Available online at http://ijdea.srbiau.ac.ir

Int. J. Data Envelopment Analysis (ISSN 2345-458X)

Vol. 11, No. 1, Year 2023 Article ID IJDEA-00422, Pages 39-48
Research Article

International Journal of Data Envelopment Analysis

Science and Research Branch (IAU)

Bargaining game model for measuring the performance of two-stage network structures

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Received 13 October 2022, Accepted 24 January 2023

Abstract

Data envelopment analysis (DEA) mainly utilizes envelopment technology to replace production functions in microeconomics. Based on this replacement, DEA is a widely used mathematical programming approach for evaluating the relative efficiency of decision-making units (DMUs) in organizations. Evaluating the efficiency of DMUs with two-stage network structures is important in management and control. The resulting two stage DEA model not only provides an overall efficiency score for the entire process, but also yields an efficiency score for each of the individual stages. In this Paper we develop Nash bargaining game model to measure the performance of DMUs that have a two-stage structure. Under Nash bargaining theory, the two stages are viewed as players. It is shown that when only one intermediate measure exists between the two stages, our newly developed Nash bargaining game approach yields the same results as applying the standard DEA approach to each stage separately. With a new breakdown point, the new model is obtained which by providing example, the results of these models are investigated. Among these results can be pointed to the changing efficiency by changing the breakdown point.

Keywords: Data envelopment analysis (DEA), Nash bargaining game model, two-stage process, intermediate measure.

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1. Introduction

Data envelopment analysis (DEA), introduced by Charnes et al, is an effective tool for measuring the relative efficiency of peer decision making units (DMUs) that have multiple inputs and multiple outputs [1]. Researchers developed two-stage network structures that the output of stage 1 is the input of stage 2. The outputs from stage 1 are referred to as intermediate measures. For example, Seiford and Zhu use a two-stage process to measure the profitability and marketability of US commercial banks. Hwang expressed two stage processes and be implemented in the banking industry [2]. Chilingerian and Sherman describe a two-stage process in measuring physician care. Kao and Hwang offered a new method of measuring the overall efficiency of such a process [3]. Chen et al. Chen et al use a weighted Additive model to summation the two stages and decompose the efficiency of the overall process [4]. For more information, readers can study [5-10]. Moreover, Liang et al. develop a number of DEA models that use the concept of game theory [11]. Also, Juan et al. develop a leader–follower model borrowed from the notion of Stackelberg games, and a centralized or cooperative game model where the combined stage is of interest [12].

In next section some preliminary results are provided. In section 3 we describe proposed model and its properties. Numerical examples are presented in section 4. Section 5 gives the conclusion of this paper.

2. Nash bargaining game model

Consider a two-stage process shown in Fig. 1. We suppose there are n DMUs and each DMUj (j = 1, 2, . . ., n) has m inputs to the first stage that denoted by $X_j = (x_{1j},\ldots,x_{mj})$ and D outputs from this stage denoted by $Z_j = (z_{1j},\ldots,z_{Dj})$. These D outputs then become the inputs to the second stage, which are referred to as intermediate measures. The s outputs from the second stage are denoted by $Y_j = (y_{1j}, y_{2j},\ldots, y_{sj})$. The constant returns to scale (CRS) model (Charnes et al), the (CRS) efficiency scores for each DMUj (j = 1, 2, . . . , n) in the first and second stages can be defined by e_j^1 and e_j^2, respectively, to get the total performance of two stage process, with using the CRS efficiency, we can define $e_j = e_j^1.e_j^2$, since [1]:

$$e_j = \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} . \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} = e_j^1.e_j^2 \quad (1)$$

The above overall efficiency definition ensures that $e_j \le 1$ from $e_j^2 \le 1, e_j^1 \le 1$, and the overall process is efficient if and only if $e_j^1 = e_j^2 = 1$ [1].

The efficiency-evaluation problem can be approached from two game theory perspectives. One is to view the two-stage process as a non-cooperative game model, in which one stage is assumed to be a leader and solved for its CRS efficiency first, and the other stage a follower, whose efficiency is computed without changing the leader’s efficiency score. The other approach is to regard the process as a centralized model, where the overall efficiency given in (2) is maximized, and a decomposition of the overall efficiency is obtained by finding a set of multipliers producing the largest first (or second) stage efficiency score while maintaining the overall efficiency score. Note that in fact, the two stages can be regarded as two players in Nash bargaining game theory. Therefore, we can approach the efficiency evaluation issue of two-stage processes by using Nash bargaining game theory directly.

Consider the set of two individuals participating in the bargaining by N={1,2}, and a payoff vector is an element of the payoff space R^2, which is the two-dimensional Euclidean space. A feasible set S is a subset of the payoff space, and a breakdown or status quo point $\vec{b}$ is an element of the payoff space. A bargaining problem can then be specified as the triple (N,S,$\vec{b}$) consisting of participating individuals, feasible set, and breakdown point. The solution is a function F that is associated with each bargaining problem (N, S,$\vec{b}$), expressed as F(N, S,$\vec{b}$). In this paper, Juan et al. demonstrated the Nash bargaining game and proves that there is one unique solution for it and the solution is Nash solution, which satisfies the above-mentioned four properties, and can be obtained by solving the following maximization problem [6]:

$$\underset{\vec{u}\in s,\vec{u}\geq\vec{b}}{Max}\prod_{i=1}^{2}(u_i-b_i) \qquad (2)$$

Where $\vec{u}$ is the payment vector for the individuals, and b_i ,u_i are the i-th components of the vector $\vec{b}$, $\vec{u}$, respectively. Note that the breakdown points or status quo represents possible payoff pairs obtained if one decides not to bargain with the other player. Consider:

$$z_d^{\max}=\max_j\left\{z_{dj}\right\},\ z_d^{\min}=\min_j\left\{z_{dj}\right\},$$

$$y_r^{\min}=\min_j\left\{y_{rj}\right\},\ y_r^{\max}=\max_j\left\{y_{rj}\right\}.$$

then

$(X_i^{\max},\ Z_d^{\min})(i=1,\ldots,m,d=1,\ldots D)$

shows the least ideal DMU in the first phase that produced the greatest amount of input and the least amount of intermediate measure.

Similarly,

$(z_d^{\max},\ y_r^{\min})(d=1,\ldots,D\ ,\ r=1\ ,\ \ldots,S)$

shows the least ideas DMU produced in the second stage, the maximum amount of intermediate measure and the lowest output. The CRS efficiency for the above two least ideal DMUs is the worst among the existing DMUs. We denote the (CRS) efficiency scores of the two least ideal DMUs in the first and second stage as θ^1_{min} and θ^2_{min}, respectively, and use θ^1_{min} and θ^2_{min} as our breakdown point.

DEA model with input-oriented, and using the formula of Nash bargaining game provided in model (2) can be expressed as a model (3):

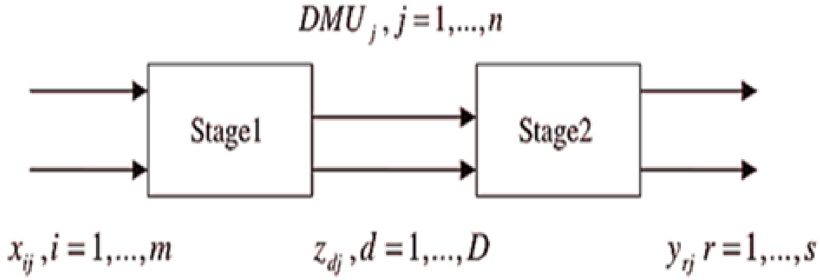

Fig. 1: Two-stage process

$$Max\,(\frac{\sum_{d=1}^{D} w_d^1 z_{do}}{\sum_{i=1}^{m} v_i x_{io}}-\theta_{\min}^1).(\frac{\sum_{r=1}^{S} u_r y_{ro}}{\sum_{d=1}^{D} w_d^2 z_{do}}-\theta_{\min}^2)$$

$$\begin{aligned} s.t.\quad & \frac{\sum_{d=1}^{D} w_d^1 z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \geq \theta_{\min}^1 \qquad (3)\\ & \frac{\sum_{r=1}^{S} u_r y_{ro}}{\sum_{d=1}^{D} w_d^2 z_{do}} \geq \theta_{\min}^2 \\ & \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1,\ j=1,...,n \\ & \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_d} \leq 1,\ j=1,...,n \end{aligned}$$

$$v_i, u_r, w_d > \varepsilon, i=1,...,m, r=1,...,s, d=1,...,D$$

Juan et al. trying to come up on with linear model by changing of variable, they reach to a parametric linear model which was equivalent with a non-linear model [6].

3. New Model with Different Breakdown Points

Zhu et al. were raised the theory of Nash bargaining game, for DMU which has two stage process [6]. They used of relative efficiency of DMU and built a virtual DMU, which in every stage has the maximum observed input and the lowest observed output. Then its CRS efficiency of virtual DMU was calculated at each step. The efficiencies obtained in both stages, constitute the breakdown point $\vec{b}$. In this paper, we will be adding a parameter Δ to the breakdown point $\vec{b}$ and review the results of the Nash bargaining game model with this new breakdown point.

Consider the (CRS) efficiency scores for each DMU_j, $(j=1,...,n)$ in the first and second stages:

$$e_j^1 = \frac{\sum_{d=1}^{D} w_d^1 z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad e_j^2 = \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d=1}^{D} w_d^2 z_{dj}} \leq 1 \quad (4)$$

It is reasonable to set w_d^1 equal to w_d^2, since the value assigned to the intermediate measures should be the same regardless of whether they are viewed as outputs from the first stage or inputs to the second stage. Then the total efficiency can be written as a product of $e_j = e_j^1.e_j^2$ where e_j^1 and e_j^2 are the individual efficiency scores of the two-stage process, since:

$$e_j = \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}}.\frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} = e_j^1.e_j^2 \quad (5)$$

The above overall efficiency definition ensures that $e_j \leq 1$ from $e_j^2 \leq 1, e_j^1 \leq 1$, and the overall process is efficient if and only if $e_j^1 = e_j^2 = 1$.

Now we will be adding the parameter Δ to the breakdown point $b = \left(\theta_{min}^1, \theta_{min}^2\right)$ obtained by Zhu et al. [6]. If we look at the terms of the transaction, a seller wants to sell his products, he gave a discount, the breakdown point is the same as discounts, now we want to know whether Δ can be added to this as discount and see if it is possible or not. So, we have:

$$e_1 \geq \theta_{\min}^1 + \Delta_1, e_2 \geq \theta_{\min}^2 + \Delta_2 \qquad (6)$$

Then the DEA model with the input-oriented for a specific DMU_o, using equations (2) and (6) are expressed as follows:

$$Max\left(\frac{\sum_{d=1}^{D} w_d^1 z_{do}}{\sum_{i=1}^{m} v_i x_{io}} - \theta_{\min}^1 - \Delta_1\right).\left(\frac{\sum_{r=1}^{S} u_r y_{ro}}{\sum_{d=1}^{D} w_d^2 z_{do}} - \theta_{\min}^2 - \Delta_2\right)$$

$$s.t.\quad \frac{\sum_{d=1}^{D} w_d^1 z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \geq \theta_{\min}^1 + \Delta_1 \qquad (7)$$

$$\frac{\sum_{r=1}^{S} u_r y_{ro}}{\sum_{d=1}^{D} w_d^2 z_{do}} \geq \theta_{\min}^2 + \Delta_2$$

$$\frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1,\ j = 1,...,n$$

$$\frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} \leq 1,\ j = 1,...,n$$

$$v_i, u_r, w_d > \varepsilon, i = 1,...,m, r = 1,...,s, d = 1,...,D$$

All constraints which are defined in model (7) are shown with the set S, S is the set of feasible solution for problem. So, the problem is defined as a triple $(\{1,2\}, S, \{\theta_{\min}^1, \theta_{\min}^2\})$.

Lemma: The feasible set S is compact and convex.

Proof: Since the feasible set S is bounded and closed in Euclidean space, then S is compact. Next, we will prove that S is also convex. Suppose $(v_1', ..., v_m', u_1', ...u_s', w_1', ..., w_D') \in S$ and $(v_1'', ..., v_m'', u_1'', ...u_s'', w_1'', ..., w_D'') \in S$. For any $\lambda \in [0,1]$ we have $\lambda v_i' + (1-\lambda)v_i'' > 0, i = 1,...,m$, $\lambda u_r' + (1-\lambda)u_i'' > 0, r = 1,...,s$ and $\lambda w_d' + (1-\lambda)w_d'' > 0, d = 1,...,D$. Since $\sum_{i=1}^{m} v_i x_{ij} > 0$ and $\sum_{d=1}^{D} w_d z_{dj} > 0$ for all j=1,…,n, the constraints in S, $\frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1$ and $\frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj}} \leq 1$ are equivalent to $\sum_{d=1}^{D} w_d z_{dj} \leq \sum_{i=1}^{m} v_i x_{ij}\ j = 1,...,n$ and $\sum_{r=1}^{S} u_r y_{rj} \leq \sum_{d=1}^{D} w_d z_{dj}\ j = 1,...,n$, respectively, for all j=1,…,n, and the constraints $\frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \geq \theta_{\min}^1 + \Delta_1$ and $\frac{\sum_{r=1}^{S} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do}} \geq \theta_{\min}^2 + \Delta_2$ are equivalent to $\sum_{d=1}^{D} w_d z_{do} \geq (\theta_{\min}^1 + \Delta_1)\sum_{i=1}^{m} v_i x_{io}$ and $\sum_{r=1}^{S} u_r y_{ro} \geq (\theta_{\min}^2 + \Delta_2)\sum_{d=1}^{D} w_d z_{do}$ respectively. Then we have

$$\sum_{d=1}^{D}[\lambda w_d' + (1-\lambda)w_d'']z_{dj} = \lambda\sum_{d=1}^{D} w_d' z_{dj} + (1-\lambda)\sum_{d=1}^{D} w_d'' z_{dj}$$

$$\leq \lambda\sum_{i=1}^{m} v_i' x_{ij} + (1-\lambda)\sum_{i=1}^{m} v_i'' x_{ij} = \sum_{i=1}^{m}[\lambda v_i' + (1-\lambda)v_i'']x_{ij}$$

and

$$\sum_{r=1}^{S}[\lambda u_r' + (1-\lambda)u_r'']y_{rj} = \lambda\sum_{d=1}^{D} u_r' y_{rj} + (1-\lambda)\sum_{d=1}^{D} u'' y_{rj}$$

$$\leq \lambda\sum_{d=1}^{D} w_d' z_{dj} + (1-\lambda)\sum_{d=1}^{D} w_d'' z_{dj} = \sum_{d=1}^{D}[\lambda w_d' z_{dj} + (1-\lambda)w'']z_{dj}.$$

Similarly, we have

$$\sum_{d=1}^{D}[\lambda w_d' + (1-\lambda)w_d'']z_{do} \geq$$

$$(\theta_{\min}^1 + \Delta_1)\sum_{i=1}^{m}[\lambda v_i' + (1-\lambda)v_i'']x_{ij}$$

and

$$\sum_{r=1}^{S}[\lambda u'_r+(1-\lambda)u''_r]y_{rj}\geq$$

$$(\theta^2_{\min}+\Delta_2)\sum_{d=1}^{D}[\lambda w'_d z_{dj}+(1-\lambda)w'']z_{dj}.$$

Therefore we have $(\lambda v'_i+(1-\lambda)v''_i,\lambda u'_r$ $+(1-\lambda)u''_i,\lambda w'_d+(1-\lambda)w''_d)\in S$, where i=1,…,m, r=1,…,s, d=1,…,D, or equivalently,

$$\lambda(v_1',...,v'_m,u'_1,...u'_s,w'_1,...,w'_D)$$

$$+(1-\lambda)(v_1'',...,v''_m,u''_1,...u''_s,w''_1,...,w''_D)\in S.$$

By changing variables, model (7) can be converted to into the following model (8).

$$\begin{aligned}
&Max\,\alpha.\sum_{r=1}^{s}\mu_{r2}y_{ro}-\theta^2_{\min}\sum_{d=1}^{D}\omega_d z_{do}-\Delta_2\sum_{d=1}^{D}\omega_d z_{do}+\Delta_2\theta^1_{\min}-\\
&\quad\theta^1_{\min}\sum_{r=1}^{s}\mu_{r2}y_{ro}+\Delta_1\theta^2_{\min}+\theta^1_{\min}\theta^2_{\min}-\Delta_1\sum_{r=1}^{s}\mu_{r2}y_{ro}+\Delta_1\Delta_2\\
&s.t.\quad \sum_{d=1}^{D}\omega_d z_{do}\geq\theta^1_{\min}+\Delta_1 \qquad (8)\\
&\quad\sum_{r=1}^{s}\mu_{r2}y_{ro}\geq\theta^2_{\min}+\Delta_2\\
&\quad\sum_{i=1}^{m}\gamma_i x_{io}=1\\
&\quad\sum_{d=1}^{D}\omega_d z_{do}=\alpha\\
&\quad\sum_{d=1}^{D}\omega_d z_{dj}-\sum_{i=1}^{m}\gamma_i x_{ij}\leq 0,\ j=1,...,n\\
&\quad\sum_{r=1}^{s}\mu_{r1}y_{rj}-\sum_{d=1}^{D}\omega_d z_{dj}\leq 0,\ j=1,...,n\\
&\quad\mu_{r1}=\alpha\mu_{r2},\ r=1,...,s\\
&\alpha,v_i,u_r,\omega_d,\mu_{r1},\mu_{r2}>\varepsilon,\\
&i=1,...,m,r=1,...,s,d=1,...,D
\end{aligned}$$

Now with regard to the above problem we have:

$$\theta^1_{min}+\Delta_1\leq\alpha=\sum_{d=1}^{D}\omega_d z_{do}\leq\sum_{i=1}^{m}\gamma_i x_{ij}=1\ (9)$$

Then $\theta^1_{min}<\theta^1_{min}+\Delta_1\leq\alpha\leq 1$ which provides both upper and lower bounds on α, and indicates that the optimal value of a represents the first-stage efficiency score for each DMU.

Thus α can be treated as a parameter within $\left[\theta^1_{\min},1\right]$. As a result, model (8) can be solved as a parametric linear program via searching over the possible α values within $\left[\theta^1_{\min},1\right]$. In computation, we set the initial value for α as the upper bound one, and solve the corresponding linear program. Then we begin to decrease a by a very small positive number ε (=0.0001 for example) for each step t, namely, $\alpha_t=1-\varepsilon\times t, t=1,2,...$ until the lower bound θ^1_{min} is reached, and solve each linear program of model (8) corresponding to α_t and denote the corresponding optimal objective value by Ω_t.

Note that not all values taken by a within $\left[\theta^1_{\min},1\right]$ lead to feasible solutions for program (8). Let $\Omega^*=\max_t \Omega_t$ and denote the specific α_t associated with Ω^* as α^*. Note that it is likely that Ω^* is associated with several α^* values. Then Ω^* associated with α^* is our solution to model (8). So $e^{1*}_o=\alpha^*\left(=\sum_{d=1}^{D}\omega^*_d z_{do}\right), e^{2*}_o=\sum_{r=1}^{s}\mu^*_{r2}y_{ro}$ and $e^*_o=e^{1*}_o.e^{2*}_o$ DMU's bargaining efficiency scores for the first and second stages and the overall process, respectively.

Table 1. Data set from bank branch.

DMU	Facilities	Long-term deposits	Current deposits	Loan deposits	Short-term deposits	Benefit payments	Profits received	Commissions	Demands	Employees
1	22257639764	6810266667	6533382471	6660488683	5279391117	352016949	202066116	56963101	3539293212	18
2	67580829850	13384460000	20653894008	6514292603	20348235001	861451731	983716703	121982833	92243450	39
3	37600865522	9847441495	11834989714	3292403437	14371902271	644834993	1090000043	35054481	487376304	26
4	48923131150	17312280000	16905824864	4527777581	13463310409	943246534	1435888500	43034496	1089553667	21
5	37528859652	8412535102	14080442643	9870287181	15598798044	597007548	482997552	81482880	6975821726	17
6	99090764221	50032670296	31210107739	6255672489	26180304041	3275820903	675980085	49370299	21986756050	37
7	28911452454	8587422983	38565140446	5164233095	31257871220	861150899	57205768	132396990	3829313873	18
8	23627756715	4457750000	5521951948	3602318900	7917362997	345435526	72180357	69388154	1981634970	10
9	18855060926	10554940000	4825960709	3735178296	18838559547	765799220	43029309	133211171	1660595686	12
10	16827555425	5740840000	7134760166	5274078690	10270725784	435968244	380507807	30655529	1008708411	11
11	27941932247	10358700000	10724354605	3323411940	11015872112	588822381	34927575	55282069	4518274284	16
12	22988545492	9485487350	58683502301	5762411733	5681617972	507788193	10308836	84673218	19614274005	19
13	14331967390	3570980000	13116402866	3787329255	8027182536	297474028	98949971	51510028	2918857883	7
14	19398186242	13141280000	11803426159	3697494936	15114276800	774715565	214872001	38653555	161449072	14
15	20968181953	7593290000	11646314795	3001501677	13075339664	528523692	10650082	78237880	1187126761	15
16	69211957637	25691547223	26346803752	23611627393	26583727896	1460532678	491004910	142728284	13835191983	38
17	54023074654	9490067836	17822839866	20839933468	14030024321	583949343	70827725	127721479	12570271723	34
18	12813794210	1078180770	7426132124	10753640577	1764529115	72560423	66685851	63833764	3789177677	11
19	80839554123	7848300000	3527946156	7329430563	74974752093	1009425815	48487435	93686315	1154714879	11
20	26972286439	2025227946	29782720443	4903913942	5108118332	179641033	28481292	60136764	2968580446	7
21	29943422636	3343930000	6366227191	3076558060	11712050371	352891860	113722736	78511660	2184456682	16
22	31538548952	3949011111	12372343907	10401839099	5800707909	258608891	66271759	107478681	7353216279	18
23	60932274982	9930400000	17092042733	4008600547	12482069129	616226525	108992266	122048764	1958931593	14
24	44934662943	2874400000	5912953675	6108992435	5639550713	219223799	101501489	57092616	933939218	9
25	47350692400	11593736660	41897725951	10345665323	26894739417	957082221	146663703	143408992	664255290	18
26	33772288181	10009895000	7911412840	6039674953	10263763947	610502081	178492325	130745475	1548674887	14
27	21727489429	14068953760	6404061069	6382317634	13589853540	810782459	10100960	100108908	604461217	14
28	26885087916	12142571773	15515134574	8578412148	16638602522	729333404	14367761	72315772	593842868	9
29	21404975399	7174550000	14056525095	5584325233	4803962975	378788560	45922330	40846651	2370014725	8
30	48468192799	11100325000	7261943470	12633088875	9400758384	571998072	20896618	174341708	23285655877	12

Table 2. Results of bank Mellat with breakdown point $\left\{\theta^1_{\min}+\Delta_1,\theta^2_{\min}+\Delta_2\right\}$.

DMU	Efficiency of stage 1	Efficiency of stage 2	Overall efficiency	
	e_0^{1*}	e_0^{2*}	$e_0^{1*}.e_0^{2*}$	α
1	0.8500	0.6772	0.5757	0.8500
2	0.4500	0.1284	0.0578	0.4500
3	1.0000	0.4296	0.4296	1.0000
4	0.8500	0.2574	0.2188	0.8500
5	0.1500	0.3505	0.0526	0.1500
6	- - - -	- - - -	- - - -	- - - -
7	- - - -	- - - -	- - - -	- - - -
8	0.4000	0.3251	0.1300	0.4000
9	- - - -	- - - -	- - - -	- - - -
10	0.5500	0.5410	0.2975	0.5500
11	0.2500	0.8177	0.2044	0.2500
12	- - - -	- - - -	- - - -	- - - -
13	0.9500	0.0374	0.0355	0.9500
14	1.0000	1.0000	1.0000	1.0000
15	0.7500	0.8560	0.6420	0.7500
16	- - - -	- - - -	- - - -	- - - -
17	- - - -	- - - -	- - - -	- - - -
18	0.9000	0.8661	0.7794	0.9000
19	0.0500	0.7362	0.0368	0.0500
20	0.8500	0.7990	0.6791	0.8500
21	1.0000	0.0261	0.0261	1.000
22	0.6500	0.0022	0.0014	0.6500
23	- - - -	- - - -	- - - -	- - - -
24	0.5000	0.6281	0.3140	0.5000
25	0.9500	0.2346	.02229	0.9500
26	0.7500	0.0164	0.0123	0.7500
27	0.9500	0.2121	0.2015	0.9500
28	0.7000	0.0316	0.0221	0.7000
29	0.7500	0.0235	0.0176	0.7500
30	0.5000	0.2279	0.1139	0.5000

4. A real example from bank branch

In this section, we apply the new Nash bargaining game on set of real data from bank branch. The data set is including 30 branches of bank branch with the four intermediate measures. The inputs to the first stage are number of employees and benefit payments. The intermediate measures connecting the two stages are types of deposits (short-term deposits, loan deposits, current deposits and long-term deposits). The outputs from the second stage are Facilities, profits received, commissions and demands. The CRS efficiency scores for the least ideal DMUs in the first and second stages are calculated as $\theta^1_{\min}=0.0452$ and $\theta^2_{\min}=0.0478$ respectively. We next begin with the initial value $\alpha=1$ in model (8), then decrease α by a small positive number

$\varepsilon = 0.0001$ for each step t, namely, $\alpha_t = 1 - 0.0001 \times t,\ t = 1, 2, ...$ until the lower bound $\theta^1_{\min} = 0.0452$ is reached. In this example, we have $(\Delta_1, \Delta_2) = (0.001, 0.001)$. Bank data set is as follows:

Note 1: The optimal value of parameter α represents the first-stage bargaining efficiency score for the corresponding DMU.

Note 2: In this table we show impossible examples with the symbol "---".

So, adding Δ to the previous breakdown point is possible. The efficiency of units with the new breakdown point will be less than or equal the efficiency of units with the Previous breakdown point. Note that the breakdown point cannot be chose arbitrarily, for example, if the CRS efficiency of each stage be used as a breakdown point, likely model (8) will be impossible. This impossibility may be due that some units are violated a number of constraints in the model (8). Finally, if the breakdown point is chosen smaller, the performance of two-stage system will increase during negotiations.

5. Conclusion

We concluded with some examples that, by adding Δ to the previous breakdown point, the efficiency of units with the new breakdown point will be less than or equal the efficiency of units with the Previous breakdown point. The chosen breakdown point cannot be arbitrarily, for example if we use the CRS efficiency for each breakdown point, likely it becomes impossible for model (8) and it may be impossible due to number of points are violated some constraints in model (8). If we increase the amount of breakdown point in Nash bargaining game, the amount of efficiency will be smaller or equal to the efficiency of the previous breakdown point, so the breakdown point (0.0) will have maximum performance of two-stage system in this model. The goal is not the best system performance during the negotiation but the goal is finding the most efficiency during the negotiation.

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