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The Presentation of an Approach of Evaluation and Ranking in Data Envelopment Analysis with Interval Data: A Case Study in the Evaluation and Ranking of Iran's Provinces in the Health and Treatment Sector

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Abstract
Today, in every society the health and treatment sector are among the most important service sectors. Therefore, it is crucial that their performance be evaluated and examined. Although the researchers have proposed many different approaches to evaluate and rank the health sectors, no precise approach for evaluating and ranking have been reported up to now. Assessing the coefficient of variation in data envelopment analysis has been extensively used as an instrument to measure the performance of decision making units and to rank them accordingly. In this study, therefore, the existing approaches were modified and two approaches were developed for interval DEA ranking based on the coefficients of variation with Interval data. These two models having none of the problems of other models, were developed to better comprehend the performance of health and treatment sectors in Iran in 2017. Conducting this study had such positive consequences as the creation of a healthy competitive atmosphere among all the medical universities to improve their performance. Another consequence might be the fact that conducting this study helped the health and treatment sectors in medical universities to be improved.

Keywords: Data Envelopment Analysis, Health Efficiency, Evaluation, Ranking, Coefficient of variance.

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1. Introduction

In recent years, there has been an increasing interest in organizational performance evaluation among researchers and managers and a big number of DEA studies with this issue has been published. Researchers and specially managers are among groups that more than other people comprehend the importance of performance evaluation and efficiency improvement. One of the major issues in the developed countries is the required resources for the health and treatment sector. Regarding the fact that in many cases there is no integrated standard for the evaluation of the hospitals as well as university hospitals (the name of an institution combining the services of a hospital with the education of medical students and with medical research) in Iran, Data Envelopment Analysis (DEA) can be an efficient instrument to deal with this issue. According to this approach, units under investigation are not compared with a previously determined standard. On the contrary, the efficiency of each unit is evaluated against the efficiency of other units. Hitherto, there has been many researchers investigating organizational efficiency utilizing DEA as their main approach of analysis. One of the concepts in DEA which is highly important both theoretically and operationally, is the units' ranking. The rank of each unit provides the decision maker managers with useful information on the priority of one unit over the others. The economic and management concept of ranking with DEA has been extensively investigated. A criterion that can be suggested for the ranking of decision making units is measuring the amount of each unit's efficiency.

DEA is a non-parametric approach based on a linear mathematical programming that has been tremendously used for the relative efficiency evaluation of the similar decision making units. The capability of this approach in the comparison of similar units with each other and also the possibility of analyzing the results have led to the increased and repeated uses of it in various fields. DEA was first presented by Charnes, Cooper and Rhodes in 1978 [1]. The first model introduced by them was CCR. This model was then continued by Banker, Charnes and Cooper with the name of BCC in 1984 [2]. In all their studies, the researchers have utilized different approaches for the organizational performance evaluation. Therefore, it can be concluded that there are many different approaches proposed and presented by the researchers. For instance, Sexton et al. [3] have suggested the cross efficiency method. In this method, at first, the index of each decision making unit is calculated n times using the obtained weights from the solution of each n problem. Then, the results with regard to the cross efficiency index for all decision making units are summarized in an $n \times n$ matrix. Each line of the matrix includes the cross efficiency index of one decision making unit. In this method, the average of the efficiency indexes of each unit is considered as the expected efficiency rank. When DEA model has multiple optimal responses, the cross efficiency model will face considerable problems. Anderson and Peterson [4] have proposed an approach (called AP) for the efficient units. In this method, the extreme efficient unit (consider k as the extreme efficient unit) is used as a model to achieve bigger efficiency in other units and to remove all their limitations until they get to the same or at least similar standards as k. To put it in another way, DMU_0 is considered as one decision making unit. In order to rank DMU_0, it should be omitted from the set of relevant production possibilities and DEA model is run for the remaining DMUs. This model might be intrusive in nature. Besides, for the DMUs having data values near to zero, instability might happen. In order to DEA with the instability problem in AP model, Sueyoshi [5] used a modified

auxiliary variables model. Like AP model, Sueyoshi also removed the under evaluation DMUs from the set of relevant production possibilities and utilized the auxiliary multiplier model for the other DMUs. This model is feasible if all the data is positive. If there is only a zero value in the data, the model will be unacceptable. Adler et al. [6] have adopted a statistical approach in ranking DEA issues. This includes common correlational analysis, linear discriminant analysis, and ratio discriminant analysis in ranking. Wang and Luo [7] have introduced an approach in ranking DMUs based on ideal and anti-ideal options are considered. They have evaluated the efficiency of each decision making unit. In this respect, the efficiency evaluation is once calculated with considering the ideal option and once with considering the anti-ideal option. Afterwards, these two values are combined and as a result the final efficiency evaluation is obtained, based on which the ranking of DMUs is conducted. In the classic models of DEA, it is assumed that there are precise values for the input and output. But considering the fact that in the real world, we normally face imprecise values and studying all the phenomena with precise values seems to be illogical. Therefore, in this study, the authors aimed to introduce two new approaches for the evaluation and ranking of decision making units based on the coefficient of variation and Interval data. These two models have none of the problems of aforementioned models.
The rest of the study includes the following sections: first, the generalities of the study and the literature review are presented (section 2); then, a model for ranking the efficient DMUs based on the coefficient of variation is suggested (section 3); next, the applicability of the suggested model is indicated (section 4); and finally, the conclusion part of this study is explained (section 5).

2. Background models

DEA is commonly used to evaluate the relative efficiency of a number of DMUs. The basic DEA model in Charnes et al. [1], called the CCR model, has led to several extensions, most notably the BCC model of Banker et al. [2]. Assume that there are n DMUs, (DMU_j: $j = 1, 2\ldots, n$) which consume m inputs ($\mathbf{x}_i$: $i = 1, 2, \ldots, m$) to produce s outputs ($\mathbf{y}_r$: $r = 1, 2, \ldots, s$). the best relative efficiency in each DMU is determined by the following model (DMUo relative efficiency).

$$\begin{aligned} &\theta_o^* = Max \sum_{r=1}^{s} u_r\ y_{ro} \\ &S.T \sum_{i=1}^{m} v_i\ x_{io} = 1 \qquad\qquad (\mathbf{1}) \\ &\sum_{r=1}^{s} u_r\ y_{rj} - \sum_{i=1}^{m} v_i\ x_{ij} \le 0 \\ &i = 1, \ldots, m, \qquad r = 1, \ldots, s \\ &j=1,\ldots,n, \qquad v_i \ge 0,\ u_r \ge 0 \end{aligned}$$

In this model, v_i is the weights of the inputs and u_r is the weights of the outputs. DMU_o is an efficient DMU when the relative efficiency in the above model equals to 1.
But the question that comes up is this: in the evaluation of several decision making units having the same amounts of efficiency, which unit has a better performance than the other units in other words, among all the efficient units, which one is the best and how these efficient units can be ranked. The researchers have proposed various approaches for the ranking of the efficient units.

3. The modification of ranking the efficient DMUs' approach based on the coefficient of variation

Bal et al. [8] have suggested a ranking approach for the efficient DMUs based on the coefficient of variation in 2008. In this section the specifics of this model are explained and its problems are stated. Suppose that U_r and V_i are the optimal weights obtained from the solution of (1) model in the evaluation of DMUs

$$\bar{u} = \frac{1}{s} \sum_{r=1}^{s} u_r \quad \textbf{(2)}$$

$$CV_{\bar{u}} = \frac{1}{\bar{u}} \sqrt{\sum_{r=1}^{s} (u_r - \bar{u})^2 / (s-1)} \quad \textbf{(3)}$$

$$\bar{v} = \frac{1}{m} \sum_{i=1}^{m} v_i \quad \textbf{(4)}$$

$$CV_{\bar{v}} = \frac{1}{\bar{v}} \sqrt{\sum_{i=1}^{m} (v_i - \bar{v})^2 / (m-1)} \quad \textbf{(5)}$$

Based on these definitions, the following model for ranking the efficient DMUs have been finally suggested by Bal et al. in 2008 [8]. This model is known as Coefficient of Variation Data Envelopment Analysis (CVDEA) in which the efficiency evaluation is as follows:

$$\alpha_o = Max \sum_{r=1}^{s} u_r\ y_{ro} - \frac{\sqrt{\sum_{r=1}^{s} \frac{(u_r-\bar{u})^2}{(s-1)}}}{\bar{u}} - \frac{\sqrt{\sum_{r=1}^{s} \frac{(v_i-\bar{v})^2}{(m-1)}}}{\bar{v}}$$

$$s.t. \quad \sum_{i=1}^{m} v_i\ x_{io} = 1 \quad \textbf{(6)}$$

$$\sum_{r=1}^{s} u_r\ y_{rj} - \sum_{i=1}^{m} v_i\ x_{ij} \le 0$$
$$j = 1, \dots, n$$

$$v_i \ge 0, \quad i = 1, \dots, m$$

$$u_r \ge 0, \quad r = 1, \dots, s$$

The above model is a non-linear model in which the coefficient of variation of input and output weights is minimized based on CCR model of ranking the efficient DMUs. It should be noted that, this model can be solved through Kuhn-Tuker model (for more detail, refer to [8]). Considering the fact that DMUs have different inputs and outputs, it can be concluded that the average of input and output weights based on (2) and (4) model has no management as well as economic justification. Therefore, Jahanshahloo et al. [9] have modified Bal et al. model in as follows. Suppose that Vi $(r = 1, \dots, m)$and U_r (r=1,…,s) are the weights obtained from solving (2) and (4) model. Using the average of input and output weights, they defined input and output coefficients of variation as follows.

$$\overline{CV}_{\bar{v}_i} = \frac{\sqrt{\sum_{i=1}^{m} (v_{ik} - \bar{v}_i)^2 / (m-1)}}{\sum_{i=1}^{m} \bar{v}_i} \quad \textbf{(7)}$$

$$\overline{CV}_{\bar{u}_r} = \frac{\sqrt{\sum_{r=1}^{s} (u_{rk} - \bar{u}_r)^2 / (s-1)}}{\sum_{i=1}^{m} \bar{u}_r} \quad \textbf{(8)}$$

According to these definitions, in an attempt to modify Bal et al. model, Jahanshahloo et al. [9] have suggested the following model for the ranking of the efficient DMUs.

$$\varphi_k = Max \sum_{r=1}^{s} u_{rk}\ y_{rk} - \frac{\sqrt{\sum_{r=1}^{s} \frac{(u_{rk}-\bar{u}_r)^2}{(s-1)}}}{\sum_{r=1}^{s} \bar{u}_r} - \frac{\sqrt{\sum_{i=1}^{m} \frac{(v_{ik}-\bar{v}_i)^2}{(m-1)}}}{\sum_{i=1}^{m} \bar{v}_i}$$

$$s.t. \quad \sum_{i=1}^{m} v_{ik}\ x_{ik} = 1 \quad \textbf{(9)}$$

$$\sum_{r=1}^{s} u_{rk}\ y_{rj} - \sum_{i=1}^{m} v_{ik}\ x_{ij} \le 0$$
$$j = 1, \dots, n, \quad (j \ne k)$$

$$v_{ik} \ge 0, \ i = 1, \dots, m$$

$$u_{rk} \ge 0, \ r = 1, \dots, s$$

But in the classic models of DEA and the other aforementioned models, there is an assumption implying that a precise value for the inputs and outputs exists. Considering the fact that in the real world we normally face imprecise data, the study of phenomena with precise data appears to be illogical. In situations like this, the models in which the efficiency of decision making units evaluated based on the imprecise results are required. Therefore, in an attempt to modify Bal et al. [8] and

Jahanshahloo et al. [9] models, the researchers in this study have introduced a model for the ranking of DMUs with Interval data. This is a non-linear model which is aimed to minimize the coefficients of variation in the input and output weights with Interval data. This model can also be considered as an approach in the ranking of the efficient DMUs.

$$\eta_k^* = Max \sum_{r=1}^{s} u_{rk} {y^U}_{rk} - \frac{\sqrt{\sum_{j=1}^{n} \frac{(u_{rj}-\bar{u}_r)^2}{(n-1)}}}{\sum_{i=1}^{m} \bar{u}_r} - \frac{\sqrt{\sum_{j=1}^{n} \frac{\left(v_{ij}-\bar{v}_i\right)^2}{(n-1)}}}{\sum_{i=1}^{m} \bar{v}_i}$$

$$s.t. \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \le 0 \quad \mathbf{(10)}$$

$$j = 1,\ldots,n \ (j \ne k)$$

$$\sum_{i=1}^{m} u_r y_{rk}^U - \sum_{i=1}^{m} v_i x_{ik}^L \le 0$$

$$\sum_{i=1}^{m} v_i x_{ik}^L = 1$$

$$v_{ik} \ge 0, \quad i = 1,\ldots,m$$

$$u_{rk} \ge 0, \quad r = 1,\ldots,s$$

in this model, η_k^* is the upper bound of DMU_0 and μ_k^* is the lower bound of DMU_0. In this model, the researchers consider the total weight difference of each DMU from the average weights. η_k^* and μ_k^* are considered as criteria for the ranking of DMU_k. The bigger η_k^* and μ_k^* values, the better the performance of DMU_k will be. For instance, if, $\eta_k^* \times \mu_k^* > \eta_j^* \times \mu_j^*$ therefore, the coefficient of variation in DMU_k weights is less than the coefficient of variation in DMU_j. As a result, DMU_k has a better ranking than DMU_j. This suggested model is more explained and understood through a practical example brought about in the next section. After elaborating on the practical example, the obtained results will be compared for the ranking of the efficient DMUs.

$$\mu_k^* = Max \sum_{r=1}^{s} u_{rk} {y^L}_{rk} - \frac{\sqrt{\sum_{j=1}^{n} \frac{\left(u_{rj}-\bar{u}_r\right)^2}{(n-1)}}}{\sum_{i=1}^{m} \bar{u}_r} - \frac{\sqrt{\sum_{j=1}^{n} \frac{\left(v_{ij}-\bar{v}_i\right)^2}{(n-1)}}}{\sum_{i=1}^{m} \bar{v}_i}$$

$$s.t. \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \le 0 \quad \mathbf{(11)}$$

$$j = 1,\ldots,n(j \ne k)$$

$$\sum_{i=1}^{m} u_r y_{rk}^L - \sum_{i=1}^{m} v_i x_{ik}^U \le 0$$

$$\sum_{i=1}^{m} v_i x_{ik}^U = 1$$

$$v_{ik} \ge 0, \ i = 1,\ldots,m$$

$$u_{rk} \ge 0, \ r = 1,\ldots,s$$

4. The Practical Example

The authors adopted a practical as well as analytical approach to this study. The overriding objective of this study was the evaluation of the health and treatment sectors in 30 provinces in Iran. Each DMU (a DMU is equivalent to a province) in this study includes two inputs and two outputs. It should be noted that the outputs are in the form Interval data as it is illustrated in table (4). (The data and indexes have been obtained from [10]).

X_1: The ratio of the number of hospital beds to the population of the province.
X_2: The ratio of the number of health centers such as hospitals, clinics, and health houses to the population of the province.
Y_1: Patients' Satisfaction of the province with the Interval data.
Y_2: The quality of giving service in the hospitals of the province which is in the form of Interval data.

Table 1: Data for the practical example

State	X1	X2	Y1	Y2
East Azarbaijan	0.0016	0.0004	[40 , 49.3]	[90 , 91.2]
Western Azerbaijan	0.0013	0.0004	[38.4 , 40.6]	[79.3 , 81.7]
Ardebil	0.0013	0.0005	[32.1 , 35.2]	[81 , 94.1]
Esfahan	0.0019	0.0003	[30.6 , 46.6]	[87 , 90.6]
Ilam	0.0019	0.0004	[37.9 , 40.9]	[75.4 , 87.4]
Busheh	0.0015	0.0003	[35.3 , 37.7]	[88.3 , 89.3]
Tehran	0.0023	0.0001	[37.6 , 78.2]	[87.1 , 89]
Chaharmahal & Bakhtiari	0.0015	0.0004	[38.8 , 41.8]	[87.6 , 85.9]
South Khorasan	0.0010	0.0005	[37.3 , 40.6]	[89.9 , 93]
Khorasan Razavi	0.0016	0.0003	[45.1 , 43.1]	[88.8 , 91.7]
North Khorasan	0.0010	0.0005	[36.8 , 36.9]	[91.4 , 93.6]
Khuzestan	0.0017	0.0003	[35.3 , 37.3]	[87.5 , 86.9]
Zanjan	0.0015	0.0006	[47.7 , 48.4]	[91.5 , 92.6]
Semnan	0.0023	0.0002	[31.8 , 35.4]	[90.9 , 91.7]
Sistan & Baluchestan	0.0009	0.0004	[27.1 , 28.8]	[84.9 , 85.8]
Fars	0.0018	0.0003	[32.9 , 35.4]	[84.6 , 86.8]
Qazvin	0.0013	0.0003	[35 , 36.4]	[87.8 , 91.6]
Qom	0.0014	0.0002	[33.3 , 34.8]	[81.2 , 91.1]
Kurdistan	0.0030	0.0011	[39.1 , 40.6]	[85.5 , 86.2]
Kerman	0.0014	0.0004	[31.7 , 36.2]	[88.1 , 90.9]
Kermanshah	0.0013	0.0004	[34.5 , 37.5]	[85.2 , 88.9]
Kohkiluyeh & Boyerahmad	0.0009	0.0006	[29.5 , 31.4]	[83.1 , 85.2]
Golestan	0.0013	0.0004	[41.2 , 42.3]	[90.7 , 92.1]
Gilan	0.0016	0.0005	[38.7 , 41.2]	[84.7 , 87]
Lorestan	0.0013	0.0004	[37.9 , 40]	[80.89 , 82.9]
Mazandaran	0.0015	0.0005	[36.5 , 37.4]	[90.9 , 92.6]
Central	0.0017	0.0004	[33.9 , 41.1]	[87.2 , 88.8]
Hormozgan	0.0012	0.0004	[56.1 , 71.9]	[90.6 , 93.1]
Hamedan	0.0014	0.0003	[59.1 , 70.7]	[80.1 ,83.4]
Yazd	0.0010	0.0004	[54.1 , 61.1]	[91.6 , 92.1

Table 2: The results obtained from utilizing interval DEA model

Unit	State	Efficient	u^*_K	l^*_K
DMU1	East Azarbaijan		0.520	0.445
DMU2	Western Azerbaijan		0.538	0.439
DMU3	Ardebil		0.876	1.570
DMU4	Esfahan		0.142	1.142
DMU5	Ilam		0.668	1.118
DMU6	Busheh		0.782	0.582
DMU7	Tehran		0.503	0.403
DMU8	Chaharmahal & Bakhtiari		0.548	0.348
DMU9	South Khorasan		0.626	0.521

DMU10	Khorasan Razavi		0.353	0.283
DMU11	North Khorasan		0.410	0.331
DMU12	Khuzestan		0.651	0.551
DMU13	Zanjan	✓	1.000	1.000
DMU14	Semnan	✓	1.000	1.000
DMU15	Sistan & Baluchestan		0.468	0.361
DMU16	Fars	✓	1.000	1.000
DMU17	Qazvin		0.529	0.449
DMU18	Qom		0.450	0.350
DMU19	Kurdistan	✓	1.000	1.000
DMU20	Kerman		0.607	0.501
DMU21	Kermanshah		0.523	0.503
DMU22	Kohkiluyeh & Boyerahmad		0.750	0.000
DMU23	Golestan		0.402	0.452
DMU24	Gilan		0.447	0.347
DMU25	Lorestan		0.343	0.363
DMU26	Mazandaran		0.567	0.467
DMU27	Markazi		0.870	0.670
DMU28	Hormozgan		0.688	0.558
DMU29	Hamedan		0.551	0.441
DMU30	Yazd		0.454	0.324

As it can be observed in table 2, Kurdistan, Fars, Semnan, and Zanjan provinces are efficient in applying interval DEA model. It should be noted that ordinary DEA models are not capable of ranking units with Interval data. Also, as it was stated in the literature review section, there were some major problems with regard to DEA model. Some of them included the inability to work with specific sets of data, inability to use all the efficient factors, instability, and finally the inability to rank all efficient DMUs. This situation creates crucial problems for the decision makers. Therefore, the senior managers of the health and treatment sector cannot plan to precisely expand the capacity of the health and hygiene services and economize the resources.

Table 3: The results obtained from utilizing the suggested model

Unit	**State**	η^*_K	v_1	v_2	u_1	u_2	Rank
DMU1	East Azarbaijan	0.7456	0.000	0.000	0.000	0.0092	
DMU2	Western Azerbaijan	0.2561	0.000	0.0036	0.000	0.1441	
DMU3	Ardebil	0.7142	0.041	0.3020	0.000	0.0003	
DMU4	Esfahan	0.7544	0.067	0.0421	0.04100	0.000	
DMU5	Ilam	0.5547	0.5208	0.000	0.000	0.000	
DMU6	Bushehr	0.5388	0.000	0.0142	0.0471	0.000	
DMU7	Tehran	0.8408	0.000	0.000	0.000	0.7701	
DMU8	Chaharmahal&Bakhtiari	0.5255	0.000	0.0001	0.0167	0.0140	
DMU9	South Khorasan	0.7409	0.2100	0.000	0.000	0.2000	
DMU10	Khorasan Razavi	0.5506	0.000	0.000	0.3281	1.8004	

DMU11	North Khorasan	0.7534	0.000	0.0944	0.344	0.0944	
DMU12	Khuzestan	0.5182	0.0704	0.000	0.000	0.000	
DMU13	Zanjan	1.0000	0.000	0.000	0.0324	0.000	3
DMU14	Semnan	1.0000	0.4800	0.000	0.000	0.0944	2
DMU15	Sistan & Baluchestan	0.3932	0.000	0.000	0.0180	0.052	
DMU16	Fars	1.0000	0.5400	0.000	0.0442	0.000	4
DMU17	Qazvin	0.5347	0.0872	0.000	0.5444	0.000	
DMU18	Qom	0.5867	0.4367	0.0231	0.000	0.000	
DMU19	Kurdistan	1.0000	0.5070	0.0214	0.000	0.000	1
DMU20	Kerman	0.2559	0.000	0.0245	0.2213	0.000	
DMU21	Kermanshah	0.6150	0.000	0.0556	0.5319	0.000	
DMU22	Kohkiluyeh&Boyerahmad	0.9090	0.0916	0.000	0.000	0.0124	
DMU23	Golestan	0.6579	0.000	0.0701	0.9410	0.0114	
DMU24	Gilan	0.5410	0.0770	0.0425	0.771	0.000	
DMU25	Lorestan	0.5400	0.000	0.2030	0.000	0.000	
DMU26	Mazandaran	0.4500	0.000	0.8004	0.2551	0.5169	
DMU27	Central	0.5924	6.082	0.0980	0.5241	0.000	
DMU28	Hormozgan	0.4877	0.000	0.000	0.0198	0.000	
DMU29	Hamedan	0.7040	0.2197	0.000	0.000	0.000	
DMU30	Yazd	0.8843	0.325	0.000	0.000	0.000	

In table 3, η_k^* for the upper bound is calculated according to (10) model. As it is demonstrated in the above table, in 2017, Kurdistan, Semnan, Zanjan, and Fars provinces have the highest amounts of η_k^*, respectively. The ranking of each unit is shown in this table.

Table 4: The results obtained from utilizing the suggested model

Unit	**State**	μ^*_K	v_1	v_2	u_1	u_2	**Rank**
DMU1	East Azarbaijan	0.5718	0.000	0.022	0.0129	0.0902	
DMU2	Western Azerbaijan	0.5972	0.000	0.0016	0.0000	0.0109	
DMU3	Ardebil	0.5793	0.000	0.0302	0.0425	0.0003	
DMU4	Esfahan	0.4400	0.0067	0.000	0.2040	0.0000	
DMU5	Ilam	0.4821	0.5008	0.000	0.0004	0.0000	
DMU6	Bushehr	0.8561	0.0232	0.000	0.0000	0.000	
DMU7	Tehran	0.4069	0.8900	0.000	0.1210	0.000	
DMU8	Chaharmahal& Bakhtiari	0.5961	0.0000	0.000	0.0001	0.000	
DMU9	South Khorasan	0.6371	0.0020	0.4151	0.01425	0.0407	
DMU10	Khorasan Razavi	0.8163	0.0000	0.000	0.0440	0.000	
DMU11	North Khorasan	0.7142	0.0054	0.000	0.0044	0.000	
DMU12	Khuzestan	0.9209	0.0080	0.0010	0.0980	0.000	
DMU13	Zanjan	0.9550	0.000	0.000	0.1500	0.000	3
DMU14	Semnan	1.0000	0.0870	0.000	0.000	0.0963	2
DMU15	Sistan& Baluchestan	0.5434	0.000	0.2001	0.1080	0.000	
DMU16	Fars	0.9582	5.540	0.7301	0.0002	0.004	4
DMU17	Qazvin	0.8843	0.0187	0.0425	0.5201	0.000	
DMU18	Qom	0.7046	0.436	0.2040	0.7204	0.000	
DMU19	Kurdistan	1	0.5710	0.0000	0.001	0.000	1
DMU20	Kerman	0.8660	0.000	0.0196	0.0000	0.000	
DMU21	Kermanshah	0.6260	0.000	0.080	0.1239	0.71174	
DMU22	Kohkiluyeh& Boyerahmad	0.6540	0.0096	0.0770	0.000	0.000	

DMU23	Golestan	0.5293	0.000	0.0000	0.0021	0.0014	
DMU24	Gilan	0.7544	0.0079	0.000	0.0701	0.000	
DMU25	Lorestan	0.5520	0.0005	0.0231	1.0425	0.000	
DMU26	Mazandaran	454.0	0.000	0.000	0.2440	0.056	
DMU27	Central	0.6881	0.0802	0.000	0.0004	0.000	
DMU28	Hormozgan	0.0503	0.000	0.000	0.0028	0.0040	
DMU29	Hamedan	0.4204	1.297	0.0452	0.000	0.0504	
DMU30	Yazd	0.1700	0.0000	0.0001	0.000	0.0120	

In Table 4 μ_k^* represents the results regarding to values for the lower bound which is calculated based on (11) model. As it is indicated in the table, Kurdistan, Semnan, Zanjan, and Fars provinces have respectively the highest amounts of η_k^*. The rankings are also shown in the table. Considering μ_k^* and η_k^* values, DMU_{19}, DMU_{14}, DMU_{13}, and DMU_{16} have gained the first, the second, the third, and the fourth ranks, respectively. These results indicate that the ranking of DMUs based on the suggested models in this study are feasible with interval data. Therefore, the senior managers in the health sector can properly plan to expand on the health and hygiene capacities and to economize the resources in order to avoid wasting them.

5. Conclusion

Using the Imprecise Data Envelopment Analysis (IDEA), the authors in the present study aimed to investigate Iran's provinces in health and treatment sectors in 2017. The results indicated that DEA could recognize the decision making units that had the best performance among other units. This can be considered as one of the appraisable capabilities of DEA model. In the classic models of data envelopment analysis, there was an assumption implying that the precise values existed for the inputs and outputs. Considering the fact that in the real world we usually face imprecise data, the study of phenomena seems to be irrational. In situations like this, the models in which the efficiency of decision making units are evaluated based on the imprecise results are required. Therefore, two different models were suggested in this study. One model was used to rank the upper bound of each decision making unit, and the other was utilized to rank the lower bound. According to the results obtained from these two suggested models, Kurdistan, Semnan, Zanjan, and Fars provinces could respectively gain the first, the second, the third, and the fourth ranks from among the other provinces. Therefore, these four provinces can be considered as a reference point in health and treatment sector for the other provinces. In fact, these results were to say that, these units utilized all their input capacity to produce a good output. Therefore, they can be selected as patterns for the other units. The senior managers of the other health and treatment sectors in other provinces should consider the managers of fars and kurdistan, Semnan, and Zanjan provinces as their reference and try to learn from their performance. One important issue in the health and treatment sectors is that in order to allocate funds to the health and treatment sectors all over the country, the performance of the best sectors should be taken into the planners' consideration. It can be concluded that, the authors had twofold objectives for conducting this study. First, they introduced the best performance unit to the senior managers in the health sector. Second, they provided the possibility of precise planning to expand the capacity of the health and hygiene services and economize the resources.

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