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Ranking of Units by Anti-Ideal DMU with Common Weights

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Abstract

Data envelopment analysis (DEA) is a powerful technique for performance evaluation of decision making units (DMUs). One of the main objectives that is followed in performance evaluation is discriminating among efficient DMUs to provide a complete ranking of DMUs. DEA successfully divides them into two categories: efficient DMUs and inefficient DMUs. The DMUs in the efficient category have identical efficiency score. But the question that raises here is in evaluation. Where several DMUs have the equal efficiency, which unit performs better and how can we rank these efficient units, Different methods have been presented for ranking the efficient units.

In this paper, we propose a method for calculating an efficiency of DMUs by comparing with the bad benchmark line. Our approach obtain common set of weights to create the best efficiency score, such that the amount of DMUs that are efficient is less than that of other models. If we have more than one efficient DMU, we can rank them by the same model and it isn't necessary to use another ranking method.

Keywords: Data Envelopment Analysis (DEA), Common Weights Analysis (CWA), Ranking, The bad benchmark.

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1. Introduction

Charnes et al. [6] introduce data envelopment analysis (DEA) to assess the relative efficiency of a homogeneous group of operating decision making units (DMUs), such as schools, hospitals, or sales outlets. The DMUs usually use a set of resources, referred to as input indices, and transform them into a set of outcomes, referred to as output indices. DEA successfully divides them into two categories: efficient DMUs and inefficient DMUs. The DMUs in the efficient category have identical efficiency score. Therefore, in these last decades several ranking methods have been invented into the DEA context (Alder, Fridman & Sinunystern [1]; Chen [7]). After the earliest ranking method, the Cross-efficiency method developed by Sexton, Silkman, and Hogan [16], a large number of papers have been developed into the DEA area such as the super-efficiency method (Anderson & Peterson [4], Banker & Chen [5], Jahanshahloo, Junior, Hossein zadeh lotfi & Akbarian [12]), the Fridman and Sinuny-stern model (Sinuny-stern,Mehrez & Barboy, [17]).

The Common Set of Weights (CSW) approach in DEA was initially introduced by Cook et al. [8] and developed by Roll et al. [15]. Hosseinzadeh Lotfi et al. [10] used the concept of MOP and the common set of weights to calculate efficiency score of all DMUs. Jahanshahloo, Memarianiet, et al. [11] offered a method based on the common weights to measure the efficiency and to rank the efficient DMUs in the two-step process. Amin and Toloo [8] presented a model in order to find the most efficient DMUs by using a common set of weights. Liu and Peng [14] searched a common set weight to evaluate the absolute efficiency of each efficient DMUs in order to give a complete ranking for all efficient DMUs. Jahanshahloo et al. [13] suggested two methods to obtain the set of common weights for ranking efficient DMUs by comparing with an ideal line and the special line. Wang and Chin [18, 19] suggested a framework for measuring cross-efficiency via the common set of weights . Wang et al. [20] proposed a new methodology based on regression analysis, which define the common weights from the angle of efficiency fitting and consider the traditional DEA efficiencies as their target efficiencies to be achieved. Davoodi and Zhiani Rezai [9] recently extended a common-weights DEA methodology involving a linear programming problem to gauge the efficiency of the DMUs with respect to the multi-objective model.

The proposed method is going to calculate an efficiency of DMUs by the bad benchmark line, will obtain common set of weights to create the best efficiency score, such that the amount of DMUs that are efficient is less than other models. If we have more than one efficient DMU, we can rank them by the same model and it isn't necessary to use another ranking method. In this paper, we review the concept of DEA framework in section 2. Common set of weights will be obtained by comparing with the bad benchmark line in section 3. Also in section 4, they will rank DMUs by CWA-efficiency. In section 5, they will bring an empirical example. Finally, section 6 will give their conclusions.

2. DEA framework

We deal with n DMUs each having m different inputs $\{x_{ij}\}$ $(i = 1, \dots, m)$ and s different outputs $\{y_{rj}\}$ $(r = 1, \dots, s)$. We denote by DMU_j $(\mathrm{x_j}, \mathrm{y_j})$ $(\mathrm{j} = 1, \dots, \mathrm{n})$ and the input/output data matrices by $\mathrm{X} = (x_{ij}) \in R^{m\times n}$ and $\mathrm{Y} = (y_{rj}) \in R^{s\times n}$ respectively. We assume $\mathrm{X} > 0$ and $\mathrm{Y} > 0$. Under the constant returns to scale (CRS) assumption, the following model measures the efficiency of DMU_O i.e. DMU under consideration [6]:

$$\theta_o^* = Max \ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \tag{1}$$

$$s.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad j = 1, \dots, n$$
$$u_r \geq \varepsilon > 0 \qquad r = 1, \dots, s,$$
$$v_i \geq \varepsilon > 0 \qquad i = 1, \dots, m.$$

Where the symbol ε is a positive Archimedean infinitesimal constant, which is used in order to avoid the appearance of zero weights. DMU_o is efficient in model (1) iff $\theta_o^* = 1.0$.
We define E=$\{j | \theta_j^* = 1.0,\ j = 1, \dots, n\}$ to represent the set of efficient DMUs. It is helpful for decision makers only to focus on the efficient $DMUs$. However, decision makers always face the problem of how to carry out a comparison among $DMUs$ on the set E.
Liu and Peng [14] present the vertical and horizontal axes set as the virtual output (weighted sum of s outputs) and virtual input (weighted sum of m inputs), respectively. Then, they define the common benchmark level as one straight line that passes through the origin with slope 1.0. $u_r\ (r = 1, 2, \dots, s)$ and $v_i\ (i = 1, 2, \dots, m)$ in the weighted sum denote the decision variables of the common weights for the rth output and ith input index, respectively. For any two DMUs, DMU_M and DMU_N, given one set of weights for each, $u_r^{'}\ (r = 1, 2, \dots, s)$ and $v_i^{'}\ (i = 1, 2, \dots, m)$, the coordinates of points M' and N' in Fig.1 are $\left(\sum_{i=1}^{m} v_i^{'} x_{iM}, \sum_{r=1}^{s} u_r^{'} y_{rM}\right)$ and $\left(\sum_{i=1}^{m} v_i^{'} x_{iN}, \sum_{r=1}^{s} u_r^{'} y_{rN}\right)$, respectively. The virtual gaps between points M' and $M^{'P}$ between N' and $N^{'P}$ on the horizontal axes are denoted by $\Delta_{M'}^{I}$, $\Delta_{N'}^{I}$, respectively and the vertical axes by $\Delta_{M'}^{O}$, $\Delta_{N'}^{O}$, respectively. So, there exists a total virtual gap $\Delta_{M'}^{I} + \Delta_{M'}^{O} + \Delta_{N'}^{I} + \Delta_{N'}^{O}$ from points (M', N') to the benchmark line. Liu and Peng [14] determined an optimal set of weights, $u_r^*\ (r = 1, \dots, s)$ and $v_i^*\ (i = 1, \dots, m)$, such that both points M^* and N^* below the ideal line could be as close to their projection points, M^{*P} and N^{*P} on the benchmark line, as possible. In other words, by adopting the optimal weights, the total virtual gaps $\Delta_{M^*}^{I} + \Delta_{M^*}^{O} + \Delta_{N^*}^{I} + \Delta_{N^*}^{O}$ from both DMUs to the benchmark line is the shortest. They formulated the common set of weights for efficient units as follows:

$$\Delta^* = Min \quad \sum_{j \in E} \left(\Delta_j^O + \Delta_j^I\right)$$
$$s.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj} + \Delta_j^O}{\sum_{i=1}^{m} v_i x_{io} - \Delta_j^I} = 1, \quad j \in E,$$
$$\Delta_j^O, \Delta_j^I \geq 0, \qquad j \in E, \qquad (2)$$
$$u_r \geq \varepsilon > 0, \qquad r = 1, \dots, s,$$
$$v_i \geq \varepsilon > 0, \qquad i = 1, 2, \dots, m.$$

Then, they transformed model (2) to a linear form.
In this paper, we are going to introduce the model that obtains an optimal set of weights such that the bad benchmark line passes through the origin and the anti-ideal point with slope g (we'll define it in the next section), and the distances of the other DMUs to this line are the furthest. In this model, the number of DMUs that are efficient is less than other models. If we have more than one efficient DMU, we can rank them by the same model and it isn't necessary to use another ranking method.

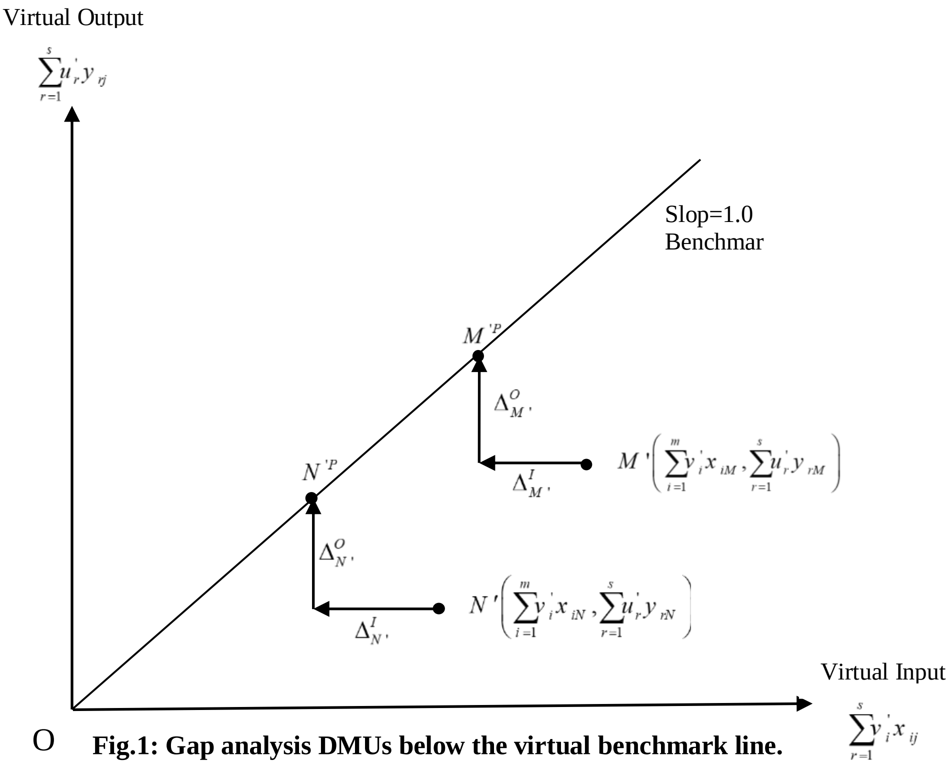

Fig.1: Gap analysis DMUs below the virtual benchmark line.

3. Common set of weights by the bad benchmark line

In conventional DEA models, each DMU in turn maximizes the efficiency score, under the constraint that none of DMU's efficiency score is allowed to exceed 1.0. In this method, according to this constraint we are going to obtain an efficiency model to calculate the best absolute efficiency score such that the total virtual gap to the bad benchmark line (we'll define it as follows) is the furthest to all DMUs. Then we can rank DMUs with the same efficiency if we will have a model with more than one DMU that has efficiency score of 1.0.

In this methodology, first we scale the input and output data by normalization; i.e. we divide ith input and rth output of all DMUs by R_i^- and R_r^+, respectively, where:

$R_i^- = \{Max\ x_{ij} \big| j = 1, \dots, n\}$
$(i = 1, \dots, m)$
$R_r^+ = \{Max\ y_{rj} \big| j = 1, \dots, n\}$
$(r = 1, \dots, s)$.

Definition 1: The anti-ideal DMU is a virtual point, which consumes the most inputs only to produce the least outputs.

That is, if we show the anti-ideal DMU with $\widetilde{DMU} = (\tilde{X}, \tilde{Y})$, i.e.

$\tilde{x}_i = Max\left\{\frac{x_{ij}}{R_i^-} \middle| j = 1, \dots, n\right\}$
$(i = 1, \dots, m)$
$\tilde{y}_r = Min\left\{\frac{y_{rj}}{R_r^+} \middle| j = 1, \dots, n\right\}$
$(r = 1, \dots, s)$.

In Fig.2 the vertical and horizontal axes are set to be the virtual output (weighted sum of s outputs) and virtual input (weighted sum of m inputs), respectively.

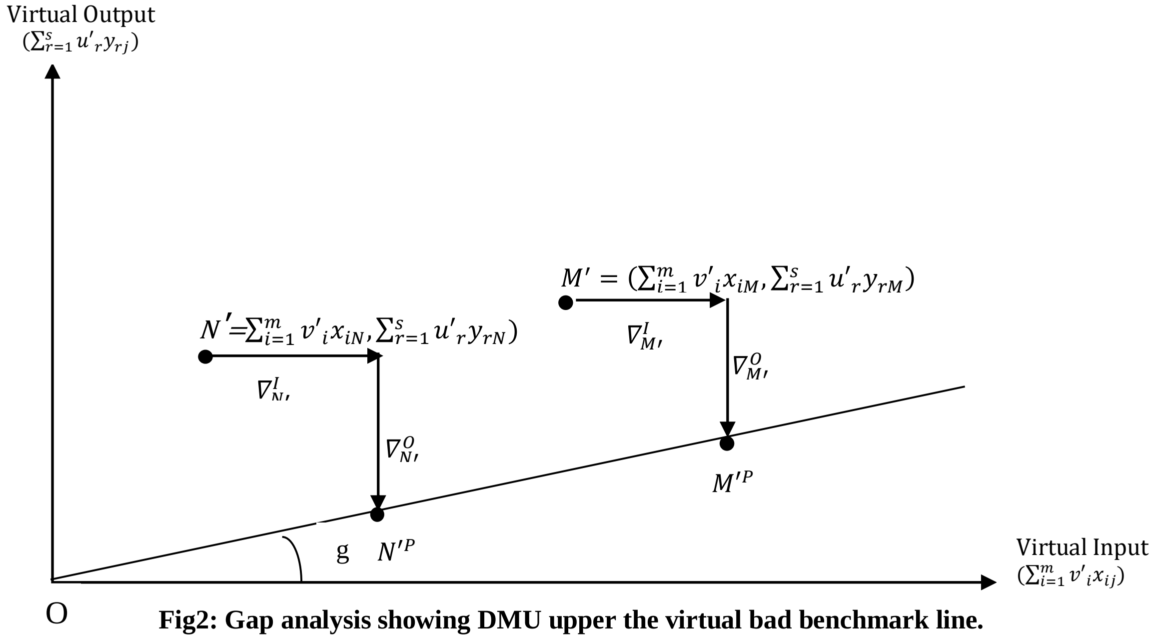

Fig2: Gap analysis showing DMU upper the virtual bad benchmark line.

Definition 2: The bad benchmark line in the virtual two-dimensions space is one straight line that passes through the origin with the slope of g that will be calculated as follows:

$$g = \frac{min\{\tilde{y}_r | r = 1, \ldots, s\}}{max\{\tilde{x}_i | i = 1, \ldots, m\}};$$

such that $\tilde{y}_r\ (r = 1, \ldots, s)$, $\tilde{x}_i (i = 1, \ldots, m)$ are outputs and inputs of the anti-ideal DMU ($\widetilde{DMU}$).

$U_r(r = 1, \ldots, s)$ and $V_i(i = 1, \ldots, m)$ in the weighted sum denote the decision variables of the common weights with the rth output and ith input index, respectively. The notation of a decision variable with superscript symbol "$'$" represents an arbitrary assigned value. For any two DMUs, DMU_M and DMU_N, if given one set of weights $U'_r(r = 1, \ldots, s)$ and $V'_i(i = 1, \ldots, m)$, then the coordinates of points M' and N' in Fig.2 are $(\sum_{i=1}^{m} v'_i x_{iM}, \sum_{r=1}^{s} u'_r y_{rM})$ and $(\sum_{i=1}^{m} v'_i x_{iN}, \sum_{r=1}^{s} u'_r y_{rN})$. The virtual gaps between points M' and M'^P on the horizontal axes and vertical axes, are denoted $\nabla_{M'}^{I}$ and $\nabla_{M'}^{O}$, respectively. Similarly, for points N' and N'^P, the gaps are $\nabla_{N'}^{I}$ and $\nabla_{N'}^{O}$. So, there exists a total virtual gap $\Delta_{M'}^{I} + \Delta_{M'}^{O} + \Delta_{N'}^{I} + \Delta_{N'}^{O}$ from points (M', N') to the bad benchmark line. We want to determine an optimal set of weights, $u_r^*(r = 1, \ldots, s)$ and $v_i^*(i = 1, \ldots, m)$, such that both points M^* and N^* upper the bad benchmark line could be as far to their projection points, M^{*P} and N^{*P} on the bad benchmark line, as possible. In other words, by adopting the optimal weights, the total virtual gaps $\Delta_{M^*}^{I} + \Delta_{M^*}^{O} + \Delta_{N^*}^{I} + \Delta_{N^*}^{O}$ from both DMUs to the bad benchmark line is the furthest. As for the constraint, the numerator is the weighted sum of outputs minus the vertical gap ∇_j^O, and the denominator is the weighted sum of inputs plus the horizontal virtual gap ∇_j^I. The constraint implies that the direction closest to the bad benchmark line is down and rightwards at the same time. Therefore, we have following model:

$$\nabla^{*}= Max \quad \sum_{j=1}^{n}\left(\nabla_{j}^{O}+\nabla_{j}^{I}\right)$$
$$s.t. \quad \frac{\sum_{r=1}^{s}u_{r}y_{rj}}{\sum_{i=1}^{m}v_{i}x_{ij}}\leq 1$$
$$j=1,\dots,n \qquad (3)$$
$$\frac{\sum_{r=1}^{s}u_{r}y_{rj}-\nabla_{j}^{O}}{\sum_{i=1}^{m}v_{i}x_{ij}+\nabla_{j}^{I}}=\text{g}$$
$$j=1,\dots,n$$
$$\sum_{r=1}^{s}u_{r}+\sum_{i=1}^{m}v_{i}=1$$
$$\nabla_{j}^{O},\nabla_{j}^{I}\geq 0$$
$$j=1,\dots,n,$$
$$u_{r}\geq \varepsilon>0 \qquad r=1,\dots,s,$$
$$v_{i}\geq \varepsilon>0 \qquad i=1,\dots,m.$$
$$v_{i}\geq \varepsilon>0 \qquad i=1,\dots,m.$$

Here, the constraint $\sum_{r=1}^{s}u_{r}+\sum_{i=1}^{m}v_{i}=1$ is added for normalization purpose and ε is positive Archimedean infinitesimal constant. It prevents weights to become zero.

Theorem 1: The model (1) is always feasible.

Proof: The model (1) is the CCR model and it is always feasible. We can find a feasible solution of model (1) that satisfy in the constraint $\sum_{r=1}^{s}u_{r}+\sum_{i=1}^{m}v_{i}=1$ (*), because the constraint (*) cut the feasible regain of model (1), therefore we suppose that there is the vector as (V',U'), that is the feasible solution of model (1) and also is feasible in the constrain (*), so:

$$\forall j \quad \frac{\sum_{r=1}^{s}u'_{r}y_{rj}}{\sum_{i=1}^{m}v'_{i}x_{ij}}\leq 1,$$
$$and$$
$$\sum_{r=1}^{s}u'_{r}+\sum_{i=1}^{m}v'_{i}=1.$$

And as well we have:

$$\forall j \ \ \nabla_{j}^{O},\nabla_{j}^{I}\geq 0,$$
$$\frac{\sum_{r=1}^{s}u\prime_{r}y_{rj}-\nabla_{j}^{O}}{\sum_{i=1}^{m}v\prime_{i}x_{ij}+\nabla_{j}^{I}}\leq \frac{\sum_{r=1}^{s}u'_{r}y_{rj}}{\sum_{i=1}^{m}v'_{i}x_{ij}}\leq 1,$$

According to the definition of g, we have $g\leq 1$, therefore:

$$\forall j \ \ \exists\nabla_{j}^{\prime O},\nabla'^{I}_{j}\geq 0,$$
$$\frac{\sum_{r=1}^{s}u'_{r}y_{rj}-\nabla_{j}^{\prime O}}{\sum_{i=1}^{m}v'_{i}x_{ij}+\nabla_{j}^{\prime I}}=g.$$

Consequently vector $(V',U',\nabla'^{I},\nabla'^{O})$ is a solution of model (3).

The ratio form of constraints in (3) can be rewritten in a linear form, formulated in the constraint of (4):

$$\nabla^{*}= Max \ \ \sum_{j=1}^{n}\left(\nabla_{j}^{I}+\nabla_{j}^{O}\right)$$
$$s.t. \quad \sum_{r=1}^{s}u_{r}y_{rj}-\sum_{i=1}^{m}v_{i}x_{ij}\leq 0$$
$$j=1,\dots,n$$
$$\sum_{r=1}^{s}u_{r}y_{rj}-\text{g}\sum_{i=1}^{m}v_{i}x_{ij}-\nabla_{j}^{O}-\text{g}\nabla_{j}^{I}=0 \qquad (4)$$
$$j=1,\dots,n$$
$$\sum_{r=1}^{s}u_{r}+\sum_{i=1}^{m}v_{i}=1$$
$$\nabla_{j}^{\mathrm{O}},\nabla_{j}^{\mathrm{I}}\geq 0 \qquad j=1,\dots,n$$
$$u_{r}\geq \varepsilon>0 \qquad r=1,\dots,s$$
$$v_{i}\geq \varepsilon>0 \qquad i=1,\dots,m.$$

Theorem 2: For all optimal weights of model (4), there isn't any DMU that is under the bad benchmark line.

Proof: Proof by contradiction, We suppose that there is the optimal weights such as(U^{*},V^{*}), that DMU_{k} is under the bad benchmark line, so, we have:

$$\frac{\sum_{r=1}^{s}u_{r}^{*}y_{rk}}{\sum_{i=1}^{m}v_{i}^{*}x_{ik}}<\text{g}.$$

Therefore, there is a $s_{k}>0$; such that we have:

$$\sum_{r=1}^{s}u_{r}^{*}y_{rk}-\text{g}\sum_{i=1}^{m}v_{i}^{*}x_{ik}+s_{k}=0 \qquad (i).$$

According to second category constrains of model (3), we have:

$$\sum_{r=1}^{s}u_{r}^{*}y_{rk}-\text{g}\sum_{i=1}^{m}v_{i}^{*}x_{ik}-(\nabla_{j}^{O*}+\text{g}\nabla_{j}^{I*})=0 \qquad (ii).$$

Therefore, (i), (ii) imply that:

$$s_{k}=-\left(\nabla_{j}^{O*}+\text{g}\nabla_{j}^{I*}\right);$$

when this equation is true if and only if $s_k = \nabla_j^{O*} + g\nabla_j^{I*} = 0$.

It shows that DMU_k is on the bad benchmark line. It is inconsistent to a absurd hypothesis, therefore, the absurd hypothesis is false, so, a assertion prove, i.e., there isn't any DMU with any optimal weights that is under optimal weights.

4. Ranking DMUs by CWA-efficiency

For ranking efficient DMUs, we use definition of the CWA efficiency score of DMU_j that Lui and Peng [5] defined as in the following equation:

$$\xi_j^* = \frac{\sum_{r=1}^{s} y_{rj} u_r^*}{\sum_{i=1}^{m} x_{ij} v_i^*} \qquad j = 1, \dots, n. \quad (**)$$

Definition 3: DMU_j is efficient if $\xi_j^* = 1$, otherwise, DMU_j is inefficient.

The numbers of DMUs with the efficiency score 1.0 By using model (4) and equation (**) are fewer than those of other efficiency methods. Sometimes DMU with $\xi_j^* = 1$ is unique. Because we use two following conditions to obtain efficient DMU.

I) The efficient DMU has to have $\xi_j^* = 1$.

II) The efficient DMU has to have the furthest distance to the bad benchmark line.

So, the numbers of DMUs which are true in conditions (I) and (II), are fewer than the numbers of DMUs which are true in the only condition (I).

In the other hand, DMUs which have $\theta^* = 1$ at model (1) are more than efficient DMUs by our proposed method. Since the most ranking methods use model (1) in their method, so, they obtain more than one efficient DMUs.

We can use our method for ranking efficient DMUs, so that we let $\nabla_j^I + \nabla_j^O$ be ∇_j $(j \in E)$, then, we can rank them by the following definition:

Definition 4: If $\xi_j^* = \xi_i^* = 1$, i.e. they are both efficient. Then the performance of DMU_j is better than DMU_i if $\nabla_j^* > \nabla_i^*$.

Also inefficient DMUs can be ranked by the following definition:

Definition 5: Performance of DMU_j is better than DMU_i, if $\xi_j^* > \xi_i^*$.

5. Empirical example

Let us rank 20 branches of bank in Iran by our proposed method. These data were previously analyzed by Amirteimoori and Kordrostami [3] are listed in Table1.

In our method, first we define an anti-ideal bank. It has maximized inputs of all of banks as in the case of m inputs and minimized outputs of all of banks as in the case of s outputs, i.e. its inputs and outputs are as follows:

$$I_1 = 1 \qquad O_1 = 0.11$$
$$I_2 = 1 \qquad O_2 = 0.184$$
$$I_3 = 1 \qquad O_3 = 0.049.$$

We calculate the amount of g as follows:

$$g = \frac{min\{0.11, 0.184, 0.049\}}{max\{1,1,1\}} = 0.049.$$

It is clear that there isn't a real bank as anti-ideal bank. The anti-ideal bank is the virtual bank. Therefore, we use our method to calculate the efficiency score of DMUs. Therefore their results are shown in the last column of Table 1.

Table 1: Data of DMUs and their CCR efficiency and Ranking by model

Branch	$\mathbf{I_1}$ Staff	$\mathbf{I_2}$ Computer	$\mathbf{I_3}$ spaces	$\mathbf{O_1}$ Deposits	$\mathbf{O_2}$ Loans	$\mathbf{O_3}$ Change	$\boldsymbol{\theta}^*_{\mathbf{CCR}}$	ξ_j^*
1	0.950	0.700	0.155	0.190	0.521	0.293	1.000	0.4382
2	0.796	0.600	1.000	0.227	0.627	0.462	0.833	0.6292
3	0.798	0.750	0.513	0.228	0.970	0.261	0.991	0.9709
4	0.865	0.550	0.210	0.193	0.632	1.000	1.000	0.5839
5	0.815	0.850	0.268	0.233	0.722	0.246	0.899	0.7072
6	0.482	0.650	0.500	0.207	0.603	0.569	0.748	0.5722
7	0.719	0.600	0.350	0.182	0.900	0.716	1.000	1.0000
8	0.785	0.750	0.120	0.125	0.234	0.298	0.798	0.2382
9	0.476	0.600	0.135	0.080	0.364	0.244	0.789	0.6109
10	0.678	0.550	0.510	0.082	0.184	0.049	0.289	0.2168
11	0.711	1.000	0.305	0.212	0.318	0.403	0.604	0.3574
12	0.811	0.650	0.255	0.123	0.923	0.628	1.000	0.9092
13	0.659	0.850	0.340	0.176	0.645	0.261	0.817	0.7818
14	0.976	0.800	0.540	0.144	0.514	0.243	0.470	0.4207
15	0.685	0.950	0.450	1.000	0.262	0.098	1.000	0.3057
16	0.613	0.900	0.525	0.115	0.402	0.464	0.639	0.5239
17	1.000	0.600	0.205	0.090	1.000	0.161	1.000	0.7988
18	0.634	0.650	0.235	0.059	0.349	0.068	0.473	0.4397
19	0.372	0.700	0.238	0.039	0.190	0.111	0.408	0.4079
20	0.583	0.550	0.500	0.110	0.615	0.764	1.000	0.8427

According to Table 1, our proposed efficiency model obtains only one DMU (DMU_7) as an efficient DMU, because$\theta_7^* = 1.000$. But seven $DMUs$ $(1, 4, 7, 12, 15, 17, 20)$ are efficient units in CCR efficiency model (see Table 1).
In the proposed method, for ranking other $DMUs$ (inefficient $DMUs$), performance of every unit is better, if it will have more score efficiency. For example, $\theta_3^* = 0.9709$ is more than other units so DMU_3 has the second rank and also $\theta_{10}^* = 0.2168$ is less than other units, so DMU_{10} has the last rank. You can see ranks of all DMUs in Table 2.

6. Conclusion

Conventional data envelopment analysis doesn't rank efficient units. In this paper, we proposed an efficiency model that often obtains only one efficient unit. For our model, first we defined the anti-ideal DMU and the bad benchmark line, and then we built the efficiency model. If we obtain more than one DMU as efficient units, we can rank them with the same model. In the other hands, If we have two $DMUs$ that are efficient$DMUs$, the performance of every unit is better, if it has the further distance from the bad benchmark line, i.e. it has the more amount of Δ^* from other units.
Advantage of our method is that we can obtain the efficiency of units with one model and also we can rank them by the same model.
As suggestion in the next paper, we can find the both line "the good and bad benchmark line" in the virtual two-dimensions pace, then determine an optimal set of weights u_r^* $(r = 1, \dots, m)$ and v_i^* $(i = 1, \dots, m)$ for calculating efficiency, such that the total virtual gaps all of DMUs to the good benchmark line is the shortest and to the bad benchmark line is the furthest.

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