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Efficiency and Return to Scale of Two-Stage network in Data Envelopment Analysis Using Additive Model

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Abstract

Data Envelopment Analysis is a nonparametric method based on the mathematical model. The concept of return to scale is one of the most important issues in economics and also in the data envelopment analysis, which includes a large part of the studies. Determining the type of return to scale (increasing, decreasing, and constant) will provide information to the manager through which he will be able to decide on how to achieve the optimal unit level under evaluation. Despite the fact that in the majority of the studies the concept of return to scale (RTS) has been investigated in radial models, this paper, in order to recognize the type of return to scale, has expressed and proved a method based on a non-radial additive model. We also developed this method for a two-stage network and in addition to inputs and outputs; we have introduced new entry intermediate measures in the intermediate products to the system. Then, we estimate and prove the type of return to scale for this network model. At the end, examples are given to examine the proposed method.

Keywords: Data Envelopment Analysis, Two-Stage Network, Return to Scale, Additive Model, Efficiency.

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1. Introduction

Evaluating the performance of organizations plays essential role in their immediate decisions. For this purpose, the efficiency and productivity of organizations should be calculated. One of the tasks of managers is to be aware of how their supervised units operate. The complexity of information, high volume of data, the impact of external factors and the effect of competing DMUs on performance, for example, inflation, unemployment, etc. are among the issues that the manager cannot make a reasonable decision to improve efficiency and productivity. Data Envelopment Analysis (DEA) is a non-parametric based on mathematical models which used to evaluate the performance of a set of decision-making units (DMUs) in a production technology with multiple inputs and outputs. In this technique, a parameter which is named the performance score has been used to evaluate the function of the decision-maker units. The efficiency of a DMU is a function of various factors such as the number of units, the amount of inputs and outputs of DMUs, the number of input and output DMUs, the type of production technology and the model which is used. One of the areas that many researchers in the science of DEA focus on is return to scale. Researchers of this science believe that return to scale is an important tool in many economic decisions, since the return to scale can provide useful information regarding the optimal DMU level. However, if we consider the unit under study as a multi-stage process with internal relations between processes, classical methods of computing returns to scale are not responsive. There is also a need for a new method for calculating the return to scale in network structures that can calculate the returns to scale of the whole process and examine the relationship between the return to the scale of the network and the return to scale of each step. Data Envelopment Analysis has been described as a method for evaluating the efficiency of homogeneous decision-maker units with multiple inputs and multiple outputs. Decision-making units can have different forms, such as hospitals, universities, banks, etc. In some cases, these DMUs have two stage network structure. The first stage uses inputs and produces outputs that these outputs form the second stage inputs. The outputs of the first stage are also named intermediate products. The second step, using the intermediate measures, produces the final output of the system. Therefore, in order to evaluate the efficiency of two-stage DMUs, it is necessary to use standard models of data envelopment analysis for the first stage, the second stage and the whole process. In this way, the whole two-step process is considered as a unit which its inputs are the first stage inputs and its outputs are the second stage outputs. This method was proposed by Sifford and Zhou [1]. Of course, in this way, a generic DMU may be efficient, while none of the first and second stages are efficient. After that, Chen and Gewick presented the data envelopment analysis model, in which the efficiency level of each stage is defined on the production possibility set of that stage. Today, managers in all organizations are demanding the optimal use of available facilities and capabilities. Therefore, the use of scientific methods to improve the performance of organizations seems necessary. Return to scale plays an important role in managerial decision making and so many researchers have studied in this field. Therefore, the proposed methods for calculating the return to scale in DEA consider these units as a black box and determine the type of return to scale of the DMU. Bencker, by using the BCC model, presented a method for determining the type of return to scale [2]. In another study, Bencker and Thrall introduced another method by using the CCR model to distinguish the returns to scale [3]. Also, Kersten and Wendin proposed an algorithm for estimating return to scale by using the FDH model, that the proposed algorithm was improved by Podinsky [4,5]. The additive model algorithm can be very useful in return to scale type because of detecting the robustness efficiency versus weak efficiency. In most of the studies that have been determined the types of return to scale, intermediate products and relationships between different parts of an investigated organization have been neglected. By these relationships, Chen defined return to scale in a network; but the definition given by him is not suitable for practical applications. Total efficiency and return to scale are calculated in a two stage network by means of an additive model [6].

2. Background.

In decision-making problems, the efficiency or good functioning of each DMU is the result of comparing its indicators with standards, that depending on whether the standards are outside or inside of the community, they are defined as absolute efficiency and relative efficiency, respectively. Also, efficacy, that is, good work, is the result of the comparison of extra-organizational indicators. Productivity is also a function of efficiency and efficacy.
One of the scientific methods in the calculation of efficiency is the use of the production function. The production function is a function that gives the highest output for each combination of inputs. In most cases, the production function is not available. An approximation of the production function is obtained by using parametric methods and nonparametric methods.

2.1 Return to Scale.

Return on scale (RTS) is one of the important issues in performance analysis. By determining the type of RTS of a decision-making unit, a manager will be able to decide whether to develop or reduce that DMU. For this purpose, at first, we need to know the relationship between the changes in their inputs and outputs. The ratio of these changes is called return to scale and is divided into three types: fixed, increasing, and decreasing. Return to scale in DEA was first evaluated by Bancer [7].
In both of these cases, by modifying the CCR model by adding the convexity constraint and introducing the BCC model, a technique for estimating RTS was proposed based on the assumption of a unique optimal solution. But unfortunately, in almost applications of DEA, for some DMUs, we can come up with an optimal multiple solution. Bunker and Trail provided BT method to measure RTS, generally (multiple solutions). In this modified method, by solving two specific linear programming models for each DMU, we find all the optimal solutions. It should be noted that the BT method only applies to technical efficient DMUs. Because for units that are technically inefficient, first of all, their performance reduction should be resolved, that is, surplus levels of sources which are used and inadequate levels of outputs which are possible by depicting inefficient DMUs on the efficiency boundary.
Consider DMU_o (under evaluated DMU) with input Xo and output Yo. The constant return to scale for DMU_o means that every multiple of inputs yields the same multiple of outputs. Besides, the increasing return to scale (decreasing) for DMU_o means that if we increase Xo in a ratio, Yo increments with a higher proportion (less) than the increase in Xo. So, it can be concluded that

determining the type of RTS for each DMU helps the manager to get the most returns by changing in the size of the inputs. Therefore, it should be noted that RTS is a local phenomenon for each DMU, that is, true in a neighborhood. It should be noted that the topic of RTS in a communication where the type of return to scale is unknown that, is, we did not accept any type of return to scale for the units is being studied. So, in this review, the production possibility set will be T_v. In the following, definitions of RTS are expressed. In the case of one input and one output, return is defined as $R=\frac{Y}{X}$. In economics, the ratio of output relative variation to input relative variation is known as elasticity. As follows:

$$e(X)=\frac{\frac{dy}{y}}{\frac{dx}{x}}$$

The elasticity definition of RTS is as follows:

(i) If $e(X)<1$, that's mean $\frac{dy}{y}<\frac{dx}{x}$ then the DMU is DRS.

(ii) If $e(X)=1$, that's mean $\frac{dy}{y}=\frac{dx}{x}$ then the DMU is CRS.

(iii) If $e(X)>1$, that's mean $\frac{dy}{y}>\frac{dx}{x}$ then the DMU is IRS.

2-2 Instructions for using $\sum_{j=1}^{n}\lambda_j^*$

Suppose that DMU_o, under evaluated DMU, is BCC- efficient. At first, we solve CCR model by ε:

$$\begin{aligned}
min \quad & \theta-\varepsilon\left(\sum_{i=1}^{m}s_i^- +\sum_{r=1}^{s}s_r^+\right) \qquad (1)\\
s.t. \quad & \sum_{j=1}^{n}x_{ij}\lambda_j+s_j^- =\theta x_{io}, i=1,...,m,\\
& \sum_{j=1}^{n}y_{rj}\lambda_j-s_r^+ =y_{ro}, r=1,...,s,\\
& \lambda_j\ge 0, \qquad j=1,...,n,\\
& s_i^-\ge 0, s_r^+\ge 0, i=1,...,m, r=1,...,s.
\end{aligned}$$

If model has unique solution, there may are three situations as follow:

1- DMU_o is belonged to CRS if $\sum_{j=1}^{n}\lambda_j^*=1$

2- DMU_o is belonged to DRS if $\sum_{j=1}^{n}\lambda_j^*>1$

3- DMU_o is belonged to IRS if $\sum_{j=1}^{n}\lambda_j^*<1$

If there are multiple optimal solution and there is λ^* subject to $\sum_{j=1}^{n}\lambda_j^*=1$, in this case, constant RTS will be found. Otherwise, we have to solve two following model:

$$\begin{aligned}
& \lambda^+ = Max \sum_{j=1}^{n}\lambda_j \qquad (2)\\
s.t. \quad & \sum_{j=1}^{n}x_{ij}\lambda_j+s_j^- =\theta x_{io}, i=1,...,m,\\
& \sum_{j=1}^{n}y_{rj}\lambda_j-s_r^+ =y_{ro}, r=1,...,s,\\
& \lambda_j\ge 0, \qquad j=1,...,n,\\
& s_i^-\ge 0, s_r^+\ge 0, i=1,...,m, r=1,...,s.
\end{aligned}$$

$$\lambda^- = Min \sum_{j=1}^{n} \lambda_j \qquad (3)$$
$$s.t. \quad \sum_{j=1}^{n} x_{ij}\lambda_j + s_j^- = \theta x_{io}, i = 1,...,m,$$
$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}, r = 1,...,s,$$
$$\lambda_j \geq 0, \qquad j = 1,...,n,$$
$$s_i^- \geq 0, s_r^+ \geq 0, i = 1,...,m, r = 1,...,s.$$

DMU (x_o, y_o) exhibits

1- IRs if $\lambda^+ < 1$
2- DRS if $\lambda^- > 1$
3- Otherwise CRS is occurred.

Banker [4], expressed that RTS is defined only for the frontier efficient points but there is no need to worry about the unit performance status under evaluation DMU, since efficiency can be achieved by depicting a DMU_o (inefficient unit) on the BCC frontier. Therefore, the type of DMU_o RTS can be determined using the above methods after the placement (X_0, Y_0) with the projection point.

3-2 Additive Model.

One of the first and most important models presented to evaluate a set of decision-maker DMUs in data envelopment analysis is an additive model, which itself is the basis of the definition of many new models in DEA. Despite the fact that in radial model, such as BCC or CCR which calculates radial efficient, detection of slack variables is done generally by the second step of measuring the efficiency, additive model directly addresses maximizes slack variables and thereby detects efficient or inefficient DMUs.

The additive model for the DMU_o evaluation is as follows:

$$Max \quad \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \qquad (4)$$
$$s.t. \quad \sum_{j=1}^{n} x_{ij}\lambda_j + s_j^- = x_{io}, \qquad i = 1,...,m,$$
$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}, \qquad r = 1,...,s,$$
$$\sum_{j=1}^{n} \lambda_j = 1,$$
$$\lambda_j \geq 0, \qquad j = 1,...,n,$$
$$s_i^- \geq 0, s_r^+ \geq 0, \qquad i = 1,...,m, r = 1,...,s.$$

DMU_o is strongly efficient if and only if the optimal value of the model (2) is zero. If the DMU_o is a technical inefficient, it can be projected to the point (X_0, Y_0) as follows:

$$(\hat{X}_O, \hat{Y}_O) = (X_O - S^{-^*}, Y_O + S^{+^*})$$

It is observed that in an additive model, the efficiency score is not calculated, but only the efficiency or inefficiency of the decision making units is determined, which is the weakness of the additive model.

3. Return to scale in two stage network structure

Assume that there are n two stage DMUs as Figure 1 , each DMU_j consumes input vector $X_j = (x_{1j}, ..., x_{mj})$ in the first stage to product $Z_j = (z_{1j}, ..., z_{dj})$ as intermediate product and input vector for the second stage of the DMU also $Y_j = (y_{1j}, ..., y_{sj})$ is the output vector.

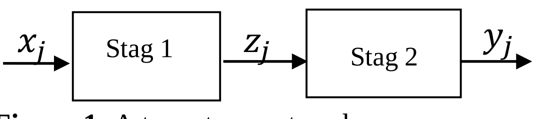

Figure 1: A two stage network

The Production Possibility Set (PPS) T_1 and T_2, corresponding to the first stage and the second one can be formulated as follow:

$$T_1 = \left\{(X,Z)|\ \sum_{j=1}^{n}\lambda_j X_j \le X, \sum_{j=1}^{n}\lambda_j Z_j \ge Z, \sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \ge 0,\ j=1,\dots,n\right\} \quad (5)$$

$$T_2 = \left\{(Z,Y)|\ \sum_{j=1}^{n}\mu_j Z_j \le Z, \sum_{j=1}^{n}\mu_j Y_j \ge y, \sum_{j=1}^{n}\mu_j = 1,\ \mu_j \ge 0,\ j=1,\dots,n\right\}.$$

Let us to show the jth DMU as (X_j, Z_j, Z_j, Y_j). The PPS, for this two-stage DMU, by adopting of the principles of observation, convexity, disposability, and variable retune to scale is defined as follows

$$T_v = \left\{(X,Y)|\ \sum_{j=1}^{n}\lambda_j X_j \le X, \sum_{j=1}^{n}\lambda_j Z_j \ge Z, \sum_{j=1}^{n}\mu_j Z_j \le Z, \sum_{j=1}^{n}\mu_j Y_j \ge Y, \sum_{j=1}^{n}\lambda_j = 1, \sum_{j=1}^{n}\mu_j = 1, \lambda_j \ge 0, \mu_j \ge 0\quad j=1,\dots,n\right\}. \quad (6)$$

Assume that each of the first and second stage as a $DMU_{1j}(X_j, Z_j)$ and $DMU_{2j}(Z_j, Y_j)$, respectively, so it is able to achieve the relative efficiency θ_1 and θ_2 associated to the first and second stage by solving following model.

$$\begin{aligned} &\theta_1^* = Min\ \ \theta_1 \quad (7)\\ &s.t.\ \ (\theta_1 X_o, Z_o) \in T_1\\ &\theta_2^* = Min\ \ \theta_2\\ &s.t.\ \ (\theta_2 Z_o, Y_o) \in T_2 \end{aligned}$$

In addition, the relative efficiency under evaluated DMU, DMU_o, is evaluated by:

$$\begin{aligned} &\theta^* = Min\ \ \theta \quad (8)\\ &s.t.\ \ (\theta X_o, Y_o) \in T_v \end{aligned}$$

We would like to measure relative efficiency of all DMUs with network two-stage structure as model presented by Chen et al. In their model, the overall efficiency score has been considered as convex composition of the first and second stage. Therefore, DMU_o is efficient if $\theta_1^* = \theta_2^* = 1$ It is explicit that if θ_1 and θ_2 are the optimal solutions of model (7) $(\lambda^*, \mu^*, \theta_1^*)$ will be a feasible solution of (8). Now, consider Q_1 and Q_2, corresponding the first stage and the second one, are the extreme directions that denoted by $(V_1^k, U_1^k, u_{01}^k)\ k=1,\dots,l_1$ and $(V_2^k, U_2^k, u_{02}^k)\ k=1,\dots,l_2$ as follow:

$$Q_1 = \left\{\begin{array}{l}(V_1,U_1,u_{01})|\ V_1 X_j - U_1 Z_j + u_{01} \ge 0,\\ j=1,\dots,n,\ V_1 \ge 0,\ U_1 \ge 0,\ u_{01} \in R^1\end{array}\right\}$$

$$Q_2 = \left\{\begin{array}{l}(V_2,U_2,u_{02})|\ V_2 Z_j - U_2 Y_j + u_{02} \ge 0,\\ j=1,\dots,n,\ V_2 \ge 0,\ U_2 \ge 0,\ u_{02} \in R^1\end{array}\right\}$$

Now, by the presented method in section 3 multiplier form of T_1 and T_2 are defined as:

$$T_1 = \left\{\begin{array}{l}(X,Z)|V_1^k X - U_1^k Z + u_{01}^k \ge 0,\\ k=1,\dots,l_1,\ \ X \ge 0,\ Z \ge 0\end{array}\right\}$$

$$T_2 = \left\{\begin{array}{l}(Z,Y)|\ V_2^k Z - U_2^k Y + u_{02}^k \ge 0,\\ k=1,\dots,l_2,\ \ Z \ge 0,\ Y \ge 0\end{array}\right\}.$$

Return to Scale in additive model:

Although, the concept of RTS is defined as the ratio of proportional variations in the output to the input in the radial model, there are two facts in practical applications:

1- The proportional changes in the input do not necessarily lead to proportional changes in the output.
2- The radial models will have failed if a manager would like to get information about the increasing the output after exerting the input changes.

Therefore, in this section, we introduce additive model to investigate the concept of return to scale.

The movement of slack variables is the objective of introduced model (9) in which assign the weight α to the inputs and output variable and then according to this, the kind of return to scale of the evaluated DMU is measured.

Max $1s_r^+ + 1s_i^-$ (9)
s.t $\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \alpha x_{\text{ip}}$ i=1,……,m
$\sum_{j=1}^{n} \lambda_j\, y_{rj} - {s^+}_r = \alpha y_{rp}$ r=1,……,s
$\sum_{j=1}^{n} \lambda_j = 1,\ \alpha \geq 0$
$\lambda_j \geq 0 \quad j = 1, \dots\dots, n$

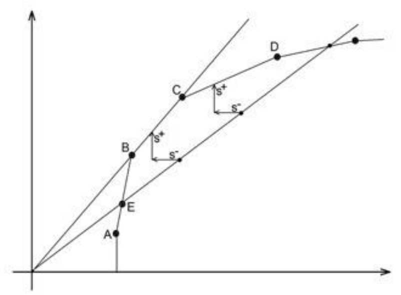

Figure 2. Determining the type of return to scale by moving slack in the additive model

Theorem 1: Consider DMU$_o$ by (X_O وY_O) is an efficient unit. There are the following conditions in order to estimate return to scale (RTS) of DMU$_o$ based on additive model (1-3):

1- The optimal objective function in additive model is greater than zero and $1 < \alpha^*$ if and only if there is increasing return to scale.
2- The optimal objective function in additive model is greater than zero and $1 > \alpha^*$ if and only if there is decreasing return to scale.
3- The optimal objective function in additive model is equal to zero and $1 = \alpha^*$ if and only if there is constant return to scale.

Prof. Suppose that the optimal objective function in additive model is non-zero and $1 < \alpha$ so (αX_O و αY_O) is inside the PPS hence it is an inefficient unit according to additive model. Therefore, by regarding to Theorem 1-3, it is increasing return to scale. Otherwise, there is at least one $1 < \alpha$ if DMU$_o$ belong to increasing RTS.
If we assume that $1 > \alpha$ or $1 = \alpha$, due to the Theorem 4-1 return to scale will be decreasing or constant, respectively. This is contrary to the premise so we will have $1 < \alpha$. Similarly, part (2) will be proved.
To prove part 3, we have: Suppose that the optimal value of the objective function for DMU$_o$ in the additive model is zero, in this case, according to the collective model, this unit is an efficient DMU. On the other hand, for each $0 < \alpha$ the amount of (αX_O وαY_O) is efficient and thus constant return to scale (CRT) has occurred. To prove the opposite of the theorem, we must prove that the optimal objective function of this DMU, in the additive model, is equal to zero. (It means that the sum of slack variables is zero).
DMU$_o$ will belong to decreasing RTS if the optimal solution is non-zero and $0 < \alpha < 1$.
Also, if $1 < \alpha$ increasing RTS is occurred and, as the result, the value of slack variables can not be non-zero, which means that the optimal value of the objective function in the additive model is zero.

4. Numerical example

In this section, we are going to get an example to better understand and analysis the above issues.

Example 1: Consider six two-stage DMUs with one input and one output as Table 1.

Table 1: the results of example (1)

DMU	1	2	3	4	5	6
X	5	4	4	5	7	10
Y	3	3	4	5	6	7
Alpha	1.67	1.3	1	0.8	0.83	0.71
Efficiency	3.32	1.67	0	0.001	0.83	2.14
Result	IRS	IRS	CRS	DRS	DRS	DRS

As the table indicates, the objective function (the sum of slacks) of DMU_3 is equal to zero and $\alpha = 1$ so this DMU is on MPSS and is the constant return to scale. As the first column in the above Table, DMU1 belongs to increasing RTS because of the
being positive its objective function and $\alpha > 1$.

Example 2: Let us to consider seven DMUs with the following inputs and outputs:

Table 2. Under evaluated unit in two stage network

	X_i	Y_r	Z'_k	Z_d
DMU_1	20	84	151	100
DMU_2	19	68	131	150
DMU_3	25	52	160	140
DMU_4	27	58	168	180
DMU_5	22	50	158	94
DMU_6	55	42	255	230
DMU_7	33	58	235	95

As we can see, the objective function for DMU_2 is equal to zero and $\alpha = 1$ so this under evaluated DMU exhibits constant RTS also DMU5 depicts increase RTS due to the positive objective function and $1 < \alpha$.

Table 3: the results of model (1)

	α	W	RTS
DMU1	1.00	53.75	IRS
DMU2	1.00	0.00	CRS
DMU3	1.38	173.74	IRS
DMU4	1.00	123.00	IRS
DMU5	1.68	213.23	IRS
DMU6	1.00	267.00	IRS
DMU7	1.44	356.47	IRS

5. Conclusion

Estimating returns to scale in data envelopment analysis is a very important issue in managerial and economic issues that we have considered in several sections. Summarize the achievements of this article as follows:

Since in the real world it may be that some manufacturing units have a network structure and within them, they include subunits, we examined how to estimate the return to scale of DMUs with a special two stage network structure by introducing a method based on the additive model solving.

In this paper, we have been discussing how to achieve the amount and type of return to scale by using the concept of elasticity which was investigated in a two stage network structure and also, an additive efficiency analysis has been used based on weighted average.

Considering the fact that most of the studies that have been done in this issue have examined the return to scale in radial models, in this paper, we have expressed and proved a method for determining the type of return to scale in a non-radial additive model. Besides, we have developed the proposed method for a two stage network, and in addition to the inputs and outputs and intermediate measures, we have added a new input in the mid-range products part. Also we have estimated and proved the type of return to scale for this network model. Finally, we have examined the proposed method in an example. Also, in empirical applications, we can establish a relationship between overall efficiency, selective weights, and output stage efficiency, which examines the total efficiency with individual stage efficiency, because overall efficiency may hide the divisions' efficiencies. After individual efficiencies were achieved when weights were changed we define the total efficiency as a function of stage efficiencies

except the weighted average of stage efficiencies. And under each set of weights, we have examined the existence of unique efficiency decomposition.

Disadvantages of the proposed method:
The proportional input and output variations are similar and both increase or decrease to the same extent. In other words, the proportional variations in inputs and outputs are not different.

Advantages of the proposed method:
Sometimes the radial models have fewer applications in real life. For this reason, in this paper, a non-radial model has been proposed and the type of return to scale of it has been investigated.

An additive method for a special two stage network with input has been proposed and has been developed in the mid-range products and the type of return to scale for this network model was also studied. In the special situation when the manager wants to calculate the same proportional variations in both inputs and outputs, he can use the proposed model. Non-radial models, the solution will obtain faster. Finally, the concepts of return to scale of undesirable data in a two stage network and return to scale of interval and inaccurate data were investigated.

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