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Research Article

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Scale Efficiency in DEA and DEA-R with Weight Restrictions

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Abstract

In data envelopment analyze (DEA) the scale efficiency in the input-oriented CCR model is less than or equal to the scale efficiency in DEA based on the fractional analysis (DEA-R). Also, the scale efficiency in case of multiple inputs and one output and vice versa the scale efficiencies are equal in DEA and DEA-R. In this paper, first, DEA-R model with weight restrictions when there is relative data is recommended. Second, the scale efficiencies in a constant return to scale are compared with each other in DEA and DEA-R models. At the end, a numerical example is provided for the proposed models with 27DMUs.

Keywords: weight restrictions, DEA, DEA-R.

1. Introduction

For the first time, Farrell (1957) suggested a model to evaluate the efficiency with one input and one output. After around two decades, Charnes et al. (1978) expanded Farrell's view and developed a model to measure the efficiency with multiple inputs and multiple outputs, and called it "data envelopment analyze (DEA)". Efficiency in DEA is defined as the weighted sum of inputs to the weighted sum of outputs. Data envelopment analyze based on the fractional analysis was presented by Despic et al. in 2007. In DEA- R the efficiency is defined as the weighted sum of the ratio of the input to the output. To determine the relative efficiency of a set, for each DMU a DEA model is solved. The vector of

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optimal weights for various units is usually different and in this case restrictions in the multiple problems should be applied in a way that both avoid zero weights in the optimal solutions and show the importance of weights. Therefore, researchers in the field of DEA examined the model of weight restrictions by adding them to other models. In the present article the DEA-R model with weight restriction is proposed in the second section and a numerical example with 27 DMUs is provided in the third section and finally, the conclusion of the article is presented in the last section.

2. DEA-R model with weight restriction

Efficiency in DEA is defined as the weighted sum of inputs to the weighted sum of outputs. The prices of v_i input and u_r output are determined by solving multiple CCR model which is of great important. In this context two problems rise: first, zero input-output weights or positive weights in front of zero inputs and outputs may lead to either zero numerator or denominator in the following problem:

$$\frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}}$$

Secondly, the use of ε to avoid zero weights in the first case do not lead to better value of objective function in the multiple model.

$$\begin{aligned}
&Max \ \sum_{r=1}^{s} u_r y_{rk} \\
&\quad s.t \ \sum_{i=1}^{m} v_i x_{ik} = 1 \qquad\qquad (1) \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, \dots, n \\
&\qquad u_r - k u_j \le 0 \qquad r = 1, \dots, s-1, \quad j = r+1, \dots, s \\
&\qquad v_i - k v_j \le 0 \qquad i = 1, \dots, m-1, \quad j = i+1, \dots, m
\end{aligned}$$

It becomes more complicated when there are weights restrictions for v_i and u_r weights. Therefore, the use of fractional analysis and DEA is suggested for solving the aforementioned problems. In DEA-R the efficiency is defined as the weighted sum of the ratio of the input to the output Overall, the model proposed by Despic et al. [1] can detect the pseudo inefficiency and also do not make any problem when using ε . Therefore, in this paper the use weight control in DEA-R models is recommended. The weight control model in input-oriented DEA-R with the assumption of constant returns to scale is suggested as follows:

$$Max\ \theta$$

$$s.t \quad \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}\left(\frac{\frac{x_{ij}}{y_{rj}}}{\frac{x_{iv}}{y_{rk}}}\right) \geq \theta \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir} = 1 \qquad (2)$$

$$w_{ir} - kw_{it} \leq 0 \qquad i = 1, \dots, m;\ r = 1, \dots, s-1;\ t = r+1, \dots, s$$

$$w_{ir} - kw_{tr} \leq 0 \qquad r = 1, \dots, s;\ i = 1, \dots, m-1;\ t = i+1, \dots, m$$

$$w_{ir} \geq 0$$

3. Numerical Example

Suppose 27 DMUs with two inputs and two outputs. Here, it should be mentioned data is taken from the paper of Shinn Sun [2]. Considering data in Table 1 and models of DEA and DEA-R in weight constraints the resulting scale efficiency of models 1 and 2 is shown in Table 2.

Table1. input and output data

Robot No.	Cost ($10,000)	Load Capacity (kg)	Velocity (m/s)	Repeatability
1	7.20	1.35	60.0	0.150
2	4.80	1.10	6.0	0.050
3	5.00	1.27	45.0	1.270
4	7.20	0.66	1.5	0.025
5	9.60	0.05	50.0	0.250
6	1.07	0.30	1.0	0.100
7	1.76	1.00	5.0	0.100
8	3.20	1.00	15.0	0.100
9	6.72	1.10	10.0	0.200
10	2.40	1.00	6.0	0.050
11	2.88	0.90	30.0	0.500
12	6.90	0.15	13.6	1.000
13	3.20	1.20	10.0	0.050
14	4.00	1.20	30.0	0.050
15	3.68	1.00	47.0	1.000
16	6.88	1.00	80.0	1.000

17	8.00	2.00	15.0	2.000
18	6.30	1.00	10.0	0.200
19	0.94	0.30	10.0	0.050
20	0.16	0.80	1.5	2.000
21	2.81	1.70	27.0	2.000
22	3.80	1.00	0.9	0.050
23	1.25	0.50	2.5	0.100
24	1.37	0.50	2.5	0.100
25	3.63	1.00	10.0	0.200
26	5.30	1.25	70.0	1.270
27	4.00	0.75	205.0	2.030

Through analyzing Table 2 and comparing the efficiency in models of CCR, DEA and DEA-R without weight constraints it can be found that DMUs of 1, 4, 10, 13, 14, 19, 20, 27 are efficient and scale efficiency is equal to one but for 21 DMU the scale efficiency of DEA is equal to 0.85154 and in DEA-R it is equal to one which shows a pseudo-inefficiency. Taking into account the weight constraints there is no relationship in DEA and DEA-R. But adding weight constraints doesn't violate the relationship between the DEA DEA-R. Overall, this idea here is as there is no relationship between scale efficiency, it is better not to use DEA-R model with weight constraints. But when there is only relative data (the ratio of inputs to outputs) model (2), it is suggested that the results reflect the scale efficiency.

Table2.comparing the efficiency in models of CCR, DEA and DEA-R

Robot No.	*Without weight constraints*		*With weight constraints*	
	DEA	DEA-R	DEA	DEA-R
1	1.00000	1.00000	1.00000	0.88889
2	0.90376	0.90724	0.22756	0.55833
3	0.52884	0.77983	0.31812	0.50740
4	1.00000	1.00000	0.13846	0.60000
5	0.59235	0.59235	0.59235	0.51389
6	0.48238	0.48389	0.08134	0.30021
7	1.00000	1.00000	0.29120	0.69318
8	0.78254	0.90594	0.52894	0.58333
9	0.37838	0.41913	0.17720	0.26414
10	1.00000	1.00000	0.34174	0.68889
11	0.67132	0.94929	0.48762	0.63958

12	0.10237	0.14069	0.10237	0.11696
13	1.00000	1.00000	0.43341	0.75000
14	1.00000	1.00000	1.00000	1.00000
15	0.61250	0.96261	0.42551	0.67878
16	0.60351	0.85746	0.60351	0.69703
17	0.40455	0.46228	0.07395	0.28437
18	0.36521	0.41463	0.18386	0.25810
19	1.00000	1.00000	0.96226	0.87589
20	1.00000	1.00000	0.03121	1.00000
21	0.85154	1.00000	0.17487	0.75199
22	0.82889	0.82889	0.06363	0.43879
23	0.69429	0.69976	0.17454	0.46250
24	0.63613	0.63963	0.16674	0.42746
25	0.55334	0.65589	0.26225	0.39431
26	0.58103	0.93724	0.48319	0.69985
27	1.00000	1.00000	1.00000	1.00000

4. Conclusion

In DEA when there is a proportion of input and output data and input and output data are not precisely identified, DEA-R is of great importance. In this paper, considering weight constraints a new model for calculating the scale efficiency is recommended when there is relative data. For future research it is recommended to calculate Malmquist index in DEA-R and DEA.

References

[1] Despic, O., Despic, M. and Paradi, J.C. (2007) DEA-R: Ratio-based comparative efficiency model, its mathematical relation to DEA and its use in applications. *Journal of Productivity Analysis*. 28: 33–44.

[2] Sun. S, W.-M. Lu, A (2005) cross-efficiency profiling for increasing discrimination in data envelopment analysis, INFOR 43 51–60.

[3] Wei, C.K., Chen, L.C., Li, R.K. and Tsai, C.H. (2011a) Using the DEA-R model in the hospital industry to study the pseudo-inefficiency problem. Expert Systems with Applications. 38: 2172–2176.

[4] Wei, C.K., Chen, L.C., Li, R.K. and Tsai, C.H. (2011b) Exploration of efficiency underestimation of CCR model: Based on medical sectors with DEA-R model. *Expert Systems with Applications*. 38: 3155–3160.

[5] Wei, C.K., Chen, L.C., Li, R.K. and Tsai, C.H. (2011c) A study of developing an input- oriented ratio-based comparative efficiency model. *Expert Systems with Applications*. 38: 2473–2477.