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A One-Model Method Based on a Relaxed Combination of Inputs for Congestion Assessment in Interval Data Envelopment Analysis

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Abstract

It is of special importance to calculate and recognize the quantity of congestion, as one of the major sources of inefficiency in different areas, and attempt to resolve it in order for reducing the costs and increasing the output. To date, various methods have been proposed for the calculation of congestion in classic data envelopment analysis (DEA) with precise input and output values while, in the real world, the input and output values are imprecise in most of the cases. The present paper proposes a new model for calculating the congestion interval for interval data in such cases that the interval inputs are not constrained to the selection of dominant projection points and, thereby, more outputs can be generated for the projection points. The proposed method is used for assessing the inefficiency and finding the values of congestion in the inputs of 20 bank branches.

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1. Introduction

Data envelopment analysis was developed, for the first time, by Charnes et al. [1] and was used as a useful tool for management and decision making. Since then, it has been astonishingly developed in terms of theorems, technology, and applications in different scientific fields. The model presented by Charnes et al. [1] is known as the CCR model. Later, Banker et al. [2] introduced a form of the CCR model with variable returns to scale (VRS) named BCC model. The classic DEA models enable us to assess the units with known and precise inputs and outputs and identify efficient units. However, in reality, often the precise data of the inputs and outputs of the units are not available and, thus, it is impossible to determine the precise numerical value for some of the inputs and outputs. This necessitates using models that can assess the efficiency of the decision-making units (DMUs) considering the imprecise data. An example of the imprecise data is the interval data, the inputs and outputs of which are defined in an interval of numbers. Many researchers have proposed various approaches for dealing with imprecise data in DEA. In general, uncertainty in the DEA literature has been reported in three continuous streams including randomized (stochastic) method, fuzzy method, and interval method. Here, we focus on the third stream, which is the one used in the present work.

Cooper et al. [3] introduced, for the first time, the term "imprecise data envelopment analysis (IDEA)", which refers to those models that have been obtained from the addition of the imprecise data (interval data) to the classic models of DEA. Lee et al. [4] studies the IDEA and developed this concept to the additive model. Zhu [5] discusses the IDEA method proposed by Cooper et al. [3] concerning the numerous scale conversions and variations of the variable, which cause a considerably higher complexity of the DEA model. Thus, he transforms the scale conversions of both precise and imprecise (interval) data to the constraints, which leads to a rapid increase in the volume of calculations. Amir Teimouri & Kord Rostami [6] expanded the method proposed by Zhu [5] for measuring the multi-component efficiency with imprecise data while maintaining the linearity of the DEA model. Despotis & Smirlis [7] developed an interval method with imprecise data in DEA by transforming a nonlinear DEA model to a linear model and defining the upper and lower bounds for the efficiency scores of the DMUs. Entani et al. [8] proposed a DEA model with interval efficiency measured both pessimistically and optimistically. Their proposed model was developed, initially, for crisp data and, then, for fuzzy and interval data. Wang et al. [9] presented a pair of DEA models needless of any changes in the variables with a constant and unit boundary for the measurement of the efficiency of the DMUs with interval input and output data. These models were developed for measuring the upper and lower bounds of the best relative efficiency of each DMU, which differs from the interval formed by the best and worst relative efficiency of each DMU.

One of the concepts of DEA is congestion. Congestion introduces an economic status and occurs when reducing in some inputs can increase the outputs. Research on congestion in DEA was commenced first by Fare & Svensson [10] in 1980. Fare et al. [11] presented a radial method in DEA for the calculation of congestion. This method was, for many years, the only method in DEA literature that served as a guideline for all the studies on congestion until Cooper et al. [12] introduced another method based on the slacks. Cooper et al. [12], by presenting examples, concluded that Fare et al.'s method [11] would fail to demonstrate the correct results. Moreover, Cooper et al. [13] proposed a unified additive model for the assessment of inefficiency and determination of congestion.

Cooper et al. [14], through a timely innovation in the data of the textile industry in China by increasing the inputs (work) and reducing the inputs (capital), could obtain helpful results for the improvement of the congestion management. These results, in general case, indicated that the application of proper changes for determining a combination of the inputs proportionate to the conditions of society would lead to increased output; therefore, obtaining a better output requires to be more flexible in changing the combination of inputs. Accordingly, Jahanshahloo & Khodabakhshi [15] presented a method with two models for determining the direction along which the changes should be applied to the inputs and also determine which inputs should be increased or decreased. Then, by solving this model for the data of the Chinese textile industry, they could obtain the input congestion. Cooper et al. [16] introduced a method that could calculate the quantity of congestion by solving only one model, in contrast to the previous methods that required solving of two models of the DEA models. Khodabakhshi [17] proposed a one-model (single-model) method for calculating the input congestion in DEA, which yielded the same results as those of the two-model method presented by Jahanshahloo & Khodabakhshi [15] and also required fewer calculations than it. The main objective of the present study is to develop the method presented in [17] for the interval congestion calculation.

The rest of the paper is organized as follows. Section (2) introduces two methods of congestion calculation with crisp data, one of which has been presented by Jahanshahloo & Khodabakhshi [15] and the other one by Khodabakhshi [17]. Section (3) addresses the one-model method presented in [17] with imprecise (interval) data. In Section (4), a numerical example is provided. And finally, Section (5) includes the conclusion.

2. Calculation of congestion with crisp data in DEA

2.1. The two-model method of congestion

In order to investigate along which direction the changes should be applied and determine which inputs should be increased or decreased, Jahanshahloo & Khodabakhshi [15] presented a two-model method as the following:

Assume that we have n DMU$_j$s (j=1, …, n). s_{11}^- and s_{12}^+ are, respectively, the slacks for increasing and reducing the ith input. Thus, in the following model, the objective function is determined such that s_{11}^- and s_{12}^+ reach their maximum and minimum value, respectively.

$$
\begin{aligned}
&\max \phi + \varepsilon\left(\sum_{i=1}^{m} s_{i1}^- + \sum_{r=1}^{s} s_r^+ - \sum_{i=1}^{m} s_{i2}^+\right)\\
&s.t \quad x_{i0} = \sum_{j=1}^{n} \lambda_j x_{ij} + s_{i1}^- - s_{i2}^+ ,\\
&\qquad i = 1, \dots, m\\
&\qquad \sum_{j=1}^{n} \lambda_j y_{ij} - \phi_0 y_{r0} - s_r^+ = 0,\\
&\qquad r = 1, \dots, s\\
&\qquad \sum_{j=1}^{n} \lambda_j = 1\\
&\qquad s_{i1}^-, s_{i2}^+, \lambda_j, s_r^+ \geq 0
\end{aligned}
\tag{1}
$$

This model is always feasible because $0 = s_{i2}^{+} = s_{i1}^{-}\left(\forall i\right)$, $\lambda_j = 0\left(j \neq 0\right)$, $s_r^{+} = 0\left(\forall r\right), \lambda_0 = 1$ and $\phi_0 = 1$ is a feasible solution for the above model. Model (1) is indeed an output-oriented BCC model except that the under-assessment DMU resources are not limited and, thus, the outputs can be improved by applying some changes to some of the inputs.

Definition 1: If the two following conditions are met, then DMU_0 will be efficient under Model (1).

1) $\phi_0^{*} = 1$

2) The optimal value of all the slacks is zero.

Model (1) is a two-step method. In the first method, we obtain the value of $\max \phi_0 = \phi_0^{*}$ regardless of the slacks. Then, in the second step, by replacing the value of ϕ_0^{*} for ϕ_0, we calculate $max \sum s_{i2}^{+} - \sum s_{i1}^{-} + \sum s_r^{+}$.

Definition 2 (Input congestion): A DMU has input congestion when the reduction in one or more inputs is associated with an increase in one or more outputs without worsening of any of the other inputs or outputs and, on the contrary, the increase in one or more inputs is associated with a reduction in one or more outputs without improvement of any of the other inputs or outputs.

Definition 3 (Technical inefficiency): A DMU is inefficient when it is possible to improve some of the inputs or outputs without worsening of the other inputs or outputs.

Technical inefficiency can be construed as a synonym with "*loss*" (wasting), thus, in presence of technical inefficiency, the improvement can be achieved needless to further utilization of the resources or needless to further production of the product.

To determine the input congestion, in addition to the two-step model (1) and finding the optimal solution $\left(\phi_0^{*}, \lambda_0^{*}, s_{i1}^{-*}, s_{i2}^{+*}, s_r^{+*}\right)$ there is another model as shown below, which is used for determining the technical inefficiency in the inputs.

$$
\begin{aligned}
& max \sum_{i=1}^{m} \delta_i^{+} \\
& s.t \quad \left(x_{i0} - s_{i1}^{-*} + s_{i2}^{+*}\right) = \sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^{+*}, \\
& \qquad i = 1, \ldots, m \\
& \qquad \phi_0^{*} y_{r0} + s_r^{+*} = \sum_{j=1}^{n} \lambda_j y_{rj}, \\
& \qquad r = 1, \ldots, s \\
& \qquad \sum_{j=1}^{n} \lambda_j = 1 \\
& \qquad \delta_i^{+} \leq s_{i1}^{-*}, \\
& \qquad i = 1, \ldots, m \\
& \qquad \delta_i^{+}, \lambda_j \geq 0
\end{aligned}
\tag{2}
$$

And finally, the congestion value of the i[th] input is defined as follows:

$$s_i^{-c} = s_{i1}^{-*} - \delta_i^{+*} \tag{3}$$

s_{i1}^{-*} and s_i^{-c} represent the total inefficiency and congestion inefficiency of the i[th] input, respectively. The technical inefficiency value is indicated by δ_i^{+*}, which has been obtained from Model (2).

2.2. The One-model method of congestion

The two-model method introduced by Jahanshahloo & Khodabakhshi [15] for congestion determination can be relaced with the one-model method proposed by Khodabakhshi [17], which is another method for determining the input congestion by the output-oriented BCC model.

Regarding the fact that $\left(\phi_0^{*}, \lambda_0^{*}, s_{i1}^{-*}, s_{i2}^{+*}, s_r^{+*}\right)$ is an optimal solution for Model (1), and also by applying

$s_i^{-c} = s_{i1}^{-*} - \delta_i^{+*}$ in (3), Model (2) can be rewritten as follows:

$$\begin{aligned}
&\max \sum_{i=1}^{m} -s_i^{-c} \\
&s.t \quad \left(x_{i0} - s_i^{-c} + s_{i2}^{+*}\right) = \sum_{j=1}^{n} \lambda_j x_{ij}, \\
&\quad i = 1,\ldots,m \\
&\quad \phi_0^{*} y_{r0} + s_r^{+*} = \sum_{j=1}^{n} \lambda_j y_{rj}, \\
&\quad r = 1,\ldots,s \\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad 0 \le s_i^{-c}, \quad i = 1,\ldots,m \\
&\quad \lambda_j \ge 0
\end{aligned} \tag{4}$$

Now, if we consider the following model:

$$\begin{aligned}
&\max \phi + \varepsilon \left(\sum_{i=1}^{m} -s_i^{-c} + \sum_{r=1}^{s} s_r^{+} - \sum_{i=1}^{m} s_{i2}^{+} \right) \\
&s.t \qquad x_{i0} = \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^{-c} - s_{i2}^{+} \\
&\qquad i = 1,\ldots,m \\
&\qquad 0 = \sum_{j=1}^{n} \lambda_j y_{rj} - \phi_0 y_{r0} - s_r^{+}, \\
&\qquad r = 1,\ldots,s \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\qquad s_i^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \ge 0
\end{aligned} \tag{5}$$

Then, the optimal solution for Model (5) will be $(\phi_0^*, \lambda_0^*, s_i^{-c*}, s_{i2}^{+*}, s^{+*}{}_r)$. It is evident that $s_{i2}^{+*}, s^{+*}{}_r, \phi^*$ is a part of the optimal solution for Model (1) and $\left(\lambda^*, s_i^{-c*}\right)$ is the optimal solution for Model (4). In other words, Model (4) is a part of the two-step solution method for Model (5).

It can be concluded that determining the congestion using the method presented in [15] two-model method requires to initially solve three problems while, in the case of using one-model method [17], even if the two-step method (5) is applied, it would be required to solve only two problems. In other words, solving 3 problems by the two-model method can be reduced to solving 2 problems by the one-model method. Therefore, it seems to be a suitable method in terms of the computational aspect.

While s_i^{-c} represents the value of the congestion of the i^th input, we will have the two following theorems:

Theorem 1: The congestion will exist if and only if, for the optimal solution $\left(\phi_0^*, \lambda_0^*, s_i^{-c*}, s_{i2}^{+*}, s^{+*}{}_r\right)$ obtained from Model (5), at least one of the following conditions is met:

1) $\phi_0^* > 1$ and there is at least one i(1≤i≤m) so that $s_i^{-c*} > 0$.

2) There is at least an r (r=1, 2, 3, …,s) for which $s_r^{+*} > 0$ and also an i (1≤i≤m) so that $s_i^{-c*} > 0$.

Theorem 2: The congestion will exist if and only if, for an optimal solution $\left(\phi_0^*, \lambda_0^*, s_i^{-c*}, s_{i2}^{+*}, s^{+*}{}_r\right)$ obtained from Model (5), there exists at least one $s_i^{-c*} > 0$ $(1 \le i \le m)$.

Proof: See Khodabakhshi [17].

3. The proposed model for the measurement of congestion with interval data

Assume that the input-output data of each DMU is included in a bounded interval, meaning that:

$$x_{ij} \in \left[\underline{x}_{ij}, \overline{x}_{ij}\right], (j = 1,\ldots,n, i = 1,\ldots,m)$$
$$y_{rj} \in \left[\underline{y}_{rj}, \overline{y}_{rj}\right], (j = 1,\ldots,n, r = 1,\ldots,s)$$

Where $\underline{X}_{ij}$ and $\underline{Y}_{rj}$ indicate the lower bound of the input ***i*** and output ***r*** of DMU$_j$, respectively. Similarly, $\overline{X}_{ij}$ and $\overline{Y}_{rj}$, respectively, represent the upper bound of the input **i** and output **r** of DMU_j, which all are positive. Models (6) and (9) are proposed for determining the interval of the congestion. Now, to find the lowest congestion value, which is represented by $\underline{s}_i^{-c*}$, the following model is used.

$$
\begin{aligned}
&\max \phi + \varepsilon\left(\sum_{i=1}^{m} -s_i^{-c} + \sum_{r=1}^{s} s_r^{+} - \sum_{i=1}^{m} s_{i2}^{+}\right) \\
&\text{s.t} \quad \sum_{j=1}^{n} \lambda_j \overline{x}_{ij} + \lambda_o \underline{x}_{io} + s_i^{-c} - s_{i2}^{+} = \underline{x}_{io}, \\
&\quad i = 1,\dots,m \\
&\quad \sum_{j=1}^{n} \lambda_j \underline{y}_{rj} + \lambda_o \overline{y}_{ro} - s^{+}_{r} = \phi \overline{y}_{ro}, \qquad (6)\\
&\quad r = 1,\dots,s \\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad s_i^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \geq 0 \\
&\quad j = 1,\dots,n, i = 1,\dots,m, r = 1,\dots,s
\end{aligned}
$$

In Model (6), DMU_0 is in the best conditions and the others in the worst conditions.

To find the lower bound of the efficiency of the under-assessment unit, the following model is used when the under-assessment unit is in the best conditions and the others in the worst:

$$
\begin{aligned}
&\max \phi + \varepsilon\left(\sum_{i=1}^{m} s_{i1}^{-} + \sum_{r=1}^{s} s_r^{+} - \sum_{i=1}^{m} s_{i2}^{+}\right) \\
&s.t \quad \sum_{j=1}^{n} \lambda_j \overline{x}_{ij} + \lambda_o \underline{x}_{io} + s_{i1}^{-} - s_{i2}^{+} = \underline{x}_{io}, \\
&\quad i = 1,\dots,m \\
&\quad \sum_{j=1}^{n} \lambda_j \underline{y}_{rj} + \lambda_o \overline{y}_{ro} - s^{+}_{r} = \phi \overline{y}_{ro}, \\
&\quad r = 1,\dots,s \qquad (7)\\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad s_{i1}^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \geq 0
\end{aligned}
$$

The optimal solution for Model (6) is $\left(\underline{\phi}_0^{*}, \lambda_0^{*}, \underline{s}_i^{-c*}, \underline{s}_{i2}^{+*}, \underline{s}_r^{+}\right)$ wherein $\underline{\phi}_0^{*}$, $\underline{s}_r^{+*}$, and $\underline{s}_{i2}^{+*}$ are a part of the optimal solution for Model (7) and $\underline{s}_i^{-c*}$can be obtained from the following model. In other words, Model (8) is a part of the two-step solution method of Model (6).

$$
\begin{aligned}
&\max \sum_{i=1}^{m} -s_i^{-c} \\
&s.t \quad \left(\underline{x}_{io} - s_i^{-c} + s_{i2}^{+*}\right) = \sum_{j=1}^{n} \lambda_j \overline{x}_{ij} + \lambda_o \underline{x}_{io}, \\
&\quad i = 1,\dots,m \\
&\quad \phi_0^{*} \overline{y}_{ro} + s_r^{+*} = \sum_{j=1}^{n} \lambda_j \underline{y}_{rj} + \lambda_o \overline{y}_{ro} \qquad (8)\\
&\quad r = 1,\dots,s \\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad 0 \leq s_i^{-c}, \ i = 1,\dots,m \\
&\quad \lambda_j \geq 0
\end{aligned}
$$

In Model (6), $\underline{s}_i^{-c*}$ takes the lowest congestion value while DMU_0 is in the best conditions and the other units in the worst conditions. Now, to find the highest congestion value, which is represented by $\overline{s}_i^{-c*}$, the following model is used, wherein the under-assessment unit is in the best conditions and the others are in the worst conditions.

$$
\max \phi + \varepsilon\left(\sum_{i=1}^{m} -s_i^{-c} + \sum_{r=1}^{s} s_r^{+} - \sum_{i=1}^{m} s_{i2}^{+}\right)
$$

$$
\begin{aligned}
s.t\ & \sum_{j=1}^{n} \lambda_j \underline{x}_{ij} + \lambda_o \overline{x}_{i0} + s_i^{-c} - s_{i2}^{+} = \overline{x}_{i0}\ , \\
& i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j \overline{y}_{rj} + \lambda_o \underline{y}_{ro} - s^{+}_{r} = \phi\ \underline{y}_{ro}\ , \\
& r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& s_i^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \geq 0 \\
& j = 1,\ldots,n, i = 1,\ldots,m, r = 1,\ldots,s
\end{aligned} \tag{9}
$$

The following model is used to find the upper bound of the under assessment unit while the under-assessment unit is in the worst conditions and the others in the best conditions.

$$
\begin{aligned}
& \max \phi + \varepsilon \left(\sum_{i=1}^{m} s_{i1}^{-} + \sum_{r=1}^{s} s_r^{+} - \sum_{i=1}^{m} s_{i2}^{+} \right) \\
s.t\ & \sum_{j=1}^{n} \lambda_j \underline{x}_{ij} + \lambda_o \overline{x}_{i0} + s_i^{-c} - s_{i2}^{+} = \overline{x}_{i0}\ , \\
& i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j \overline{y}_{rj} + \lambda_o \underline{y}_{ro} - s^{+}_{r} = \phi\ \underline{y}_{ro}, \\
& r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& s_i^{-}, s_{i2}^{+}, \lambda_j, s_r^{+} \geq 0 \\
& j = 1,\ldots,n, i = 1,\ldots,m, r = 1,\ldots,s
\end{aligned} \tag{10}
$$

The optimal solution for Model (9) is $\left(\overline{\phi}_0^{*}, \lambda_0^{*}, \overline{s}_i^{-c*}, \overline{s_{i2}}^{+*}, \overline{s}_r^{+}\right)$ wherein $\overline{\phi}_0^{*}$, $\overline{s}^{+*}{}_r$ and $\overline{s_{i2}}^{+*}$ are a part of the optimal solution for Model (10) and $\overline{s}_i^{-c*}$ has been obtained from Model (11). In other words, Model (11) is indeed a part of the two-step solution method of Model (9).

$$
\begin{aligned}
& \max \sum_{i=1}^{m} - s_i^{-c} \\
s.t\ & \sum_{j=1}^{n} \lambda_j \underline{x}_{ij} + \lambda_o \overline{x}_{i0} + s_i^{-c} - s_{i2}^{+} = \overline{x}_{i0}\ , \\
& i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j \overline{y}_{rj} + \lambda_o \underline{y}_{ro} - s^{+}_{r} = \phi\ \underline{y}_{ro}\ , \\
& r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& 0 \leq s_i^{-c},\ \ i = 1,\ldots,m
\end{aligned} \tag{11}
$$

In order for the congestion assessment of DMU_0 with interval data, $\left[\underline{s_i}^{-c}.\ \overline{s_i}^{-c}\right]$ is introduced as the congestion interval (CI).

Theorem2: The congestion will exist if and only if, for the optimal solutions $\left(\underline{\phi}_0^{*}, \lambda_0^{*}, \underline{s_i}^{-c*}, \underline{s_{i2}}^{+*}, \underline{s}_r^{+}\right)$ and $\left(\overline{\phi}_0^{*}, \lambda_0^{*}, \overline{s}_i^{-c*}, \overline{s_{i2}}^{+*}, \overline{s}_r^{+}\right)$ that have been obtained from Models (6) and (9), respectively, at least one of the following conditions is met:

a) $\overline{\phi}_0^{*}$ is unequal to one and there exists at least one i(1≤i≤m) so that $\overline{s}_i^{-c*}$>0.

b) There exists at least one ***r***, (r=1, 2, …,s) so that $\overline{s}_r^{+*} > 0$, and also there exists one i (1≤i≤m) so that $\overline{s}_i^{-c*} > 0$.

Proof: It is straightforward according to [17].

4. Numerical example

The DMUs investigated in the present work are related to 20 banks with 3 inputs, namely payable loans, personnel, and non-performing, and 5 outputs, including the total sum of four main deposits, other deposits, loans granted, received interest, and fee. The given data were considered as interval data, which are presented in

Tables (1) and (2). Table (1) includes the input data of the banks. In this table, the columns 2, 4, and 6 indicate the lower bounds and the columns 3, 5, and 7 indicate the upper bounds of the inputs of the banks. Similarly, in Table (2), the even columns contain the lower bounds of the outputs of the banks and the lower bounds of the outputs are shown in columns 3, 5, 7, 9, and 11. Now, considering the data provided in Tables (1) and (2), we assess and calculate the congestion interval of the banks using the proposed method.

Table 1: Input data of 20 banks

DMU	$\underline{x}_{1j}$	$\overline{x}_{1j}$	$\underline{x}_{2j}$	$\overline{x}_{2j}$	$\underline{x}_{3j}$	$\overline{x}_{3j}$
1	5007.37	9613.37	36.29	36.86	87243	87243
2	2926.81	5961.55	18.8	2019	9945	12120
3	8732.7	17752.25	25.74	27.17	47575	50013
4	945.93	1966.39	20.81	22.54	19292	19753
5	8487.07	17521.66	14.16	14.8	3428	3911
6	13759.35	27359.36	19.46	19.46	13929	15657
7	587.69	1205.47	27.29	27.48	27827	29005
8	4646.39	9559.61	24.52	25.07	9070	29983
9	1554.29	3427.89	20.47	21.59	412036	413902
10	17528.31	36297.54	14.84	15.05	8638	10229
11	2444.34	4955.78	20.42	20.54	500	937
12	7303.27	14178.11	22.84	23.19	16148	21353
13	9852.15	19742.89	18.47	21.83	17163	17290
14	4540.75	9312.24	22.83	23.96	17918	17964
15	3039.58	6304.01	39.32	39.86	51582	55136
16	6585.81	13453.58	25.57	26.52	20975	23992
17	4209.18	8603.79	27.59	27.95	41960	43103
18	1015.52	2037.82	13.63	13.93	18641	19354
19	5800.38	11875.39	27.12	27.26	19500	19569
20	1445.68	2922.15	28.96	28.96	31700	32061

Table 2: Output data of 20 banks

DMU	$\underline{y}_{1j}$	$\overline{y}_{1j}$	$\underline{y}_{2j}$	$\overline{y}_{2j}$	$\underline{y}_{3j}$	$\overline{y}_{3j}$	$\underline{y}_{4j}$	$\overline{y}_{4j}$	$\underline{y}_{5j}$	$\overline{y}_{5j}$
1	2696995	3126798	263643	382545	1677519	1853365	106634.76	125740.28	965.97	6957.33
2	340377	440355	95978	117659	377309	390302	32396..65	37836.56	304.67	479.4
3	1027546	1061260	37911	503089	1233548	1822028	96842.33	108080.01	2285.03	3174
4	1145235	1213541	229646	268460	468520	542101	32362.8	39273.37	207.98	510.93
5	390902	395241	4924	12136	129752	142873	12662.71	14165.44	63.32	92.3
6	988115	10873392	74133	1111324	507502	574355	153591.3	72257.28	480.16	869.52
7	144906	165818	180530	180617	288513	323721	40507.97	45847.48	176.58	370.81
8	408163	416416	405396	486431	1044221	1071812	56260.09	73948.09	4654.71	5882.53
9	335070	410427	337971	449336	1584722	1802942	176436.81	189006.12	560.26	2506.67
10	700842	768593	14378	15192	2290745	2573512	662725.21	791463.08	58.89	86.86
11	641680	696338	114183	241081	1579961	2285079	117527.58	20773.91	1070.81	2283.08
12	453170	481943	27196	29553	245726	275717	35757.83	42790.14	375.07	559.85
13	553167	574989	21298	23043	425886	431815	145652.24	50255.75	438.43	836.82
14	309670	342598	20168	26172	124188	126930	8143.79	11948.04	936.62	1468.45
15	286149	317186	149183	270708	787959	810088	106798.63	11962.3	1203.79	4335.24
16	321435	347848	66169	80453	360880	379488	89971.47	165524.22	200.36	399.8
17	618105	835839	244250	404579	9136507	9136507	33036.79	41826.51	2781.24	4555.42
18	248125	320974	3063	6330	26687	26687	9525.6	10877.78	240.04	274.7
19	640890	679916	490508	684372	2946797	2946797	66097.16	95329.87	961.56	1914.25
20	109948	120208	14943	17495	297674	297674	21991.53	27934.19	282.73	471.22

Table 3: Results obtained from the proposed method for the applied example

DMU	φ	s^-_{i1}	s^-_{i2}	s^-_{i3}	s^+_{i1}	s^+_{i2}	s^+_{i3}
1	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
2	[1 2.85]	[0 0]	[0 1993.7]	[0 0]	[0 1919.44]	[0 0]	[0 0]
3	[1 1.88]	[0 0]	[0 0]	[0 0]	[0 8340.64]	[0 0]	[0 46.67]
4	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
5	[1 1.07]	[0 0]	[0 0]	[0 0]	[0 88.54]	[0 0]	[0 0]
6	[1 1.05]	[0 0]	[0 0]	[0 0]	[0 22635.52]	[0 0]	[0 0]
7	[1 1.34]	[0 0]	[0 6.77]	[0 0]	[0 0]	[0 0]	[0 0]
8	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
9	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
10	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
11	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
12	[1 2.84]	[0 0]	[0 2.14]	[0 0]	[0 1200.99]	[0 0]	[0 0]
13	[1 2.12]	[0 0]	[0 1.32]	[0 0]	[0 7560.10]	[0 0]	[0 0]
14	[1 3.70]	[0 0]	[0 0.15]	[0 0]	[0 6202.13]	[0 0]	[0 0]
15	[1 2.21]	[0 0]	[0 18.61]	[0 0]	[0 0]	[0 0]	[0 0]
16	[1 3.78]	[0 0]	[0 3.97]	[0 0]	[0 4371.61]	[0 0]	[0 0]
17	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
18	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
19	[1 1]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]	[0 0]
20	[1 5.37]	[0 0]	[0 2.74]	[0 0]	[0 0]	[0 0]	[0 0]

The columns 3-8 in Table (3) show the changes that can be imposed on the combination of the inputs. For example, the number of personnel of the 7th unit (i.e. 7th bank) should be reduced because, in the fourth column in Table (3), s^-_{i2} is non-zero. The 5th and 6th units can increase their granted loans due to having a non-zero $s_{i1}{}^+$. Also, the 2nd unit should reduce its personnel and increase the granted loans. Based on the data given in Table (3), it can be found out that, due to the consideration of a free combination of the inputs, the proposed model is capable to produce the outputs more than or equal to the observed outputs by merely imposing some limited changes on some of the inputs.

As can be inferred from Table (4), Units 1, 4, 8, 9, 10, 11, 17, 18, and 19 are efficient and the congestion values of the 1st, 2nd, and 3rd inputs of these units are zero. But the other units are inefficient. It can be seen in Table (3) that the 2nd unit is inefficient with congestion since, in Table (4), the upper bound values of the congestion relevant to the granted loans and personnel (s^c_{i1} and s^c_{i2}) are non-zero implying that the presence of congestion has resulted from the excessive use of the personnel and the huge granted loans. Such a status stands true for Units 12, 13, 14, and 16. The 3rd unit has congestion since its efficiency is unequal to 1 and there exists at least one input with non-zero congestion interval. In Banks 3, 5, and 6, granting the huge loans has resulted in the congestion (the upper bound of S^c_{i1} is non-zero). The only bank in which the congestion has been caused merely by the use of excessive personnel is the 20th bank. In Bank 7, the excessive use of all the three inputs has incurred the congestion. In all the banks with congestion, the output can be increased by eliminating the congestion.

Table 4: Congestion values (quantities)

DMU	S_{i1}^{c}	S_{i2}^{c}	S_{i3}^{c}
1	[0 0]	[0 0]	[0 0]
2	[0 3838.88]	[0 1993.71]	[0 0]
3	[0 45271.05]	[0 0]	[0 0]
4	[0 0]	[0 0]	[0 0]
5	[0 177.09]	[0 0]	[0 0]
6	[0 45271.05]	[0 0]	[0 0]
7	[0 1.34]	[0 6.77]	[0 6.77]
8	[0 0]	[0 0]	[0 0]
9	[0 0]	[0 0]	[0 0]
10	[0 0]	[0 0]	[0 0]
11	[0 0]	[0 0]	[0 0]
12	[0 2401.97]	[0 2.14]	[0 0]
13	[0 15120.19]	[0 1.32]	[0 0]
14	[0 12404.26]	[0 0.15]	[0 0]
15	[0 0]	[0 18.61]	[0 0]
16	[0 8743.23]	[0 3.97]	[0 0]
17	[0 0]	[0 0]	[0 0]
18	[0 0]	[0 0]	[0 0]
19	[0 0]	[0 0]	[0 0]
20	[0 0]	[0 2.74]	[0 0]

5. Conclusion and suggestions

In the currently existing models in DEA, the maximum output is obtained by the minimum possible input value of the under-assessment DMU_0. However, sometimes, a higher output value is achieved by applying quite a few changes to some of the input elements.

In this paper based on the one- model method proposed in [17] for crisp data, a model for calculating the interval congestion for interval data has been proposed, and the applicability of the proposed method was shown for assess and calculate the congestion interval of the 20 banks. This method can be developed for network structures with various imprecise data such as random and fuzzy data.

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