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Two-stage network-structured production process in the presence of undesirable intermediate measures

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Abstract

The importance of the issue of environmental pollution is so great that the attention of many researchers has drawn attention to the recycling issue in recent decades. Since a proper evaluation can improve the performance of a system in various situations, the presentation of an appropriate method that can evaluate a production system correctly is necessary. Therefore, first, we introduce a two-stage network structure consists of undesirable intermediate measures. Then, by considering the weak disposability assumption to handle desirable and undesirable outputs, and also links between the subsections of the process, a non-radial model to evaluate the performance of the proposed structure is introduced. The proposed model is developed from the perspective of the centralized approach. Also, it is defined based on the Russell-based model. The introduced model can calculate the efficiency of the whole system without the need to evaluate the efficiency of its subsections. A real case on an industrial production system of 30 regions in China is used to illustrate the proposed model.

Keywords: Two-stage process, Undesirable output, Weak disposability, intermediate measure, Russell measure.

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1. Introduction

These days, many countries focus on the protection of natural resources and the reduction of environmental pollution. Industrial factories are one of the important agents that create environmental pollution such as toxic gasses and wastewater. Accordingly, some researchers tried to resolve this problem by recycling. Therefore, the good performance of recycling systems can be a proper pattern to other systems to improve environmental problems. A non-parametric method is utilized to evaluate the performance of systems with multiple inputs and outputs is data envelopment analysis (DEA). Generally, the traditional DEA models are defined as radial and non-radial. Also, the model objectives are considered based on decreasing inputs or increasing outputs of systems. Russell measure is one of the defined measures to evaluate the relative efficiency of decision-making units that do not require the importance of the attributes of inputs and outputs.

In 1985, Färe et al. [1] defined the Russell graph measure of technical efficiency as a combination of the input and output Russell measures technical efficiency in an additive way. Then, Pastor et al. [2] separately averaged the input and output efficiency and introduced a new efficiency in a ratio form by combining efficiencies. Their proposed model was called the enhanced Russell graph measure. As we know, conventional models of data envelopment analysis do not consider the internal structure, while this issue can affect the performance evaluation of the system. Therefore, in 2000, Färe and Grosskopf [3] introduced network data envelopment analysis (NDEA). Afterwards, many studies were conducted on network structures evaluation, especially two-stage structures, that here are some of them.

Chen and Zhu [4], by the link between the subsections of the two-stage structure, developed a model to evaluate the efficiency of the two-stage processes under the assumption of variable returns to scale. Since the inefficiency of the two stages may include leads to the overall efficiency in some cases, Kao and Hwang [5] modified this problem and introduced a new model with a multiplicative function. Chen et al. [6] introduced an additive model for assessing the efficiency of two-stage processes. In the proposed model, performance improvement in one stage leads to the inefficiency of other stages. So, Liang et al. [7] resolved this problem by presenting two models as a centralized and leader-follower model. In the following, we will have an overview of some studies on network structures including undesirable factors focused on environmental issues and industrial productions in recent years.

Wu et al. [8] introduced an additive DEA model to evaluate the efficiency of the two-stage network structures with undesirable intermediate measures. They applied the proposed approach to industrial production in 30 provincial-level regions in China. Based upon the centralized approach, Wu et al. [9] developed a model to evaluate the efficiency of the two-stage structures with shared inputs and undesirable intermediate measures. They used their proposed model to assess the performance of industries in China. Wu et al. [10] proposed a new model, based on the Nash bargaining game, to assess the relative efficiency of the two-stage structures with undesirable outputs. Also, they applied their proposed model to evaluate the sustainable manufacturing of a set of iron and steel makers in China. Li et al. [11] defined a model under the assumption of variable returns to scale for two-stage structures with feedback flows and undesirable outputs and evaluated the performance of the ecological system of some regions in China. Zhou et al. [12], first, introduced a mixed network structure two-stage with undesirable intermediate measures. Then, they estimated the performance of WUWT systems based on the slacks-based measure model. The WUWT systems consist of the water use sector (WU) and the water treatment sector (WT). Also, based on the non-cooperative approach, Chen et al. [13] developed a model for a two-stage network structure with undesirable elements and evaluated the environmental efficiency of Chinese industrial water systems. Hu et al. [14] used a weighted superposition method and defined a model to evaluate the two-stage processes with undesirable outputs and feedback variables. Then, they analyzed the performance of oil production and oilfield wastewater treatment of 13 centers. Nematizadeh et al. [15] introduced a directional distance function model for evaluating the performance of two-stage feedback processes with undesirable factors. They calculated the relative efficiency of ecological systems of 31 regions of China. Moreover, Nematizadeh et al. [16] introduced a mixed network structure with shared inputs and undesirable intermediate measures and extended the slacks-based measure models to evaluate the performance of the proposed system and their ranking. They applied their proposed approach for assessing the performance of the treatment water system of some regions of China. Wang et al. [17] proposed a stochastic DEA model to evaluate the environmental efficiency of the industrial system consists of undesirable factors.

Given the importance of the two-stage network structures researchers are always trying to present a proper method to evaluate such structures.

The main purpose of the paper is to provide a model for the introduced two-stage structure that is not dependent on the important attribute of inputs and outputs. Therefore, we introduce a non-radial model based on the Russell measure model such that a connection between the subsections is considered. The proposed model is developed from the perspective of the centralized approach. Also, due to the presence of undesirable factors, the weak disposability assumption of Shephard [18] is used to handle desirable and undesirable outputs. The proposed approach can evaluate the relative efficiency of the whole system and its subsections, simultaneously. To analyses the proposed method, we used a real case on industrial productions in China.

The rest of this paper is organized as follows: In Section 2, we briefly review the weak disposability assumption and the Russell measure model. In Section 3, the proposed model to estimate the relative efficiency of the two-stage structure with the undesirable intermediate measure introduces. Section 4 applies the suggested method to assess the performance of the industrial production of 30 regions in China. Finally, conclusions appear in Section 5.

2. Preliminaries

In this section, a brief review of the weak disposability assumption and the enhanced Russell graph efficiency measure are explained.

2.1 Weak disposability assumption

Assume that there are k DMUs, and each $DMU_k : k = 1,...,K$ consists of input vector $x_k = (x_{1k},...,x_{nk}) \geq 0$, desirable and undesirable output vectors $v_k = (v_{1k},...,v_{mk}) \geq 0$ and $w_k = (w_{1k},...,w_{jk}) \geq 0$, respectively. The production technology set can be represented by:

$$T = \{(x,v,w) \mid (v,w) \text{ can be produced by } x\}$$

Definition 1. Outputs (v,w) are weakly disposable if and only if $(x,v,w) \in T$ and

$0 \le \theta \le 1$ implies $(x, \theta v, \theta w) \in T$. (see Shephard [18])

Based upon the weak disposability assumption, Kuosmanen [19] considered the non-uniform contraction factor θ^k for each $DMU_k: k = 1,\dots,K$, and introduced the production technology set in the following non-linear form:

$$
\begin{aligned}
T = \{(x,v,w):& \\
&\sum_{k=1}^{K} \lambda^k x_n^k \le x_n^o, && n = 1,\dots,N,\\
&\sum_{k=1}^{K} \theta^k \lambda^k v_m^k \ge v_m^o, && m = 1,\dots,M,\\
&\sum_{k=1}^{K} \theta^k \lambda^k w_j^k = w_j^k, && j = 1,\dots,J,\\
&\sum_{k=1}^{K} \lambda^k = 1,\\
&0 \le \theta^k \le 1,\ \lambda^k \ge 0, && k = 1,\dots,K\}.
\end{aligned}
\tag{1}
$$

Then, by considering $\lambda^k = \rho^k + \mu^k$ whereby $\rho^k = \theta^k \lambda^k$ and $\mu^k = (1-\theta^k)\lambda^k$, he converted the non-linear technology (1) into the following linear form:

$$
\begin{aligned}
T = \{(x,v,w):& \\
&\sum_{k=1}^{K} (\rho^k + \mu^k)\, x_n^k \le x_n^o, && n = 1,\dots,N,\\
&\sum_{k=1}^{K} \rho^k v_m^k \ge v_m^o, && m = 1,\dots,M,\\
&\sum_{k=1}^{K} \rho^k w_j^k = w_j^k, && j = 1,\dots,J,\\
&\sum_{k=1}^{K} (\rho^k + \mu^k) = 1,\\
&\rho^k,\ \mu^k \ge 0, && k = 1,\dots,K\}.
\end{aligned}
\tag{2}
$$

Given the above technology, the relative efficiency of each unit under evaluation can be obtained by minimizing inputs and undesirable outputs or maximizing outputs.

2.2 Enhanced Russell graph efficiency measure

Consider k DMUs which each $DMU_k : k = 1,\dots,K$ consists of input vector $x_k = (x_{1k},\dots,x_{nk}) \ge 0$ and output vector $v_k = (v_{1k},\dots,v_{mk}) \ge 0$. Also, assume the following production technology set under constant returns to scale assumption as follows:

$$
\begin{aligned}
T = \{(x,v):& \\
&\sum_{k=1}^{K} \lambda^k x_n^k \le x_n^o, && n = 1,\dots,N,\\
&\sum_{k=1}^{K} \lambda^k v_m^k \ge v_m^o, && m = 1,\dots,M,\\
&\lambda^k \ge 0, && k = 1,\dots,K\}.
\end{aligned}
\tag{3}
$$

In technology (3), λ^k is an intensity variable. Also, the inputs and outputs constraints are written based on the free disposability assumption.

Given the technology (3), Pastor et al. [2] introduced a ratio efficiency to evaluate the performance of DMU_o as follows:

$$
\begin{aligned}
R_e^* = Min\ & \frac{\frac{1}{N}\sum_{n=1}^{N} \alpha_n}{\frac{1}{M}\sum_{m=1}^{M} \varphi_m}\\
s.t.\ & \\
&\sum_{k=1}^{K} \lambda^k x_n^k \le \alpha_n x_n^o, && n = 1,\dots,N,\\
&\sum_{k=1}^{K} \lambda^k v_m^k \ge \varphi_m v_m^o, && m = 1,\dots,M,\\
&0 < \alpha_n \le 1, && n = 1,\dots,N\\
&\varphi_m \ge 1, && m = 1,\dots,M,\\
&\lambda^k \ge 0, && k = 1,\dots,K.
\end{aligned}
\tag{4}
$$

The introduced non-radial model is called the enhanced Russell graph efficiency measure. Also, this model is defined based on the Russell graph measure of technical efficiency by Färe et al. [1].

Model (4) can be formulated under variable returns to scale assumption by adding the convexity constraint

$\sum_{k=1}^{K}\lambda^{k}=1$. The constraints $0<\alpha_n\leq 1$ and $\varphi_m\geq 1$ are the requirements for dominance. Also, the above model does not require the importance of the attributes of inputs and outputs. The objective function is a ratio between the average efficiency of inputs and the average efficiency of outputs. Minimizing the objective function means that the numerator is minimized and the denominator is maximized, simultaneously.

If R_e^* is the optimal objective value to model (4), the efficient unit is defined as follows:

Definition 2. Unit under evaluation of DMU_o is efficient if and only if $R_e^*=1$.

It is noteworthy that $0<R_e^*\leq 1$.

In optimality, $R_e^*=1$ is equivalent to $\alpha_n^*=\varphi_m^*=1\ (n=1,...,N,\ m=1,...,M)$.

3. Russell-based model for two-stage structures

This section introduces a Russell-based model to evaluate the relative efficiency of the two-stage structure with the undesirable intermediate measure in terms of the centralized approach such that connecting between subsections of the two-stage process is considered.

So, suppose that there are k DMUs, and each $DMU_k : k=1,...,K$ has a two-stage structure as can be seen in Figure 1.

Figure 1 illustrates each DMU is composed of two subsections in series so that the first stage consumes the input vector $x_k=(x_{1k},...,x_{nk})\geq 0$ to produce two types of outputs: the final desirable output vector $v_k=(v_{1k},...,v_{mk})\geq 0$ and the undesirable output vector $w_k=(w_{1k},...,w_{jk})\geq 0$. The second stage uses its own inputs $w_k=(w_{1k},...,w_{jk})\geq 0$ and the dedicated input vector $z_k=(z_{1k},...,z_{tk})\geq 0$ to produce the final desirable output vector $y_k=(y_{1k},...,y_{rk})\geq 0$. Note that the undesirable intermediate measure w_k plays the output role for the first stage and the input role for the second stage, simultaneously.

According to the technology defined by Kuosmanen [19], and also the proposed model of pastor et al. [2], we provide the following model to evaluate the relative efficiency of the two-stage structure as Figure 1:

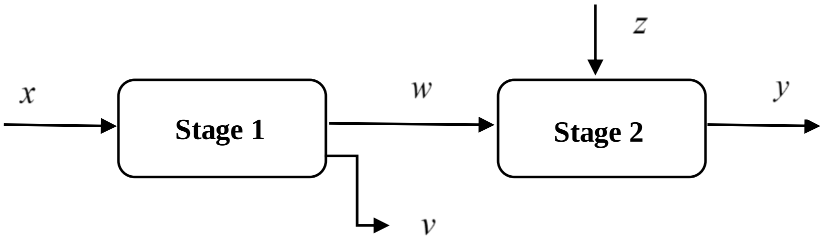

Figure 1- The structure of the two-stage process

$$\pi_o^{*Overall}=\text{Min}\frac{\frac{1}{N+J+T}\left(\sum_{n=1}^{N}\alpha_n+\sum_{j=1}^{J}\gamma_j+\sum_{t=1}^{T}\delta_t\right)}{\frac{1}{M+R}\left(\sum_{m=1}^{M}\beta_m+\sum_{r=1}^{R}\varphi_r\right)}$$

$s.t.$ (5)

Stage 1 constraints:

$$\sum_{k=1}^{K}(\rho^k+\mu^k)\,x_n^k\le\alpha_n x_n^o,\qquad n=1,\dots,N,$$
$$\sum_{k=1}^{K}\rho^k v_m^k\ge\beta_m v_m^o,\qquad m=1,\dots,M,$$
$$\sum_{k=1}^{K}\rho^k w_j^k=\gamma_j w_j^o,\qquad j=1,\dots,J,$$

Stage 2 constraints:

$$\sum_{k=1}^{K}(\rho^k+\mu^k)\,z_t^k\le\delta_t z_t^o,\qquad t=1,\dots,T,$$
$$\sum_{k=1}^{K}\rho^k w_j^k=\gamma_j w_j^o,\qquad j=1,\dots J,$$
$$\sum_{k=1}^{K}\rho^k y_r^k\ge\varphi_r y_r^o,\qquad r=1,\dots,R,$$

Generic constraints:

$$\sum_{k=1}^{K}(\rho^k+\mu^k)=1,$$
$$\rho^k,\ \mu^k\ge0,\qquad k=1,\dots,K,$$
$$0<\alpha_n,\delta_t,\ \gamma_j\le1,\qquad \textit{for all } n,\ j,\ t,$$
$$\beta_m,\ \varphi_r\ge1,\qquad \textit{for all } m,\ j,\ r.$$

Model (5) is the Russell efficiency measure model under variable returns to scale assumption for the two-stage structure as Figure 1. The objective function is the ratio between the average efficiency of inputs and intermediate measure to the average of outputs. Also, minimizing the objective function means that the numerator is minimized and the denominator is maximized, simultaneously. In the above model, $0<\alpha_n,\ \gamma_j,\delta_t\le1$ and $\beta_m,\varphi_r\ge1$ denote contraction and expansion coefficients of the inputs and undesirable and desirable outputs, respectively.

Note that the overall performance of the two-stage structure in the centralized approach achieves when its subsections work in coordination. Also, ρ^k, μ^k are the same non-negative intensity variables in two stages.

Notably, the undesirable intermediate measure w_k plays the output role for the first stage and the input role for the second stage, simultaneously. This leads to costs for both components of the system and thus the system as a whole. Therefore, it is rational that w_k decreases for both stages to strike a balance for the whole system.

It can be seen that model (5) is a non-linear programming model that can be converted into a linear programming problem by using the Charnes-Cooper transformation (see [20]). Therefore, let $\frac{1}{\frac{1}{M+R}\left[\sum_{m=1}^{M}\beta_m+\sum_{r=1}^{R}\varphi_r\right]}=\tau(>0)$, also set the symbols $\alpha_n\tau=\bar{\alpha}_n$, $\gamma_j\tau=\bar{\gamma}_j$, $\delta_t\tau=\bar{\delta}_t$, $\beta_m\tau=\bar{\beta}_m$, $\varphi_r\tau=\bar{\varphi}_r$, $\rho^k\tau=\bar{\rho}^k$, $\mu^k\tau=\bar{\mu}^k$, $\forall n,\ j,\ t,\ m,\ r$.

Then, model (5) can be transformed into the following linear program:

$$\pi_o^{*Overall}=Min\frac{1}{N+J+T}\left(\sum_{n=1}^{N}\bar{\alpha}_n+\sum_{j=1}^{J}\bar{\gamma}_j+\sum_{t=1}^{T}\bar{\delta}_t\right)$$

$s.t.$ (6)

Stage 1 constraints:

$$\sum_{k=1}^{K}(\bar{\rho}^k+\bar{\mu}^k)\,x_n^k\le\bar{\alpha}_n x_n^o,\qquad n=1,\dots,N,$$
$$\sum_{k=1}^{K}\bar{\rho}^k v_m^k\ge\bar{\beta}_m v_m^o,\qquad m=1,\dots,M,$$
$$\sum_{k=1}^{K}\bar{\rho}^k w_j^k=\bar{\gamma}_j w_j^o,\qquad j=1,\dots,J,$$

Stage 2 constraints:

$$\sum_{k=1}^{K}(\bar{\rho}^k+\bar{\mu}^k)\,z_t^k\le\bar{\delta}_t z_t^o,\qquad t=1,\dots,T,$$
$$\sum_{k=1}^{K}\bar{\rho}^k w_j^k=\bar{\gamma}_j w_j^o,\qquad j=1,\dots J,$$
$$\sum_{k=1}^{K}\bar{\rho}^k y_r^k\ge\bar{\varphi}_r y_r^o,\qquad r=1,\dots,R,$$

Generic constraints:

$$\sum_{k=1}^{K}(\bar{\rho}^k+\bar{\mu}^k)=\tau,$$
$$\frac{1}{M+R}\left[\sum_{m=1}^{M}\bar{\beta}_m+\sum_{r=1}^{R}\bar{\varphi}_r\right]=1,$$
$$0<\tau\le 1$$
$$\bar{\rho}^k,\ \bar{\mu}^k\ge 0,\qquad k=1,\dots,K,$$
$$0<\bar{\alpha}_n,\ \bar{\gamma}_j,\ \bar{\delta}_t\le\tau,\qquad for\ all\ n,\ j,t$$
$$\bar{\beta}_m,\ \tilde{\varphi}_r\ge\tau,\qquad for\ all\ m,r.$$

Suppose $(\bar{\alpha}_m^*,\bar{\beta}_n^*,\ \bar{\gamma}_j^*,\bar{\delta}_t^*,\ \bar{\varphi}_r^*)$ is an optimal solution to model (6). Since τ^* is positive, we can obtain an optimal solution to model (5) from $\alpha_n^*=\frac{\bar{\alpha}_m^*}{\tau}$, $\beta_n^*=\frac{\bar{\beta}_n^*}{\tau}$, $\gamma_j^*=\frac{\bar{\gamma}_j^*}{\tau}$, $\delta_t^*=\frac{\bar{\delta}_t^*}{\tau}$, $\varphi_r^*=\frac{\bar{\varphi}_r^*}{\tau}$, $\rho_k^*=\frac{\bar{\rho}_k^*}{\tau}$, $\mu_k^*=\frac{\bar{\mu}_k^*}{\tau}$. Therefore, the optimal objective value of $\pi_o^{*Overall}=\frac{1}{N+J+T}\left(\sum_{n=1}^{N}\alpha_n^*+\sum_{j=1}^{J}\gamma_j^*+\sum_{t=1}^{T}\delta_t^*\right)$ denotes the overall efficiency for unit under evaluation DMU_o and $0<\pi_o^{*Overall}\le 1$.

Also, the Russell efficiency measure of each subsection is calculated as follows:

$$\pi_o^{*Stage\,1}=\frac{\frac{1}{N+J}\left(\sum_{n=1}^{N}\alpha_n^*+\sum_{j=1}^{J}\gamma_j^*\right)}{\frac{1}{M}\sum_{m=1}^{M}\beta_m^*}\qquad(7)$$

$$\pi_o^{*Stage\,2}=\frac{\frac{1}{J+T}\left(\sum_{j=1}^{J}\gamma_j^*+\sum_{t=1}^{T}\delta_t^*\right)}{\frac{1}{R}\sum_{r=1}^{R}\varphi_r^*}\qquad(8)$$

Which $0<\pi_o^{*Stage\,1}\le 1$ and $0<\pi_o^{*Stage\,2}\le 1$ are the optimal efficiency scores of each subsection of the unit under evaluation DMU_o.

Definition 3. The unit under evaluation DMU_o is called the overall efficient if and only if $\pi_o^{*Overall}=1$; Otherwise, it is inefficient.

Definition 4. The first and second subsections of the unit under evaluation DMU_o is said to be efficient if and only if $\pi_o^{*Stage\,1}=1$ and $\pi_o^{*Stage\,2}=1$, respectively; Otherwise, there are inefficient.

Theorem 1. The unit under evaluation DMU_o is called overall efficient $(\pi_o^{*Overall}=1)$ if and only if both subsections of DMU_o are efficient, i.e. $\pi_o^{*Stage\,1}=\pi_o^{*Stage\,2}=1$.

Proof. Consider unit under evaluation DMU_o is overall efficient, i.e. $\pi_o^{*Overall}=1$. In optimality, $\pi_o^{*Overall}=1$ is equivalent to $\bar{\alpha}_m^*=\bar{\beta}_n^*=\bar{\gamma}_j^*=\bar{\delta}_t^*=\bar{\varphi}_r^*=1\ \forall m,n,j,t,r$. Therefore, according to expressions (7) and (8), we have $\pi_o^{*Stage\,1}=1$ and $\pi_o^{*Stage\,2}=1$ means that DMU_o is efficient in both subsections. Similarly, the converse of the theorem is true.

4. Case Study

Although the increase in the level of gross domestic products (GDP) of a country causes the economic growth of a country, it causes irreparable damage to the environment. Therefore, in recent years, many countries have tried to solve this problem. One of the proposed solutions is a recycling system for the manufacturing industry. In this regard, industries that show better performance can be good examples to pattern other industries. Evaluating the performance of manufacturing industries can play an important role in improving these conditions. In recent years, China is one of the countries that has made great efforts in this direction.

it should be noted that manufacturing industries that, in addition to industrial production, are responsible for recycling materials such as polluted water and toxic gases have a network structure. Therefore, in this section, the performance of 30 industrial production centers from different regions of China which have a two-stage structure following Figure 2 is estimated.

- **Stage 1: Production stage**

  Inputs:

  Labor (x_1), Energy (x_2), Capital (x_3),

  Undesirable outputs:

  Wastewater (w_1), Waste gas (w_2), Solid waste (w_3),

  Final desirable output:

  Gross industrial products (v),
- **Stage 2: Pollution treatment stage**

  Inputs:

  Investment (z), Wastewater (w_1), Waste gas (w_2), Solid waste (w_3),

  Desirable outputs:

  Recycled materials (y).

It is noteworthy that wastewater (w_1), waste gas (w_2), and solid waste (w_3) are intermediate measures and they have output and input roles for the first stage and the second stage, simultaneously. The statistical summary of consumed inputs and produced outputs are collected in Table 1. (see Wu et al. [8])

The performance of 30 Chinses industrial production centers and their subsections are evaluated by model (6). The results obtained are depicted in Figure 3.

Three lines in Figure 3 shows the efficiency of the whole two-stage structure and each of its subsections. It seems that there are scatters among the performance of various regions especially the performance of the second stage. Also, it can be seen that overall efficient regions are also efficient in their subsections. This is exactly according to theorem 1. The details of the performance of the whole system and subsections of 30 regions are given in Table 2.

Columns 2, 3, and 4 in Table 3, respectively, show the efficiency of the whole system and their subsections that obtain by model (6) and expressions (7), (8). As stated in definitions 3 and 4, a unit is overall or partial efficient if and only if the efficiency score of that unit is equal to one. Accordingly, we continue the discussion.

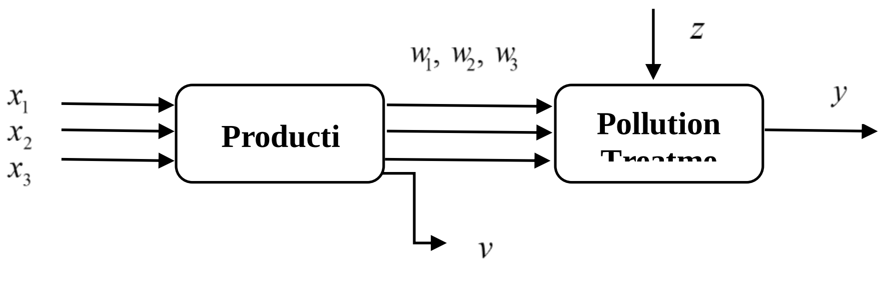

Figure 2- The structure of two-stage industrial production process

Table 1- The statistical summary of data

Indicates	Min	Max	Mean	Std. Dev.
x_1	12.44	1568	318.0933	359.6899
x_2	1359	34808	12983.7	8172.431
x_3	1621.38	66134.06	19752.22	16928.9
v	1381.25	92056.48	22234.27	24745.89
w_1	5782	217426	71219.53	61693.49
w_2	1360	56324	17305.07	12169.53
w_3	212	31688	8031	6635.777
z	0.41153	36.4491	11.06698	8.211797
y	3.16232	286.3867	59.28278	65.01208

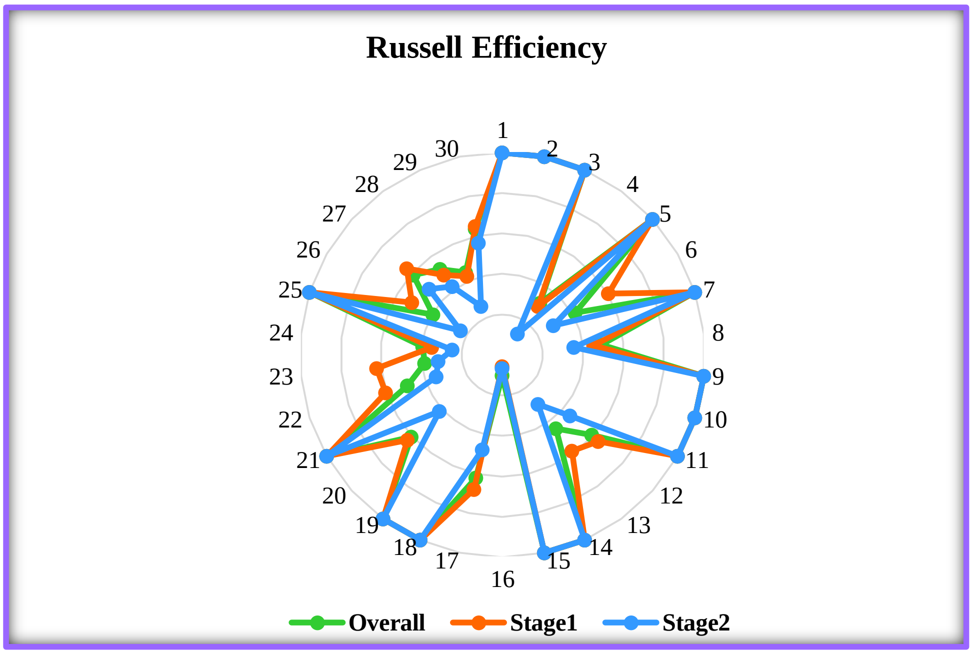

Figure 3- Russell efficiency of the two-stage process

Table 2- The results obtained the proposed model

Regions	$\pi_o^{*Overall}$	$\pi_o^{*Stage\,1}$	$\pi_o^{*Stage\,2}$
1	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000
4	0.3165	0.2981	0.1288
5	1.0000	1.0000	1.0000
6	0.4148	0.6040	0.2909
7	1.0000	1.0000	1.0000
8	0.4926	0.4521	0.3549
9	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000
12	0.5928	0.6377	0.4505
13	0.4490	0.5874	0.2995
14	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000
16	0.1016	0.0576	0.0660
17	0.6222	0.6797	0.4777
18	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000
20	0.6049	0.6296	0.4170
21	1.0000	1.0000	1.0000
22	0.4923	0.6061	0.3444
23	0.3856	0.6239	0.3201
24	0.3962	0.3490	0.2486
25	1.0000	1.0000	1.0000
26	0.3969	0.5165	0.2368
27	0.5918	0.6364	0.4870
28	0.5217	0.4889	0.4184
29	0.4463	0.4264	0.2621
30	0.6369	0.6494	0.5661

In the proposed approach, industrial units 1, 2, 3, 5, 7, 9, 10, 11, 14, 15, 18, 19, 21, and 25 are overall efficient. According to theorem 1, these units are overall efficient because their subsections are efficient. The rest of the units are not overall efficient. The lowest overall efficiency score, 0.1016, belongs to unit 16. Also, the lowest efficiency score for the first and the second stage (production and pollution treatment stage) are 0.0576 and 0.0660, respectively. These scores belong to unit 16.

A comparison of the performance of the first and second subsections shows that the second stage is generally less efficient than the first stage. Since total performance is determined by the performance of its subdivisions, it is clear that the performance of the second stage has a

greater effect on overall performance. The standard deviation also shows this well.
The standard deviation of the efficiency score for the whole system and each component is also 0.2834, 0.2691, and 0.3445, respectively, which shows that the scattering of efficiency score of the second stage (pollution treatment stage) is more than the entire system and the first stage (production stage). It should be noted that this will not necessarily happen in all two-stage structures.

5. Conclusion

In recent years, increasing industrial pollution has led to environmental problems. Therefore, researchers have drawn attention to the issue of recycling. The performance evaluation of industrial systems can detect its strength and weakness. Hence, an appropriate model that can evaluate such systems properly is essential.
The main goal of this paper is to present a model which can evaluate the performance of industrial systems consisting of production and recycling sections. A model that does not require the importance of the attributes of inputs and outputs. Moreover, a model that can establish proper communication between system subsections.
According to this, first, a two-stage structure that consists of undesirable intermediate measures is introduced. Then, based upon the Russell measure model, a non-radial model under variable returns to scale assumption is defined for assessing the relative efficiency of the proposed structure. In the suggestion model, the weak disposability assumption to handle desirable and undesirable outputs is considered. Finally, to illustrate the suggested model, a real application in the field of manufacturing industries of 30 regions of China is used.

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