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Finding common weights in DEA using a compromise solution approach

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Abstract

The weights generated by the common weights approach provide a common criterion for ranking the decision-making units (DMUs) in data envelopment analysis (DEA). Existing common weights models in DEA are either very complicated or unable to produce a full ranking for DMUs. This paper proposes a new compromise solution model to seek a common set of weights for full ranking for DMUs. The maximum inefficiency scores calculated from the standard DEA model are regarded as the anti-ideal solution for the DMUs to avoid. A common set of weights that produces the vector of inefficiency scores for the DMUs furthest to the anti-ideal solution is sought. The discrimination power of the new model is tested using two numerical examples and its potential application for fully ranking DMUs is illustrated.

Keywords: Common weights, Data envelopment analysis, Ranking

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1. Introduction

Data envelopment analysis (DEA) has been widely applied to measure the relative efficiency for a group of homogeneous decision-making units (DMUs) with multiple inputs and multiple outputs such as schools, hospital, bank branches and so on [1]. The main idea of the traditional DEA model (CCR and BCC model) is to select for each DMU a set of input and output weights that can maximize the ratio of its sum of weighted outputs to its sum of weighted inputs while keeping all the DMUs' ratios not greater than 1[2]. The maximum ratio of each DMU is defined as its efficiency value and a DMU is said to be DEA efficient if its efficiency score is equal to 1. But the evaluation of DMUs based on the selection of different set of weights is unacceptable. Therefore, some researchers by linking DEA with some other techniques such as multi-objective programming have developed models to generate a set of common weights for evaluating and ranking the DMUs .Cook, Roll [3] and Roll, Cook [4] first noted the use of common weights for efficiency evaluation in the background of applying DEA for efficiency evaluation of high way maintenance units. This idea was furtherly developed by Ganley and Cubbin [5] and Roll and Golany [6]. They proposed several methods to select common sets of weights based on some different aspects, such as looking for central values for all the weights, maximizing the average (sum) efficiency of all DMUs and maximizing the number of efficient DMUs. Sinuany-Stern and Friedman [7] proposed a non-linear model to generate a set of common weights based on the best separation of the units into efficient and inefficient as given by DEA. Kao and Hung [8] suggested a model based on the common set of weight techniques with the goal of making the DMUs' efficiencies closest to the efficiency that is generated by traditional models in DEA. Inspired by Kao and Hung [8] model, Zohrehbandian, Makui [9] propose an improved linear DEA model by introducing an MDCM model to generate common-weights selection for the DMUs. Liu, Peng [10] transform each efficient DMU into a virtual DMU with one input and one output that its input is the weighted sum of DMU inputs and its output is the weighted sum of DMU outputs. They proposed a linear programming model to select a set of common weights with the purpose of minimizing the total distance between the virtual DMUs and the benchmarking line. Saati, Hatami-Marbini [11] and Sun, Wu [12] proposed two common weight models. These models minimize (maximize) the DMUs' distances to the ideal (anti-ideal) DMU. Dong, Chen [13] attempted to use the concept of satisfaction degree to propose a DEA-based approach for measuring the efficiencies of the DMUs. Wu, Chu [14] noted that the degree of satisfaction is actually the relative distance between the minimum efficiency and the maximum efficiency, and this relative distance is termed as degree of satisfaction. Wang, Liu [15] employed the concept of DEA cross-efficiency to develop a novel common set of weights method to minimize the distance between self-evaluation efficiency and peer-evaluation efficiency.

Other researches have incorporated other techniques into DEA for common-weights selection, such as multiple-objective integer programming[16], preference analysis[17], Shannon's entropy [18] and prospect theory[19]. Owing to the advantages in efficiency evaluation and ranking, DEA common-weight evaluation approaches have been widely applied in many areas, such as fixed resources allocation [20], technology selection [21, 22] and centralized allocation of emission permits[23].

In this paper, we propose the compromise solution approach to generate common weights under the DEA framework. The maximum inefficiency score calculated from the standard DEA model is the target for each DMU to avoid. The DMUs select a common set of weights which yields the furthest distance between the vector of inefficiency scores calculated from this set of weights and the target. Section 2 Section 2 briefly reviews the CCR model developed by Charnes, Cooper [24] and provides a basis for assessing the target efficiencies of DMUs. New model is introduced in Section 3. Section 4 demonstrates the models with two data sets. Conclusions are given in Section 5.

2. CCR model

Suppose we have a set of n DMUs that each $DMU_j\,(j=1,2,...,n)$ produces s different outputs indexes $Y_j=(y_{1j},y_{2j},...,y_{sj})\in\mathbb{R}_+^s$ from m different inputs indexes $X_j=(x_{1j},x_{2j},...,x_{mj})\in\mathbb{R}_+^m$, where $\mathbb{R}_+^s$ and $\mathbb{R}_+^m$ are two sets of nonnegative numbers. The efficiency of DMU_j is as follows:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \qquad (1)$$

Where $u_r\,(r=1,2,...,s)$ and $v_i\,(i=1,2,...,m)$ are the rth output and ith input weights respectively.

According to [24], the best relative efficiency of each DMU can be measured by the following CCR model, which was named by the acronym of the three authors:

$$\begin{aligned}
E_p^* = Max\ & \sum_{r=1}^{s} u_r y_{rp} \\
s.t.\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0,\ j=1,2,...,n \\
& \sum_{i=1}^{m} v_i x_{ip} = 1, \qquad (2) \\
& u_r \ge 0, r=1,2,...,s \\
& v_i \ge 0, i=1,2,...,m
\end{aligned}$$

Where DMU_p is DMU under evaluation and E_p^* is referred the relative efficiency of DMU_p. Model (2) can also be expressed equivalently in the following deviation variable form:

$$\begin{aligned}
E_p^* = Min\ & \alpha_p^p \\
s.t.\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \alpha_j^p = 0,\ j=1,2,...,n \\
& \sum_{i=1}^{m} v_i x_{ip} = 1, \qquad (3) \\
& u_r \ge 0, r=1,2,...,s \\
& v_i \ge 0, i=1,2,...,m \\
& \alpha_j^p \ge 0, j=1,2,...,n
\end{aligned}$$

Where α_p^p is the deviation variable for DMU_p. DMU_p is efficient if and only if $\alpha_p^{p*}=0$. If DMU_p is not efficient, then its efficiency score is $1-\alpha_p^{p*}$ that α_p^p can be regarded as a measure of inefficiency [25].

3. New model for common weights

Let $\alpha_j^{p*}(j=1,2,...,n)$ be optimal value of α_j^p in model (3) for evaluation DMU_p. So α_p^{p*} be the (CCR) inefficiency of DMU_p and $\alpha_j^{p*}(j \underset{\ne p}{=} 1,2,...,n)$ be the (CCR) inefficiency of DMU_j under evaluation DMU_p. According to the above mentioned, we construct the cross-inefficiency matrix as following form:

Table1: Cross-inefficiency evaluation for n DMUs

DMU_p		Inefficiencies			
		α_1^p	α_2^p	$\cdots$	α_n^p
Target DMU	1	α_1^1	α_2^1	$\cdots$	α_n^1
	2	α_1^2	α_2^2	$\cdots$	α_n^2
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	n	α_1^n	α_2^n	$\cdots$	α_n^n

Using table 1, we define the following terms:

$$\alpha_j^{\max} = \max_{i=1,2,\dots,n} \alpha_j^i$$
$$\alpha_j^{\min} = \min_{i=1,2,\dots,n} \alpha_j^i \qquad (4)$$

Using (4), we build the following new model for estimating the common weights for the n DMU:

$$\begin{aligned} \max \ & \min_{j=1,2,\dots,n} \left\{ \frac{\alpha_j^{\max} - \alpha_j}{\alpha_j^{\max} - \alpha_j^{\min}} \right\} \\ s.t. \ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \alpha_j = 0, \ j = 1,2,\dots,n \\ & \sum_{j=1}^{n} \sum_{i=1}^{m} v_i x_{ij} = 1 \qquad (5) \\ & u_r \geq 0, r = 1,2,\dots,s \\ & v_i \geq 0, i = 1,2,\dots,m \\ & \alpha_j \geq 0, \ j = 1,2,\dots,n \end{aligned}$$

The ratio form of objective function in (5) can be rewritten in a linear form, formulated in the constraints and objective function of (6).

$$\begin{aligned} \max \ & \delta \\ s.t. \ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \alpha_j = 0, \ j = 1,2,\dots,n \\ & \sum_{j=1}^{n} \sum_{i=1}^{m} v_i x_{ij} = 1 \qquad (6) \\ & \alpha_j + \left(\alpha_j^{\max} - \alpha_j^{\min} \right) \delta \leq \alpha_j^{\max}, \ j = 1,2,\dots,n \\ & u_r \geq 0, r = 1,2,\dots,s \\ & v_i \geq 0, i = 1,2,\dots,m \\ & \alpha_j \geq 0, \ j = 1,2,\dots,n \end{aligned}$$

The CWA efficiency score of DMU_j is defined as the following form:

$$\gamma_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} \qquad (7)$$

Where $u_r^* (r = 1,2,\dots,s)$ and $v_i^* (i = 1,2,\dots,m)$ is optimal solution of (6).

Definition 1. The performance of DMU_j is better than DMU_j if $\gamma_j^* > \gamma_i^*$.

Definition 2. If $\gamma_j^* = \gamma_i^* < 1$, i.e. they are both CWA-inefficient , then the performance of DMU_j is better than DMU_j $\alpha_j^* < \alpha_i^*$.

Definition 3. If $\gamma_j^* = \gamma_i^* = 1$, then the performance of DMU_j is better than DMU_j if $\eta_j^* > \eta_i^*$

that η_j^* is optimal value of j[th] dual variable correspond with the following constraints of model (6)[10].

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \alpha_j = 0, \ j = 1,2,\dots,n$$

4. Numerical examples

In this section two numerical examples are to be examined using the proposed new model to test its discrimination capability and to illustrate its potential application in fully ranking DMUs.

Example 1. Consider the example investigated by[8], who examined the efficiencies of 17 forest districts (DMUs) in Taiwan in terms of four inputs and three outputs that were defined as follows:

Inputs: x_1:budget in US dollars, x_2:initial stocking in cubic meters, x_3:labor in number of employees, x_4:land in hectares.

Table 2: Input and output data in example 1.

DMU	Inputs				Outputs		
	x_1	x_2	x_3	x_4	y_1	y_2	y_3
1	51.62	11.23	49.22	33.52	40.49	14.89	3166.71
2	85.78	123.98	55.13	108.46	43.51	173.93	6.45
3	66.65	104.18	257.09	13.65	139.74	115.96	0
4	27.87	107.6	14	146.43	25.47	131.79	0
5	51.28	117.51	32.07	84.5	46.2	144.99	0
6	36.05	193.32	59.52	8.23	46.88	190.77	822.92
7	25.83	105.8	9.51	227.2	19.4	120.09	0
8	123.02	82.44	87.35	98.8	43.33	125.84	404.69
9	61.95	99.77	33	86.37	45.43	79.6	1252.62
10	80.33	104.65	53.3	79.06	27.28	132.49	42.67
11	205.92	183.49	144.16	59.66	14.09	196.29	16.15
12	82.09	104.94	46.51	127.28	44.87	108.53	0
13	202.21	187.74	149.39	93.65	44.97	184.77	0
14	67.55	82.83	44.37	60.85	26.04	85	23.95
15	72.6	132.73	44.67	173.48	5.55	135.65	24.13
16	84.83	104.28	159.12	171.11	11.53	110.22	49.09
17	71.77	88.16	69.19	123.14	44.83	74.54	6.14

Table 3: Results of proposed model in example 1

DMU	CCR	Rank	α_j^*	γ_j^*	η_j^*	Rank
1	1.0000	1	0	1	0.2207	2
2	1.0000	1	0.0078	0.858	0	4
3	1.0000	1	0.0417	0.6138	-0.0205	12
4	1.0000	1	0.0015	0.9567	0	3
5	1.0000	1	0	1	0.6011	1
6	1.0000	1	0.0092	0.8528	-0.1213	5
7	1.0000	1	0.0066	0.8143	-0.0836	7
8	1.0000	1	0.0204	0.6559	-0.168	10
9	1.0000	1	0.0073	0.8183	-0.4239	6
10	0.9403	10	0.0146	0.7008	0	9
11	0.9346	11	0.062	0.4047	0	16
12	0.8290	12	0.0136	0.7199	0	8
13	0.7997	13	0.0579	0.4614	0	15
14	0.7733	14	0.0149	0.6219	0	11
15	0.7627	15	0.0268	0.5092	0	14
16	0.7435	16	0.0602	0.2926	0	17
17	0.6873	17	0.0225	0.5582	0	13

Outputs: y_1: main product in cubic meters, y_2: soil conservation in cubic meters, y_3: recreation in number of visits. Table 2 shows the input and output data of the 17 forest districts.

The efficiency scores of the 17 forest districts calculated from the Model (3) is shown in the second column of Table 3. As can be seen, the model (3) identifies DMU1 through DMU9 as DEA efficient units. In order to rank the nine DEA efficient units, we use results of model (6), to rank them that are shown in the fourth through the seventh columns of Table 3.

As can be seen in table 3, DMU_5 and DMU_{16} took first place and last place, respectively.

Example 2. This example is taken from [26, 27] and is about ranking Twelve flexible manufacturing systems (FMSs) were evaluated in terms of two inputs and four outputs that were defined as follows:

Inputs: x_1: annual operating and depreciation costs measured in units of one hundred thousand dollars, x_2: the floor space requirements of each specific system measured in thousands of square feet.

Outputs: y_1: the improvements in qualitative benefits, y_2:WIP, y_3: average number of tardy jobs and y_4 :average yield.

Table 4 shows the input and output data of the 12 FMSs.

Table 4: Input and output data in example 2

FMS	Inputs		Outputs			
	x_1	x_2	y_1	y_2	y_3	y_4
1	17.02	5	42	45.3	14.2	30.1
2	16.46	4.5	39	40.1	13	29.8
3	11.76	6	26	39.6	13.8	24.5
4	10.52	4	22	36	11.3	25
5	9.5	3.8	21	34.2	12	20.4
6	4.79	5.4	10	20.1	5	16.5
7	6.21	6.2	14	26.5	7	19.7
8	11.12	6	25	35.9	9	24.7
9	3.67	8	4	17.4	0.1	18.1
10	8.93	7	16	34.3	6.5	20.6
11	17.74	7.1	43	45.6	14	31.1
12	14.85	6.2	27	38.7	13.8	25.4

Table 5: Results of proposed model in example 2

DMU	CCR	Rank	α_j^*	γ_j^*	η_j^*	Rank
1	1	1	0.015	0.8718	0	8
2	1	1	0.0157	0.8595	-0.2387	9
3	0.9824	9	0.0093	0.8946	0	7
4	1	1	0	1	0.8507	2
5	1	1	0.0021	0.9686	-1.5206	4
6	1	1	0.0006	0.9872	0	3
7	1	1	0	1	1.9490	1
8	0.9614	10	0.008	0.9047	0	5
9	1	1	0.0048	0.895	-1.1894	6
10	0.9536	11	0.0113	0.8484	0	10
11	0.9831	8	0.0231	0.8183	0	11

12	0.8012	12	0.0269	0.7497	0	12

The CCR efficiency scores for 12 FMSs in the second columns in table 5 show that 7 of 12 FMSs are efficient, so we cannot find any difference between them for ranking. Thus we use model (6) for further distinction. The results of this model for evaluation of 7 FMSs are shown in the fourth through the seventh columns of Table 5.

As can be seen from Table 5, DMU_7 and DMU_{12} took first place and last place, respectively.

5. Conclusion

For assessment of all the DMUs on the same scale, this paper examines the application of the compromise solution approach for generating a common set of weights under the DEA framework. The maximum inefficiency score calculated from the standard DEA model is the anti-ideal solution for each DMU to avoid. The common set of weights that is able to produce a vector of inefficiency scores furthest to the anti-ideal solution is desired. This vector of inefficiency scores is called the compromise solution. This work computes common weights using the max-min model. Based on the results of the examples in Section 4, the proposed model's discrimination power is acceptable.

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