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The approach of the goal programming to solve the problem of multi-criteria data envelopment analysis and its application in decision voting

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Abstract

Increasing the discrimination power the of data envelopment analysis method and choosing appropriate weights is one of the important issues in data envelopment analysis. One of the ways to overcome this problem is to use multi-objective data coverage analysis. In multi-objective problems, the goal of the objective functions is usually contradictory to each other, so it is not possible to find an optimal solution for all the objective functions simultaneously. In this article, we use the ideal programming approach to solve the problem of multi-criteria data envelopment analysis, and then we compare the presented method with previous methods in the framework of preferential voting.

Keywords: Data envelopment analysis, multi-objective programming, goal programming, preferential voting.

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1. Introduction

Data envelopment analysis is a linear programming method for evaluating a set of decision-making units, which was first proposed by Charnes [1]. The relative efficiency value is obtained from the ratio of the weighted sum of the output to the weighted sum of the inputs. If the total number of inputs and outputs is large compared to the number of decision making units, many efficient DMUs may be introduced. Li & Reeves (1999) stated that in addition to the weakness in distinguishing between decision-making units, the selection of inappropriate weights is another weakness of DEA [2]. In order to increase the discrimination power of DEA methods and also to find appropriate weights for inputs and outputs, they introduced a multi-criterion model (MCDEA) according to the CCR model, and in a numerical example, they compared the results of their proposed model with the cross-efficiency model. They compared Since in multi-objective problems it is usually not possible to find a solution that optimizes all the objective functions at the same time, therefore multi-objective problems look for a non-dominated solution. Bal & Orcku (2002) presented a model for solving the multi-objective problem proposed by Li & Reeves (1999) using ideal programming. They applied Lexico's optimal programming method to solve the multi-objective GPDEA problem [2-3]. Bal et al. (2010) converted the multi-objective GPDEA problem into a single-objective problem by using the weighted sum method and introduced a non-dominated solution by minimizing the sum of deviations [4]. Ghasemi et al. (2014) stated that the introduced GPDAEA-CCR and GPDEA-BCC models are invalid and introduced a model named bi-objective weighted method to solve the mentioned multi-objective problem [5].

2- Previous studies

Charnes et al. (1978) proposed the CCR model for evaluation in the following form [1]:

$$
\begin{aligned}
& Max\ UY_o && (1)\\
& s.t.\quad UY_j - VX_j \le 0 \quad ; j=1,\dots,n\\
& \qquad VX_o = 1\\
& \qquad U \ge 0, V \ge 0
\end{aligned}
$$

In the above model, if the value of the objective function is equal to one, the unit under evaluation is efficient, but in many cases, when the ratio of the total number of inputs and outputs to the number of decision-making units is high, the data envelopment analysis technique is not able to distinguish between DMUs, which means that more It introduces the decision maker from a single unit. To increase the power of differentiation, various methods have been proposed, including the cross-efficiency method or the use of super-efficiency models. Li and Reeves (1999) proposed a multi-objective model and claimed that their model not only increases the discrimination power of the DEA method, but also obtains reasonable weights for inputs and outputs without having prior information about the weights. They proposed the following multi-objective data envelopment analysis (MCDEA) model [2]:

$$
\begin{aligned}
& Min\ d_o \left(or\ \ Max \sum UY_o \right) && (2)\\
& Min\ M\\
& Min\ \sum_j d_j\\
& s.t.\quad UY_j - VX_j + d_j = 0 \quad ; j=1,\dots,n\\
& \qquad VX_o = 1\\
& \qquad M - d_j \ge 0 \quad ; j=1,\dots,n\\
& \qquad U \ge 0, V \ge 0\\
& \qquad d_j \ge 0 \quad ; j=1,\dots,n
\end{aligned}
$$

Considering that $UY_o - VX_o + d_0 = 0$ and $VX_o = 1$, therefore, the minimization of is equivalent to the objective function of d_0 model (1), that is, the maximization of UY_o. In the second category of limitations, i.e. $UY_j - VX_j + d_j = 0 \ (j = 1,...,n)$, d_j are distance multipliers that should be reduced. The second and third objective function is the minimization of $d_j \ (j = 1,...,n)$ s with infinite soft and one soft, respectively.

Bal and Orcku (2002) proposed the use of ideal programming to solve the above multi-objective problem and introduced the ideal multi-objective data envelopment analysis model (GPMCDEA), then minimized the deviation from the introduced ideals with the help of the lexicographic method [3]. Bal et al. (2010) instead of using Lexico's ideal programming, with the help of the weighted sum method, they turned the ideal multi-objective problem into a single-objective problem, and by considering the relative importance of all objectives equally, the sum of the deviations from the ideals They designed the modeling both when the principle of constant returns to scale exists and when this principle is not established and named the models GPDEA-CCR and GPDEA-BCC respectively [4]. Ghasemi et al. (2014) talked about the problems of these two models for the first time and proposed a two-objective model which they called (BiO-MCDEA) [5]. Then Rubem et al. (2016) stated five inconsistencies for the two models of Bal et al. (2010) and finally proposed the following model (WGP-MCDEA-CCR) using ideal programming [6]:

$$
\begin{aligned}
&Min \ \ a = \{\lambda_1 d_1^+ + \lambda_2 d_2^+ + \lambda_3 d_3^+\} && (3)\\
&s.t. \ \ UY_j - VX_j + d_j = 0 && ; j = 1,...,n\\
&\quad VX_o = 1 &&\\
&\quad M - d_j \geq 0 && ; j = 1,...,n\\
&\quad d_o - d_1^+ + d_1^- \leq g_1 &&\\
&\quad M - d_2^+ + d_2^- \leq g_2 &&\\
&\quad \sum_{j=1}^{n} d_j - d_3^+ + d_3^- \leq g_3 &&\\
&\quad U \geq 0, V \geq 0 &&\\
&\quad d_j \geq 0 && ; j = 1,...,n\\
&\quad d_1^+, d_2^+, d_3^+, d_1^-, d_2^-, d_3^- \geq 0 &&
\end{aligned}
$$

which are g_1, g_2, g_3, and goals $d_o, M, \sum_k d_k$ respectively, which are determined by the manager. The less the deviation from the ideal, the better, so the goal is to minimize the variables of deviation from the ideal, and which has been turned into a single-objective problem with the help of the weighted sum method.

3- Weaknesses of the WGP-MCDEA-CCR model

Rubem et al. (2016) considered the values of g_1, g_2, g_3 as the ideals of $d_o, M, \sum_k d_k$, respectively, and assigned the determination of these parameters to the manager. They have indicated that it is $d_o \in [0,1]$ and suggested a value of $g_1 = 1$ for the related ideal parameter.

The first weakness in model (3) is that if $d_o = 1$, considering that $UY_o - VX_o + d_0 = 0$ and $VX_o = 1$, then $UY_o = 0$. This is one of the inconsistencies in Bal et al.'s (2010) model.
The second weakness is that model (3) does not guarantee $\exists j;\, UY_j \neq 0$, in other words, model (3) may have an optimal solution that $\forall j;\, U^* Y_j = 0$.
They have entrusted the determination of parameters g_1, g_2, g_3 as an ideal to the manager, but the limits of g_2, g_3 and cannot be calculated, and what values should be considered for them is entirely up to the manager, and it is obvious that the optimal answer is to choose the parameters of the ideal. It is sensitive. The uncertainty of how to determine the goals of g_1, g_2, g_3 will cause confusion for decision makers (managers). Table 1 shows the input and output values for 6 decision-making units, the sixth and seventh columns show the results of model 1 and model 3, respectively. As you can see, considering the ideal values in the seventh column, the output weighted sum for DMU_1 is zero.
Considering that $d_o \leq 1$, considering $g_1 = 1$ as ideal, the non-negative variable d_1^+ in condition $d_o + d_1^- - d_1^+ \leq 1$ will be equal to zero in every optimal solution. Therefore, according to goal $g_1 = 1$, the value of d_1^+ in the objective function will always be zero, see the eighth column of table 1.
As we have shown in the above table, model (3) is not qualified to evaluate in all cases.

Table 1: Input and output values and efficiency values using model (3)

DMUs	I1	I2	O1	O2	The efficiency value obtained from the model (1)	The efficiency value obtained from model (3) per $g_1 = 1, g_2 = 1, g_3 = 6$	d_1^+
1	8	4	10	3	0.88	0.00	0
2	4	5	11	5	1	1.00	0
3	7	8	8	2	0.45	0.44	0
4	5	7	14	2	0.93	0.93	0
5	3	6	12	4	1	1.00	0
6	7	3	9	4	1	0.20	0

4- Suggested method

In this section, we propose an ideal programming model to solve the multi-objective problem (2). From the limitations of model (2) $UY_o - VX_o + d_o = 0$ and $VX_o = 1$, the result is $UY_o = 1 - d_o$. According to this stipulation, the closer d_o is to zero, the closer the efficiency value of DMU_o is to one. Therefore, we consider the value of zero as the ideal of d_o. Therefore, if variables d_1^-, d_1^+ are values of deviation from the ideal, we will have condition $d_o + d_1^- - d_1^+ = 0$. On the other hand, $d_o \geq 0$ is always $d_1^- = 0$. Therefore, in the end, the adverb will be

$d_o - d_1^+ = 0$. So that the goal is to minimize the variable deviation from the ideal d_1^+.

In the category of restrictions $\frac{UY_j}{VX_j} + d_j = 1$, if $d_j = 0$, then $\frac{UY_j}{VX_j} = 1$ means DMU_j works, so it is desirable to have: $\forall j;\ d_j = 0$.

In the second objective function of model (2) $M = \max_j d_j$ and considering that $\forall j;\ d_j = 0$ is desirable, so we consider $M = 0$ as ideal. It means $M + d_2^- - d_2^+ = 0$. which are d_2^-, d_2^+ deviation variables from the ideal. On the other hand, $\forall j;\ d_j \geq 0$ and $M \geq 0$, therefore, the variables of deviation from the ideal of d_2^- will always be equal to zero. Therefore, the relevant adverb is considered as $M - d_2^+ = 0$.

In the third objective function of model (2) It means $\sum_j d_j$, we also consider the ideal of zero. Therefore, we have $\sum_j d_j + d_3^- - d_3^+ = 0$, of which d_3^-, d_3^+ are deviation variables from the ideal. On the other hand, $\sum_j d_j \geq 0$ because $\forall j;\ d_j \geq 0$. Therefore, always $d_3^- = 0$. So the relevant adverb is considered as $\sum_j d_j - d_3^+ = 0$. Pay attention that if $d_3^+ = 0$, then $\sum_j d_j = 0$ means $\forall j;\ d_j = 0$, resulting in $\forall j;\ \frac{UY_j}{VX_j} = 1$. In other words, if there is an optimal solution that is $d_3^{+*} = 0$, then all the decision making units will be efficient.

According to the above explanations, the ideal programming model proposed to solve the multi-objective problem (2) is as follows:

$$
\begin{aligned}
Min\ \ & d_1^+ + d_2^+ + d_3^+ && (4)\\
s.t.\ \ & UY_j - VX_j + d_j = 0 && ;\ j = 1,\dots,n\\
& VX_o = 1 &&\\
& M - d_j \geq 0 && ;\ j = 1,\dots,n\\
& d_o - d_1^+ = 0 &&\\
& M - d_2^+ = 0 &&\\
& \sum_{j=1}^{n} d_j - d_3^+ = 0 &&\\
& U \geq 0, V \geq 0 &&\\
& d_j \geq 0 && ;\ j = 1,\dots,n\\
& d_1^+, d_2^+, d_3^+ \geq 0 &&
\end{aligned}
$$

5- Using the proposed method in preferential voting

In preferential voting, each voter chooses a subset of r, $(r = 1,..,k)$ out of n candidates in order of priority, if v_{kj} represents the number of k-th priority votes for the j-th candidate, the overall desirability index is defined as $Z_j = \sum_{r=1}^{k} w_r v_{rj}$. It is clear that the weight or relative importance of each priority must be greater than its next priority, i.e. $w_1 > w_2 > \dots > w_k$. Cook and Kress

(1990) proposed to transform these weight constraints into a linear form:

$$w_r - w_{r+1} \geq d(r,\varepsilon) \quad r = 1,\dots,k-1$$
$$w_k \geq d(k,\varepsilon)$$

$d(r,\varepsilon): N \times R^+ \to R^+$ is called detection intensity function and it is non-negative, also ε is called detection factor. $d(r,\varepsilon)$ is the minimum distance between the r-th priority weight and the $r+1$ -th priority weight. The model provided by Cook and Kress (1990) is:

$$Z_p(\varepsilon) = Max \quad \sum_{r=1}^{k} w_r v_{rp} \qquad (5)$$
$$s.t. \quad \sum_{r=1}^{k} w_r v_{rj} \leq 1 \qquad j = 1,\dots,n$$
$$w_r - w_{r+1} \geq d(r,\varepsilon) \quad r = 1,\dots,k-1$$
$$w_k \geq d(k,\varepsilon)$$

Cook and Kress (1990) ranked six candidates with a recognition intensity function of $d(r,\varepsilon) = \dfrac{\varepsilon}{r}$ and $\varepsilon = 10^{-3}$ using preferential voting. Table 2 shows the prioritization of 20 voters related to the data of Cook and Kress (1990) article.

The proposed model has been implemented for the data in Table 2. The efficiency values obtained from model (4) for these six candidates are displayed in the second column of Table 3. The third column of Table 3 shows the results of the Cook and Kress (1990) model. As you can see, compared to the second column, the second column has introduced fewer candidates, so the proposed model compared to the Cook and Kress (1990) model of It has a higher discrimination power.

Table 2: Data from the article by Cook and Kress (1990).

candidate	The number of votes in the first priority	The number of votes in the second priority	The number of votes in the third priority	The number of votes in the fourth priority
A	3	3	4	3
b	4	5	5	2
c	6	2	3	2
d	6	2	2	6
e	0	4	3	4
f	1	4	3	3

Table 3: The results of the proposed method

candidate	Efficiency values obtained from the model (4)	Efficiency values from Cook and Kress (1990)
a	0.81235781	0.81
b	1.00000000	1
c	0.81279531	1
d	1.00000000	1
e	0.68701719	0.68
f	0.68720469	0.68

6-Conclusion

In this article, we theoretically discussed the model presented by Rubem et al (2016). This model may have solved some of the problems of the previous models, but we showed that it is not qualified for evaluation in all cases. Then, we presented our model with a new formulation based on ideal programming, which, in addition to the improvements of the Rubem et.al (2016) model, also solved its problems. We showed this with a numerical example in the preferential voting problem.

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