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Resource Allocation through Context-dependent data envelopment analysis

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Abstract

System designs, optimizing resource allocation to organization units, is still being considered as a complicated problem especially when there are multiple inputs and outputs related to a unit. The algorithm presented here will divide the frontiers obtained with DEA. In this way, we investigate a new approach for resource allocation.

Keywords: DEA, context-dependent, budget allocation, cost-efficiency, efficiency.

1. Introduction

Our literature review indicates that to this day in the evaluation of decision making units (DMUs) all DEA models have been used: many models are given only the amounts of inputs and outputs, each DMU corresponding to the considered inputs and outputs selects its best set of weights; the values of weights vary from one DMU to another. Each DMU's performance score is calculated by the DEA models which range between zero and one that provides its relative degree of efficiency. Moreover, the sources and amounts of inefficiency in each input and output for every DMU are also identified through the models. The subject of debate here is the possibility of further exploiting of available data to develop DEA-type models which help DMUs design their optimal systems and not just evaluate their existing systems.

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In fact, the purpose of this paper is to show the kind of budget allocations to DMUs as a result of which we have maximum revenue and minimum cost. Budget allocation is the distribution and division of services and facilities among people and existing programs. In this process, the allocation is done through Context-dependent data envelopment analysis [4]. In each level we calculate respective $\begin{pmatrix} X \\ Y \end{pmatrix}$ and find the frontier which if budget B is allocated to DMUs in this level, maximum revenue and minimum cost will be resulted.

1. Discussion and summery

1.1. Determining Levels

Assume that DMU_j (j=1… n) produces the outputs $Y_j = (y_{1j},...,y_{sj})$ by consuming $X_j = (x_{1j},...,x_{mj})$ the inputs [3]. And also suppose that $J^l = \{DMU_j; j=1,...,n\}$ is all DMUs set. We define $J^{l+1} = J^l - E^l$ that $E_j = \{DMU_j; J^l = 1 \mid \phi^*(l,k) = 1\}$ and $\phi^*(l,k)$ is the optimal solution value for the following linear programming problem:

$$
\begin{aligned}
\phi^*(l,k) = \quad & \underset{\lambda_j,\, \phi(l,k)}{\text{Max}} \ \phi(l,k) \\
\text{s.t.} \quad & \sum_{j \in F(J^L)} \lambda_j x_j \le x_k \\
& \sum_{j \in F(J^L)} \lambda_j y_j \ge \phi(l,k) y_k; \\
& \lambda_j \ge 0, \quad j \in F(J^L).
\end{aligned}
\tag{1}
$$

Where (x_k, y_k) present input and output vector with respect to DMU_k, respectively. $j \in F(J^L)$ shows that $DMU_j \in J^l$, F(.) proposed a correspondence between a set of DMUs and the analogue common indices set.

Model (1) is an output oriented CCR [5], where L=1 and defines DMUs which are in the first level efficiency frontier. DMUs which are in E^l set define L^{th}-efficient level. When L=2 the model (1), after removing DMUs in the first level, gives us DMUs in the second efficient level.

In this method, multiple efficient levels are recognized. These efficient levels will be obtained by the following algorithm:

Step1: set L=1, assess all the DMUs set, J^1, through the model (1) DMUs which are efficient in the first level can be reached.

Step2: Leave out DMUs that are considered in step1. $J^{l+1} = J^{l} - E^{l}$ (Stop when $J^{l+1} = \varnothing$)

Step3: Consider a new subset of inefficient DMUs through the model (1) we gain a new set of efficient DMUs E^{l+1} (The new efficient frontier.)

Step4: Let L=L+1. Go to step2.

Stopping rule: $J^{l+1} = \varnothing$, the algorithm stops.

1.2. Measuring profit efficiency

Now we consider profit efficiency model that helps to optimize DMUs system design. Let's suppose DMU (x,y) present benchmark DMU that with inputs $X = (x_1, x_2, ..., x_m)^T > 0$ corresponding with cost $C = (c_1, c_2, ..., c_m)^T > 0$ and output vector $Y = (y_1, y_2, ..., y_s)^T > 0$ corresponding with prices $P = (p_1, p_2, ..., p_s)^T > 0$. Note that both input vector x and output vector y are now variables. The calculation of maximum attainable profit, within the DEA framework, is the starting point of a profit analysis which can be done by using the model shown in [4]:

$$
\begin{aligned}
\text{Max} \quad & \xi = \sum_{r=1}^{s} P_r y_r - \sum_{i=1}^{m} C_i x_i \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = x_i \quad , i = 1, ..., m; \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = y_r \quad , r = 1, ..., s \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} \quad , i = 1, ..., m; \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad , r = 1, ..., s \\
& \sum_{j=1}^{n} \lambda_j = 1; \quad \lambda_j \ge 0, j = 1, ..., n
\end{aligned} \tag{2}
$$

Where P_r and w_i are, respectively, the price of r^{th} output and cost of it h input DMU_j , and the rest of the notation is as previously defined. Model (2) assures us to reach maximization profit value.

Therefore, for different weights (P_r , w_i), different price and cost is possible. So Profit efficiency of DMU_o is:

$$PE = \frac{\sum_{r=1}^{s} P_r y_{ro} - \sum_{i=1}^{m} C_i x_{io}}{\sum_{r=1}^{s} P_r y_r^* - \sum_{i=1}^{m} C_i x_i^*} ;$$

2. Budget Allocation through Context-dependent DEA

One of the main activities of management is making a strategy that is possible to deploy. In organizations that the strategic management has not been implemented resource allocation is based on politics and personal factors. In strategic oriented organizations resource is allocated on the basis of the preferences which are determined through the annual purposes.

Budget provides the possibility of allocating the limited resources based on planning preferences. One of the big barriers is the unsuccessful connection between administrative programs and specifying priorities regarding allocating budget to enduring guideline programs.

The model mentioned above, implicitly could make the information of budget resource portfolio available for DMUs, such as an optimum budget and budget congestion.

In this section the purpose is to introduce an algorithm which helps us to fairly allocate the total available budget B among DMUs. For comparison, the allocation is done by Context-dependent DEA method. Supposing that the total budget B is available we follow this algorithm:

Step1: Find the efficient level as mentioned in section 2.1. Let these efficient level's names $E^1, E^2, ..., E^L$.

Step2: In each level, distinctly calculate (x,y) in model (2). (For more information see section 2.2 and Fig 3.1) Suppose the profit value for each level; $E^1, E^2, ..., E^L$, respectively, is $\xi^1, \xi^2, ..., \xi^L$.

Step3: For finding the percentage allocation of budget B in each efficient level this indices will be used:

$$\eta_j = \frac{B\xi_j}{\sum_{j=1}^{L} \xi_j} \quad ; j = 1, ..., L \ (3),$$

Which $\frac{B\xi_1}{\sum_{j=1}^{L} \xi_j} + \frac{B\xi_2}{\sum_{j=1}^{L} \xi_j} + ... + \frac{B\xi_n}{\sum_{j=1}^{L} \xi_j} = B(\frac{\xi_1}{\sum_{j=1}^{L} \xi_j} + \frac{\xi_2}{\sum_{j=1}^{L} \xi_j} + ... + \frac{\xi_n}{\sum_{j=1}^{L} \xi_j}) = B$

η_j is the allocation rate of constant budget B to DMUs in efficient level E^j as if constant budget B is fairly allocated to all DMUs.

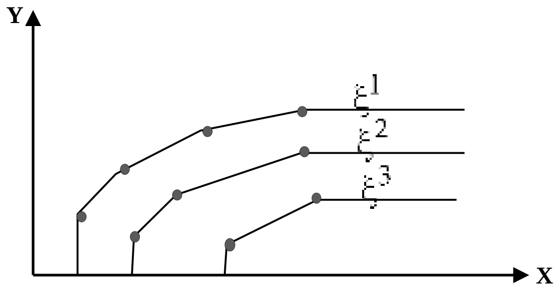

Fig. 3.1.levels and revenue efficiency value

Table 1.Inputs and outputs.

Inputs	Outputs
Payable interest	The total sum of four main deposits
Personnel	Other deposits
Non-performing loans	Loans granted
	Received interest
	Fee

Table 2.Input-data for the 20 bank branches.

DMU_j	x_{1j}	x_{2j}	x_{3j}
1	5007.37	36.29	87243
2	2926.81	18.8	9945
3	8732.7	25.74	47575
4	945.93	20.81	19292
5	8487.07	14.16	3428
6	13759.35	19.46	13929
7	587.69	27.29	27827
8	4646.39	24.52	9070
9	1554.29	20.47	412036
10	17528.31	14.84	8638
11	2444.34	20.42	500
12	7303.27	22.87	16148
13	9852.15	18.47	17163
14	4540.75	22.83	17918
15	3039.58	39.32	51582
16	6585.81	25.57	20975
17	4209.18	27.59	41960
18	1015.52	13.63	18641
19	5800.38	27.12	19500
20	1445.68	28.96	31700

Table 3.Output-data for the 20 bank branches.

DMU_j	y_{1j}	y_{2j}	y_{3j}	y_{4j}	y_{5j}
1	3126798	382545	1853365	125740.28	6957.33
2	440355	117659	390203	37836.56	749.4
3	1061260	503089	1822028	108080.01	3174
4	1213541	268460	542101	39273.37	510.93
5	395241	12136	142873	14165.44	92.3
6	1087392	111324	574355	72257.28	869.52
7	165818	180617	323721	45847.48	370.81
8	416416	486431	1071812	73948.09	5882.53
9	410427	449336	1802942	189006.12	2506.67
10	768593	15192	2573512	791463.08	86.86
11	696338	241081	2285079	20773.91	2283.08
12	481943	29553	275717	42790.14	559.85
13	574989	23043	431815	50255.75	836.82
14	342598	26172	126930	11948.04	1468.45
15	317186	270708	810088	111962.3	4335.24
16	347848	80453	379488	165524.22	399.8
17	835839	404579	9136507	41826.51	4555.42
18	320974	6330	29173	10877.78	274.7
19	679916	684372	3985900	95329.87	1914.25
20	120208	17495	308012	27934.19	471.22

3. Application

Now we will consider the branches of one of the Iran's commercial banks with 3 inputs and 5 outputs (see Table2 and Table3) as our DMUs. The inputs are payable interest, personnel and non-performing loans and the outputs are the total sum of four main deposits, other deposits, loans granted, received interest and fee (see table1). These data were collected in 2005. First based on the algorithm in section 2.1 we find three levels:

Level1 = {1,4,6,7,8,9,10,11,15,17,19}

Level2 = {2, 3, 5, 14, 16, 18, 20}

Level3 = {12, 13}

Consider the data given in Table 1 and Table 2. Let B = 1,000,000, C = (4, 2, 5) and P = (6, 7, 5, 4, 8). The unified model (2) is a linear program and thus can be solved by any LP algorithm. So we have:

Profit efficiency in first frontier is: 29,075,477.720000

Profit efficiency in second frontier is: 2,334,938.880000

Profit efficiency in third frontier is: 5,888,766.984000

As shown above the first frontier has the maximum revenue and minimum cost value but logically it is not a good way to allocate all the budget to the first level because DM wants to have fair judgment in the society. Not only the main aim of DM is having fair distribution of budget so that all the DMUs have a sufficient resource but also ensures that the maximum profit will satisfy DM.

We contribute 3 levels in allocation in this way:

$$\frac{1,000,000\times 29,075,477.720000}{29,075,477.720000+2,334,938.880000+5,888,766.984000}=\frac{1,000,000\times 29,075,477.720000}{37,299,183.584}=779,520$$

$$\frac{1,000,000\times 2,334,938.880000}{29,075,477.720000+2,334,938.880000+5,888,766.984000}=\frac{1,000,000\times 2,334,938.880000}{37,299,183.584}=62,600$$

$$\frac{1,000,000\times 5,888,766.984000}{29,075,477.720000+2,334,938.880000+5,888,766.984000}=\frac{1,000,000\times 5,888,766.984000}{37,299,183.584}=157,880$$

It means that the best way to allocate budget in this example is giving 779,520 of budget B to the first level and 62,600 of budget B to the second level and 157,880 of budget B to the second level.

4. Conclusion

We can use the above approach, wherever there is a human need, to allocate a limited budget to a number of teams or some of the activities so that the case which is chosen from a variety of combinations of allocations will have the most output value. Applying the optimization models allows all the various allocation cases to be considered and the most optimized of them which is based on objective function to be selected. The presented algorithm will enable us to allocate the limited budget to all DMUs. This was properly examined in a case study in an Iranian bank. Although obviously the first frontier has the largest proportion of budget, the rest of levels are fairly allocated.

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