inputs/outputs data. However, ordinal preference information can be converted into interval data in order to use models (1) and (2) easily, even in these situations.

Strong ordinal preference information, such as $x_{ij} > x_{ik}$ can be expressed as $x_{ij} \geq \eta_i x_{ik}$. Here, $\eta_i > 1$ parameter is on the degree of preference intensity and it is offered by a decision maker [2].

There are the following ordinal relations for scale transformation for strong ordinal preference information $x_{i1} > x_{i2} > \ldots > x_{in}$:

$$1 \geq \hat{x}_{i1}; \quad \hat{x}_{ij} \geq \eta_i \hat{x}_{i,j+1}, \quad j = 1,\ldots,n-1; \quad \hat{x}_{in} \geq \sigma_i \tag{3}$$

Where σ_i is a small positive number, which indicates the proportion of possible minimum value $\{x_{ij} \mid j = 1,\dots,n\}$ to the possible maximum value. A decision maker can offer its approximation. This number is called ratio parameter. In addition, η_i is a preference intensity parameter, which is true in $\eta_i > 1$ relation that is offered by a decision maker. The permissible interval obtained for each $\hat{x}_{ij}$ ($j = 1,\dots,n$) is obtained as follows [2]:

$$\sigma_r \le \chi_r^{1-n} \text{ with } \hat{y}_{rj} \in [\sigma_r \chi_r^{n-j}, \chi_r^{1-j}], \quad j = 1,\dots,n, \tag{4}$$

It should be noted that σ_i is used as a lower bound for the normalized ordinal preference information and it should be true in the following condition:

$$\sigma_i \in (0, x_{in} / x_{i1}], \tag{5}$$

In addition, η_i should also be true in the following condition:

$$\eta_i \in (1, x_{i1} / x_{i2}]. \tag{6}$$

Using the above scale transformation and estimating the permissible intervals, all the strong ordinal preference information are converted into the interval data and therefore they can be integrated into models (1) and (2).

3. A preference degree approach for comparing and ranking efficiency intervals

As the efficiency score for each supplier is specified by an interval, a simple -but practical- ranking approach is needed to compare and rank interval numbers. Several approaches have been developed for ranking interval numbers; however, they all have some drawbacks, especially when the interval numbers have identical center and different widths. They are all unable to differentiate between these numbers. We use the degree of preference developed by Wang et al. [3] to compare and rank suppliers' efficiency interval. This approach is summarized as follows.

Assume that $a = [a^L, a^U]$ and $b = [b^L, b^U]$ are two interval numbers. The rate by which an interval number is bigger than another interval number is called its ***degree of preference***. Based on this, we have the following definitions and properties.

Definition 1. The degree of preference of a over b (or $a > b$) is defined as

$$P(a > b) = \frac{\max(0, a^U - b^L) - \max(0, a^L - b^U)}{(a^U - a^L) + (b^U - b^L)} \tag{7}$$

The degree of preference of a over b (or $a > b$) can also be defined in a similar way. That is:

$$P(b > a) = \frac{\max(0, b^U - a^L) - \max(0, b^L - a^U)}{(a^U - a^L) + (b^U - b^L)} \tag{8}$$

It is clear that $P(a > b) + P(b > a) = 1$ and $P(a > b) = P(b > a) \equiv 0.5$ when $a = b$, i.e. $a^L = b^L$ and $a^U = b^U$.

Definition 2. If $P(a > b) > P(b > a)$, then it is said that a is superior to b to the degree of $P(a > b)$ this is denoted by $a \overset{P(a>b)}{\succ} b$; If $P(a > b) = P(b > a) = 0.5$ then it is said that a is indifferent to b, and is denoted by $a \sim b$; If $P(b > a) > P(a > b)$ then it is said that a is inferior to b to the degree $P(b > a)$, and is shown by $a \overset{P(b>a)}{\prec} b$.

Property 1. $P(a > b) = 1$ if and only if $a^L \ge b^U$.

Property 2. If $a^L \ge b^L$ and $a^U \ge b^U$, then $P(a > b) \ge 0.5$ and $P(b > a) \le 0.5$.

Property 3. If b is nested in a, i.e. $a^L \le b^L$ and $a^U \ge b^U$, then $P(a > b) \ge 0.5$ if and only if $\frac{a^L + a^U}{2} \ge \frac{b^L + b^U}{2}$.

Property 4. If $P(a > b) \ge 0.5$ and $P(b > c) \ge 0.5$, then $P(a > c) \ge 0.5$.

Ranking process is shown as below:

***Step 1**:* Calculate the matrix of degrees of preference:

$$M_P = \begin{array}{c} \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{array} \begin{array}{c} \begin{array}{cccc} \theta_1 & \theta_2 & \cdots & \theta_n \end{array} \\ \begin{bmatrix} - & p_{12} & \cdots & p_{1n} \\ p_{21} & - & \cdots & p_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & \cdots & - \end{bmatrix} \end{array} \tag{9}$$

Here, we have:

$$p_{ij} = P(\theta_i > \theta_j) = \frac{\max(0, \theta_i^U - \theta_j^L) - \max(0, \theta_i^L - \theta_j^U)}{(\theta_i^U - \theta_i^L) + (\theta_j^U - \theta_j^L)}, \quad i, j = 1, \ldots n; i \ne j. \tag{10}$$

Step 2. Find a row in the matrix of degrees of preference in which all elements except the diagonal element are greater than or equal to 0.5. If this row corresponds with θ_i, then θ_i is the most preferred interval number.

Step 3. Remove row i and column i (and therefore θ_i) from the matrix. In the reduced matrix, if θ_j is the most preferred interval numbers among the remaining intervals, then θ_j receives the second place and is shown as $\theta_i \overset{p_{ij}}{>} \theta_j$, if $p_{ij} > 0.5$, or as $\theta_i \sim \theta_j$, if $p_{ij} = p_{ji} = 0.5$.

Step 4. For further analysis, remove row j and column j from the reduced matrix and continue the process until the remaining intervals are ranked.

We will show the above ranking in the numerical example of the next section.

4. Revision of the numerical example proposed by Farzipoor Saen [1]

The set of data for this example was adopted from Farzipoor Saen [1], which contains some indices on 18 suppliers. DEA inputs include total cost of shipments (x_1), distance (x_2), and supplier reputation (x_3). *"Distance"* is generally considered as a nondiscretionary input variable. *"Supplier reputation"* is included as a qualitative input. *"Supplier reputation"* is measured by an ordinal scale so that, for instance, supplier reputations (18) and (17) have the highest rank and the lowest ranks, respectively. DEA output is the number of bills received from the supplier without errors (y_1). Table 1 shows inputs and output of the supplier. In this example $\varepsilon = 10^{-4}$ is considered as the non-Archimedes infinitesimal.

Table 1: Inputs, output, converted ordinal data and efficiency interval for 18 suppliers

Supplier number (DMU)	Inputs			Output	Converted Ordinal Data of Supplier Reputation	Optimistic Efficiency Interval
	x_{1j}	x_{2j}	x_{3j}	y_1		
1	253	249	5	[50, 65]	[0.01216, 0.53032]	[0.187, 0.292]
2	268	643	10	[60, 70]	[0.01551, 0.67684]	[0.248, 0.327]
3	259	714	3	[40, 50]	[0.01103, 0.48102]	[0.272, 0.451]
4	180	1809	6	[100, 160]	[0.01276, 0.55684]	[0.994, 1.000]
5	257	238	4	[45, 55]	[0.01158, 0.50507]	[0.167, 0.267]
6	248	241	2	[85, 115]	[0.01050, 0.45811]	[0.303, 0.624]
7	272	1404	8	[70, 95]	[0.01407, 0.61391]	[0.508, 0.696]
8	330	984	11	[100, 180]	[0.01629, 0.71068]	[0.328, 0.662]
9	327	641	9	[90, 120]	[0.01477, 0.64461]	[0.277, 0.482]
10	330	588	7	[50, 80]	[0.01340, 0.58468]	[0.177, 0.393]
11	321	241	16	[250, 300]	[0.02079, 0.90703]	[0.821, 1.000]
12	329	567	14	[100, 150]	[0.01886, 0.82270]	[0.293, 0.434]
13	281	567	15	[80, 120]	[0.01980, 0.86384]	[0.286, 0.401]
14	309	967	13	[200, 350]	[0.01796, 0.78353]	[0.609, 1.000]
15	291	635	12	[40, 55]	[0.01710, 0.74622]	[0.215, 0.261]
16	334	795	17	[75, 85]	[0.02183, 0.95238]	[0.248, 0.285]
17	249	689	1	[90, 180]	[0.01000, 0.43630]	[0.369, 0.979]
18	216	913	18	[90, 150]	[0.02292, 1.00000]	[0.455, 0.679]

[1] Ranking such that 18 ≡ highest rank, …, 1 ≡ lowest rank ($x_{3,18} > x_{3,16} \ldots > x_{3,17}$).

To convert strong ordinal preference information into interval data, assume that preference intensity parameter and ratio parameter are estimated as $\eta_3 = 1.05$ and $\sigma_3 = 0.01$, respectively. Using the technique explained in the earlier section, the interval estimate for reputation of each supplier can be achieved, which is shown in the fourth column of Table 1. To convert strong ordinal preference information into interval data, Farzipoor Saen [1] had assumed that preference intensity parameter on strong ordinal preference information was estimated as $\eta_3 = 1.12$. It is clear that condition $x_{3j} \geq 1.12 x_{3,j+1}$ is not true for both successive ranks. For instance, it is clear that $x_{34} = 6 \geq 1.12 \times x_{31} = 1.12 \times 5 = 5.6$ is true; however, $x_{38} = 11 \ngeq 1.12 \times x_{32} = 1.12 \times 10 = 11.2$ is not true. Therefore, further attentions should be paid to choose η_i s. According to the remarks made in the earlier section, $\eta_3 \in (1,1.0588]$ and $\sigma_3 \in (0, 0.0556]$.

By executing DEA models (1) and (2), the scores of suppliers' optimistic efficiency interval are shown in the fifth column of Table 1. According to Table 1, suppliers 4, 11 and 14 are optimistic efficient. The remaining fifteen suppliers with relative efficiency scores less than one are considered as the optimistic non-efficient ones. It should be noted that Farzipoor Saen [1] identified suppliers 4, 6, 11, 14, and 17 as the optimistic efficient ones. According to the optimistic efficiency interval achieved in the fifth column of Table 1 of this note and the fifth column of Table 1 in Farzipoor Saen's [1] article it becomes clear that, except suppliers number 13 and 18, there are obvious differences among all suppliers in the upper bound of optimistic efficiency interval.

Finally, Table 2 reports ranking optimistic efficiency interval of 18 suppliers achieved using degrees of preference. According to Table 2, eighteen suppliers are ranked in terms of optimistic efficiency interval as follows:

$$\mathrm{DMU}_4 \overset{96.76\%}{\succ} \mathrm{DMU}_{11} \overset{68.60\%}{\succ} \mathrm{DMU}_{14} \overset{63.04\%}{\succ} \mathrm{DMU}_{17} \overset{59.02\%}{\succ} \mathrm{DMU}_7 \overset{58.50\%}{\succ} \mathrm{DMU}_{18}$$
$$\overset{62.90\%}{\succ} \mathrm{DMU}_8 \overset{54.81\%}{\succ} \mathrm{DMU}_6 \overset{65.97\%}{\succ} \mathrm{DMU}_9 \overset{54.62\%}{\succ} \mathrm{DMU}_{12} \overset{50.62\%}{\succ} \mathrm{DMU}_3 \overset{56.12\%}{\succ} \mathrm{DMU}_{13}$$
$$\overset{78.87\%}{\succ} \mathrm{DMU}_2 \overset{51.02\%}{\succ} \mathrm{DMU}_{10} \overset{57.14\%}{\succ} \mathrm{DMU}_{16} \overset{69.01\%}{\succ} \mathrm{DMU}_1 \overset{50.99\%}{\succ} \mathrm{DMU}_{15} \overset{64.38\%}{\succ} \mathrm{DMU}_5$$

Here, $\mathrm{DMU}_4 \overset{96.76\%}{\succ} \mathrm{DMU}_{11}$, i.e. DMU_4 performance is better than DMU_{11} as much as 96.76%. As far as optimistic perspective is concerned, it is clear that the DMU_4 has the best performance and following positions are taken by DMU_{11} and DMU_{14}, respectively.

5. Conclusion

Decision making to choose the best supplier among an extensive set of suppliers is an important issue in production study. The present note shows a computational error in Farzipoor Saen's [1] article in calculating the value of preference intensity parameter, which is used in converting ordinal preference information into interval data. Another verification test was carried out to achieve correct efficiency interval of suppliers. Finally, approach based on degrees of preference was used to compare and rank optimistic efficiency intervals of suppliers.

Table 2: Degree of preference matrix for optimistic efficiency interval achieved based on models (1) and (2) and their rankings

DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Rank
1	–	0.2391	0.0704	0.0000	0.6098	0.0000	0.0000	0.0000	0.0484	0.3594	0.0000	0.0000	0.0273	0.0000	**0.5099**	0.3099	0.0000	0.0000	16
2	0.7609	–	0.2132	0.0000	0.8939	0.0600	0.0000	0.0000	0.1761	**0.5102**	0.0000	0.1545	0.2113	0.0000	0.8960	0.6810	0.0000	0.0000	13
3	0.9296	0.7868	–	0.0000	1.0000	0.2960	0.0000	0.2398	0.4531	0.6954	0.0000	0.4938	**0.5612**	0.0000	1.0000	0.9398	0.1039	0.0000	11
4	1.0000	1.0000	1.0000	–	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.9676**	1.0000	1.0000	0.9849	1.0000	1.0000	1.0000	1.0000	1
5	0.3902	0.1061	0.0000	0.0000	–	0.0000	0.0000	0.0000	0.0000	0.2857	0.0000	0.0000	0.0000	0.0000	0.3562	0.1387	0.0000	0.0000	18
6	1.0000	0.9400	0.7040	0.0000	1.0000	–	0.2279	0.4519	**0.6597**	0.8340	0.0000	0.7165	0.7752	0.0211	1.0000	1.0000	0.2739	0.3101	8
7	1.0000	1.0000	1.0000	0.0000	1.0000	0.7721	–	0.7050	1.0000	1.0000	0.0000	1.0000	1.0000	0.1503	1.0000	1.0000	0.4098	**0.5850**	5
8	1.0000	1.0000	0.7602	0.0000	1.0000	**0.5481**	0.2950	–	0.7143	0.8834	0.0000	0.7768	0.8374	0.0731	1.0000	1.0000	0.3104	0.3710	7
9	0.9516	0.8239	0.5469	0.0000	1.0000	0.3403	0.0000	0.2857	–	0.7262	0.0000	**0.5462**	0.6125	0.0000	1.0000	0.9669	0.1387	0.0629	9
10	0.6406	0.4898	0.3046	0.0000	0.7143	0.1660	0.0000	0.1166	0.2738	–	0.0000	0.2781	0.3212	0.0000	0.6782	**0.5714**	0.0279	0.0000	14

11	1.0000	1.0000	1.0000	0.0324	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	–	1.0000	1.0000	**0.6860**	1.0000	1.0000	0.7997	1.0000	2
12	1.0000	0.8455	**0.5062**	0.0000	1.0000	0.2835	0.0000	0.2232	0.4538	0.7219	0.0000	–	0.5781	0.0000	1.0000	1.0000	0.0866	0.0000	10
13	0.9727	**0.7887**	0.4388	0.0000	1.0000	0.2248	0.0000	0.1626	0.3875	0.6788	0.0000	0.4219	–	0.0000	1.0000	1.0000	0.0441	0.0000	12
14	1.0000	1.0000	1.0000	0.0151	1.0000	0.9789	0.8497	0.9269	1.0000	1.0000	0.3140	1.0000	1.0000	–	1.0000	1.0000	**0.6304**	0.8862	3
15	0.4901	0.1040	0.0000	0.0000	**0.6438**	0.0000	0.0000	0.0000	0.0000	0.3218	0.0000	0.0000	0.0000	0.0000	–	0.1566	0.0000	0.0000	17
16	**0.6901**	0.3190	0.0602	0.0000	0.8613	0.0000	0.0000	0.0000	0.0331	0.4286	0.0000	0.0000	0.0000	0.0000	0.8434	–	0.0000	0.0000	15
17	1.0000	1.0000	0.8961	0.0000	1.0000	0.7261	**0.5902**	0.6896	0.8613	0.9721	0.2003	0.9134	0.9559	0.3696	1.0000	1.0000	–	0.6283	4
18	1.0000	1.0000	1.0000	0.0000	1.0000	0.6899	0.4150	**0.6290**	0.9371	1.0000	0.0000	1.0000	1.0000	0.1138	1.0000	1.0000	0.3710	–	6

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