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Non-Cooperative Procedure in Two-Stage Systems with Shared Inputs and Final Outputs in the First Stage: A Slacks-Based Measure Model

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Abstract

In the present world, calculating the efficiency of systems with an internal structure, such as two-stage systems, is principally imperative. Conventional Data envelopment analysis (DEA) is a non-parametric approach that measures the efficiency of comparable black box systems. There is a weakness in traditional DEA that does not survey the internal structure of systems. Hence, for evaluating the efficiency of the systems with internal structure, Network DEA (NDEA) was presented. Two-stage systems are a special case of network systems. Many procedures were suggested to measure the efficiency of these systems such as slacks-based measure (SBM) approach. In this paper, we will focus on two-stage systems with shared inputs between stages and final output in stage 1 and we shall propose non-cooperative models to calculate the efficiency of these systems based on SBM procedure. Also, we shall indicate the overall efficiency of these systems as the product of the efficiency scores of stages. In the end, to explain the proposed approach, a numerical example will be presented.

Keywords: Data envelopment analysis, Efficiency, DEA, Two stage system, Non-cooperative, Slacks -based measure.

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1. Introduction

Data Envelopment Analysis (DEA) is a mathematical programming for evaluating the relative efficiency of decision making units (DMUs) that is introduced by Charnes, Cooper, and Rhodes [1]. In the real life problems there are some systems that have two-stage structure. The traditional DEA, measures the efficiency of this system, regardless of its internal structure. So, this is one of the disadvantages of the traditional DEA model. In order to calculate the efficiency of such systems, some authors applied two-stage DEA methods. DMUs with two- stage structure are used in many of recent studies in the field of DEA [2,3].

Recently, a number of radial and non-radial models have surveyed the efficiency of these systems. Seiford and Zhu [4] introduced models that evaluate the overall efficiency and efficiency of stages of two-stage systems independently. Tone [5], introduced a non-radial model slacks based measured (SBM) that measures the efficiency of black box systems. By considering the relationship of stages, Kao and Hwang [3] suggested a model that compute the efficiencies of two-stage systems under the constant return to scale assumption. Likewise, Tone and Tsutsui [6] introduced a model to evaluate the efficiency of systems with internal processes. Also, in their proposed model, the overall efficiency of the system is decomposed to the weighted average of the processes that the weight of each process is defined by the decision maker (DM). Then, Kao [2] extends their model and proposed a network SBM model wherein the overall efficiency of the system is equal to the weighted average of the efficiency of processes that weight of the p-th process, is recognized as ratio of the efficiency of p-th process to sum of the efficiency of the total processes. Recently, a number of researchers have looked at systems that have extended two-stage structure (such as two-stage systems with shared input, shared output etc. (see [7-16]). Jahangard and Mahboubi [17] presented a non-cooperative approach based on game theory and SBM approach to measure the efficiency of a two-stage network structure. Also, Esfidani et al. [18] introduced a new procedure for measuring the efficiency of multi-period two-stage systems. They also, presented efficiency change indexes to identify the status of the efficiency from a period to another period. In order to measure the efficiency of multi-period two-stage systems with no-radial model, a slacks-based measure approach is presented by Esfidani et al. [19]. Amirteimoori et al. [9] proposed a non-radial model with considering the weak disposability assumption to handle desirable and undesirable outputs and measure the efficiency of two-stage systems. A new approach for centralized resource allocation based on the proposed SBM model of DEA is presented by Seyfpanah [20]. Also, Salahi et al. [21] presented an additive slack-based measure model to measure the overall efficiency and efficiency of stages of two-stage structure with shared inputs and feedback.

In this paper, we use the non-cooperative approach and suggest models based on slacks-based measure, that evaluate the efficiency of two-stage systems with shared inputs between stages and final output in stage 1.

This paper is organized as follows:

In section 2, we describe a two-stage system with shared inputs and final output in stage 1. Then in section 3, we suggest the models that evaluate the efficiencies of stages and overall efficiency by using non-cooperative approach. Finally, the proposed approach is illustrated by the real data of resin manufacturing companies that extracted from Khodakarami et al. [22].

2. Preliminary

This section will describe a non-radial slacks-based measure (SBM) model to calculate the efficiencies of two-stage systems.

Suppose that there are n DMUs with two-stage structure. Stage 1 of $DMU_j, j=1,...,n$ consumes the input vector $x^1{}_{ij}\,(i=1,...,m)$ and shared input vector $x^{1'}{}_{gj}\,(g=1,...,m')$ to generate intermediate products $z_{dj};d=1,...,D$ and final output vector $y^1{}_{hj}\,(h=1,...,s')$. Then stage 2 uses these intermediate products and the shared input vector $x^{2'}{}_{gj}\,(g=1,...,m')$ to produce output vector $y^2{}_{rj}\,(r=1,...,s)$.

In figure 1 the structure of a two-stage system is shown.

3. The Non-cooperative model

In this section, according to the concept of leader-follower, we will suggest the models to measure the overall efficiency and efficiency of stages for each DMU_o (DMU under evaluation) based on slacks-based measure.

For this purpose, stage 1 and stage 2 are determined as the leader and follower stages, respectively, by assuming that stage 1 is the most important stage from the point of view of the decision maker (DM). Therefore, we develop an SBM model to measure the efficiency of stage 1. Therefore, the following model is proposed:

$$\rho_o^{1(leader)}=\min\ \frac{1-\frac{1}{m+m'}\left(\sum_{i=1}^{m}\frac{s_i^-}{x^1{}_{io}}+\sum_{l=1}^{m'}\frac{k_l^-}{x^{1'}{}_{lo}}\right)}{1+\frac{1}{D+s'}\left(\sum_{d=1}^{s}\frac{t_d^+}{z_{do}}+\sum_{h=1}^{s'}\frac{f_h^+}{y^1{}_{ho}}\right)}$$

$$s.t$$

$$\begin{aligned}
&\sum_{j=1}^{n}\lambda_j x^1{}_{ij}+s_i^-=x^1{}_{io} && i=1,...,m\\
&\sum_{j=1}^{n}\lambda_j x^{1'}{}_{lj}+k_l^-=x^{1'}{}_{lo} && l=1,...,m' \quad (1)\\
&\sum_{j=1}^{n}\lambda_j z_{dj}-t_d^+=z_{do} && d=1,...,D\\
&\sum_{j=1}^{n}\lambda_j y^1{}_{hj}-f_h^+=y^1{}_{ho} && h=1,...,s'\\
&\lambda_j\geq 0, && j=1,...,n\\
&s_i^-,k_l^-\geq 0 && i=1,...,m \quad l=1,...,m'\\
&t_d^+,f_h^+\geq 0 && d=1,...,D \quad h=1,...,s'
\end{aligned}$$

Wherein, λ_j is intensity vector of $DMU_j, j=1,...,n$ in stage1.

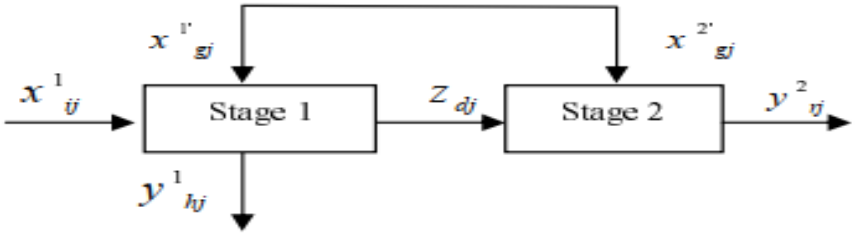

Figure1. Two-stage system

Definition1. DMU_o is efficient if and only if $\rho_o^{I}=1$.

And also, the following model is introduced to measure the efficiency of DMU_o in stage 2:

$$\rho_o^{\mathrm{II}(follower)} = \min \frac{1-\frac{1}{D+m'}\left(\sum_{d=1}^{D}\frac{t_i^-}{z_{do}}+\sum_{l=1}^{m'}\frac{k_l^{'-}}{x^{2'}_{lo}}\right)}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y^{2}_{ro}}}$$

$$s.t$$

$$\begin{aligned}
&\sum_{j=1}^{n}\mu_j x^{2'}_{lj}+k_l^{'-}=x^{2'}_{lo} && l=1,\dots,m'\\
&\sum_{j=1}^{n}\mu_j z_{dj}+t_d^-=z_{do} && d=1,\dots,D \qquad (2)\\
&\sum_{j=1}^{n}\mu_j y^{2}_{rj}-s_r^+=y^{2}_{ro} && r=1,\dots,s\\
&\sum_{j=1}^{n}\lambda_j x^{1}_{ij}+s_i^-=x^{1}_{io} && i=1,\dots,m\\
&\sum_{j=1}^{n}\lambda_j x^{1'}_{lj}+k_l^-=x^{1'}_{lo} && l=1,\dots,m'\\
&\sum_{j=1}^{n}\lambda_j z_{dj}-t_d^+=z_{do} && d=1,\dots,D\\
&\sum_{j=1}^{n}\lambda_j y^{1}_{hj}-f_h^+=y^{1}_{ho} && h=1,\dots,s'\\
&\sum_{j=1}^{n}\lambda_j z_{dj}=\sum_{j=1}^{n}\mu_j z_{dj} && d=1,\dots,D\\
&\frac{1-\frac{1}{m+m'}\left(\sum_{i=1}^{m}\frac{s_i^-}{x^{1}_{io}}+\sum_{l=1}^{m'}\frac{k_l^-}{x^{1'}_{lo}}\right)}{1+\frac{1}{D+s'}\left(\sum_{d=1}^{s}\frac{t_d^+}{z_{do}}+\sum_{h=1}^{s'}\frac{f_h^+}{y^{1}_{ho}}\right)}=\rho_o^{\mathrm{I}(leader)}\\
&\lambda_j,\mu_j\geq 0 && j=1,\dots,n\\
&s_i^-,k_l^-,k_l^{'-},t_d^-\geq 0 && i=1,\dots,m \quad l=1,\dots,m' \quad d=1,\dots,D\\
&s_r^+,t_d^+,f_h^+\geq 0, && r=1,\dots,s \quad d=1,\dots,D \quad h=1,\dots,s'
\end{aligned}$$

Note that, this model measures the efficiency of stage 2, while the efficiency of stage 1 is unchanged.

Definition2. DMU_o is efficient in stage 2 if and only if $\rho_o^{\mathrm{II}}=1$.

Therefore, the overall efficiency of system is as follows:

$$\rho_o^{overall}=\rho_o^{\mathrm{I}(leader)}\times\rho_o^{\mathrm{II}(follower)}$$

Let $(\lambda_j^*,\mu_j^*,s_i^{-*},k_l^{-*},k_l^{'-*},t_d^{-*},s_r^{+*},t_d^{+*},f_h^{+*})$ be an optimal solution of model (2). We define the following frontier projection point for inefficient DMU_o:

$$\begin{cases}
\tilde{x}^1_{io}=x^1_{io}-s_i^{-*}\\
\tilde{x}^{1'}_{lo}=x^{1'}_{lo}-k_l^{-*}\\
\tilde{x}^{2'}_{lo}=x^{2'}_{lo}-k_l^{'-*} \qquad (3)\\
\tilde{y}^1_{ho}=y^1_{ho}+f_h^{+*}\\
\tilde{y}^2_{ro}=y^2_{ro}+s_r^{+*}\\
\tilde{z}_{do}=\sum_{j=1}^{n}\lambda^*_j z_{dj}=\sum_{j=1}^{n}\mu^*_j z_{dj}
\end{cases}$$

Definition3. DMU_o is overall efficient if and only if $\rho_o^{overall}=1$.

Theorem1. For each DMU_o, $\rho_o^{overall}=1$ if and only if $\rho_o^{\mathrm{I}(leader)}=\rho_o^{\mathrm{II}(follower)}=1$.

Proof. According to $\rho_o^{overall}=\rho_o^{\mathrm{I}(leader)}\times\rho_o^{\mathrm{II}(follower)}$, the proof is obvious.

4. Case Study

In this section, we illustrate the introduced approach using the real data of resin manufacturing companies that extracted from Khodakarami et al. [14]. They consider a two-stage structure for any company. Inputs, intermediate products, and outputs that they used are listed in the following Table:

Table 1: Inputs, Outputs and intermediate measures

Inputs	Intermediate measures	Outputs
Annual cost	Desposits	Transections
Annual personnal turnover	-	Loans
Invironmental costs	-	Profits

Table 2: The results of Models (1), (2)

DMU	Overall Efficiency	Efficiency of stage1	Efficiency of stage 2
1	0.38	0.38	1
2	0.58	1	0.58
3	1	1	1
4	0.39	1	0.39
5	0.33	1	0.33
6	1	1	1

The efficiencies of these companies based on the results of the proposed models are listed in Table 2.

The columns 2, 3 and 4 of Table 2 indicate the overall efficiencies and the efficiency of stages 1, 2 respectively.

In stage 1, $DMU_2, DMU_3, DMU_4, DMU_6$ are efficient. But, DMU_4 is inefficient in stage 2. The efficient companies in stage 2 are DMU_1, DMU_3, DMU_5 DMU_6.

Therefore, DMU_3, DMU_6 are overall efficient. Then, the best and worst efficiency score belong to DMU_2, DMU_5 respectively.

5. Conclusion

Two-stage systems are a special case of network systems. Conventional DEA models treat the network systems as black boxes, regardless of their internal structure. In the recent years, many of network non-radial DEA models have been proposed that measure the efficiency of two-stage systems such as network slacks-based measure models. In this paper, we considered two-stage systems wherein a part of intermediate measures goes from stage 1 to stage 2 and we have shared input for both stages. We suggested the models to measure the overall efficiency and efficiency of stages of these systems with slacks-based measure and non-cooperative conditions. We define that the overall efficiency of the system to product of the efficiencies of stages. Finally, we used the real data of resin manufacturing companies to illustrate the suggested model.

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