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A Novel Approach for Efficiency Measurement in AHP with Mixed Criteria and Its Comparison with DEA

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Abstract

Multi-Criteria Decision-Making (MCDM) problems often involve a combination of qualitative and quantitative criteria for evaluating alternatives. A major challenge in such problems is employing a methodology that can simultaneously and effectively process both types of criteria. While Data Envelopment Analysis (DEA) is a well-established method for efficiency evaluation in quantitative-based problems, its inherent limitations in handling qualitative criteria reduce its applicability in more complex decision scenarios. In this study, we introduce a novel efficiency measurement approach using the Analytic Hierarchy Process (AHP) and establish a mathematical comparison with the CCR-DEA model. The proposed method integrates pairwise comparisons and weight derivations for qualitative and quantitative criteria, enabling efficiency analysis without requiring the conversion of qualitative data into numerical values. The findings demonstrate that AHP not only provides a meaningful efficiency assessment in the presence of mixed criteria but also exhibits structural similarities to DEA under specific conditions. This highlights the potential of AHP as a complementary tool for MCDM problems where both qualitative and quantitative factors play a crucial role.

Keywords: Analytic Hierarchy Process, Data Envelopment Analysis, Multi-Criteria Decision-Making, Efficiency Analysis.

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1. Introduction

In modern decision-making environments, evaluating alternatives based on multiple criteria is a fundamental yet challenging task. Many real-world problems require assessing a combination of both qualitative and quantitative factors, making the selection of an appropriate evaluation method crucial. Among the most widely used approaches for efficiency assessment are the Analytic Hierarchy Process (AHP) and Data Envelopment Analysis (DEA). While DEA is traditionally designed for problems involving strictly quantitative inputs and outputs, AHP provides a structured framework for handling both qualitative and quantitative criteria. However, a comprehensive and systematic approach that effectively integrates these methods for efficiency measurement remains an open research challenge.

AHP, introduced by Saaty, is a multi-criteria decision-making (MCDM) technique that employs pairwise comparisons to derive relative weights for criteria and alternatives. This method has gained widespread recognition across diverse fields, including management, engineering, and economics, due to its ability to process subjective judgments and combine them with numerical data. Over the years, various enhancements to AHP have been proposed, including hybrid models integrating artificial intelligence, optimization techniques, and fuzzy logic to improve decision accuracy in complex scenarios [1].

On the other hand, DEA, first proposed by Charnes, is a non-parametric approach for assessing the relative efficiency of decision-making units (DMUs). It evaluates how effectively each unit converts multiple inputs into desirable outputs, making it a powerful benchmarking tool in operations research and performance evaluation. One of DEA's main advantages is its ability to handle multiple inputs and outputs without requiring an explicit mathematical function. However, DEA struggles with qualitative criteria, as it inherently relies on precise numerical values, requiring qualitative factors to be artificially converted into quantitative measures—an approach that can introduce bias and distort efficiency results [2].

In DEA-based efficiency analysis, a DMU's performance is evaluated relative to a set of comparable units. Inputs may include labor, capital, and raw materials, while outputs typically represent measurable outcomes such as production volume or revenue. However, in many practical settings, qualitative aspects such as customer satisfaction, innovation, or service quality play a critical role in determining efficiency. Traditional DEA models are not well-equipped to handle these non-numerical attributes, limiting their applicability in decision-making contexts where qualitative criteria are crucial.

This study proposes a new efficiency measurement approach using AHP to overcome DEA's limitations in handling mixed qualitative-quantitative criteria. Unlike DEA, which requires data transformation for qualitative factors, AHP inherently incorporates both types of criteria without information loss. The paper provides a rigorous mathematical formulation of efficiency within the AHP framework and systematically compares it with the CCR-DEA model. By analyzing their structural similarities and differences, we demonstrate that under specific conditions, AHP-based efficiency measurement aligns closely with DEA results, validating its applicability as both an alternative and a complementary methodology. The findings suggest that AHP can serve as an effective decisionsupport tool in multi-criteria efficiency assessment, particularly in scenarios where qualitative criteria play a significant role alongside quantitative factors.

2. Literature Review

2.1. Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP), introduced by Saaty, is a widely applied multi-criteria decision-making (MCDM) method designed to handle complex decision problems by structuring them hierarchically. This method enables decision-makers to systematically evaluate alternatives based on a combination of qualitative and quantitative criteria through pairwise comparisons. One of the key strengths of AHP is its ability to incorporate subjective judgments into a mathematical framework, making it particularly useful in decision-making scenarios where qualitative factors play a significant role [1].

Several studies, including Vaidya & Kumar et al. and Saaty 2008, have employed AHP for decision problems where criteria include both cost (undesirable) and benefit (desirable) factors. In these studies, alternatives are prioritized based on their relative performance, ensuring that those with higher benefits and lower costs receive better rankings [3,4]. Moreover, Ishizaka highlighted that while qualitative criteria in AHP are assessed through expert pairwise comparisons, quantitative criteria can either be directly incorporated or normalized before analysis to maintain consistency [5].

Recognizing the need to enhance AHP's capability in efficiency measurement, researchers have explored hybrid models that integrate AHP with other MCDM techniques such as TOPSIS and DEA. For instance, Cheng et al. (2013) proposed AHP-DEA models, where qualitative criteria are weighted through AHP, while DEA processes quantitative inputs and outputs. These studies indicate that AHP can compensate for DEA's limitation in handling qualitative data, making it a viable complementary approach for efficiency evaluation [6].

A key challenge in efficiency analysis is the simultaneous consideration of cost and benefit criteria, which is particularly relevant in cases where both tangible and intangible factors influence decision outcomes. Studies by Wang et al. and Jablonsky have compared AHP-based and DEA-based efficiency assessments, demonstrating that AHP provides a more flexible framework by allowing the inclusion of qualitative factors without requiring numerical conversion. Their findings suggest that while DEA is well-suited for benchmarking homogeneous decision-making units (DMUs) based on measurable inputs and outputs, AHP extends applicability to scenarios where qualitative judgments are essential [7,8].

Given these insights, AHP presents itself as a powerful alternative or complementary method to DEA in efficiency evaluation, particularly in decision-making problems where qualitative and quantitative factors must be considered simultaneously. This research aims to build on these foundations by introducing a novel AHP-based efficiency metric and comparing its performance with CCR-DEA model, providing a structured framework for efficiency assessment in multi-criteria environments.

2.2. Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a powerful non-parametric mathematical programming technique that is widely used to assess the relative efficiency of decision-making units (DMUs) with multiple inputs and outputs. First, it introduced by Charnes, Cooper 1978. DEA evaluates the performance of a set of

DMUs without the need for a predefined functional relationship between inputs and outputs. Instead, it constructs an efficient frontier that represents the best possible performance achievable by the DMUs in the dataset [2].

The core principle behind DEA is to evaluate the efficiency of a given DMU by comparing it to a reference set of the most efficient units. Efficient DMUs are located on the frontier, while inefficient DMUs are those that are below this frontier. DEA calculates a technical efficiency score for each unit, representing how well it transforms inputs into outputs relative to the best-performing units in the dataset.

DEA is based on the following key characteristics:

1. Multiple Inputs and Outputs: DEA handles problems where multiple inputs and outputs are involved, making it particularly useful in complex decision-making scenarios.

2. Non-Parametric Nature: Unlike other methods such as regression, DEA does not require a specific functional form for the relationship between inputs and outputs, which allows it to model complex, non-linear relationships without relying on any assumption about the production technology.

3. Returns to Scale: DEA can be applied under two main assumptions about returns to scale: Constant Returns to Scale (CRS): As used in the original CCR model, where outputs increase proportionally with inputs. Variable Returns to Scale (VRS): As seen in the BCC model, which accounts for scenarios where outputs do not scale proportionally to inputs, providing greater flexibility for more heterogeneous units.

DEA's flexibility and non-parametric nature make it widely applicable across diverse industries such as healthcare, education, banking, manufacturing, and public administration, where performance measurement is critical. Notable studies in these areas, such as bank efficiency (Coelli et al.) and healthcare sector analysis (Rossetti et al.), highlight its robustness in comparing and evaluating units with multiple dimensions of performance [9,10].

Despite its wide application, one of the main limitations of DEA is its exclusive reliance on quantitative data. It is particularly effective in settings where all performance indices can be quantitatively measured, such as production rates, financial outcomes, or service delivery times. However, qualitative criteria, such as customer satisfaction, service quality, or employee morale, which are increasingly important in decision-making, cannot be directly included in DEA models. This limitation has been recognized in the literature, where various studies (e.g., Färe et al., [11]) have pointed out the challenge of incorporating subjective, qualitative inputs into DEA-based models.

Given these limitations, there has been a growing interest in combining DEA with other methodologies to better address both quantitative and qualitative aspects of decision-making. Notable among these is the integration of DEA with the Analytic Hierarchy Process (AHP), which allows for the incorporation of qualitative judgments through pairwise comparisons and weighted aggregation. This hybrid approach leverages the strengths of AHP in handling qualitative data while utilizing the rigor of DEA in evaluating the efficiency of DMUs.

Our research introduces the efficiency of AHP through a novel approach, compares it with the efficiency of DEA, and demonstrates that there are similarities between these two methods. The proposed model does not require the conversion of qualitative data into quantitative metrics, ensuring a more accurate and context-sensitive evaluation of DMUs in decision problems that involve both qualitative and quantitative criteria. This combined approach is positioned as a complementary methodology for multi-criteria decision-making in environments where both types of criteria are integral to the evaluation process.

2.2.1. Efficiency

Consider a set of n alternatives or DMUs (decision-making units), where the vectors $x_j = (x_{1j}, ..., x_{mj})$ represent the input values and $y_j = (y_{1j}, ..., y_{sj})$ represent the output values for DMU j, for $y_j \geq 0$,, $x_j \neq 0$ .The efficiency $j = 1,...,n$, $x_j \geq 0$, $y_j \neq 0$ of DMU is then defined as the ratio of the weighted sum of the outputs to the weighted sum of the inputs:

$$Efficiency\ DMUj = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \qquad (1)$$

where x_{ij} denotes the value of input i for DMU j , and y_{rj} represents the value of output r for DMU j. The weights u_r and v_i correspond to the outputs and inputs, respectively, and they are determined through the optimization model.

2.2.2. CCR-DEA Basic Model

In the CCR model, it is assumed that returns to scale are constant, and its fundamental formulation is expressed as a fractional programming problem:

$$\begin{aligned} \max \ \theta_k &= \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \\ S.t. \ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad j = 1,...,n, \qquad (2) \\ & u_r \geq 0, v_i \geq 0, \quad r = 1,...,s; i = 1,...,m, \end{aligned}$$

where model (2) converted to linear model (3) by Charnes-Cooper transformations.

$$\begin{aligned} \max \ \theta_k &= \sum_{r=1}^{s} u_r y_{rj} \\ S.t. \ & \sum_{i=1}^{m} v_i x_{ij} = 1, \qquad (3) \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1,...,n, \\ & u_r \geq 0, v_i \geq 0, \quad r = 1,...,s; i = 1,...,m. \end{aligned}$$

If the objective function in model (3) results in a value of 1, then DMU k is considered efficient. Conversely, if the efficiency value is less than 1, DMU k is deemed inefficient, and improvements can be achieved by reducing inputs, increasing outputs, or a combination of both.

2.2.3. Ranking

In this section, we focus on ranking efficient DMUs using the Anderson-Peterson [12] model. To do so, the efficient alternative is removed from the set of all alternatives, and the CCR model is used to evaluate it. The Anderson-Peterson model is formulated as follows:

$$\begin{aligned} \max \ \theta_k &= \sum_{r=1}^{s} u_r y_{rk} \\ S.t. \ & \sum_{i=1}^{m} v_i x_{ik} = 1, \qquad (4) \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1,...,n; j \neq k, \\ & u_r \geq 0, v_i \geq 0, \ r = 1,...,s; i = 1,...,m. \end{aligned}$$

The value of the objective function is greater than or equal to 1. The alternative with the highest objective function value is ranked higher.

3. Proposed Method

In this paper, to improve evaluation accuracy and avoid the complexities of directly combining qualitative and quantitative criteria in a hierarchical process, we propose using two separate hierarchical processes (as shown in Figures 1 and 2). Initially, an independent hierarchical process is used to determine the weights of the alternatives based on the input criteria. Subsequently, another independent hierarchical process is carried out for the output criteria. In this method, both sets of criteria (input and output) can be either qualitative or quantitative, but they cannot simultaneously be treated as purely quantitative, as this would make data envelopment analysis (DEA) a more appropriate method for evaluating alternative efficiency.

In the second step, the results from both processes obtained in the first step (the weights of the alternatives corresponding to input and output criteria) are combined to provide a final efficiency evaluation. As a result, a comprehensive ranking of the alternatives based on their efficiency is derived. This approach ensures that each set of criteria is analyzed independently and accurately, while preventing the issues related to the direct combination of qualitative and quantitative criteria in a single process. Ultimately, the efficiency index of each alternative is calculated by considering the weighted combination of both input and output criteria.

The step of the AHP for n alternatives and several criteria are as follows:

Step 1. Define the Goal: The decision-making goal is defined.

Step 2. Define the Criteria: The different criteria used to evaluate the alternatives are identified.

Step 3. Construct Pairwise Comparison Matrices: For each criterion, the alternatives are compared against each other, and pairwise comparison matrices are constructed.

Step 4. Calculate the Weights: The weights for each alternative are calculated based on the comparisons using mathematical methods.

Step 5. Aggregate the Weights: The weights for each alternative across all criteria are combined, and ultimately, a ranking of the alternative is created.

Assume there are n alternatives, each associated with m cost (input) criteria and s profit (output) criteria. The AHP efficiency of alternative i, $i=1,\dots,m$, is given by:

$$E_{AHP}^{i}=\frac{W_{out,\,i}}{W_{in,\,i}} \tag{5}$$

where $W_{out,\,i}$ and $W_{in,\,i}$ represent the weights of alternative i relative to the output (profit) and input (cost) criteria, respectively.

Theorem: Prove that the efficiency of AHP, calculated as the ratio of output weights to input weights, is equivalent to the absolute efficiency in DEA.

Proof: Let us consider n alternatives, each with m quantitative input criteria and s qualitative output criteria. Let matrix X represent the input data matrix, and matrix A represent the pairwise comparison matrix for the input criteria, normalized column-wise. Suppose that v is the weight vector derived from the row average of the input criteria weights in matrix A. Thus, Xv is the weighted vector of the alternatives with respect to the input criteria.

Let P_k denote the pairwise comparison matrix between the alternatives for each output criterion k. Assume that matrix is normalized row-wise, and the resulting weight vector is y_k, $k=1,...,s$. Similarly, let matrix Q represent the pairwise comparison matrix between the outputs criteria, where its row-normalized form results in the weight vector u. Consequently, the weights of the alternatives relative to the output criteria is given by Yu, where is the matrix of output weights $Y=[y_1,...,y_k,...,y_s]$.

Now, referring to the definition of AHP efficiency for alternative i, calculated as the ratio of output weights to input weights, we express it as:

$$E_{AHP}^{i}=\frac{Yu}{Xv}=\frac{\sum_{k=1}^{s}y_{ik}u_k}{\sum_{j=1}^{m}x_{ij}v_j},$$

Where y_{ik} denotes the weight of alternative i relative to output criterion k and u_k is the weight of output criterion k.

Also, x_{ij} denotes the weight of alternative i relative to input criterion j and v_j is the weight of output criterion j.

By examining equation (1), it becomes apparent that the structure of the numerator and denominator in AHP efficiency closely mirrors the design of the DEA-CCR efficiency formula. Specifically, if we assume that the output weights in AHP, (u_k) are analogous to the output weights in DEA-CCR, (u_r) and the input weights in AHP, (v_j) correspond to the input weights in DEA-CCR, (v_i) the structure of the formulas in both models is identical. Therefore, the two models can be regarded as functionally equivalent. Although in AHP, x_{ij} and y_{ik} represent the weight of alternative i relative to inputs and outputs criterion, but in DEA-CCR (model 3), x_{ij} andboth are input y_{rj} and output values which are normalized in solving model. In this scenario, these values can be interpreted as the input and output weights for each decision-making unit (alternative) in the context of the respective input and output criteria.

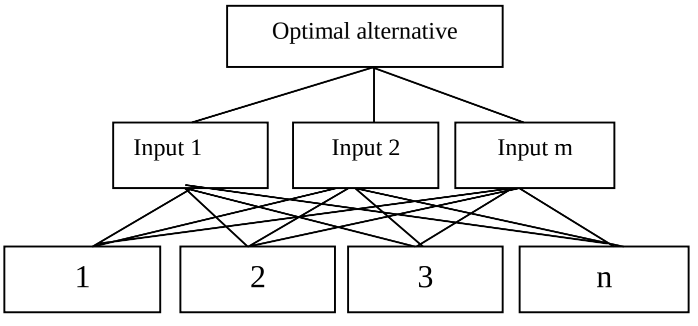

Fig. 1. Hierarchical structure based on the input criterion

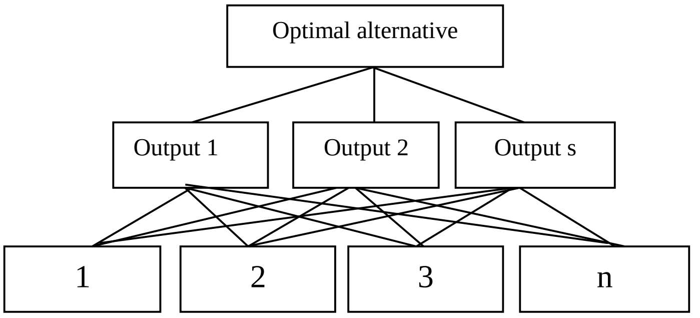

Fig. 2. Hierarchical structure based on the output criterion

4. Numerical Example:

Consider four bank branches, each evaluated using three quantitative input criteria and three qualitative output criteria. The input criteria are training hours for personnel, training costs, and number of personnel, while the output criteria are public relations, creativity, and discipline. The data for the input criteria is shown in Table 1. Since the output criteria are qualitative, pairwise comparison matrices are used to express their priorities as perceived by the decision-maker. These matrices are employed in Tables 6 through 13. For the purpose of dimensionless scaling of the data, the sum of the column vectors has been employed.

To determine the weights of the alternatives, two hierarchical processes are conducted: one for the inputs and one for the outputs. Initially, the hierarchical process for evaluating the alternatives relative to the quantitative input criteria is performed. Since the input criteria are quantitative, no pairwise comparisons are necessary. As a result, the weights of the alternatives based on input criteria are presented in Table 2.

Additionally, the relative priorities of input criteria, as well as their normalized values along with the weights of each criterion, are presented in Tables 3 and 4 respectively.

Table 1. Quantitative Inputs for Alternatives

Inputs	Number of personnel	(per thousand) Training costs	Training hours
A	10	11	12
B	15	10	8
C	10	7	11
D	12	14	10

Table 2. Weights of Alternatives Based on Criteria of Personnel Number, Training Costs, and Training Hours

Alternatives	Personnel count	Personnel-basd wieghts	Training costs (per thousand)	Training cost weights	Training hours	Training hours weights
A	10	0.213	11	0.262	12	0.293
B	15	0.319	10	0.238	8	0.195
C	10	0.213	7	0.167	11	0.268
D	12	0.255	14	0.333	10	0.244
Total	47	1	42	1	41	1

Table 3. Pairwise Comparison Matrix for Input Criteria

	Personnel count	Training costs	Training hours
Personnel count	1	2	$1/2$
Training costs	$1/2$	1	3
Training hours	4	$1/3$	1
Total	5.5	3.333	4.25

Table 4. Normalized Pairwise Comparison Matrix and Average Weight of Each Criterion

	Personnel count	Training costs	Training hours	Weights
Personnel count	8.182	0.6	0.059	0.28
Training costs	0.091	0.3	0.706	0.366
Training hours	0.727	0.1	0.235	0.354
Total	1	1	1	1

The final weights of the alternatives based on the input criteria, which are determined by summing the products of each criterion's weight and the corresponding alternative's weight for that criterion, are shown in Table 5.

Next, we proceed with the hierarchical analysis for the alternatives based on output criteria. Since the outputs are qualitative, the decision-maker determines the relative priority of the units with respect to the output criteria. The results of this analysis are provided in Tables 6 through 13.

Table 5. Weights of Alternatives Based on Input Criteria

Alternatives	Weights
A	0.237
B	0.256
C	0.287
D	0.220

Table 6. Pairwise Comparisons of Alternatives Based on the Public Relations Criterion

Option priority for public relations	A	B	C	D
A	1	2	8	3
B	$1/2$	1	6	2
C	$1/8$	$1/6$	1	4
D	$1/3$	$1/2$	$1/4$	1
Total	1.958	3.66	15.25	10

Table 7. Normalized Weight of Each Alternative Relative to the Public Relations Criterion

Option priority for public relations	A	B	C	D	Weights
A	0.510	0.546	0.524	0.3	0.472
B	0.255	0.271	0.393	0.2	0.280
C	0.063	0.045	0.065	0.4	0.143
D	0.170	0.136	0.016	0.1	0.105

Table 8. Pairwise Comparisons of Alternatives Relative to the Creativity Criterion

Option priority for creativity	**A**	**B**	**C**	**D**
A	1	$1/3$	$1/4$	2
B	3	1	$1/2$	$1/2$
C	4	2	1	3
D	$1/2$	2	$1/3$	1
Total	8.5	6.33	2.083	6.5

Table 9. Normalized Weights of Alternatives Relative to the Creativity Criterion

Option priority for public relations	A	B	C	D	Weights
A	0.11	0.052	0.12	0.307	0.147
B	0.352	0.157	0.24	0.076	0.249
C	0.470	0.315	0.48	0.461	0.431
D	0.058	0.315	0.16	0.15	0.173

Table 10. Pairwise Comparisons of Alternatives Relative to the Discipline Criterion

Option priority for discipline	A	B	C	D
A	1	$1/4$	$1/6$	2
B	4	1	$1/3$	1
C	6	3	1	2
D	$1/2$	1	$1/3$	1
Total	11.5	5.25	1.833	7

Table 11. Normalized Weight of Each Alternative Relative to the Discipline

Option priority for discipline	**A**	**B**	**C**	**D**	**Weights**
A	0.086	0.047	0.090	0.285	0.127
B	0.347	0.190	0.181	0.142	0.217
C	0.521	0.571	0.545	0.428	0.517
D	0.043	0.190	0.181	0.142	0.139

Table 12. Priority of Output Criteria Relative to Each Other

Priority of output criteria	**public relations**	**creativity**	**discipline**
Public relations	1	3	2
Creativity	$1/3$	1	$1/4$

Discipline	$^1/_2$	4	1
Total	1.833	8	3.25

Table 13. Calculation of Criterion Weights Based on Output Indicators

Priority of output criteria	Public relations	Creativity	Discipline	Weights
Public relations	0.546	0.375	0.615	0.512
Creativity	0.182	0.125	0.077	0.128
Discipline	0.273	0.5	0.308	0.360

Table 14. Weight of Each Alternative Relative to the Output Criteria

Alternatives	Weights
A	0.306
B	0.254
C	0.317
D	0.123

The hierarchical process continues by determining the priority of the output criteria, which is shown in Table 12. The calculation of the weights for each criterion based on the output indicators is presented in Table 13.

Using the criterion weights from Table 13 and the weights derived from the pairwise comparisons relative to the criteria in Tables 7, 9, and 11, the final weight of each alternative based on the output criteria is shown in Table 14.

Next, we calculate the efficiency of each alternative, considering both the output and input weights from Tables 14 and 5. The calculated efficiencies are as follows:

$$E_{AHP}^{A}=\frac{0.306}{0.237}=1.291,\quad E_{AHP}^{B}=\frac{0.254}{0.256}=0.992,$$
$$E_{AHP}^{C}=\frac{0.317}{0.287}=1.105,\quad E_{AHP}^{D}=\frac{0.123}{0.220}=0.559$$

The results indicate that Alternative A has the highest efficiency and is ranked first, Alternative C is ranked second, Alternative B is ranked third, and Alternative D has the lowest efficiency, ranked last.

5. Efficiency Evaluation Using Data Envelopment Analysis (DEA)

In this section, the alternatives from the given example are evaluated using the Data Envelopment Analysis (DEA) method. It is evident that DEA models rely on quantitative input and output indicators. The inputs are quantitative; however, instead of using qualitative outputs, the weights obtained for the alternatives relative to each output criterion are employed. These inputs are normalized, and the quantitative outputs are presented in Table 15. The efficiency of the alternatives is computed using the CCR model, as shown in Table 16.

Table 15. Inputs and Outputs for Efficiency Evaluation of Four Alternatives

Alternatives	Personnel-basd wieghts	Training cost weights	Training hours weights	Public relations weights	Creativity weights	Discipline weights
A	0.213	0.262	0.293	0.472	0.147	0.127
B	0.319	0.238	0.195	0.280	0.249	0.217

C	0.213	0.167	0.268	0.141	0.431	0.517
D	0.255	0.333	0.244	0.105	0.173	0.139

Table 16. Final Efficiency of Each Alternative Based on the CCR Model

Alternatives	CCR efficiency	Super efficiency	Ranking
A	1	1.8531	2
B	1	1.0494	3
C	1	2.9178	1
D	0.52	0.4332	4

6. Comparison of AHP and DEA in Efficiency Evaluation

In this research, two prominent methods—Hierarchical Analysis (AHP) and Data Envelopment Analysis (DEA)—were compared for evaluating and ranking alternatives. In the AHP method, Alternative A emerged as the most efficient, while in the DEA-CCR model, Alternatives A, B, and C were recognized as efficient, with only Alternative D being classified as inefficient. For ranking the efficient alternatives, the Anderson-Peterson (AP) model was employed, placing Alternative C in the first position and Alternative A second. This discrepancy demonstrates that each method evaluates efficiency criteria in fundamentally different ways. Nevertheless, in both methods, Alternative B was consistently ranked third and Alternative D ranked fourth, signaling a degree of convergence between the results.

Using pairwise comparisons, AHP processes qualitative and quantitative criteria separately and calculates the efficiency index based on the ratio of output weight to input weight. Conversely, DEA is an optimization-based mathematical model that determines efficiency by maximizing the output-to-input ratio. A key limitation of the DEA-CCR model, when dealing with qualitative data is its inability to directly process such data, necessitating the conversion of qualitative values into quantitative ones. This transformation may diminish the accuracy of the results. Furthermore, the DEA-CCR model may classify multiple alternatives as efficient without providing sufficient differentiation, particularly when the number of alternatives is smaller than the total number of input and output criteria (usually, the number of alternatives must exceed three times the sum of input and output criteria). This necessitates models like AP for more precise ranking.

Despite this, AHP's flexibility in evaluating qualitative criteria yields a more transparent and definitive ranking, while, DEA excels in purely quantitative contexts. The difference in the ranking of Alternatives A and C between the two methods underscores how the chosen data processing approach influences the results. However, the consistent ranking of Alternatives B and D across both methods indicates a level of coherence in the overall ranking, which can guide decision-makers in selecting the most suitable method.

7. Conclusions

The primary objective of this study was to assess the efficiency of the Analytic Hierarchy Process (AHP) when addressing both qualitative and quantitative criteria in alternative evaluation. To achieve this, two distinct hierarchical processes were employed for quantitative and qualitative criteria, ensuring a more accurate assignment of weights to each alternative. This approach facilitates a more comprehensive evaluation of alternatives in complex situations involving multiple criteria, eliminating the need to convert qualitative data into quantitative forms, which could compromise the precision of the results. We have also demonstrated that a similar structure relationship exists between the AHP method and the DEA technique.

In comparison to the DEA-CCR method, which primarily relies on quantitative data for efficiency evaluation, the results revealed that DEA cannot process qualitative data effectively. Moreover, in DEA, multiple units can be classified as efficient simultaneously, necessitating additional models for ranking. On the other hand, AHP provided a more precise evaluation by using pairwise comparisons for both qualitative and quantitative criteria, enabling it to highlight significant differences in the ranking of alternatives. Ultimately, in this study, the alternatives were assessed using both AHP and DEA methods, with AHP proving to offer more reliable and accurate results, particularly in scenarios where both qualitative and quantitative criteria are present.

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