Available online at http://ijdea.srbiau.ac.ir
Int. J. Data Envelopment Analysis (ISSN 2345-458X)
Vol. 1, No. 3, Year 2013 Article ID IJDEA-00131, 9 pages
Research Article

International Journal of Data Envelopment Analysis
Science and Research Branch (IAU)

Measuring congestion in data envelopment analysis with common weights

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Received 25 July 2013, Accepted 22 August 2013

ABSTRACT

Congestion of economic concepts Occurs when at least one output is increasing at a decreasing an output, No Improvement in other inputs or outputs. So congestion is some inefficiency and it is necessary to determine the congestion of units and their congestion is given. In this paper, the congestion of units using a common weighting model of Liu et al. Based on a comparison of the inputs are unknown. This judgment is a high- congestion units. The only way to solve a linear programming model is capable of detecting congestion in all units.

Keywords: Data envelopment analysis; Common weights analysis; Ranking; Congestion

1. Introduction

Congestion is said to occur when the output that is maximally possible can be increased by reducing one or more inputs without improving any other inputs or outputs. Conversely, congestion is said to occur when some of the outputs that are maximally possible are reduced by increasing one or more inputs without improving any other inputs or outputs. Fare and Svensson Proportional to the congestion by varying rule were defined and developed.[7]. Fare and Grosskopf to determine the functional role of input in the proposed density.[5,6]. Then Brocket et al [2] and cooper et al Have developed a new method based on DEA to obtain input congestion. Other methods proposed by other scientists to evaluate congestion but in this paper, we pay to a method that is presented by Noura et al. Our work is based on the following definition of density is given by Cooper.

Definition 1. A process in which at least one input decreases at increasing output without worsening other inputs or outputs, or vice versa, is called congestion. When congestion occurs, resulting in increased output with no improvement in one or more input other input and output decrease.

Calculated to obtain the congestion and the concentration of decision-making units are removed, has two advantages: 1.If it is removed from the material input and material input costs, the cost is low.

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2. By definition, the congestion has been reduced output, after removing the compression force output increases.

The DEA model to estimate the performance of multiple units of different decisions, different sets of weights are achieved. So the basic DEA models of the same weight will usually vary from one unit to another decision that seems illogical. The idea of a common set of weights that were first introduced by Cook et al, (1990) this objection can be overcome.[4]. Rool et al (1991) to evaluate this idea Maintenance units with DEA used.[12]. Cook et al (1991) chose to help spread the weight of a ranking member of the DEA solution was bound Close the gap between the upper and lower bound on the weights used.[3]. Ganely et al (1992) proposed another common weighting methods for ranking DMUs, they used Ranking of units for maximum efficiency of the total of all credits.[8]. Sinuany et al in (1998) calculated the most common weights for inputs and outputs given the same rating scale developed in all units.[13]. Jahanshahloo et al. (2005)from the BCC model based on multiple multi-objective fractional programming, to generate common weights used to rank.[9].

Liu et al In (2008) consider a model number of a linear model to calculate the weight of all the units combined expressed and it was used for ranking.[10].In this paper, the density of units using a common weight of Liu et al Based on a comparison of the inputs are unknown this judgment is a high-density units, and the only way to solve a linear programming model is capable of detecting congestion in all units.

Sections of this paper is as follows: In Section 2, the method for measuring the density of states is Noura et al. Section 3, a common weight method for measuring the congestion is described. In Section 4, presents examples to illustrate the proposed method and compare Noura and his colleagues describe the method of measuring congestion. And at the end, the results are expressed.

2. Congestion measured by comparing the input

Noura and his colleagues have proposed a method for measuring the congestion at which they use. Comparing the measured densities of the inputs. Suppose we have n DMUs with m inputs and s outputs, and that the vectors $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{sj})^T$ denote the input and output values of $DMU_j, j = 1, \dots, n$, respectively. First, we solve the output-oriented BCC (Banker, Charnes, Cooper) model [1], which assumes variable returns to scale (VRS), in order to obtain the efficiency of each DMU.

$$\emptyset_o^* = \text{Max } \emptyset_o + \varepsilon\left(\sum_{r=1}^{s} s_{ro}^{+} + \sum_{i=1}^{m} s_{io}^{-}\right)$$

s.t.

$$\sum_{j=1}^{n} x_{ij}\lambda_j + s_{io}^{-} = x_{io}, \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j + s_{ro}^{+} = \emptyset_o y_{ro}, \quad r = 1, \dots, s \qquad (1)$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\left(\lambda_j, s_{ro}^{+}, s_{io}^{-}\right) \geq 0, \quad j = 1, \dots, n, \quad r = 1, \dots s, \quad i = 1, \dots, m$$

In (1), $\varepsilon > 0$ is a Non-Archimeadean element smaller than any positive real number. We solve Model (1), above, for each $DMU_j, j = 1, \dots, n$, and obtain the optimal solution: $(\emptyset^*, \lambda^*, s^{+^*}, s^{-^*})$.
Denoting the $\emptyset^*$ corresponding to DMU_j by$\emptyset_j^*$, we define set E as follows:

$$E = \left\{j \middle| \emptyset_j^* = 1\right\} \qquad (2)$$

Among the DMUs in set E, there exists at least one, say DMUl, that has the highest consumption in its first input component compared with the first input component of the remaining DMUs of set E. That is to say,

$$\exists\ (l \in E)\ \text{s.t.}\ \forall\ (j \in E) \rightarrow\ x_{1l} \geq x \qquad (3)$$

We denote x_{1l} by x_1^*. We then find, again, among the DMUs in E, a DMU, say DMU_t, that has the highest consumption in its second input component compared to the remaining DMUs in E. In other words,

$$\exists\ (t \in E)\ \text{s.t.}\ \forall\ (j \in E) \rightarrow\ x_{2t} \geq x_{2j} \qquad (4)$$

We denote x_{2t} by x_2^*. In a similar manner, for all input components i=1,…,m. we can identify a DMU in E whose ith input consumption is higher than that of all other DMUs in the set. We denote such an input by $x_i^*, i = 1, \dots, m$. Note that $x_1^*, x_2^*, \dots, x_m^*$ need not necessarily be selected from a single DMU. We now define congestion as follows:

Definition 2. Congestion is present if and only if, in an optimal solution $(\emptyset^*, \lambda^*, s^{+^*}, s^{-^*})$ of (1) for DMU_o. at least one of the following two conditions is satisfied:
(i) $\emptyset^* > 1$, and there is at least one $x_{io} > x_i^*, i = 1, \dots, m$.
(ii) There exists at least one $s_r^{+^*} > 0$ $(r = 1, \dots, s)$, and at least one $x_{io} > x_i^*, i = 1, \dots, m$.

We denote the amount of congestion in the ith input of DMU_o by $s_i^{c'}$ where $x_{io} > x_i^*$ and define it as:

$$s_i^{c'} = x_{io} - x_i^* \qquad (5)$$

Congestion is considered not present when $x_{io} \leq x_i^*$ and $s_i^{c'} = 0$. The sum of all $s_i^{c'}$ is the amount of congestion in DMU_o.

3. The proposed method

In the conventional DEA model, every decision-making unit itself tries to gain the maximum performance. It should be noted that none of the units are fully operational decisions, can not be greater than one. Decision makers always directly measure the maximum performance level of the decision maker considers common to all units. It helps the benchmark level, Liu and Peng presented a method for generating common weights. Liu and Peng approach is to model the shape of the objective function to minimize the sum of all virtual distance units to the decision criteria with The limitation fraction the numerator of which is the weighted sum of the output gap as well as the virtual vertical the denominator is a weighted sum of inputs minus the horizontal distance between the virtual and the ratio is equal to a number one. This limitation is due to the upward movement of the left is close to the baseline. The resulting ratio is equal to one, which means that the baseline point is reached. In this method it is assumed that the decision maker is standard on all units.[10]. According to the above description, the proposed model and Peng Liu for generating common weights in data to scale assumptions about the nature of the output variables (VRS) are as follows:

$$\begin{aligned}
&\text{Min } \Delta = \sum_{j=1}^{n} \Delta_j^I + \Delta_j^O \\
&\text{s.t. } \frac{\sum_{j=1}^{n} v_i x_{ij} + v_0 - \Delta_j^I}{\sum_{j=1}^{n} u_r y_{rj} + \Delta_j^O} = 1, \quad j=1,\dots,n, \\
&\quad \Delta_j^I, \Delta_j^O \geq 0, \qquad j = 1,\dots,n \\
&\quad u_r \geq \varepsilon > 0, \qquad r = 1,\dots,s \\
&\quad v_i \geq \varepsilon > 0, \qquad i = 1,\dots,m
\end{aligned} \tag{6}$$

We can rewrite the fractional shape above to the liner shape:

$$\begin{aligned}
&\text{Min } \Delta = \sum_{j=1}^{n} \Delta_j^I + \Delta_j^O \\
&\text{s.t. } \sum_{j=1}^{n} v_i x_{ij} + v_0 - \sum_{j=1}^{n} u_r y_{rj} - \Delta_j^O - \Delta_j^I = 0, \quad j=1,\dots,n, \\
&\quad \Delta_j^I, \Delta_j^O \geq 0, \quad j = 1,\dots,n \\
&\quad u_r \geq \varepsilon > 0, \quad r = 1,\dots,s \\
&\quad v_i \geq \varepsilon > 0, \quad i = 1,\dots,m
\end{aligned} \tag{7}$$

If assume that $\Delta_j = \Delta_j^I + \Delta_j^O$ we have liner programing that is below:

Min $\Delta = \sum_{j=1}^{n} \Delta_j$

s.t. $\sum_{j=1}^{n} v_i x_{ij} + v_0 - \sum_{j=1}^{n} u_r y_{rj} - \Delta_j = 0$, j=1,…,n,

$\Delta_j \geq 0, \quad j = 1, \dots, n$ (8)

$u_r \geq \varepsilon > 0, \; r = 1, \dots, s$

$v_i \geq \varepsilon > 0, \; i = 1, \dots, m$

The set of E defined as fallow:

E={j|$\Delta_j^* = 0$} (9)

Comparison of methods for measuring the Congestion of the input, the most amount of each component is calculated to each entry.

Congestion is defined as follows.

Definition 3. Congestion occurs if and only if the optimal solution $(\Delta_o^*, u^*, v^*, v_0^*)$ The model (8) for DMU_o following condition is satisfied: There is at least one i such that $x_{io} > x_i^*$, $\Delta_o^* > 0$

We denote the amount of congestion in the ith input of DMU_o by $s_i^{c'}$ where $x_{io} > x_i^*$ and define it as:

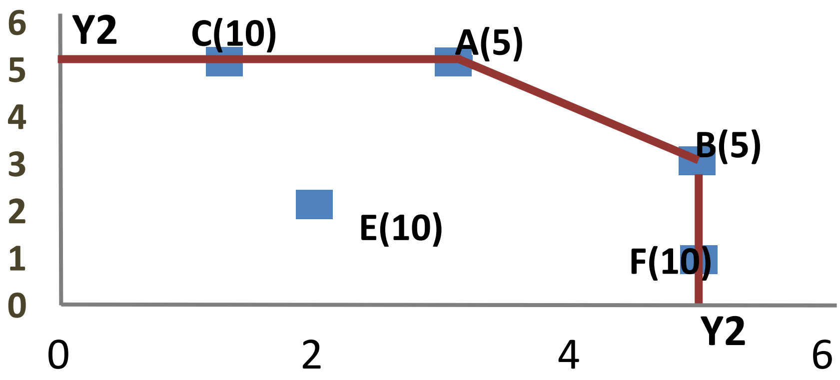
$s_i^{c'} = x_{io} - x_i^*$

Congestion is considered not present when $x_{io} \leq x_i^*$ and $s_i^{c'} = 0$. The sum of all $s_i^{c'}$ is the amount of congestion in DMU_o.

4. Numerical examples

1.4. Example 1

With one input and two output five decisions is shown in Figure 2.



Input each DMU's name is in parenthesis. With the performance of DMUs we have E={A,B,C,F} , so $x^* = 10 \geq x_j \; \forall j (j \in E)$.

Requirement for compression is inefficient, so according to the method congestion measured by comparing the input, density is calculated as follows:

$$\emptyset_E^* > 1, x_E = 10, \qquad s_1^{c'} = x_E - 10 = 0.$$

So this method does not detect any DMU fund is exposed to condensation,
While the form of, for example, can DMUa DMUc to move, Such that the first output is increased from 1 to 3 units ,while the input is reduced from 10 to 5 and the output does not change. However, these examples are solved with the proposed method.
Consider Figure 2. Solving the model (8) we have: E= {A,B} So

$x^* = 5 \geq x_j \; \forall j (j \in E)$.

According to the common weight method for measuring the congestion, congestion is calculated as follows:

$$\Delta_E^* > 0, x_E = 10, \qquad s_1^{c'} = x_E - 5 = 5.$$

$$\Delta_F^* > 0, x_F = 10, \qquad s_1^{c'} = x_F - 5 = 5.$$

$$\Delta_C^* > 0, x_C = 10, \qquad s_1^{c'} = x_C - 5 = 5.$$

As seen in way of measure of congestion with common weight determines the congestion of all the units, and in this way to determine the congestion for all DMU will only solve a model, so in way of measure of congestion with common weight, Calculations significantly decrease.

2.4. Example 2

Tableau of (1) show 5 unit of determiner with 2 inputs and 2 outputs, and result of (1) model.

With focus in performance of DMUs , in the last column, and tray to the way of measure of congestion based on comparison inputs, we have: E={A,B,C} so X*=(4,6)The DMU_a in first input have congestion and the amount of congestion is one and do not distinguish units C and E that have congestion.

However, these examples are solved with the proposed method.

Spot the data of table 1.

Solving the model (8) we have: E= {A,B} so X*=(4.0,4.0)

According to the common weight method for measuring the congestion, congestion is calculated as follows:

Table 1,

The inputs and outputs of units of determiner.

$\emptyset_o^*$	O_2	O_1	I_1	I_2	DMU
1	2	2	2	2	A
1	4	4	4	4	B
1	2	4	6	4	C
2	2	2	6	6	D
3.5	1	1	5	3.5	E

$$\Delta_E^* > 0, x_E = (3.5,5.0), \qquad s_1^{c'} = x_E - (4.0,4.0) = (-0.5,1.0).$$

$$\Delta_D^* > 0, x_D = (6.0,6.0), \qquad s_1^{c'} = x_F - (4.0,4.0) = (2.0,2.0).$$

$$\Delta_C^* > 0, x_C = (4.0,6.0), \qquad s_1^{c'} = x_C - (4.0,4.0) = (0.0,2.0).$$

As can be seen, units C, D, and E have congestion, so according to the way of measure of congestion with common weight, specify all units that have congestion.

5. Deduce

in this paper we distinguished units that have congestion with used of weight of Liu et al, and based on compred inputs with high congestion, increase, and this way is able to distinction of congestion in all units, just with solve one liner programing model. Therefore, this method greatly reduces the computational. The method for measuring the congestion of Noura and her colleagues studied And found that this approach to detect problems at an input and an output, not congestion But in the higher-congestion units are able to detect some, Using common weight model is able to identify the units that are in compression.

References:

[1] R.D. Banker, A. Charnes, W.W. Cooper, Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis, Management Science, 30 (1984) 1078–1092.

[2] P.L. Brockett, W.W. Cooper, H.C. Shin, Y. Wang, Inefficiency and congestion in Chinese production before and after the 1978 economic reforms, Socio-Economic Planning Sciences, 32 (1998) 1-20.

[3] W.D. Cook, M. Kress, A multiple criteria decision model with ordinal preference data, European Journal of Operational Research, 54 (1991) 191-198.

[4] W.D. Cook, M. Kress, Data envelopment model for aggregating preference ranking, Management Science, 36 (1990) 1302-1310.

[5] R. Färe, S. Grosskopf, Measuring congestion in production, Journal of Economics, 43 (1983) 257-271.

[6] R. Färe, S. Grosskopf, When can slacks be used to identify congestion. An answer to W. W. Cooper, L. Seiford, J. Zhu, Socio-Economic Planning Sciences, 35 (2001) 1-10.

[7] R. Färe, L. Svensson, Congestion of production factors, Econometrica, 48 (1980) 1745-1753.

[8] J.A. Ganley, J.S. Cubbin, Public sector efficiency measurement: applications of data envelopment analysis, Amsterdam, North-Holland, (1992).

[9] G.R. Jahanshahloo, A. Memariani, F. Hosseinzadeh Lotfi, H.Z. Rezai, A note on some of DEA models and finding efficiency and complete ranking using common set of weights, Applied Mathematics and Computation, 166 (2005) 265-281.

[10] F.H.F. Liu, H.H. Peng, Ranking of units on the DEA frontier with common weights, Computers Operations Research, 35 (2008) 1624–1637.

[11] A.A. Noura, F. Hosseinzadeh Lotfi, G.R. Jahanshahloo, S. Fanati Rashidi, R.P. Barnett, A new method for measuring congestion in data envelopment analysis,Socio- Economic Planning Sciences, 44 (2010) 240-246.

[12] Y. Roll, W. Cook, B. Golany, Controlling factor weights in data envelopment analysis, IIE Transactions, 24 (1991) 1-9.

[13] Z.S. Stern, L. Friedman, A Slacks-based Measure of Efficiency in Data Envelopment Analysis, European Journal of Operational Research., 111 (1998) 470-478.