Available online at http://ijdea.srbiau.ac.ir

Int. J. Data Envelopment Analysis (ISSN 2345-458X)

Vol.8, No.3, Year 2020 Article ID IJDEA-00422, 14 pages
Research Article

International Journal of Data Envelopment Analysis

Science and Research Branch (IAU)

Influence of undesirable output factor on efficiency determination in DEA: A Case study of hospital emergency Tehran

A. Monzeli[1], B. Daneshian*[1], G. Tohidi[1], M. Sanei[1], S. Razavyan[2]

[1] Department of Mathematics, Islamic Azad University, Central Tehran Branch, Iran
[2] Department of Mathematics, Islamic Azad University, South Tehran Branch, Tehran Iran

Received 2 March 2020, Accepted 19 June 2020

Abstract

One of the major concerns of managers in the field of health is the optimal allocation of manpower and resources; So in this study, to the quality of provided services in this area not to be damaged, we have used the data envelopment analysis (DEA) model to determine the efficiency of hospital emergency departments and possible improvements. Traditional DEA models do not seek to reduce undesirable outputs and increase undesirable inputs, so in this study, decision-making units (DMUs) effect on efficiency has also been investigated in addition to determining the efficiency of decision-making units (DMUs) with the presence of some undesirable output components. To do this, first, the set of proper production possibilities has been defined according to the problem assumptions and while examining the performance and ranking with Andersen-Petersen and Super-SBM models, a new method has been provided to determine the unfavorable performance of some output components in decision-making units. We have specified the effect of undesirable output on determining efficiency. We have also provided a real example of 30 hospital emergencies for 5 desirable inputs and 4 desirable outputs and one undesirable output, solved that example, and determined the efficiency score.

Keywords: Data Envelopment Analysis, Undesirable output, Efficiency.

* Corresponding author Email: be_daneshian@yahoo.com

1. Introduction

Medical care is a complex configuration that includes primary, secondary and post-care (AL-Refaie, et al., 2014) [1] and hospitals are one of the most important organizations in this field, the most crowded part of them is the emergency room. It is considered the most vital element in the treatment system (Farzaneh Khalqabad et al., 2019) [2].

The emergency department of the hospital is a unit that works 24 hours a day, 365 days a year to provide treatment for all emergency, semi-emergency and non-emergency patients quickly. This department receives, and treats more than 33 million patients in the country every year. Overcrowding in hospital emergencies is a global issue (at the moment because of COVID 19 virus spreading) that has become a major concern due to the increasing number of patients, facing too complicated cases and limitation of available resources for hospitals and can delay the provision of emergency services to patients. (Kuo, Leung, et al., 2018) [3].

Data envelopment analysis is one of the most popular methods for determining efficiency, and the efficiency boundary based on the concept of the condition of defective units was first proposed by Pareto, an Italian economist, in 1927. This concept was used in production by Kopman in 1951 and was also introduced by Farrell to evaluate performance in 1957 (Cooper et al. 2002).

In 1987, Charnes, Cooper, and Rhodes identified efficient boundaries using linear planning and used them to determine productivity [4]. they used both the output-axis and input-axis models by this method. Although these two models are not the only ones used, they are still the most popular DEA model. Many researchers use the DEA method to determine the efficiency boundary and efficiency evaluation (Jahanshahloo and Hosseinzadeh Lotfi, Adel Azar, and ... a 2005) [5].

In many practical issues, some inputs of decision-making units' increase maybe result in efficiency reduction and their reduction maybe efficiency increase; such as waste recycling operations, scrap metal and, glass, etc., where it is necessary to decrease the undesirable inputs to improve the level of efficiency. or some of the outputs of decision-making units may be such that increasing these outputs reduces efficiency and decreases it increases efficiency. Consider the waste of a factory or patients' deaths in hospitals and doctors and nurses' dismissals in training centers, should be reduced as an undesirable output to increase efficiency. Undesirable outputs are generally desirable products and therefore the output can be improved only by reducing them.

There are some methods for importing undesirable outputs into the DEA that can be divided into two categories:

1. Direct methods
2. Indirect methods.

In indirect methods, undesirable inputs and outputs in each decision-making unit are converted into desirable inputs and outputs by a descending uniform function and then the unit's efficiency is evaluated using DEA standard models, such as the method by Kopman in 1951, the method by Gallin and Roll in 1989 for importing undesirable outputs and Seiford's method for importing undesirable inputs and outputs in 1990.

Direct ones are methods that use assumptions in the production possibility set so that they are used in evaluating desirable input and output. Like the method was proposed by Farrel in 1989.

From 1980, productivity efficiency evaluating has been investigated in presence of undesirable outputs.

One of the first studies conducted by Petman in 1983, was the investigation and use of undesirable outputs in productivity efficiency evaluation. Also, this study was discussed by Kewes and colleagues in the discussion of production indicators in 1982, which also included desirable and undesirable outputs. Thus, in 1983, Petman in completing a study based on an undesirable output value scheme, determined the estimated shadow prices of the contract instead of market prices. A few years later, In 1993, Fire and colleagues, proposed another alternative method for estimating shadow prices using the distance function. A few years later, in 1997, Chang and his colleagues proposed another way to evaluate the productivity and efficiency of undesirable outputs. They proposed the use of the directional distance function to increase the desirable outputs and reduce the undesirable data simultaneously. Dyson Skill and colleagues (2001) and Seyford and Zhou (2002) have also proposed various methods for examining undesirable outputs and inputs.

In 2009, Tone and colleagues proposed an efficiency measurement model based on auxiliary variables called SBM.

Undesirable outputs are generally desirable products; therefore, they can be reduced only by an accompanying reduction in the second product. To understand this concept, the shadow price of an undesirable output must be negative and on the opposite, it must be positive for desirable output. Based on these conditions, kao et al. in a paper presented a data envelopment analysis model that allows the production units under evaluation to determine the shadow price for both desirable and undesirable outputs to maximize the measured efficiency score. The proposed model satisfies the assumption of the poor usability of outputs. It is also shown that there is a directional function model in a group that has been widely used in undesirable outputs modeling. However, unlike conventional measures of directional distance, the proposed model can provide an efficiency rate in the range of zero and one for easy comparison among inefficient produced units [6].

Another strategy is that the total losses of fixed outputs and sometimes the undesirable outputs of other DMUs are commensurate with their actual outputs, and this damage depends on the actual outputs and reduces output (inputs increase) (Yang et al. 2014).

In addition to the previous model, in 2008, Linus and colleagues solved the problem of one-dimensional output, and finally in 2014, with the expansion of that model, introduced the situation where there was an undesirable output with a fixed value.

In 2017, Kao et al [6]. Developed two previous models to reduce the impact of undesirable outputs, and in 2019 Chiang Kao presented another article evaluating efficiency in the presence of undesirable outputs by changing the shadow price [6].

In 2016, Dakpo, Jeanneaux et al. and Pham and Zelenyuk in 2019 presented a critical review of technological models with undesirable output, most of them are multidimensional technologies.

Estimation of the margin of non-parametric production limits parameters in the presence of undesirable outputs has been proposed by Victor Podinovski in 2019 [9].

Qingxian An et.al. in 2019 An Application to the Environmental Efficiency of China's Regional Industry, used modified Distance Friction Minimization Model with Undesirable Output [6].

In 2019, Jie Wu et al. studied the environmental efficiency measurement of thermoelectric power plants using an efficient frontier DEA approach with fixed-sum undesirable output [9].

In 2020, Malin Song et al. studied accident deaths as an undesirable output in the

production and evaluation of the safety efficiency of coal mines in Chinese [10].
In 2020, Barnabé Walheer studied output, input, and undesirable output interconnections in data envelopment analysis using convexity and returns-to-scale [12].
In 2020, Shiwei Yu et al. assessed the environmental efficiency of one province in China by an improved network of data envelopment analysis models with undesirable output [12].
In 2020, Roberto Gómez- Calvet et al. evaluated European Energy Efficiency Evaluation based on the use of super-efficiency under undesirable outputs in SBM models [13].
In this research, we have presented the production possibility set by the concept of undesirable output. Then, like the output, we have examined the efficiency of decision-making units of hospital emergency units as a case study.

2. Background

Suppose we have n observations on n DMUs with input and output vectors (x_j, y_j) for j = 1, 2,..., n. Let $x_j = (x_1, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{sj})$ All $x_j \in R^m$ and $y_j \in R^s$ and $x_j > 0$, $y_j > 0$ *for j = 1, 2,... n.* The input matrix *X* and output matrix *Y* can be represented as $X = [x_1, \dots, x_j, \dots, x_n]$ and $Y = [y_1, \dots, y_j, \dots, y_n]$ Where X is an $(m \times n)$ matrix and *Y* an $(s \times n)$ matrix.
The production possibility set T is generally defined as

$$T = \{(x, y) | x \ \ can\, produce\, y\} \quad (1)$$

In DEA, the production possibility set under a Variable Return to Scale (VRS) technology is constructed form the observed data (x_j, y_j) for j = 1, 2, ..., n as follows:

$$T = \left\{ (x, y) \middle| \begin{array}{l} x \geq \sum_{j=1}^{n} \lambda_j x_j ,\, y \leq \sum_{j=1}^{n} \lambda_j y_j, \\ \lambda_j \geq 0, \sum_{j=1}^{n} \lambda_j = 1, j = 1, \dots, n \end{array} \right\} (2)$$

In the absence of undesirable factors when a $DMU_o, o\epsilon\{1,\ 2, \dots ,\ n\}$ is under evaluation, we can use the following BCC model:

$$\begin{array}{l} \min\ \theta \\ \quad \theta x_o - X\lambda \geq 0 \\ \quad Y\lambda \geq y_o \\ \quad \lambda \geq 0 \end{array}$$

Corresponding to each output *y*, *L(y)* is defined as the following:

$$L(y_j) = \{x | (x, y_j) \in T\} \quad (4)$$

In fact, $L(y_j)$ is a function that y_j portrays to a subset of inputs so that inputs can produce y_j.
Now suppose that some inputs are undesirable so input matrix *X* can be represented as $X = (X^g, X^b)^T$, where $X^g = (x_{1j}^g, \dots, x_{m_1 j}^g),\ j = 1, \dots, n$ and $X^b = (x_{1j}^b, \dots, x_{m_1 j}^b)$ $j = 1, \dots, n\ are\ (m_1 \times n)\ and\ (m_2 \times n)$ matrixes that represent desirable (good) and undesirable (bad) inputs, respectively.
And similarly, suppose that some outputs are undesirable so outputs. Matrix *Y* can be represented as $Y = (Y^g, Y^b)^T$, where $Y^g = (Y_{1j}^g, \dots,\ Y_{s_1 j}^b), j = 1, \dots, n$ and $Y^b = (Y_{1j}^b, \dots, Y_{s_2 j}^b), j = 1, \dots, n$ are $(s_1 \times n)\ and\ (s_2 \times n)$ matrixes that represent. Desirable (good) and undesirable (bad) inputs, respectively.

Definition 1: Let DMU of $(x_1^g, x_1^b, y_1^g, y_1^b)$ is dominant to DMU of $(x_2^g, x_2^b, y_2^g, y_2^b)$ if $(x_1^g \leq x_2^g, x_1^b \geq x_2^b)$ and $(y_1^g \geq y_2^g, y_1^b \leq y_2^b)$ the unequal be strict at least in a component. So that,

$$\begin{pmatrix} -x_1^g \\ x_1^b \\ y_1^g \\ -y_1^b \end{pmatrix} \geq \begin{pmatrix} -x_2^g \\ x_2^b \\ y_2^g \\ -y_2^b \end{pmatrix}$$

Definition 2: DMU_o is efficient if in T there is no DMU to be dominant over it.
We consider the properties of the Production Possibility Set as the following:

(1) T is convex.
(2) T is closed.
(3) The monotony property of desirable inputs and outputs. So that,

$$\forall u \in R_{+}^{m_1}, v \in R_{+}^{s_1},$$
$$(x^g, x^b, y^g, y^b) \in T$$
$$\Rightarrow (x^g + u, x^b, y^g - v, y^b) \in T$$

This is not necessarily established for undesirable factors, because in this case, T has no efficient DMU.

We can define the Production Possibility Set T satisfying (1) through (3) by

$$T = \{(x^g, x^b, y^b, y^g)|$$
$$x^g \geq \sum_{j=1}^{n} \lambda_j x_j^g, x^b = \sum_{j=1}^{n} \lambda_j x_j^b, y^b = \sum_{j=}^{n} \lambda_j y_j^b, y^g \leq \sum_{j=1}^{n} \lambda_j y_j^g$$
$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, ..., n$$

3.Measures of Efficiency Using Undesirable Output

In input oriented data, to determine the efficiency of the unit under evaluation, we seek to improve the unit under evaluation by reducing the desirable input and increasing the undesirable input. And in output oriented data, we increase desirable output and decrease the undesirable output. Farell (1989) introduced a model to increase and decrease desirable and undesirable output, respectively, But his model had a problem and it was nonlinear form.

$[TR\beta]$ method introduced by Ali and Seiford (1990) increase desirable output and decrease undesirable outputs simultaneously, but the problem with this method is that the efficiency size depends on β amount. increase will increase efficiency score for inefficient decision-making units [14].

There are some other methods such as [WD] and [MLT] that were introduced by Far (1989) and Galony and Roll (1989) respectively that. in some methods such as [WD] and [MLT] decrease undesirable outputs only accompanies with decreasing desirable outputs . However, we believe that efficiency improvements are achieved if the desirable output increases or the undesirable output decreases. This will be investigated in nature and output oriented in the presence of undesirable output.

3.1 Andersen-Petersen method

Data envelopment analysis evaluates the relative efficiency of decision-making units but does not allow ranking of efficient units. In fact, if the number of efficient units is large, this will be problematic. In this approach, the main idea is that the under evaluation unit is compared with a linear combination of other units. so that the unit itself is not considered. For the input model, these results can reach values that are efficient with respect to DMUo, then use these values to rank DMUs, and delete some (but not all) of the connections that occur for efficient DMUs. (Cooper, Seiford & Tone, 2006). This model is presented in (Andersen & Petersen, 1993) as follows [15]:

$$Min\ E_j - \delta\left(e'^{S^-} + e'^{S^+}\right)$$
$$Z; S^-; S^+ \geq 0$$
$$st$$
$$E_j X_j = \sum_{\substack{k=1 \\ k \neq j}}^{n} z_k X_k + s^-$$
$$Y_j = \sum_{\substack{k=1 \\ k \neq j}}^{n} z_k X_k + s^+$$

X_j is the next m vector and Y_j is the next s vector for the j unit; E_j Is a scalar that is used to define the input vector of the j decision maker unit in the output vector output of the j decision maker unit within the reference technology.

Z is an incremental vector denoting an increase in the unit k, δ is a very small non-

Archimedean value, and e' is the linear vector (1, ...,1) suitable for the dimension. This method can be used to rank between efficient and inefficient units, calculate the distance of the unit understudy from the linear combination of other units that do not include the unit understudy, and calculate these distances radially (if an input increases in one dimension and does not decrease other dimensions).

3.2 Super-SBM method

For purposes such as the possibility of eliminating DMUs, the Andersen-Petersen measurement is considered as a defect in the behavior of mild variables; It is also considered because it does not provide the behavior of mild variables with fixed units for us (Cooper et al., 2006) [4].

This model (set of production possibilities) has been introduced in the research of Cooper et al. As follows:

$$p(x_o, y_o) = \left\{ (\bar{x}, \bar{y}) \middle| \bar{x} \geq \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j x_j \,,\, \bar{y} \leq \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j y_j \,,\, \bar{y} \geq 0, \lambda \geq 0 \right\} \quad (7)$$

In addition, by defining a subset of $\bar{p}(x_o, y_o)$ we will have $p(x_o, y_o)$:

$$\bar{p}(x_o, y_o) = p(x_o, y_o) \cap \{\bar{x} \geq x_o \,,\, \bar{y} \leq y_o\}$$

Now to determine the distance between (x_o, y_o) and $(\bar{x}, \bar{y})$, which are both members of $\bar{p}(x_o, y_o)$, the following index is defined. This index cannot be less than 1:

$$\delta = \frac{\frac{1}{m}\sum_{j=1}^{m} \frac{\bar{x}_j}{x_{io}}}{\frac{1}{s}\sum_{r=1}^{s} \frac{\bar{y}_r}{y_{ro}}} \quad (8)$$

Therefore, the SBM super-efficiency model is presented as follows:

$$Min \qquad \delta = \frac{\frac{1}{m}\sum_{j=1}^{m} \frac{\bar{x}_j}{x_{io}}}{\frac{1}{s}\sum_{r=1}^{s} \frac{\bar{y}_r}{y_{ro}}}$$

$$St$$

$$\bar{x} \geq \sum_{j=1,\neq 0}^{n} \lambda_j x_j \quad (9)$$

$$\bar{y} \leq \sum_{j=1,\neq 0}^{n} \lambda_j y_j$$

$$\bar{x} \geq x_o \,, \bar{y} \leq y_o \;,\; \lambda \geq 0$$

Where the distance between the decision units (x_o, y_o), $(\bar{x}, \bar{y})$, x_j and y_j are the input and output variables in the decision unit j.

Uniformity and stability in measurement, considering surplus input variables and output shortage variables in calculating efficiency along with the need to perform efficiency calculation based on the reference set, are the features of this method.

The different methods available for ranking may offer similar results, but the use of several methods gives the decision maker the flexibility to choose a result that adjust to reality and other considerations that are not considered in the model. (For example cost here).

3.3 Nature of the output

Suppose $DMU_o = (x_o^g, x_o^b, y_o^g, y_o^b)$ be unit under evaluation, corresponding to the input $x_o = (x_o^g, x_o^b)$ and for outputs set $p(x_o^g, x_o^b)$ is defined as follows:

$$p(x_o^g, x_o^b) = \left\{ (y^g, y^b) \middle| (x_o^g, x_o^b, y^g, y^b) \in T \right\}$$

and we consider the subset of $p(x_o^g, x_o^b)$ as :

$$\partial^p p(x_o^g, x_o^b) = \left\{ (y^g, y^b) \middle| \begin{array}{l} \forall (u,v) \geq 0, (u,v) \neq 0 \\ \quad \Rightarrow \quad (y^g + u, y^b - v) \notin p(x_o^g, x_o^b) \end{array} \right\} \quad (10)$$

That $\partial^s L(y_o^g, y_o^b)$ includes all inputs of the efficient DMUs which can produce(y_o^g, y_o^b).

The model to evaluate the efficiency of DMUo with the most decrease of y_o^g and the most increase of y_o^b is as follows:

$$NE^d(x_o, y_o) = sup\{\mathcal{B} | y_o + \mathcal{B}d \in p(x_o)\}$$

Where $d = (d^g, d^b)$ indicate the direction of unit under evaluation such that $d^g \in R_+^{s_1}$ and $d \in R_-^{m_2}$ leads to increase desirable output and decrease undesirable output.

In this research, we direct the desirable outputs to the efficient boundary in a radial direction; Thus: $d^g = y_o^g$

We also reduce the undesirable outputs in the radial direction, i.e.: $d^I = -y_o^g$

Therefore, according to the definition we have:

$$\begin{aligned} \beta_o^* = Max \quad & \beta_o \\ st. \quad & \\ & \sum_{j=1}^{n} \lambda_j x_j^g + s^- = x_o^g \\ & \sum_{j=1}^{n} \lambda_J x_j^b = x_o^b \qquad\qquad (11) \\ & \sum_{j=1}^{n} \lambda_j y_j^g - s^+ = y_o^g + \beta_o d_o^g \\ & \sum_{j=1}^{n} \lambda_j y_j^b = y_o^b + \beta_o d_o^b \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0 \;\; for all \;\; j = 1, \ldots, n \end{aligned}$$

Theorem 1: The DMUo in model (11) is efficient if and only if

1) $\beta_o^* = 1$

2) All slacks are zero for all optimal solutions.

Theorem 2: If β_o^* be optimal solution of model (11) in DMU_o then

$$(y_o^* + \beta_o^* d_o^g + s^{+^*}, y_o^b + \beta_o^* d_o^b) \in \partial^p p(x_o^g, x_o^b)$$

3.4 Case Study

Since that a large percentage of patients go to public hospitals, they usually waste a lot of time due to the formation of long queues and lack of resources, and sometimes due to these reasons, some critically ill patients die, and these issues cause dissatisfaction of the patient's relatives. In the present study, the emergency center of public and private hospitals in Tehran province has been selected as a research sample and data were collected in one month and the queue time has been collected through the queuing device. In order to have enough decision-making units, the simulation output was also used. The purpose of this study is practical and data nature was considered quantitatively, which is evaluated in the following example using basic models and designed models.

3.5 Input and Output Indicators and Data

Considering a large percentage of patients go to public hospitals, they usually waste a lot of time because of the formation of long queues and lack of resources. In this research, 30 emergency centers of Tehran hospitals were selected for our research population. The purpose of this study is practical and the data considered quantitatively. The data of this study were collected by attending the emergency department and spending our time with the patients come in a day (three-shift wards). We were working around the clock, all days and night long, an entire 24-hours a day to collect and arrange the statistical data then we have calculated the average case of data. In the emergency department, after the patients' arrival, the triage divided patients into five levels based on their feeling, and in triage form, the lower level of patients show us the worse condition. Level 1, 2, and 3 patients need to go to specialists, and Levels 4 and 5 should go to a general practitioner for other treatment processes such as outpatient treatment, short-term hospitalization, or the need to go to a specialized ward for treatment. System inputs and outputs (emergency wards) were determined based on face-to-face observation and the above explanations. The most important factor is the human resource in the treatment process such as specialist physicians, general practitioners, and nurses. Also, the patient waiting time in line, facilities, and equipment, the most important equipment for hospitalizing is a hospital bed, it's urgent for one's needs to use treatment service.

We considered two types of standard for this system output, the first one is based on the patients' numbers who recovered and the second indicator was the length of stay of recovery or the patient's presence in the emergency room. Long queues in the wards indicate the efficiency or inefficiency of the treatment system in people's minds. The shorter queue in the service cab is more helpful for recover patients quickly. Therefore, according to hospital managers and experts expertise, the number of specialist physicians, general practitioners, nurses, hospital beds, and waiting time in the treatment queue were considered as a system input and patients who recuperate, the time that patients stay in service cab considered to three periods, less than 4 hours, between 4 up to 12 hours and more than 12 hours. Those periods determined as the desired outcome. Death rate is considered an unfavorable outcome.

3.6 Example

We consider 30 decision-making units with 5 entrances including a number of nurses, general practitioner, specialist doctor or emergency medicine, number of hospital beds and waiting time for patients to produce 4 desirable outputs including improved and discharged from hospital, outpatient treatment in time less than 4 hours, stay between 4 and 12 hours and the number of hospitalizations more than 12 hours and an unfavorable output or number of deaths in the hospital. These decision-making units are described in Table 1.

Table 1- Data of hospital emergency departments

	Inputs	undesirable output	Outputs

DMU_S	Nurse X_1	Doctor X_2	Specialist doctor X_3	BedX_4	waiting timeX_5	death y^b	Cleared persons Y_1	Stay less than 4 hours Y_2	Stay between 4 to 12 hours Y_3	Stay more than 12 hours y_4
DMU_1	18	1	1	38	27	2	1155	295	265	32
DMU_2	19	2	1	41	15	3	1254	338	305	30
DMU_3	21	2	2	42	17	1	1259	325	261	28
DMU_4	19	2	1	39	21	4	1244	320	263	29
DMU_5	20	2	1	40	25	2	1254	323	271	29
DMU_6	22	2	2	42	34	7	917	125	169	22
DMU_7	21	2	1	41	26	3	1245	332	237	28
DMU_8	21	2	1	41	18	2	1254	323	270	28
DMU_9	20	2	1	40	19	1	1204	340	265	27
DMU_{10}	20	2	1	39	17	1	1254	315	270	29
DMU_{11}	20	2	1	39	18	2	1260	324	272	29
DMU_{12}	19	1	2	39	29	4	944	192	246	30
DMU_{13}	18	1	2	38	29	5	985	194	240	28
DMU_{14}	19	1	2	40	30	1	1085	295	226	32
DMU_{15}	19	1	2	39	30	7	764	162	116	20
DMU_{16}	20	1	2	41	31	5	691	150	244	19
DMU_{17}	20	1	2	42	31	3	994	192	246	28
DMU_{18}	20	1	2	41	25	4	931	201	256	29
DMU_{19}	21	1	2	42	26	2	941	188	274	28
DMU_{20}	20	1	2	41	25	1	1145	284	275	27
DMU_{21}	21	1	2	41	25	5	948	193	212	32
DMU_{22}	18	1	1	38	27	4	994	305	266	28
DMU_{23}	20	1	2	39	26	2	941	245	246	27
DMU_{24}	19	1	2	39	27	3	984	189	274	29
DMU_{25}	20	1	2	41	27	2	948	193	247	28
DMU_{26}	22	2	2	43	14	1	1259	335	271	30
DMU_{27}	23	2	2	44	32	6	1015	224	261	24
DMU_{28}	21	2	1	42	13	1	1370	365	322	35
DMU_{29}	22	2	1	42	15	2	1244	320	270	29
DMU_{30}	23	2	2	44	14	5	1154	314	272	31

Table 2 - Results of the implementation of two models for emergency departments of hospitals

Super-SBM		Andersen-Petersen		DMUs	Super-SBM		Andersen-Petersen		DMUs
rank	Efficiency	rank	Efficiency		Efficiency	rank	Efficiency	Rank	
14	1	26	0.8873	16	1/00052	5	1.162	2	1
15	1	22	0.9146	17	1/00067	4	1.0469	4	2
7	1/00047	13	0.9818	18	1	12	0.919	19	3
6	1/0005	10	0.9986	19	1	12	0.9997	9	4
3	1/00072	3	1.0683	20	1/00033	9	0.9611	15	5
15	1	5	1.0463	21	1	12	0.6693	29	6
1	1/00324	6	1.0339	22	1	12	0.9318	18	7
8	1/00049	21	0.9156	23	1	12	0.9377	16	8
11	1/00006	7	1.0193	24	1	13	0.9781	14	9
6	1/00092	20	0.9169	25	1	13	0.9857	12	10
16	1	25	0.904	26	1	13	0.9905	11	11
18	1	28	0.7801	27	1	13	0.9375	17	12
2	1/00136	1	1.327	28	1	14	0.9048	24	13
10	1/00017	23	0.908	29	1/00285	2	1	8	14
17	1	27	0.8663	30	1	14	0.6615	30	15

According to the data in Table 2, in the analysis of the Andersen-Petersen model, the emergency departments of hospitals 28, 2 , and 1, respectively, have the best services and are ranked higher in terms of efficiency, and hospitals 15 and 6, as well as 16 and 27, have the lowest efficiency and need to investigate the input and output factors of these wards and according to the output results of Gomez software, the high waiting time for patients in these wards and a low number of recoveries are the reasons for inefficiency. Therefore, to improve the efficiency of the ward, hospital officials should reduce the waiting time by increasing the efficiency of the staff, so that many patients will be treated and discharged.

In the analysis of the Super-SBM model, the efficiency of the units is almost the same as the previous model. In this model, units 22, 14, and 20 have the highest efficiency, respectively, and 27, 30, and 21 have the lowest efficiency. ward nurses should be motivated to increase service delivery. There is low efficiency in 27 emergency rooms due to nurse's and doctors' underemployment, and they must increase the efficiency of the system by moving low-performing staff.

Table 3 - Results of the proposed model for emergency departments of hospitals

DMUs	β_o^*	BCC	DMUs	β_o^*	BCC	DMUs	β_o^*	BCC
DMU_1	1	1	DMU_{11}	0.9905	1	DMU_{21}	0.9663	1
DMU_2	0.9994	1	DMU_{12}	0.8457	0.9375	DMU_{22}	0.9859	1
DMU_3	1	0.919	DMU_{13}	0.7548	0.9048	DMU_{23}	1	1
DMU_4	0.8997	1	DMU_{14}	1	1	DMU_{24}	0.9959	1
DMU_5	0.969	0.9711	DMU_{15}	0.5568	0.6615	DMU_{25}	0.9759	0.9169
DMU_6	0.5782	0.6693	DMU_{16}	0.7895	0.8873	DMU_{26}	1	0.919
DMU_7	0.9108	0.944	DMU_{17}	0.9036	0.9146	DMU_{27}	0.7801	0.8106
DMU_8	0.9377	0.9405	DMU_{18}	0.9818	1	DMU_{28}	1	1
DMU_9	1	1	DMU_{19}	1	0.9986	DMU_{29}	0.9658	0.908
DMU_{10}	1	1	DMU_{20}	1	1	DMU_{30}	0.7324	0.8857

According to the results of Tables 2 and 3, the efficiency of each unit has been determined and ranked based on the DEA classical and proposed models. It is determined by the comparison of the results of these tables those units that have the most unfavourable outputs such as, DMU_2, DMU_4, etc, their efficiency has been decreased and even changed from efficient to inefficient, and vice versa. In addition, some units such as DMU_3, DMU_{19}, DMU_{23} and DMU_{26}, which were inefficient in the output-driven BCC model with efficiency on a variable scale, have been changed from inefficient to efficient due to low rate of mortality in these hospitals.

DMU2 was efficient in BCC-O and Anderson Peterson models, its inputs include 19 nurses, 2 general practitioners, 1 specialist and 41 beds with a waiting time of 15 minutes. Unfavorable output or mortality are three persons, its efficiency has been reduced in the proposed model. DMU_1 is almost similar to this unit, but its unfavorable output is less. Also, because it has less human factor than DMU_2, the waiting time or queuing is longer. However, because the number of services provided by this unit to patients for more than 12 hours is more, its efficiency has not changed in the proposed model. In the proposed model, the inefficiency of DMU_2 is due to the poor performance of human resources, and if they reduce one unit of nurse and three units of hospital bed with the same output, its efficiency will be maintained.

Consider DMU_6. Patients wait 34 minutes for treatment. Although human resources are more than other inputs, but it has the highest mortality. This inefficiency is due to shortage or underwork of human resources in the classical and proposed models. Its causes should be investigated by managers to prevent deaths caused by lack of timely care, prolongation of treatment bureaucracy or increase staff motivation, and so on.

DMU_{27} is similar to DMU_6, despite the fact that the number of human resources and beds are standard, but it has poor performance and mortality is higher in this unit. So, it is clear that the motivation of employees is low in this hospital according to the number of inputs and longer waiting time. We gave the conclusion gained by using DEA models to the internal manager of the ward. He believed that one of the factors in this case was the existence of the planning force and the nurses who were somehow punished or they feel that they have been transferred to this section for punishment, so they do not do their duty on time and it causes more time in the treatment process. Therefore, according to the explanations, we conclude that the higher the unfavorable output in the units under evaluation, the lower the efficiency and vice versa, these factors also affect on determining the efficiency. Aaccording to the real example of hospital emergency, the proposed model is acceptable and these cases is true in the model.

4.Research innovation

These days because of Corona spread, serious concern among managers is the optimal allocation of staff and resources in health service. So that while maintaining the quality of medical services they try to reduce the death rate and prevent long-term hospitalization, or long queue of patients in emergencies. For these reasons, some patients die in critical condition, and these issues cause dissatisfaction among the patient's relatives.

Since the importance of the health system, it is not possible to stop the system for changes. Therefore, in the present study, we evaluate department efficiency and ranking them by using the classical DEA model, then we use this model in society practically. Unpleasant output (death rate) makes a negative view on people who judge treatment service. People related the outcome to dereliction of emergency or treatment service.

If this model uses operationally in the treatment system of all hospitals. Managers can consider the needs of manpower, facilities, and other resources scientifically. Hence satisfaction of the people about the hospital treatment staff will be increased. The approach presented in this research can also be used to optimize other departments and help senior managers to have a better apperceive of the impact of changes in the system.

5. Conclusion

Reducing the waiting time of patients in the hospital emergency, identifying queuing points, identifying important sources and combining resources and reducing patient mortality were some of the objectives of this study. Our proposed models in this study determine the efficiency of decision-making units, assuming that some of their output components may be undesirable. Real numerical examples show that these models ensure that the presence of undesirable output factors is effective in determining the efficiency of the decision-making units under evaluation and are compared with a unit corresponding to the efficient boundary set. It is possible to improve decision-making units and push them towards the efficient boundary by decreasing undesirable output and increasing desirable output.

References

[1] Al-Refaie, A., Fouad,R. H., Li, M.H., and Shurrab, M.(2014). Applying simulation and DEA to improve performance of emergency department in a jordanian hospital. Simulation Modelling Practice and Theory, 41, 59-72.

[2] Farzaneh,Km.,Ali Soltani, Nazari Shirkouhi, S., Azadeh, M., & Moosakhani, S. (2019). A uniqua Mathematical Framework for optimizing patient satisfaction in emergency departments. Iranian Journal of Management studies, 12(2), 81-105.

[3] Kuo, Y, H., Leung, J, M., Graham, C, A., Tsoi, K, K., & Meng, H, M,. (2018). Using simulation to assess the impacts of a fast-track system for hospital emergency services. Journal of advanced mechanical design, systems, and manufacturing, 12(3), JAMDSMOO73.

[4] Charnes, A. cooper, W. W. & Rhodes, E. (1978). Measuring the efficiency of decision- making units. European journal of operational research, 2(6),429-444.

[5] Jahanshahloo, G. R. Hosseinzadeh Lotfi, F. & Zohrehbandian, M. (2005a) finding the piecewise linear frontier production function in data envelopment analysis. Applied Mathematics and computation, 163(1), 483-488.

[6] Kao, C., & Hwang, S. N. (2019). Efficiency evaluation in the presence of undesirable outputs: the most favorable shadow price approach. Annals of Operations Research, 278(1-2), 5-16.

[7] Victor V. Podinovski (2019), Direct estimation of marginal characteristics of nonparametric production frontiers in the presence of undesirable outputs, European Journal of Operational Research, S0377-2217(19)30428-X.

[8] Qingxian An· Xiangyang Tao· Bo Dai· Jinlin Li (2019) Modified Distance Friction Minimization Model with Undesirable Output: An Application to the Environmental Efficiency of China's Regional Industry, Springer Science+Business Media, LLC, Computational Economics, 019-26 .

[9] Jie Wu. Panpan Xia· Qingyuan Zhu· Junfei Chu (2019) Measuring environmental efficiency of thermoelectric power plants: a common equilibrium efficient frontier DEA approach with fixed-sum undesirable output, Annals of Operations Research (2019) 275:731–749.

[10] Song. Malin, Wang.J, Zhao.J , Baležentis.T , Shen. Z. (2020) Production and safety efficiency evaluation in Chinese coal mines: accident deaths as undesirable output, Ann Oper Res 291:827–845.

[11] Barnabé Walheer (2020) Output, input, and undesirable output interconnections in data envelopment analysis: convexity and returns-to-scale, convexity and returns-to-scale. Ann Oper Res 284, 447–467.

[12] Shiwei Yu, Jie Liu, Longxi Li1(2020) Evaluating provincial eco-efficiency in China: an improved network data envelopment analysis model with undesirable output, Springer, Environmental Science and Pollution Research 6886–6903.

[13] Roberto Gómez-Calvet, David Conesa, Ana Rosa Gómez-Calvet, and Tortosa-Ausina, (2020) European Energy Efficiency Evaluation Based on the Use of Super-Efficiency Under Undesirable Outputs in SBM Models, Advances in

Efficiency and Productivity II, International Series in Operations Research & Management Science (ISOR, volume 287).

[14] Seiford LM, Zhu J (2002) Modeling undesirable factors in efficiency evaluation. Eur J Oper Res 142(1):16–20 132(2):400–410.

[15] Cooper, W, W., Seiford, L. M., & Tone, K. (2006). Introduction to data envelopment analysis and its uses: with DEA-solver software and references, Springer science & Business Media.