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Predicting the Efficiency of Decision-Making Unit by Using Piecewise Polynomial Extrapolation in Different Times

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Abstract

In this article, we will estimate efficiency amount of decision-making unit by offering the continuous piecewise polynomial extrapolation and interpolation by CCR model input-oriented on the assumption that it is constant returns to scale in different times. And finally, we will estimate efficiency amount of decision-making unit indifferent times by offering an example.

Keywords: Decision-Making Unit, Efficiency, Returns to Scale, Extrapolation and Interpolation.

1. Introduction

Charnes et al. [1] established data envelopment analysis (DEA) on the basis of the calculation of decision-making units (DMUs) efficiency. If we calculate the efficiency of a DMU by DEA models on the assumption that it is constant returns to scale in different times, we reach the conclusion that the amounts of efficiency for given DMU is a number which is greater than zero and smaller or equal to one; but if one uses interpolation the resulted interpolant may not be restricted to this region, i.e. it may have values greater than one or less than zero.

In the article, we solve the problem by offering a continuous piecewise polynomial interpolation and extrapolation and then the efficiency of a given unit can be predicted via this interpolant.

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2. Basic idea

In this section we, first review the basis of interpolation and then we offer a continuous piecewise interpolant for prediction of amount of efficiency in decision-making unit.

1.2 The interpolation problem

For data $\{(x_0, y_0),\dots,(x_n, y_n)\}$ of function $f(x)$ that, $x_0 < x_1 < \dots < x_n$ there is a polynomial from n maximum degree such as $p(x)$ so that:

$$P(x_i) = y_i = f(x_i) = f_i \quad (1)$$

For each $i = 0,1,\dots,n.$

That we can obtain from the different methods including Newton divided-difference method [2]. As follows:

$$P(x) = f[x_0] + f[x_0, x_1](x - x_0) + \dots + f[x_0,\dots,x_n](x - x_0)\dots(x - x_{n-1}) \quad (2)$$

But in cases where $f(x)$ (maybe) is a bounded function, the interpolant may reach values outside of the range of the original function. This is the case where occurs in polynomial interpolation. To overcome this problem we will offer a new interpolant which is also bounded.

2.2 bonded interpolant

Suppose that for each $x \in [x_0, x_n]$ *we have*; $f(x) \in [m', M']$ and also, suppose that for each $x \in [x_0, x_n]$ we have $P(x) \in [m, M]$ (It shown in the Fig.1). Now we definite $Q(x)$ to be a function from $[m, M]$ into $[m', M']$, then $Q(P(x))$ would be a function from $[x_0, x_n]$ into $[m', M']$ which interpolates our data $\{(x_0, y_0),\dots,(x_n, y_n)\}$ in which; $y_i = f(x_i)$ for each $i = 0,1,\dots,n.$ So it suffices to define $H(x) = Q(P(x))$, then H would be the desired interpolant which is bounded.

$$y_i = H(x_i) = Q(P(x_i)) = Q(y_i) \quad (3)$$

So

$$y_i = Q(y_i),\ \text{i}=0,1,\dots,\text{n} \quad (4)$$

Now we define $Q(x)$ as follows:

$$Q(x)=\begin{cases}\dfrac{M'-m'}{m-m'}(x-m')+m' & x\in[m,m']\\ x & x\in\left[m',M'\right]\\ \dfrac{m'-M'}{M-M'}(x-M')+M' & x\in[M',M]\end{cases} \qquad (5)$$

In this case, $H(x)$ would be:

$$H(x)=\begin{cases}\dfrac{M'-m'}{m-m'}\left(P(x)-m'\right)+m' & x\in A\\ P(x) & x\in B\\ \dfrac{m'-M'}{M-M'}\left(P(x)-M'\right)+M' & x\in C\end{cases} \qquad (6)$$

Where

$$\begin{aligned}A&=\left\{x\in\left[x_0,x_n\right]:\ P(x)\in\left[m,m'\right]\right\},\\ B&=\left\{x\in\left[x_0,x_n\right]:\ P(x)\in\left[m',M'\right]\right\},\\ C&=\left\{x\in\left[x_0,x_n\right]:\ P(x)\in\left[M',M\right]\right\}.\end{aligned} \qquad (7)$$

It is clearly that $H(x)$ defined in phrase (6) is an interpolant for the given data which is bounded to the range of $f(x)$ other interpolants may be used instead of present interpolation. For obtaining extrapolation, it is enough to obtain the range of present interpolation over given interval and extrapolation is achieved by placement of the obtained amounts of M, m in (5) relation.

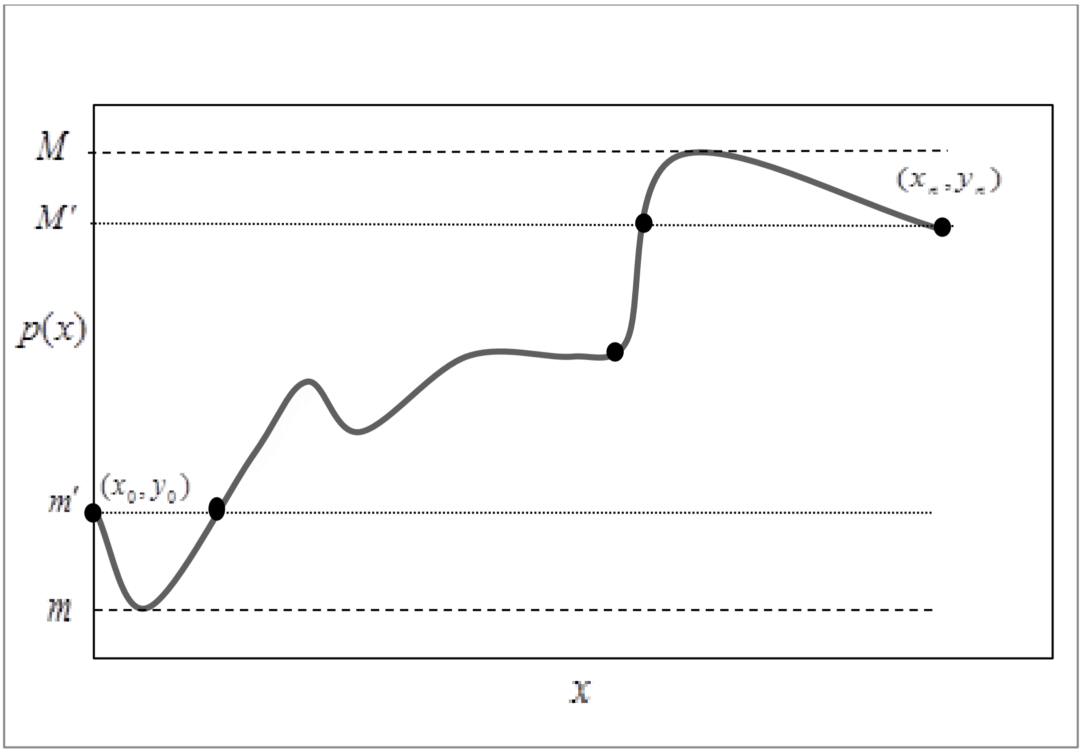

Figure 1. Range $f(x):[m', M']$, Range $P(x):[m, M]$, (x_i, y_i) for each i: node

3.2 Using interpolation and extrapolation for estimating efficiency

Suppose that we have n $DMU_j^t = \left(x_j^t, y_j^t\right), (j=1,2,\cdots,n)$ in the time period t, (t=1,…,T) where each of their y_j^t output vectors is produced by consuming x_j^t input vector. Suppose returns to scale is constant returns to scale in different times and for DMU_p, efficiency over time $t \in \{1,...T\}$ is equal to θ_p^t and also suppose that a polynomial interpolation $p(x)$ is used for the data $\{(1,\theta_p^1),...,(T,\theta_p^T)\}$ so $P(t)=\theta_p^t$ for each t. Suppose $p(x)$ ranges over interval $[1,T]$ equals to $[m, M]$. On the other hand, we know that the value of efficiency never concludes zero, now we consider ε as a very small positive real number and we define $Q(x)$ to be a function (as follows) from $[m, M]$ into $[\varepsilon, 1]$ so that the interpolation data would be $\{(1,\theta_p^1),...(T,\theta_p^T)\}$ therefore, $Q(x)$ according to relation (5) equals to;

$$Q(x)=\begin{cases}\dfrac{1-\varepsilon}{m-\varepsilon}(x-\varepsilon)+\varepsilon & x\in[m,\varepsilon]\\ x & x\in[\varepsilon,1]\\ \dfrac{\varepsilon-1}{M-1}(x-1)+1 & x\in[1,M]\end{cases} \qquad (8)$$

Where

$$\begin{aligned}A&=\{x\in[1,T]:P(x)\in[m,\varepsilon]\},\\ B&=\{x\in[1,T]:P(x)\in[\varepsilon,1]\},\\ C&=\{x\in[1,T]:P(x)\in[1,M]\}.\end{aligned} \qquad (9)$$

Now if we set $H(x)=Q(P(x))$, then, $H(x)$ is an interpolation for high given and its range is greater than or equals to ε and smaller than or equals to one and satisfies the following criterion:

$$H(x)=\begin{cases}\dfrac{1-\varepsilon}{m-\varepsilon}(P(x)-\varepsilon)+\varepsilon & x\in A\\ P(x) & x\in B\\ \dfrac{\varepsilon-1}{M-1}(P(x)-1)+1 & x\in C\end{cases} \qquad (10)$$

Whereas if we want to use given function for extrapolation. It is enough to obtain $p(x)$ range over discussed interval and place ε as a very small positive real number and we estimate efficiency value regarding to $H(x)$ criterion in the given time. But, pay attention whereas T be a number very great, because we prevent to increase the degree of $p(x)$ polynomial, it is enough that we use cubic spline [3] (quadric spline or linear spline) and considering that in spline interpolation for obtaining extrapolation function, it is enough to use the intervals itself criterion for extrapolation in the first and last sub-interval. Not being increasing $p(x)$ degree cause is that by increasing the amount of T there is probability that the amounts near to zero are obtained for estimated efficiency amount because efficiency amount is less than one and playing high number in computer may be considered as zero possibility.

3. Numerical example

We consider three decision- making units in seven time periods, x input produces y output and in table (1) these amounts have been determined in different times.

Table 1: Three decision making units during seven time periods

j	DMU_1	DMU_2	DMU_3
1	(1,2)	(2,3)	(3,4)
2	(3/2,3)	(1/2,2)	(2,4)
3	(8/3,5)	(3,7/2)	(1,3)
4	(32/3,4)	(2,3)	(4,5)
5	(10,1)	(1,10)	(2,3)
6	(4,5)	(6,7)	(8,9)
7	(1,1)	(2,3/2)	(3,5/2)

Then, following data are obtained by calculating the efficiency for first decision making unit in different times by using CCR model oriented input [1], for high unit during given times that they have summarized in Table 2

Table 2: The Efficiency of Units

Time	Efficiency
1	1
2	0.5
3	0.625
4	0.25
5	0.01
6	1
7	1

Therefore, by finding polynomial interpolation in mentioned points, we have:

$$p(x) = 1 - 0.5(x-1) + 0.3125(x-1)(x-2)$$
$$-0.1875(x-1)(x-2)(x-3)$$
$$+0.073325(x-1)(x-2)(x-3)(x-4)$$
$$-0.0108(x-1)(x-2)(x-3)(x-4)(x-5)$$
$$-0.00345(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$$

Which for estimating the amount of efficiency in $t = 6.5, t = 8$ times, it is enough to obtain $p(x)$ range over $\left[1,8\right]$ interval, which equals to $\left[-11.7592, 1.5766\right]$ interval. And by placing $\varepsilon = 0.001$, we have:

$$H(x) = \begin{cases} -0.0849\left(p(x)-1\right) + 0.001 & x \in A \\ p(x) & x \in B \\ -1.7326\left(p(x)-1\right)+1 & x \in C \end{cases}$$

In which A, B, C were the same thing that were expressed previously. Therefore, we have:

$Q(8) = 0.9994,$

$Q(6.5) = 0.0419$

Conclusion

In the paper, we have studied to make proper interpolation and extrapolation for estimating the efficiency of decision-making in different times. Because in some cases, the collection of data in determined time may be problematic for us and it is not possible ever, this method will be proper. This method can be used for obtaining interpolation and extrapolation of continuous real function in the compressed interval either. And also, this method recommends the first interpolation in the way that interpolation is situated considered range.

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