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Portfolio Performance Evaluation in a Modified Mean-Variance-Skewness Framework with Negative Data

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Abstract

The present study is an attempt toward evaluating the performance of portfolios using mean-variance-skewness model with negative data. Mean-variance non-linear framework and mean-variance-skewness non- linear framework had been proposed based on Data Envelopment Analysis, which the variance of the assets had been used as an input to the DEA and expected return and skewness were the output. Conventional DEA models assume non-negative values for inputs and outputs. However, we know that unlike return and skewness, variance is the only variable in the model that takes non-negative values. This paper focuses on the evaluation process of the portfolios in a mean-variance-skewness model with negative data. The problem consists of choosing an optimal set of assets in order to minimize the risk and maximize return and positive skewness. This method is illustrated by application in Iranian stock companies and extremely efficiencies are obtained via mean-variance-skewness non-linear framework with negative data for making the best portfolio. The finding could be used for constructing the best portfolio in stock companies, in various finance organization and public and private sector companies.

*Keywords***:** Portfolio, Data Envelopment Analysis (DEA), Skewness, Efficiency, Negative data.

1. Introduction

In financial literature, a portfolio is an appropriate mix investments held by an institution or private individuals. Evaluation of portfolio performance has created a large interest among employees also academic researchers because of huge amount of money are being invested in financial markets. The

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theory of mean – variance, Markowitz [11] is considered the basis of many current models and this theory is widely used to select portfolios. This model is due to the nature of the variance in quadratic form. . Due to quadratic form, a study by Arditti [1], Kane [9] and Ho and Cheung [7] shown that investors prefer skewness which means that utility functions of investors are not quadratic. Other problem in Markowitz model is that increasing the number of assets will be developed the covariance matrix of asset returns and will be added to the content calculation. Due to these problems sharp one-factor model is proposed by Sharp [16]. This method reduces the number of calculations required information for the decision. Data envelopment analysis (DEA) has proved the efficiency for assessing the relative efficiency of Decision Making Units (DMUs) that employing multiple inputs to produce multiple outputs (Charnes et al [4]). Morey and Morey [13] proposed mean – variance framework based on Data Envelopment Analysis, which the variance of the portfolios is used as an input to the DEA and expected return is the output. Joro& Na [8] introduced mean - variance – skewness framework and skewness of returns are also considered as an output. Briec et al. [3] introduced shortage function. This shortage function obtains an efficiency measure that looks for improvements in both mean and skewness and decreases in variance. Kerstence et al. [10] introduced a geometric representation of the MVS frontier related to a new tool introduced in the literature by Briec. Mhiri and prigent [12] analyze the portfolio optimization problem by introducing the higher moments of the main financial index returns. In new models instead of estimating the whole efficient frontier, only the projection points of the assets are calculated. In these models are used a non-linear DEA-like framework where the correlation structure among the units is taken into account. Conventional DEA models assume non-negative values for inputs and outputs. These models cannot be used for the case in which DMUs include both negative and positive inputs and/or outputs. Poltera et al. [14] consider a DEA model which can be applied in the cases where input/ output data take positive and negative values.The other models solve negative data such as Modified slacks-based measure model (MSBM) [2006], semi-oriented radial measure (SORM) [2010] and etc. The portfolio optimization problem is a well-known problem in financial real world. The investor's objective is to get the maximum possible return on an investment with the minimum possible risk. Also the investors prefer to maximize positive skewness. Since there are a large number of assets to invest in, this objective leads to select the best assets via mean-variance-skewness non-linear model with negative data.

The rest of the paper is organized as follows: Section 2 briefly reviews the portfolio performance literature. Section 3 explainsmean-variance RDM and mean-variance-skewness RDM non-linear models. Section 4 presents computational results using Iranian stock companies data and finally conclusions are given in section 5.

2. Background

Portfolio theory to investing is published by Markowitz [11]. This approach starts by assuming that an investor has a given sum of money to invest at the present time. This money will be invested for a time as the investor's holding period. The end of the holding period, the investor will sell all of the assets that were bought at the beginning of the period and then either consume or reinvest. Since portfolio is a collection of assets, it is better that to select an optimal portfolio from a set of possible portfolios. Hence the investor should recognize the returns (and portfolio returns), expected (mean) return and standard deviation of return. This means that the investor wants to both maximize expected return and minimize uncertainty (risk). Rate of return (or simply the return) of the investor's wealth from the beginning to the end of the period is calculated as follows:

$$\text{Return} = \frac{(\text{end-of-period wealth}) - (\text{beginning-of-period wealth})}{\text{beginning-of-period wealth}} \qquad (1)$$

Since Portfolio is a collection of assets, its return r_p can be calculated in a similar manner. Thus according to Markowitz, the investor should view the rate of return associated to any one of these portfolios as what is called in statistics a random variable. These variables can be described expected the return (min or $\bar{r}_p$) and standard deviation of return. Expected return and deviation standard of return are calculated as follows:

$$\bar{r}_p = \sum_{i=1}^{n} \lambda_i \bar{r}_i \ , \ \sigma_p = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \Omega_{ij} \right]^{1/2} \qquad (2)$$

Where:

n=the number of assets in the portfolio

$\bar{r}_p$ =The expected return of the portfolio

λ_i =The proportion of the portfolio's initial value invested in asset i

$\bar{r}_i$ =The expected return of asset i

σ_p = The deviation standard of the portfolio

Ω_{ij} = The covariance of the returns between asset i and asset j

In the above, optimal portfolio from the set of portfolios will be chosen that maximum expected return for varying levels of risk and minimum risk for varying levels of expected return(Sharp [17]).

Data Envelopment Analysis is a nonparametric method for evaluating the efficiency of systems with multiple inputs and multiple outputs. In this section we present some basic definitions, models and

concepts that will be used in other sections in DEA. They will not be discussed in details. Consider DMU_j, ($j = 1,...,n$) where each DMU consumes m inputs to produce s outputs. Suppose that the observed input and output vectors of DMU_j are $X_j = (x_{1j},...,x_{mj})$ and $Y_j = (y_{1j},...,y_{sj})$ respectively, and let $X_j \geq 0$ and $X_j \neq 0$, $Y_j \geq 0$ and $Y_j \neq 0$. A basic DEA formulation in input orientation is as follows:

$$
\begin{aligned}
&\min\ \theta - \varepsilon\Big(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\Big) \\
&s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad i = 1,...,m, \qquad (3) \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro} \quad r = 1,...,s, \\
&\quad \lambda \in \Lambda, \\
&\quad s^+, s^- \geq 0, \\
&\quad \varepsilon \geq 0
\end{aligned}
$$

Where λ is a n-vector of λ variables, s^+ as-vector of output slacks, s^- an m-vector of input slacks and set Λ is defined as follows:

$$
\Lambda = \begin{cases} \{\lambda \in R_+^n & \text{with constant returns to scale,} \\ \{\lambda \in R_+^n, 1\lambda \leq 1\} & \text{with non-increasing returns to scale ,} \\ \{\lambda \in R_+^n, 1\lambda = 1\} & \text{with variable returns to scale} \end{cases} \qquad (4)
$$

Note that subscript ‘o’ refers to the unit under the evaluation. A DMU is efficient iff $\theta = 1$ and all slack variables s^-, s^+ equal zero; otherwise it is inefficient (Charnes et al. [4]). In the DEA formulation above, the left –hand sides in the constraints define an efficient portfolio. θ is a multiplier defines the distance from the efficient frontier. The slack variables are used to ensure that the efficient point is fully efficient. This model is used for asset selection. The portfolio performance evaluation literature is vast. In recent years these models have been used to evaluate the portfolio efficiency. Also in the Markowitz theory, it is required to characterize the whole efficient frontier but the proposed models by Joro & Na do not need to characterize the whole efficient frontier but only the projection points.

The distance between the asset and its projection which means the ratio between the variance of the projection point and the variance of the asset is considered as an efficiency measure (θ). In this framework, there is n assets, λ_j is the weight of asset j in the projection point, r_j is the expected return of asset j, μ_o and δ_o^2 are the expected return and variance of the asset under evaluation respectively. Efficiency measure θ can be solved via following model:

$$\begin{aligned} \min \quad & \theta - \varepsilon(s_1 + s_2) \\ s.t. \quad & E\left[\sum_{j=1}^{n} \lambda_j r_j\right] - s_1 = \mu_o, \\ & E\left[(\sum_{j=1}^{n} \lambda_j (r_j - \mu_j))^2\right] + s_2 = \theta \delta_o^2 \\ & \sum_{j=1}^{n} \lambda_j \le 1 \quad \forall \lambda \ge 0 \end{aligned} \qquad (5)$$

Model (5) is revealed by the non-parametric efficiency analysis Data Envelopment Analysis (DEA). Joro and Na [8] extended the described approach in (5) into mean-variance-skewness framework where κ_o is the skewness of the asset under evaluation. The efficiency measure θ can be solved through using the following model:

$$\begin{aligned} \min \quad & \theta - \varepsilon(s_1 + s_2 + s_3) \\ s.t. \quad & E\left[\sum_{j=1}^{n} \lambda_j r_j\right] - s_1 = \mu_o, \\ & E\left[(\sum_{j=1}^{n} \lambda_j (r_j - \mu_j))^2\right] + s_2 = \theta \delta_o^2 \\ & E\left[(\sum_{j=1}^{n} \lambda_j (r_j - \mu_j))^3\right] - s_3 = \kappa_o \\ & \sum_{j=1}^{n} \lambda_j \le 1 \quad \forall \lambda \ge 0 \end{aligned} \qquad (6)$$

Model (6) projects the asset with the efficient frontier by fixing the expected return and skewness levels and minimizing the variance.

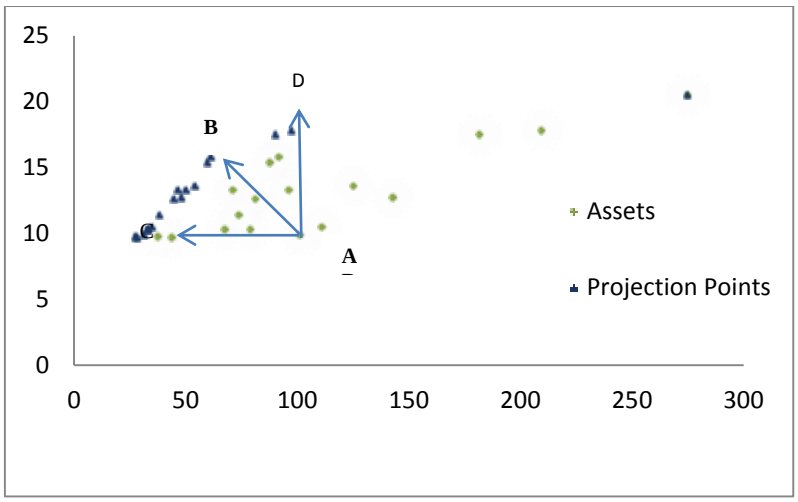

Fig 1. Different projections (input oriented, output oriented, combination oriented)

Fig 1 illustrates different projection that consist of input oriented, output oriented and combination oriented in models of data envelopment analysis. C is the projection point obtained via fixing expected return and minimizing variance, B via maximizing return and minimizing variance simultaneously, and D via fixing variance and maximizing return.

In the conventional DEA models, each $DMU_j\,(j=1,...,n)$ is specified by a pair of non-negative input and output vectors $(x_j, y_j) \in R_+^{m+s}$, in which inputs $x_{ij}\,(i=1,...,m)$ are utilized to produce outputs, $y_{rj}\,(r=1,...,s)$. These models cannot be used for the case in which DMUs include both negative and positive inputs and/or outputs. Poltera et al.[14] consider a DEA model which can be applied in the cases where input/ output data take positive and negative values. Rang Directional Measure (RDM) model proposed by Polera et al. goes as follows:

$$
\begin{aligned}
&\max \quad \beta \\
&st \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \beta R_{io} \qquad i=1,...,m, \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \beta R_{ro} \qquad r=1,...,s, \qquad (7) \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1, \\
&\qquad \lambda_j \ge 0 \qquad\qquad j=1,...,n.
\end{aligned}
$$

Ideal point (I) within the presence of negative data, is :

$I = (\max_j \{y_{rj} : r=1,...,s\}, \min_j \{x_{ij} : i=1,...,m\}$ where

$$R_{io} = x_{io} - \min_j \{x_{ij} : j = 1,...,n\}, \quad i = 1,...,m,$$
$$R_{ro} = \max_j \{y_{rj} : j = 1,...,n\} - y_{ro,} \quad r = 1,...,s. \qquad (8)$$

The other models solve negative data such as Modified slacks-based measure model (MSBM), Emrouznejad [6], semi-oriented radial measure (SORM), Sharp et al. [15] and etc.

3. Modified models in the presence of negative data

In model (6) if return and skewness is considered positive then the results are correct, hence the problem can happen only if return and skewness can take both positive and negative values. As we know that unlike return and skewness, variance is only a non-negative number. Also, Bhattacharyya et al. [2] predicted that for assets with negative expected returns, expected return will be a declining and convex function of skewness. Assume the basic problem is to select a portfolio from n financial assets. A portfolio $\lambda = (\lambda_1,...,\lambda_n)$ is a vector of proportions in each of these n financial assets with $\sum_{i=1}^{n} \lambda_i = 1$. Excluding short sales, one must impose the condition $\lambda_i \geq 0$ for all $i \in \{1,...,n\}$. In general, the set of admissible portfolio is written as follows:

$$\Im = \{\lambda \in R^n : \sum_{i=1}^{n} \lambda_i = 1, \lambda_i \geq 0\}.$$

Assets are characterized by an expected return $E[r_i]$. By a covariance matrix Ω with

$$\Omega_{ij} = \mathrm{cov}(r_i, r_j) = E[(r_i - E[r_i])(r_j - E[r_j])] \qquad (9)$$

And by a co-skewness tensor of rank three Λ with:

$$\Lambda_{ij} = E[(r_i - E[r_i])(r_j - E[r_j])(r_k - E[r_k])] \qquad (10).$$

Then, we have:

$$E[r(\lambda)] = \sum_{i=1}^{n} \lambda_i E[r_i]$$
$$Var[r(\lambda)] = E[(r(\lambda) - E[r(\lambda)])^2] = \sum_{i,j=1}^{n} \lambda_i \lambda_j \Omega_{ij} \qquad (11)$$
$$Sk[r(\lambda)] = E[(r(\lambda) - E[r(\lambda)])^3] = \sum_{i,j,k=1}^{n} \lambda_i \lambda_j \lambda_k \Lambda_{ij}$$

To condense notation, the function $\phi : \Im \to R^3$ defined by:

$$\phi(\lambda) = (E[r(\lambda)], Var[r(\lambda)], Sk[r(\lambda)])$$

is introduced to represent the expected return, variance and skewness of a given portfolio λ .In the reminder, an element of R^3 is called a MVS point. Thus, a MVS point can be the image by ϕ of a portfolio, or any arbitrary point in this three-dimensional space. It is useful to define the MVS image of $\Im$ as the image $\phi(\Im)$, with

$\phi(\Im) = \{\phi(\lambda); \lambda \in \Im\}$.

This set can be extended by defining a MVS disposal representation set via:

$DR = \phi(\Im) + (R_- \times R_+ \times R_-)$.

For the purpose of gauging portfolio efficiency, a subset of this representation set the weakly and strongly efficient frontier must be defined as:

Definition 1: In the MVS space, the weakly efficient frontier is defined as:

$\partial^w(\Im) = \{(M,V,S) \in DR; (-M',V',-S') < (-M,V,-S) \Rightarrow (M',V',S') \notin DR\}$.

Definition 2: In the MVS space, the strongly efficient frontier, is defined as:

$\partial^s(\Im) = \{(M,V,S) \in DR; (-M',V',-S') \le (-M,V,-S) \text{and} (-M',V',-S') \ne (-M,V,-S) \Rightarrow (M',V',S') \notin DR\}$

Extremely, we present following non-linear mean-variance RDM model on the basis of negative data:

$$\begin{aligned} \max \quad & \beta \\ \text{s.t.} \quad & E\left[\sum_{j=1}^{n} \lambda_j r_j\right] \ge \mu_o + \beta R_{\mu o} \\ & E\left[(\sum_{j=1}^{n} \lambda_j (r_j - \mu_j))^2\right] \le \sigma_o^2 - \beta R_{\sigma_o^2} \\ & \sum_{j=1}^{n} \lambda_j = 1 \qquad \lambda \ge 0 \end{aligned} \tag{12}$$

Ideal point (I) within the presence of negative data, is

$$\begin{aligned} & I = (\min_j\{\sigma_j^2\}, \max_j\{\mu_j\}) \text{ where,} \\ & R_{\mu o} = \max_j\{\mu_j : j = 1,...,n\} - \mu_o \\ & R_{\sigma_o^2} = \sigma_o^2 - \min_j\{\sigma_j^2 : j = 1,...,n\}. \end{aligned} \tag{13}$$

The above model can be expressed as following:

$$\begin{aligned} \max \quad & \beta \\ \text{s.t.} \quad & E[r(\lambda)] \ge \mu_o + \beta R_{\mu o} \\ & Var[r(\lambda)] \le \sigma_o^2 - \beta R_{\sigma_o^2} \\ & \sum_{j=1}^{n} \lambda_j = 1 \qquad \lambda \ge 0 \end{aligned} \tag{14}$$

Also, we present following non-linear mean-variance-skewness RDM model on the basis of negative data:

$$\begin{aligned} \max \quad & \beta \\ s.t. \quad & E\left[\sum_{j=1}^{n}\lambda_j r_j\right] \geq \mu_o + \beta R_{\mu_o} \\ & E\left[\left(\sum_{j=1}^{n}\lambda_j (r_j - \mu_j)\right)^2\right] \leq \delta_o^2 - \beta R_{\delta_o^2} \\ & E\left[\left(\sum_{j=1}^{n}\lambda_j (r_j - \mu_j)\right)^3\right] \geq \kappa_o + \beta R_{\kappa_o} \\ & \sum_{j=1}^{n}\lambda_j = 1 \quad \lambda_j \geq 0 \end{aligned} \tag{15}$$

Ideal point (I) within the presence of negative data, is

$$\mathrm{I} = (\min_{\mathrm{j}}\{\sigma_{\mathrm{j}}^2\}, \max_{\mathrm{j}}\{\mu_{\mathrm{j}}, \kappa_{\mathrm{j}}\})$$

Where

$$\begin{aligned} \mathrm{R}_{\mu_{\mathrm{o}}} &= \max\left\{\mu_{\mathrm{j}}\right\} - \mu_{\mathrm{o}} \\ \mathrm{R}_{\delta_{\mathrm{o}}^2} &= \delta_{\mathrm{o}}^2 - \min\left\{\delta_{\mathrm{o}}^2\right\} \\ \mathrm{R}_{\kappa_{\mathrm{o}}} &= \max\left\{\kappa_{\mathrm{j}}\right\} - \kappa_{\mathrm{o}} \end{aligned} \tag{16}$$

The above model can be expressed as following:

$$\begin{aligned} \max \quad & \beta \\ s.t. \quad & E\left[r(\lambda)\right] \geq \mu_o + \beta R_{\mu_o} \\ & Var\left[r(\lambda)\right] \leq \delta_o^2 - \beta R_{\delta_o^2} \\ & Sk\left[r(\lambda)\right] \geq \kappa_o + \beta R_{\kappa_o} \\ & \sum_{j=1}^{n}\lambda_j = 1 \quad \lambda_j \geq 0 \end{aligned} \tag{17}$$

The methodology in this paper starts with asset selection via performance evaluation in presence of negative data. The data used for this methodology is from 20 Iranian stock companies. In many cases similar to this example there are a lot of assets. It is better that starts with asset selection via performance evaluation. The choice of the asset can be random or discrete. The random choice of assets is usually biased and do not promise an optimum portfolio; hence it is more rational to have an objective choice while selecting the assets to be included in the portfolio. Performance evaluation is calculated by using models 14 and 17.

4. Application in Iranian Stock Companies

We illustrate our approach in non-linear mean-variance-skewness model for a data set 20 Iranian stock companies. A list of stocks used is provided in Table 1. In this report, there is expected return, variance, skewness of stocks which expected return and skewness are considered as output and variance is as input. The example is received from Iranian stock companies and is about portfolio performance evaluation in a mean-variance-skewness RDM framework. Thus, we know that unlike return and skewness, variance is the only variable in the model that takes non-negative values. In the analysis, the variance of the stocks is used as an input to the DEA and expected return and skewness are used as output.

Table1. Descriptive statistics of the Iranian stock companies

Iranian stock companies	Expected return	Variance	Skewness
VNVIN	7.285	6.534	-1.503
VPARS	7.388	10.474	0.789
VBHMN	-2.193	3.720	-0.470
VPASAR	10.853	4.256	0.813
DGABR	12.517	32.259	0.540
STRAN	9.052	70.764	1.488
FBAHNR	52.511	57.497	-0.6
FMLI	-3.676	19.609	-0.851
FVLAD	3.537	21.496	-1.03
KCHINI	7.57	67.378	0.591
VTVSA	6.896	14.171	-0.270
VLSAPA	1.888	29.002	-.964
VNFT	18.737	42.133	-1.314
VTGART	1.302	12.419	-0.947
VKHARZM	1.231	1.611	-1.048
VSAKHT	14.741	11.429	-0.922
KHSAPA	3.896	25.358	-0.853
VSINA	2.967	4.856	0.499
RTKG	32.677	28.464	0.487
VBMLT	2.022	1.56	-0.589

Linear MV and linear MVS with RDM model are calculated at table 2. Also, non linear MV efficiency measure and non-linear MVS efficiency measure using models 14 and 17 are calculated at table 2.

Table2. Efficiency measure of the Iranian stock companies

Iranian stock companies	Non-linear mean-variance RDM model (model 14)	MV RDM linear	Non-linear mean-variance-skewness RDM model (model 17)	MVS RDM linear
VNVIN	0.46	0.89	0.46	0.89
VPARS	0.41	0.83	0.43	0.91
VBHMN	0.39	0.81	0.43	0.81
VPASAR	0.5	1	0.54	1
DGABR	0.31	0.65	0.38	0.78
STRAN	0.21	0.41	0.34	1
FBAHNR	1	1	1	1
FMLI	0.31	0.62	0.33	0.62
FVLAD	0.31	0.66	0.34	0.66
KCHINI	0.22	0.43	0.32	0.66
VTVSA	0.37	0.77	0.40	0.77
VLSAPA	0.28	0.59	0.31	0.59
VNFT	0.32	0.63	0.39	0.65
VTGART	0.35	0.72	0.37	0.72
VKHARZM	0.43	1	0.46	1
VSAKHT	0.45	0.94	0.48	0.94
KHSAPA	0.3	0.63	0.33	0.63
VSINA	0.4	0.83	0.43	0.83
RTKG	0.51	1	0.68	1
VBMLT	0.43	0.65	0.47	0.65

Table 2 represents the calculated and compared the results of efficiency of model 14 and model 17 to linear MV RDM model and linear MVS RDM model. As seen in Table 2, model 14 and model 17 scores are as a conservative estimate of the linear MV RDM and linear MVS RDM scores. In this example all the linear DEA scores are greater than the non-linear model.Also, we compare the results linear MV RDM and linear MVS RDM. Because of, skewness equation is increased the results are not better. But, we know that the RDM model gives inefficiency score. Therefore, the efficiency score linear MVS RDM model is greater than linear MV RDM. But, those having positive skewness, have more efficiency increasing. For example, STRAN having the most positive skewness, Firstly linear MV RDM efficiency score has 0.41 and linear MVS RDM efficiency score ,secondly becomes 1. But those having negative skewness, don't change in efficiency score. Non-linear MVS RDM efficiency

scores are become nearly better than non-linear MV RDM efficiency scores. But those having positive skewness, have more efficiency increasing. For example, STRAN and KCHINE.
The results are obtained by General Algebraic Modeling System (GAMS) software.

5. Conclusion

This paper introduced a measure for portfolio performance using non-linear mean-variance-skewness RDM model. Joro and Na had proposed models for evaluating portfolio efficiency in which Data Envelopment Analysis model was employed. In these models was used a non-linear DEA-like framework where the correlation structure among the units was taken into account. We have applied model 14, and model 17 with return and skewness as output and the variance as the input to 20 stocks. The detailed results are presented in Table 2. In the numerical example is also observed that compared with linear, these models are highly exact in all the units, that is, all the linear DEA scores are greater than the non-linear models. This means that the DEA frontier is always dominated via the non-linear modified mean-variance frontier. But, those having positive skewness, have more efficiency increasing. For example, STRAN having the most positive skewnes. Firstly, linear MV RDM efficiency score has 0.41 and linear MVS RDM efficiency score, Secondly becomes 1. But those having negative skewness, don't change in efficiency score. Non-linear MVS RDM efficiency scores are nearly better than non-linear MV RDM efficiency scores. But, those having positive skewness, have more efficiency increasing. For example, STRAN and KCHINE.

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