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Sensitivity analysis for efficiency security margin of medical sciences hospitals in Iran using data envelopment analysis

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Abstract

Part of data envelopment analysis, is the analysis of the sensitivity of a set of efficient units to changes in input and output values. In general, the first issue that sensitivity analysis addresses are the sensitivity of the amount of performance to each of the factors affecting performance. Hospitals are known as the largest and most expensive operating units of the health care system and account for a high percentage of health sector resources. Therefore, performance evaluation in these units is very important. Therefore, in order to increase the efficiency and effective use of resources and inputs and reduce hospital costs, a sensitivity analysis algorithm is presented to determine the excess amounts of inputs in inefficient hospitals. This algorithm also determines the security margin of the efficiency of efficient hospitals, in the sense that if the efficiency of efficient hospitals is threatened by improving the performance of inefficient hospitals, it will provide them with a security margin. For this purpose, using data envelopment analysis models, the efficiency of 15 hospitals of Iran University of Medical Sciences was evaluated, of which 6 were inefficient and 9 were efficient. Unlike the resources allocated to hospitals, there was a significant gap between the growth of available resources and the resources needed in this department, which was determined by the sensitivity analysis algorithm of excess amounts of inputs for example in Farshchian inefficient hospital. That hospital managers can achieve maximum efficiency by reducing inputs and better allocating resources.

Keywords: Sensitivity analysis, Performance security margin, Data envelopment analysis, Universities of Medical Sciences of Iran.

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1. Introduction

Health care is one of the basic and vital needs of any society, so that different countries of the world, consider their most successful services in optimizing and providing appropriate health services. Better resource allocation and better use of resources in the health sector is very important. Todays, international organizations working to promote the health of all countries in the world believe that what threatens the health of developing countries the most is the problems in resource management rather than the lack of health budgets. Therefore, efficiency evaluation is the first step in evaluating the performance of different health care sectors.

First, some research has been done in this field and then the articles of sensitivity analysis are reviewed.

In a study, Ghaderi et al. in [1] examined the efficiency of hospitals in Iran University of Medical Sciences using the data envelopment analysis method. In their article, they examined and analyzed the technical efficiency of the hospitals of Iran University of Medical Sciences from the method of data envelopment analysis during the years 2000-2004. For this purpose, they used the input-axis cover form with variable scale efficiency and used four outputs, namely outpatient admission, day-hospitalization, occupational bed and number of surgeries, and four inputs, namely number of beds, nursing staff, total medical staff and other personnel. The result was the elimination of surplus manpower in the form of a comprehensive plan based on the results of the data envelopment analysis method, which plays a major role in reducing the costs of hospitals and health care.

Rezapour et al. [2] in their study evaluated the economic efficiency of educational and medical centers of Iran University of Medical Sciences in 2015 using artificial neural network. They used a feeder multilayer prospectron network with LM optimization function and sigmoid tangent transfer function in the middle layer and linear function in the output layer. According to the results of one of the factors reducing economic efficiency in educational and medical centers, low utilization rate and occupancy rate of hospital beds was obtained so that the inactivity of hospital centers with full capacity and low utilization rate of active beds caused Costs are imposed and consequently economic efficiency is reduced .Asefzadeh and Rezapour [3] in their article evaluating the economic efficiency, allocation and distribution of resources by considering the amount of output in the educational and medical centers of Qazvin University of Medical Sciences in 1998-2007 .

They recorded human resource production information, capital resource data, and concluded that because human and capital resources impose huge costs on hospitals, it is necessary for medical center officials to properly plan and assess the need for principles. Absorb input as well as identify ways that affect their performance and identify and improve positive performance indicators to optimally use the data to determine the desired production capacity. Tabibi et al. [4] considering the importance of medical records in monitoring evaluation and planning to improve the quantity and quality of services and the lack of knowledge of the university about its status, in a study to determine the performance of medical records in specialized teaching hospitals of Iran University of Medical Sciences and Health Services. The study method was descriptive in 2001, which resulted in moderate performance of the medical records department of the studied hospitals. They recommended the preparation and development of specific instructions for each unit, the employment of specialized manpower, the holding of retraining courses, the optimal allocation of resources, equipment and space required for each unit, as well as the continuous evaluation of the performance of medical records departments to improve performance.

In recent years, the subject of sensitivity analysis and stability of DEA models has been widely studied. In the first DEA sensitivity analysis research Charens et al. [5] tested the change in single output by updating the optimal base inverse matrix in the DEA model. Also, Charens and neralich [6] investigated the sensitivity of the DEA additive model where it maintains sufficient performance conditions. Charens et al. [7] developed the sensitivity analysis technique and classification on the DEA super efficiency model, so that a relative change simultaneity in all inputs and outputs DMUs under evaluation are assumed. Seiford and Zhu [8] determined in two separate workshops to establish the effectiveness of data envelopment analysis and once again measured efficiency can be disintegrate into two components of perturbation to a frontier testing DMU and other DMUs. In another study Meters et al. [9] measured the stability of a set of DMUs. They did this using a unity pattern and employing pre-determined perturbations and using trial and error.

Jahanshahloo et al. [10] developed a new sensitivity analysis approach for a category DMUs and found the stability radius for all efficient DMUs. They achieved the stability radius for all efficient DMUs by combining a number of DEA classic models, provided that the efficiency score of the efficient DMUs remain unchanged. In spite of Meters et al.'s paper they found the maximum quantity of perturbations of data so that all first level efficient DMUs remain at the same level.

Jahanshahloo et al. [11] in a study, found radius of stability for all decision-making units, with interval data. In that approach, organization classification remains unchanged under perturbations of the interval data. Agarwal et al. [12] employed sensitivity analysis to know how sensitive the solution values and efficiency scores of the DMUs are to the numerical observations. In that paper, they proposed a new model of sensitivity analysis in data envelopment analysis (DEA) that examines the robustness of DEA efficiency scores by changing the reference set of the decision-making units (DMUs). The model is also used for ranking the efficient DMUs and to identify the outliers on the frontier. The results of numerical example showed that they are not sensitive to the efficient regions. In an article Jahanshahloo et al. [13] presented a sensitivity analysis method for CCR, BCC and additive models when changes are considered for a specific efficient DMU and assumed constant for other DMUs. They are obtained a stability region using PPS supporting hyper plan, before and after missing the unit under evaluation from observations set.

Zamani and Borzouei [14] in their paper addressed issue of sensitivity of efficiency classification of variable returns to scale (VRS) technology for enhancing the credibility of data envelopment analysis (DEA) results in practical applications when an additional decision-making unit (DMU) needs to be added to the set being considered. Furthermore, their study determines a stability region for the additional DMU within which, in addition to efficiency classification, the efficiency score of a specific inefficient DMU is preserved. This stability region is simply specified by the concept of defining hyperplanes of production possibility set of VRS technology and the corresponding half spaces. Also, Jahanshahloo et al. [15] in a paper presented a simple but important correction of the Cooper [16] model to classify and find the stability radius and the range of changes to efficient and inefficient DMUs. The modified version identifies the efficiency classifications of DMUs and the range of change of inputs and outputs of all DMUs by solving only one model, but the number of problems that it solves is less than other models. He et al. [17] in their paper proposed an approach to analyze the sensitivity and stability radius. By assuming that the data vary within a bounded interval, all of the decision-making units (DMUs) can be classified as E++, E+, and E−. To determine how sensitive these classifications are to possible data perturbations, they developed programs to calculate the stability radius within which the percentage data variation does not change the class of efficiency unit. In addition, the data changes are applied to not only the DMU that is being evaluation but also all of the DMUs and the various input and output subsets. Daneshvar et al. [18] presented a stability region extension specifically for DMUs that are located in the interface of weakly efficient and efficient frontier. Neralić and Wendell [19] extended fundamental results on metric sensitivity in Data Envelopment Analysis. Specifically, they showed how to obtain a larger radius of stability for a decision-making unit (DMU) by exploiting knowledge about its variability, and how to enlarge a DMU's region of stability to a nonsymmetric hyperbox. In the other paper Azizi et al. [20] presented optimistic and pessimistic perspectives for obtaining an efficiency evaluation for the DMU under consideration with imprecise data. Additionally, slacks-based measures of efficiency are used for direct assessment of efficiency in the presence of imprecise data with slack values. Finally, the geometric average of the two efficiency values is used to determine the DMU with the best performance. Banker et al. [22] have studied Sensitivity and stability for Banker's model (1998) of Stochastic Data Envelopment Analysis (SDEA). They have obtained in the case of the DEA model, necessary and sufficient conditions to preserve the efficiency of efficient decision-making units (DMUs) and the inefficiency of inefficient DMUs for different perturbations of data in the model. The cases of perturbations of all inputs, of perturbations of output and of the simultaneous perturbations of output and all inputs are considered [21]. Neralić and,Wendell [23] presented an algorithmic approach to sensitivity in Data Envelopment Analysis for the CCR and Additive models. Specifically, their algorithm gives sufficient conditions that preserve the efficiency of a decision-making unit (DMU) under arbitrary perturbations of the inputs and/or outputs of the DMUs. Then they illustrate the results for the Additive model. Banihashemi et al. [24] said that a specific inefficient DMU can scarcely reach to the efficient frontier and achieving the score 1 in efficiency but it can easily obtain an efficiency score of α, that α is a constant which is usually closed to 1 and defined by the decision maker. There upon, they found a region which named Improvement Region (IR) for a specific inefficient DMU which can obtain at least an efficiency score of α. In this region the inefficient DMU which is under evaluation can satisfy the decision maker and also it can be improved itself to gain a new efficiency score. Shirouyehzad et al. [25] proposed a modified model to analyze sensitivity of inefficient units. In other words, their sensitivity analysis model was proposed for inefficient DMUs based on the management coefficients. In their model, instability radius of each inefficient DMU is determined in terms of variations in inputs and outputs. Because that is important issues for a decision maker is to select an appropriate way to move an inefficient DMU toward efficiency frontier. Also, management coefficients are applied to determine the possibility of variations in inputs and outputs variables.

Proposed management coefficient in their model may help managers to improve organization condition through the enhanced coefficients related to inputs and outputs. Yakob and Isa [26] in their study performed several tests to ensure the stability of the relative efficiency obtained from the DEA. These tests were demonstrated on DEA efficiency scores of risks and investment management function of life insurers and tactful operators. Several stability tests performed in their paper on the illustrative data show a stable efficient frontier. The test also indicated that the efficiency score is reliable in discriminating between efficient and inefficient decision-making units (DMUs). Therefore, it can be concluded that the DEA model used is appropriate in furnishing a comprehensive guide towards the best practices that other firms might adopt and worst practices that other firms should avoid. In turn, the managerial decision-making can be made with more confidence.

The efficiency security margin (ESM) provides useful insights into how improvements in one decision-making unit (DMU) may affect the performance of others, especially in systems with both efficient and inefficient units. Ehdaei and Mehrgan [27] introduced the ESM concept, focusing on output increases to improve efficiency and assess potential threats to other units' efficiency. While output expansion is important, their method overlooked input reduction, which is equally critical in resource-sensitive sectors like healthcare.

Building on the work of Ehdaei and Mehrgan [27], we propose a refined version of the efficiency security margin, addressing both output expansion and input reduction. This balanced approach not only enhances performance by increasing outputs but also considers optimizes resource use, reducing costs and improving overall efficiency. This dual focus is especially important in sectors like healthcare, where efficiency must be maximized while controlling costs.

Our method applies this improved framework to conduct sensitivity analysis across 20 hospitals in the medical sciences, offering deeper insights into how changes in one unit can impact the performance of others.

Hospitals, as key components of the health system, play a crucial role in health economics. With rising costs, medical centers must employ economic analysis to optimize resource use. Despite the allocated resources, a significant gap exists between available and needed resources. This study evaluates the efficiency of hospitals at Hamadan University of Medical Sciences using data envelopment analysis, identifying efficient and inefficient hospitals to help managers maintain performance. The algorithm also identifies excess inefficient inputs, enabling cost reduction and improved efficiency.

This article is organized as follows: Section 2 reviews basic DEA models for measuring optimistic and pessimistic efficiencies. Section 3 considers absolute and relative efficiency security margins and the proposed algorithm with a numerical example. Section 4 applies the algorithm to a medical sciences hospital for ranking DMUs. Finally, Section 5 presents the conclusions.

2. DEA models for measurement of the optimistic efficiencies

Assume that there are n decision making units for evaluation, and each DMU consist of m inputs and s outputs. We define $x_{ij}, (i = 1, \dots, m)$ and $y_{rj}, (r = 1, \dots, s)$ as the input and output values of

$DMU_j, (j = 1, \dots, n)$, all of which are known and positive. Based on the definition of efficiency, the efficiency of DMU_j is defined as:

$$RE_p = \frac{\frac{uy_p}{vx_p}}{max\left\{\frac{uy_j}{vx_j}, j=1,\dots,n\right\}} \qquad (1)$$

Maximizing the above fraction, gives the best relative efficiency or optimistic efficiency and minimizing it, gives the worst relative efficiency or pessimistic efficiency. In order to differentiate between the efficiency of DMU_j from the other DMUs, Charens et al. [28] presented a known CCR model that measures the best relative efficiency of DMUs. In the model (2), the subscript o indicates the DMU under evaluation., $u_r, (r = 1, \dots, s)$ and $v_i, (i = 1, \dots, m)$ are the weights for the rth output and the ith input respectively and ε is a non-Archimedean infinitesimal.

$$\begin{aligned} &\text{Max } \theta_o = \sum_{r=1}^{s} u_r y_{ro} \\ &\text{s.t} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \\ &j = 1, \dots, n \\ &\sum_{i=1}^{m} v_i x_{io} = 1 \qquad (2) \\ &u_r, v_i \ge \epsilon, \ \ r = 1, \dots, s\,; \\ &i = 1, \dots, m. \end{aligned}$$

If there are a set of positive weights $v_i^*, (i = 1, \dots, m)$ and $u_r^*, (r = 1, \dots, s)$ that make $\theta_0^* = 1$, then DMU_o is called DEA-efficient or optimistic efficient; Otherwise, it is called DEA-non-efficient or optimistic non-efficient. All DEA-efficient DMUs define an efficiency frontier [29].

In model (2), when we say $\theta_0^* = 1$, DMU_oworks in the radial sense, it means that along the radial, the inputs contract by one ratio or the outputs expand by one ratio. If $\theta_0^* < 1$ then DMU_o is not on the efficient frontier, then DMU_o is inefficient. We use one-dimensional output and two-dimensional input data that is shown in Figure 1 to illustrate the efficiency frontier. four DMUs namelyDMU_M, DMU_A, DMU_D, and DMU_N define an efficiency frontier.

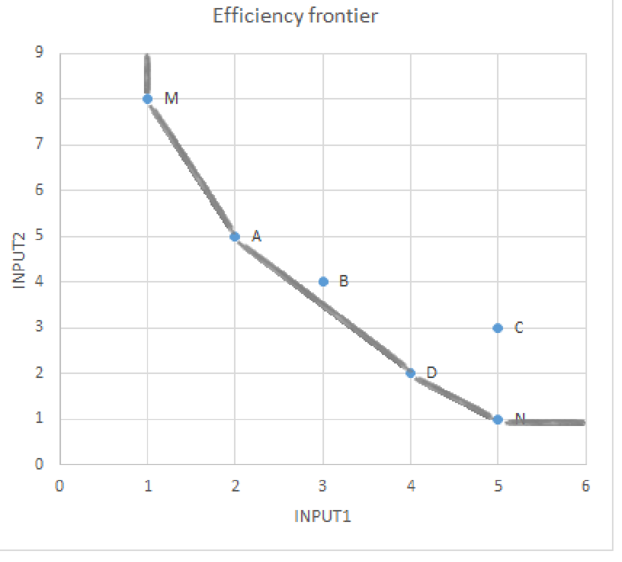

Figure 1 DMUs with the optimistic frontier

3. Determining the optimistic efficiency security margin of DMUs

In general, first topic that sensitivity analysis concerns are the sensitivity of the amount of efficiency to any factors affecting efficiency, and how obtains sufficient condition to keep efficiency of DMUs the presence of those factors. Despite the numerous and various studies in sensitivity analysis, what should be paid more attention to it, is the subject to determine the efficiency security margin of DMUs which concerns effectiveness improvement the performance of each DMUs on the other DMUs in DEA.

According to the proposed algorithm, when the status of an efficient decision-making unit is measured in relation to each of the other units, its absolute performance security margin is obtained and the closest threat to its performance position is identified. But if we measure only one unit, the security margin of relative efficiency is obtained for the same unit. The efficient unit is called the unit under evaluation and the inefficient unit is called the compared unit. The security margin DMU_i relative to DMU_j is defined as follows:

Definition 1. The security margin is defined for DMU_i regard to DMU_j as the maximum (minimum) measure of output (input) for all the inputs and outputs of DMU_j simultaneously so that not to decrease the efficiency ofDMU_j. Evidently can't define the efficiency security margin of a DMU regarding itself; this means always$i \neq j$. The subscript i, j indicate the DMU under evaluation and comparison unit respectively.

Remark. Since the performance of security margin of DMUs are variant to inputs and outputs variations, so when we evaluate to inputs variations, we call it input oriented performance security margin and when evaluate to outputs variations, we call it output oriented performance security margin.

According to the Figure 1, we consider the optimistic frontier with DMUs namelyDMU_D,DMU_A, DMU_M, and DMU_N. We want to know, how to calculate the relative efficiency security margin of DMU_A regarding DMU B with two-dimensional input and one-dimensional output data and also how find out the absolute efficiency security margin forDMU_A? The proposed algorithm by model (3) improves the performance of the DMU_B -which called compared unit- on the other hand, it monitors the performance of theDMU_A. While the efficiency of DMU under evaluation has not reduced, increases the performance of theDMU_B. First stage that the efficiency of the$DMU_A$is reduced, is equal to its optimistic efficiency security margin. The performance improvement level of the comparison unit to this stage, is answer of the problem. First, we present the algorithm and then answer the above question.

3.1 The proposed new algorithm to determine the efficiency security margin of DMUs with optimistic frontier

Improvement of performance for DMUs always is the probable and goal subject. Therefore, each DMUs should be careful about performance of the other competitive DMUs. The distance of each DMU has in terms of efficiency with other DMUs, creates a security margin for its performance. Now, there are some questions: what is the security margin of the DMU under evaluation for improvement and retain its performance? How much improvement of the other inefficient DMUs can cause to decrease performance of the efficient DMUs? Which the closest DMU can threat the security margin of efficient DMUs?

Answering these questions, necessitates the concept of absolute and relative efficiency security margin. Presented algorithm in this section, obtains the relative security margin of the DMUs. In order to identify the nearest DMU that threat the DMU_K, after calculation the relative efficiency security margin of DMU_k (for 1≤t≤n, t $\neq$ k), we must determine the minimum value of them, that is called the absolute efficiency security margin of DMU_k that is defined as:

ARESM[2](k)=min{RESM(k,t);1≤t≤n ,t≠k}

In step one, using one of the classic DEA models, the efficiency of all decision-making units is obtained. DMU_k is the performance unit under evaluation that aims to find the safety margin of its performance, and DMU_t is called the compared unit, which can be any of the inefficient units. To find the absolute efficiency margin, each inefficient unit is selected as the benchmark unit, and according to ARESM(k) formula, the nearest unit that threatens the efficiency of efficienct DMU_k is selected. To ensure consistency with prior work, we have used symbols from [27] framework.

Step1: With one of DEA models, compute the optimistic efficiency of all DMUs, consists of DMU_k and DMU_t ;

Step 2: Save amount of $E(k)$ in $\tilde{E}(k)$ as the initial value the efficiency of DMU_k;

In the second step, the efficiency score DMU_k is maintained as the first value obtained at E˜ (k).

Step 3: Save For $r = 1, \ldots, s$ and $i = 1, \ldots, m$ the values of $y(r,t)$ and $x(i,t)$ are in $\tilde{y}(r,t)$ and $\tilde{x}(i,t)$ respectively;

In the third step, the initial values of input and output of the measurement unit are maintained at x˜ (i, t) and y˜ (r, t), respectively, where the index t represents the compared unit.

Step 4: Set the amount of α equals to $\alpha = \delta = 0.01$

The value of α - the amount of data perturbation - with the value of δ - the percentage of data variation - is set to 0.01 in the first stage of perturbation.

α is the coefficient variations of inputs and outputs and δ is the percent of data variations.

The new values of the input and output data after applying the perturbation value of α are calculated as follows in step5:

Step 5: Compute the new values of inputs and outputs for $r = 1, \ldots, s$ and $i = 1, \ldots, m$, as follows:

$y(r,t) = \tilde{y}(r,t).(1+\alpha)$, r=1,…,s

$x(i,t) = \tilde{x}(i,t)(1-\alpha)$, i=1,…,m

Step 6: As in Step 1, compute the (optimistic, pessimistic) efficiency of DMU_k and save in $\tilde{E}(k)$;

In step 6, like step 1, the relative score efficiency of DMU_k with the new input and output is calculated and saved in E˜ (k).

Step 7: If $E(k)$ is less than $\tilde{E}(k)$, then go to step 9; otherwise perform the next step;

In step7, the values of E(k) and E˜(k) are compared. If E(k) is the initial performance value of DMU_k (unit under evaluation) is less than E˜(k), we go to step 9 and get the relative security margin of DMU_k relative to DMU_t, which is calculated as a percentage. This is where the algorithm stops. Otherwise, we go to step 8.

Step 8: Set α= α +δ and run fifth step again;

Step 9: Express the efficiency security margin of DMU_k regarding DMU_t as percent:

RESM(k,t)=100.(α-δ)

The following is a numerical example of the ARESM of DMU_A according to the steps of the algorithm to determine the closest threat to its performance.

Example 1. Consider optimistic frontier consist of 6 DMUs in Figure 1. DMU_D, DMU_B, DMU_A are in efficient frontier and DMU_C is inefficient.

[2] Absolute Efficiency Security Margin

The main purpose is determining the closest DMU that threat the performance of theDMU_A. To this end, we must compute the relative efficiency security margin of DMU_A to other DMUs, and then obtain absolute performance security margin of it. So, we compute the relative efficiency security margin of the DMU_A regarding DMU_Bfirstly. DMU_B and DMU_A are the compared unit and unit under evaluation respectively. The scores efficiency of DMUs are calculated using model (2). The results of the algorithm's steps in the six stages are shown in able 1: From table 1, we find out after sixth order of performing algorithm, is imperiled the performance security margin of theDMU_A, and its position is occupied by theDMU_B. As soon as DMU_A loses its efficiency, algorithm stops. In fact, first stage where reduces the efficiency of DMU_A is equivalent to performance security margin of it, that is computed by the following formula as percentage. The measure of improvement performance of compared unit to this stage**,** is the answer of the problem.

RESM (A, B) =100(α-δ) =100(0.07-0.01) =6%

Also, the score efficiency of DMU_C, reduced from 0.762 to 0.714. This means that the improvement performance of DMU_B affected the performance of DMU_C and it was far from the efficient frontier too. In continuation, we compute the relative performance security margin of the DMU_A regarding $DMU_C$and show the results in table 2.

Consider the rows of theDMU_A and DMU_C. DMU_Cimproves its performance. DMU_A also resists not to miss its efficient position. In stage 5 of the algorithm, score efficiency of the DMU_C leads to one and be efficient. But DMU_A maintains its efficiency. This process continues until in the tenth stage of the algorithm, score efficiency of the DMU_A leads to 0.8303. Here algorithm stops and obtains the optimistic relative efficiency security margin of the DMU_A regardingDMU_C. Looking at the rows of theDMU_B and DMU_D, we find out their efficiency affected the improvement performance of the DMU_C and have reduced.

RESM (A, C) =100(α-δ) =100(0.1-0.01) =9%

In the last stage, we must measure the performance security margin of DMU_AregardingDMU_D. The results of the algorithm's steps are shown in table 3.

Table 1 Sensitivity analysis to determine the efficiency security margin ofDMU_AregardingDMU_Bin six stages of the algorithm

DMUs	Eff	Eff-Stage1	Eff-Stage2	Eff-Stage3	Eff-Stage4	Eff-Stage5	Eff-Stage6
DMU A	1	1	1	1	1	1	0.98
DMU B	0.941	0.96	1	1	1	1	1
DMU C	0.762	0.762	0.762	0.754	0.743	0.729	0.714
DMU D	1	1	1	1	1	1	1

Table 2 Sensitivity analysis to determine the efficiency security margin of DMU_A regarding DMU_C in ten stages of the algorithm

DMUs	Efficiency	Eff-Stage1	Eff-Stage2	Eff-Stage3	Eff-Stage4	Eff-Stage5	Eff-Stage6	Eff-Stage7	Eff-Stage8	Eff-Stage9	Eff-Stage10
DMU A	1	1	1	1	1	1	1	1	1	1	0.8303
DMU B	0.941	0.9412	0.9412	0.9412	0.9412	0.9290	0.8771	0.8166	0.7481	0.6727	0.5535
DMU C	0.762	0.7773	0.8090	0.8591	0.9307	1	1	1	1	1	1
DMU D	1	1	1	1	1	1	0.9850	0.8563	0.7295	0.6092	0.4982

Table 3 Sensitivity analysis to determine the efficiency security margin of DMU_A regarding DMU_D in eight stages of algorithm

DMUs	Eff	Eff-stage1	Eff-stage2	Eff-stage3	Eff-stage4	Eff-stage5	Eff-stage6	Eff-stage7	Eff-stage8
DMU A	1	1	1	1	1	1	1	1	0.9722
DMU B	0.941	0.9334	0.9178	0.8940	0.8623	0.8226	0.7745	0.7187	0.6482
DMU C	0.762	0.7482	0.7216	0.6827	0.6343	0.5786	0.5173	0.4534	0.3889
DMU D	1	1	1	1	1	1	1	1	1

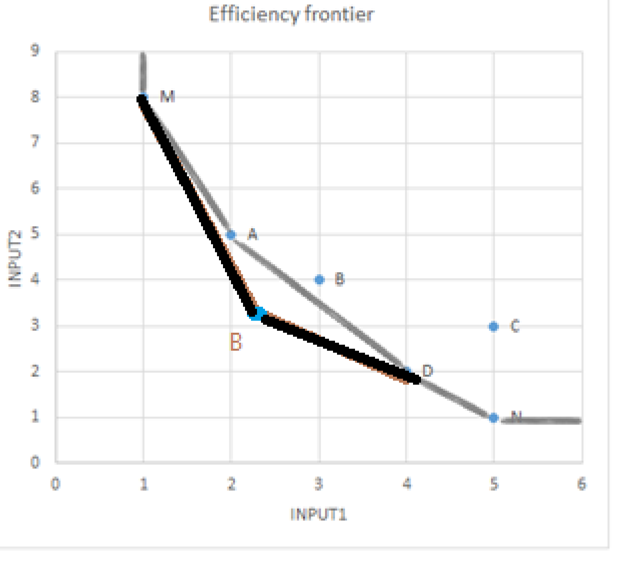

Figure 2- New PPS shows DMU_B as closest threat of DMU_A after running algorithm

The performance improvement level of the compared unit, to this stage is equal to answer of the problem.

RESM (A, D) =100(α-δ) =100(0.08-0.01) =7%

Looking at Table3, it is evident that the DMU_D, with improvement its performance, reduces the efficiency of the DMU_A in stage 8 and its score is being 0.9722. The score efficiency of the DMU_B decreases from 0.941 to 0.6482 in stage8, and it is far from the efficient frontier. Also, the score efficiency of DMU_C and DMU_B affected the algorithm and decreases from 0.762 to 0.3889. The relative performance security margin of the DMU_A in relation to other DMUs was measured. Now we can answer that question:" what is the absolute performance security margin of the DMU_A?" The absolute performance security margin of DMU_A obtains as follows:

ARESM (A) =min {RESM (A, t); 1≤t≤n ,t≠A}

The above formula identifies the closest DMU which threats the efficiency of the DMU under evaluation.

RESM (A, B) =100(α-δ) =100(0.07-0.01) =6%
RESM (A, C) =100(α-δ) =100(0.1-0.01) =9%
RESM (A, D) =100(α-δ) =100(0.08-0.01) =7%
RAESM(A)=min{6%,9%,7% }=6%

The number 6% is belong to DMU_B, that means the closest threat and main rival of DMU_A isDMU_B. Providing this information help the managers of large and small firms that in the competitive market with the remarkable technological advancement, recognize their companies, and not neglect the performance of the others.

In the new PPS in Figure 2 is shown DMU_B as closest threat of unit under evaluation A.

In Section 4, for a real sample of 15 hospitals of Iran University of Medical Sciences, the proposed algorithm is used. For this purpose, Shahid Beheshti Hospital is selected as the unit under evaluation to calculate its efficiency security margin. On the other hand, Inefficient Hospital Farshchian, is selected as compared unit to improve its performance. Finally, the efficiency security margin of Shahid Beheshti Hospital relative to Farshchian Hospital is determined.

4. Sensitivity analysis of hospitals of Medical Sciences in IRAN

In this example we consider fifteen hospital Medical Sciences Iran university with two inputs and three outputs. The inputs include the number of employees (including physicians, nursing staff and other employees) and the number of active beds as capital. Also, the outputs which include the number of patients, the number of operations and the occupied day beds, in 2018 year. Those information and efficiency with efficient and inefficient hospitals are shown in table 4.

The following are some of the input and output concepts:

Expert Medico: A person who has succeeded in obtaining a specialist degree in a medical specialty after completing a specialized medical assistant course.

General practitioner: A person with a Ph.D. degree in general.

Active couch: The couch that is ready to accept the patient and is prepared, namely, the existence of specialized facilities, manpower, equipment and other resources for the patient " s use and restore health and diagnosis, and include beds of the exam room, treatment room, recovery, physical therapy and beds of staff in the dormitory.

Fixed - bed: An official bed approved by the hospital has permission for operation.

The number of occupied beds: it is the number of beds occupied that are relative to the bed of the day to the active bed in a given period, which is obtained by multiplying by 100, per cent of the daily bed occupation.

According to table 4, nine hospitals have a technical efficiency of one and 6 hospitals have an efficiency of less than one, which identifies efficient and inefficient hospitals. In the studied hospitals, Imam Hassan Hospital has the lowest efficiency score, so that it does not use more than 35% of production factors optimally. The other 5 inefficient hospitals have an average of 0.933. This means that 67 of their capacity units have been left unused. This means that they can reduce their costs by approximately 7%.

Table 4 Inputs and outputs and the score efficiency of fifteen hospitals

Hospitals	I1	I2	O1	O2	O3	Efficiency
Beasat	1497	491	412180	48854	131357	1.0000
Beheshti	606	248	89109	5756	76364	1.0000
Sina	696	334	182127	24316	94739	1.0000
Fatemieh	596	118	89755	13675	43977	1.0000
Emam Hasan	127	34	84535	623	2040	0.5436
Emam Hossein	438	131	68839	6107	36562	0.8575
Mehr	327	111	108438	5598	26815	0.8375
Emam Reza	324	78	215405	7206	18117	1.0000
Ayat Allah Bahari	85	19	109327	503	2835	1.0000
Shahid Heydari	86	30	94809	1368	1367	1.0000
Alimoradiyan	443	188	180908	14987	51086	1.0000
Valiaasr1	365	105	219271	8126	24999	0.9545
Valiaasr2	223	99	159999	4236	24499	1.0000
Ghaeem	433	133	240060	5284	33804	0.9453
Farshchian	649	163	133375	1013	45964	0.8516

According to table 4, nine hospitals have a technical efficiency of one and 6 hospitals have an efficiency of less than one, which identifies efficient and inefficient hospitals. In the studied hospitals, Imam Hassan Hospital has the lowest efficiency score, so that it does not use more than 35% of production factors optimally. The other 5 inefficient hospitals have an average of 0.933. This means that 67 of their capacity units have been left unused. This means that they can reduce their costs by approximately 7%.

In data envelopment analysis method is based on minimization of production factors for each of the inputs, to the level of optimal values, therefore, we use the proposed sensitivity analysis algorithm to find out how much the hospitals needs to reduce their consumption from the input to be efficient and so that they can significantly reduce their costs.

Now, in order to increase the efficiency of Farshchian Hospital, which according to table 9 has the most use in inputs, and to implement the sensitivity analysis algorithm, we consider Farshchian Inefficient Hospital as a compared unit and Beheshti Hospital as a unit under evaluation to determine the extent of Farshchian Inefficient Hospital. It must reduce its consumption of inputs to reach the efficiency frontier, which can significantly reduce its costs. On the other hand, the efficiency security margin of efficient Shahid Beheshti Hospital should be obtained. We run the algorithm in 9 stages, the results are shown in table 5.

Table 5 Sensitivity analysis of performance hospitals with performing the security margin algorithm in nine stages

Hospitals	EFF Stage1	EFF Stage2	EFF Stage3	EFF Stage4	EFF Stage5	EFF Stage6	EFF Stage7	EFF Stage8	EFF Stage9
Beasat	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Beheshti	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9761
Sina	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Fatemieh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Emam Hasan	0.5436	0.5436	0.5436	0.5436	0.5436	0.5436	0.5436	0.5436	0.5436
Emam Hossein	0.8575	0.8575	0.8575	0.8575	0.8575	0.8511	0.8399	0.8062	0.7771
Mehr	0.8375	0.8375	0.8375	0.8375	0.8375	0.8320	0.8179	0.8004	0.7808
Emam Reza	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Ayat Allah Bahari	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Heydari	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Alimoradiyan	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Valiaasr1	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545	0.9545	0.9531
Valiaasr2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Ghaeem	0.9453	0.9453	0.9453	0.9453	0.9453	0.9336	0.9118	0.8872	0.8588
Farshchian	0.8602	0.8778	0.9049	0.9426	1.0000	1.0000	1.0000	1.0000	1.0000

Looking at table 5, it is evident that inefficient Farshchian hospital with decreasing inputs improves its performance and be efficient in stage five. In fact, first input of Farshchian hospital reach to 557 from 649 and second input reach 140 from 163. In other hand Beheshti hospital in nine stage misses its efficiency, and leads to 0.9761. This means that inefficient unit Farshchian, with improvement its performance, imperils the position of unit Beheshti. First stage that reduces the efficiency of Beheshti hospital, is equivalent to its performance security margin, where occurs in nine stages. Also, we find out the efficiency of the other hospitals, affected the improvement performance of Farshchian hospital. The relative efficiency security margin of Farshchian unit is obtained from the following formula as percentage. In fact, improvement performance compared unit, is the solution of the problem up to this stage.

RESM (Farshchian, Beheshti) $=100(\alpha-\delta) =100(0.09-0.01) =8\%$

Finally, relative efficiency security margin of shahid Beheshti hospital regard to Farshchian hospital was obtained 8%. Also, we ranked these hospitals after and before algorithm that result was shown in table 6:

In table 6, the efficiency of hospitals is calculated based on the Anderson-Petersen method. Also, the ranking of hospitals before and after the perform of algorithm has been obtained. As it can be seen, Farshchian inefficient hospital, which had a surplus of inputs and we reduced them according to the algorithm, has risen from 13th to 2nd rank, and efficient hospital Shahid Beheshti has become one grade worse.

Table 6- Efficiency and Ranking of hospitals after and before running algorithm

DMUs	Super Efficiency Before algorithm	Super Efficiency after algorithm	Rank before algorithm	Rank after algorithm
Beasat	1.1178	1.1178	6	7
Beheshti	1.0458	0.9776	9	10
Sina	1.1635	1.1635	3	4
Fatemieh	1.3512	1.2950	2	3
Emam Hasan	0.5436	0.5436	15	15
Emam Hossein	0.8575	0.7780	12	14
Mehr	0.8375	0.7814	14	13
Emam Reza	1.1419	1.1419	4	5
Ayat Allah Bahari	1.8642	1.8642	1	1
Heydari	1.0886	1.0886	7	8
Alimoradiyan	1.0675	1.0675	8	9
Valiaasr1	0.9545	0.9532	10	11
Valiaasr2	1.1199	1.1199	5	6
Ghaeem	0.9453	0.8601	11	12
Farshchian	0.8516	1.3557	13	2

Table 7**-** Comparison of efficiency results of hospitals before and after the perform of the algorithm

Hospital	Initial efficiency	Final efficiency	Result
Beasat	1.0000	1.0000	Unchanged
Beheshti	1.0000	0.9761	Decrease
Sina	1.0000	1.0000	Unchanged
Fatemieh	1.0000	1.0000	Unchanged
Emam Hasan	0.5436	0.5436	Unchanged
Emam Hossein	0.8575	0.7771	Decrease
Mehr	0.8375	0.7808	Decrease
Emam Reza	1.0000	1.0000	Unchanged
Ayat Allah Bahari	1.0000	1.0000	Unchanged
Heydari	1.0000	1.0000	Unchanged
Alimoradiyan	1.0000	1.0000	Unchanged
Valiaasr1	0.9545	0.9531	Decrease
Valiaasr2	1.0000	1.0000	Unchanged
Ghaeem	0.9453	0.8588	Decrease
Farshchian	0.8602	1.0000	Increase

Also, the ranking results of hospitals under the influence of algorithm performing and improving the efficiency of inefficient Farshchian hospital were obtained. Ayatollah Bahari has remained stable and Farshchian has improved. The ranking of Besat, Shahid Beheshti, Sina, Fatemieh, Imam Hossein, Imam Reza, Heidari, Alimoradian, Valiasr 1, Valiasr 2 and Ghaem hospitals has deteriorated the summary of these results and the effect of improving the performance of Farshchian Hospital in the last stage of the performing of the sensitivity analysis algorithm on other hospitals can be seen in table7.

5. Conclusion

In this paper we explained sensitivity analysis and presented a new and important topic of sensitivity analysis called relative efficiency security margin of DMUs with efficient frontier and a numerical example were used to illustrate the advantages and capabilities of our proposed sensitivity analysis algorithm. Then we perform the proposed sensitivity analysis algorithm for a real case of fifteen hospitals of Medical Sciences in Iran with two inputs and three outputs. After running sensitivity analysis algorithm results were shown in table 5. Farhchian hospital must reduce first input up to 92 units and second input up to 23 units to be efficient.

According to the column six and stage five of algorithm in table 5, Farhchian hospital being efficient and its score efficiency reach to one. This means that out of 649 first inputs, 92 are surplus and out of 163 second inputs, 23 are surplus and it is necessary for managers to apply these changes to improve performance, which is one of the strengths of the algorithm.

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