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Cross efficiency for fuzzy data envelopment analysis

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Abstract

Data envelopment analysis (DEA) is a non-parametric technique to measure the relative efficiencies of a set of decision-making units (DMUs) with common crisp inputs and outputs. Ranking of DMUs is of great importance in DEA. One of the methods for ranking DMUs is to obtain cross efficiency. The cross-efficiency method was developed as a DEA extension to rank DMUs with the main idea being to use DEA to do peer evaluation, rather than in pure self-evaluation mode. In this paper, we propose cross efficiency for DMUs with fuzzy data and use the efficiency scores to rank the fuzzy DMUs. We consider the input and output values as triangular fuzzy numbers. Our proposed model is based on input and output α-cuts. We then elaborate on the model with a numerical example.

Keywords: Fuzzy data, Cross efficiency, Data envelopment analysis (DEA)

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric technique to measure the relative efficiencies of a set of decision-making units (DMUs) with common crisp inputs and outputs. The cross-efficiency method was developed as a DEA extension to rank DMUs with the main idea being to use DEA to do peer evaluation, rather than in pure self-evaluation mode. Cross efficiency has been further investigated by [1]. There are mainly two advantages of the cross-evaluation method. It provides an ordering among DMUs, and it eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from application area experts. Cross efficiency evaluation has been used in various applications, e.g., efficiency evaluations of nursing homes [2], R&D project selection preference voting, and others. Some researchers have proposed several fuzzy models to evaluate the efficiency of DMUs with fuzzy data. [3] have used a-cut approach to measure the efficiency by BCC model t4J with fuzzy data. In this paper, we investigate cross efficiency for fuzzy data. The proposed model is based on input and output α-cuts. In most α-cut based methods, the resulting model is solved by comparing two intervals, i.e., interval of left-hand side and interval of right-hand side of each equality/ inequality constraints. The rest of the paper is organized as follow. In Section 2, we explain the CCR model with fuzzy data and compute efficiency intervals. In Section 3, we propose cross efficiency with fuzzy data, which is then used to rank fuzzy DMUs. In Section 4, we use a numerical example to elaborate on the proposed model.

2. DEA model with fuzzy data

Assume that we have a set of n fuzzy DMUs, each with m fuzzy inputs and s fuzzy outputs [4]. We consider the input and output values to be triangular fuzzy numbers, so that the ith input and the rth output of DMUj (j = 1, 2, …, n) are denoted by [5]

$$\tilde{x}_{ij} = (\overset{L}{x}_{ij}, \overset{M}{x}_{ij}, \overset{R}{x}_{ij}), (i = 1, 2, ..., m)$$

$$\tilde{y}_{rj} = (\overset{L}{y}_{rj}, \overset{M}{y}_{rj}, \overset{K}{y}_{rj}), (r = 1, 2, ..., s)$$

respectively. To evaluate DMUP, we indicate the α-cuts (α ∈ [0,1]) of the fuzzy inputs and outputs of DMUP as follows[6].

$$\tilde{x}\overset{\alpha}{ip} = \begin{bmatrix} (1-\alpha)\, x\overset{L}{ip} + \alpha\, x\overset{M}{ip}, \\ (1-\alpha)\, x\overset{R}{ip} + \alpha\, x\overset{M}{ip} \end{bmatrix}, \quad (1)$$
$$(i = 1, 2, ..., m), (\alpha \in [0,1])$$

$$\tilde{y}\overset{\alpha}{_{rp}} = \begin{bmatrix} (1-\alpha)\, \overset{L}{y}_{rp} + \alpha\, \overset{M}{y}_{rp}, \\ (1-\alpha)\, \overset{R}{y}_{rp} + \alpha\, \overset{M}{y}_{rp} \end{bmatrix}, \quad (2)$$
$$(r = 1, 2, ..., s), (\alpha \in [0,1])$$

Therefore, the input and output values of DMU_P are intervals and its efficiency is expressed as an interval, denoted by $(\alpha \in [0,1]), \theta_p^{\alpha} = [\theta_p^{L\alpha}, \theta_p^{R\alpha}]$, which is obtained by the following two models[7].

The scores obtained by the above two models are the best and the worst efficiency scores for DMUP, respectively; that is to say, in Model (3), DMUP is in its best state while the other DMUs are in their worst state, and the situation is the other way round in Model (4).

$$\theta_p^{R\alpha} = Min\theta \tag{3}$$
$$s.t.\ (1-\alpha)\sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} x_{ij}^{R} + \alpha \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} x_{ij}^{M} \leq (\theta - \lambda_{ip})\left[(1-\alpha) x_{ip}^{L} + \alpha x_{ip}^{M}\right], (\forall i = 1,..,m)$$
$$(1-\alpha)\sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} y_{rj}^{L} + \alpha \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} y_{rj}^{M} \geq (1-\lambda_{rp})\left[(1-\alpha) y_{rp}^{R} + \alpha y_{rp}^{M}\right], (\forall_r = 1,...,s)$$
$$\lambda_{jp} \geq 0 \quad \forall j = 1,...,n$$

$$L_\alpha \tag{4}$$
$$\theta_p = Min\ \theta$$
$$s.t.\ (1-\alpha)\sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} x_{ij}^{L} + \alpha \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} x_{ij}^{M} \leq (\theta - \lambda_{ip})\left[(1-\alpha) x_{ip}^{R} + \alpha x_{ip}^{M}\right], (\forall_i = 1,2...,m)$$
$$(1-\alpha)\sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} y_{rj}^{R} + \alpha \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_{jp} y_{jr}^{M} \geq (1-\lambda_{rp})\left[(1-\alpha) y_{rp}^{L} + \alpha y_{rp}^{M}\right], (\forall_r = 1,2...,s)$$
$$\lambda_{jp} \geq \forall_j = (1,2,...,n)$$

$$Max \quad E_{pp}^{R\alpha} = \sum_{r=1}^{s} u_{rp}\left[(1-\alpha) y_{rp}^{R} + \alpha y_{rp}^{M}\right] \tag{5}$$
$$s.t.\ \sum_{r=1}^{s} u_{rp}\left[(1-\alpha) y_{rj}^{L} + \alpha y_{rj}^{M}\right] - \sum_{i=1}^{m} v_{ip}\left[(1-\alpha) x_{ij}^{R} + \alpha x_{ij}^{M}\right] \leq 0,\ \forall_j = 1,000,n, j \neq p$$
$$\sum_{r=1}^{s} u_{rp}\left[(1-\alpha) y_{rp}^{R} + \alpha y_{rp}^{M}\right] - \sum_{i=1}^{m} v_{ip}\left[(1-\alpha) x_{ip}^{L} + \alpha x_{ip}^{M}\right] \leq 0$$
$$\sum_{i=1}^{m} v_{ip}\left[(1-\alpha) x_{ip}^{L} + \alpha x_{ip}^{M}\right] = 1$$
$$u_{rp} \geq 0, \forall r = 1,...,s, v_{ip} \geq 0\ , \forall i = 1,...,m$$

3. Cross efficiency with fuzzy data

In this section, we intend to explain cross efficiency for fuzzy DMUs [8]. Assume that we have n fuzzy DMUs with triangular fuzzy inputs and outputs. We consider the α-cuts of inputs and outputs [9]. Cross efficiency is based on the multiplier form of the CCR model [10]. Suppose we are going to evaluate DMUP.

$$Max \quad E_{pp}^{L\alpha} = \sum_{r=1}^{s} u_{rp}\left[(1-\alpha)y_{rp}^{L} + \alpha\, y_{rp}^{M}\right] \tag{6}$$

$$s.t. \quad \sum_{r=1}^{s} u_{rp}\left[(1-\alpha)\, y_{rj}^{R} + \alpha\, y_{rj}^{M}\right] - \sum_{i=1}^{m} v_{ip}\left[(1-\alpha)\, x_{ij}^{L} + \alpha\, x_{ij}^{M}\right] \le 0, \forall\, j = 1,\dots,n,\, j \ne p$$

$$\forall_j = 1,\dots,n,\, j \ne p$$

$$\sum_{r=1}^{s} u_{rp}\left[(1-\alpha)\, y_{rp}^{L} + \alpha\, y_{rp}^{M}\right] - \sum_{i=1}^{m} v_{ip}\left[(1-\alpha)\, x_{ip}^{R} + \alpha\, x_{ip}^{M}\right] \le 0$$

$$\sum_{i=1}^{m} v_{ip}\left[(1-\alpha)x_{ip}^{R} + \alpha\, x_{ip}^{M}\right] = 1$$

$$u_{rp} \ge 0, \forall r = 1,\dots,s, v_{ip} \ge 0 \;\; , \forall i = 1,\dots,m$$

The multiplier models for evaluating DMUP are the duals of Models (3) and (4), as follows.

Suppose that the optimal solution to Model (5) is $(u_{rp}^{R\alpha}, v_{ip}^{R\alpha})$ and that of Model (6) is $(u_{rp}^{L\alpha}, v_{ip}^{L\alpha})$ Using the weights obtained from Models (5) and (6) in evaluating DMU_P, we have the following efficiency interval for DMU_j (j = 1, 2, …, n).

$$E_{pj}^{\alpha} = \left[E_{pj}^{L\alpha},\, E_{pj}^{R\alpha}\right], \forall\, j = 1,2,\dots,n\;, \alpha \in [.,1]$$

Considering the fact that E_{pj}^{α} is an interval and that Models (5) and (6) are used to evaluate

DMU_P, we have all the possible cases as follows.

(8)

$$A = Min\left\{\sum_{r=1}^{s} u_{rp}^{L\alpha} y_{rj}^{L\alpha}, \sum_{r=1}^{s} u_{rp}^{R\alpha} y_{rj}^{L\alpha}\right\}, \forall\, j = 1,\dots,n, \alpha \in [0,1]$$

$$B = Max\left\{\sum_{r=1}^{s} u_{rp}^{L\alpha} y_{rj}^{R\alpha}, \sum_{r=1}^{s} u_{rp}^{R\alpha} y_{rj}^{R\alpha}\right\}, \forall\, j = 1,\dots,n, \alpha \in [0,1]$$

$$C = Min\left\{\sum_{i=1}^{m} v_{ip}^{L\alpha} x_{ij}^{L\alpha}, \sum_{i=1}^{m} v_{ip}^{R\alpha} x_{ij}^{L\alpha}\right\}, \forall\, j = 1,\dots,n, \alpha \in [0,1]$$

$$D = Max\left\{\sum_{i=1}^{m} v_{ip}^{L\alpha} x_{ij}^{R\alpha}, \sum_{i=1}^{m} v_{ip}^{R\alpha} x_{ij}^{R\alpha}\right\}, \forall\, j = 1,\dots,n, \alpha \in [0,1]$$

Then the efficiency of DMU_j (j = 1, …, n) is defined as[11]:

$$E_{pj}^{L\alpha} = \frac{A}{D}, E_{pj}^{R\alpha} = \frac{B}{C}, \forall\, j = 1,2\dots,n\;, \alpha \in [0,1]$$

We now calculate the mean of all (j = 1, 2, …, n)

E_{pj}^{α} for each DMU_j (j = 1, …, n).

$\bar{E}_j^{\alpha}$ denotes the cross-efficiency score of DMU_j (j = 1, …, n) at the α ∈ [0,1] level[12].

Now, for each DMU_j (j = 1, 2, …, n) we define:

$$R_j = \frac{1}{n}\max\left\{E_{pj}^{R\alpha}\right\} - \frac{1}{n}Min\left\{E_{pj}^{L\alpha}\right\} \;,(j = 1,\dots,n)$$

Then, we rank the DMUs based on the R_j (j = 1, …, n) obtained, such that the DMU_j with a greater R_j (j = 1, …, n) has a better rank [13]. Since there might be multiple optimal solutions to Models (5) and (6) in evaluating DMU_n, we use the following two models to select the optimal solution.

Table 1: α-cut of fuzzy DMUs

	1	2	3	4	...	n
1	$E^{\alpha}_{١١}=\left[E^{\alpha L}_{١١},E^{\alpha R}_{١١}\right]$	$E^{\alpha}_{١٢}=$	$E^{\alpha}_{١٣}=$	$E^{\alpha}_{١۴}=$	...	$E^{\alpha}_{١n}=$
2	$E^{\alpha}_{٢١}=\left[E^{\alpha L}_{٢١},E^{\alpha R}_{٢١}\right]$	$E^{\alpha}_{٢١}=$	$E^{\alpha}_{٢١}=$	$E^{\alpha}_{٢١}=$	...	$E^{\alpha}_{٢١}=$
. .	$E^{\alpha}_{٣١}$				...	.
n	$E^{\alpha}_{n١}=\left[E^{\alpha L}_{n١},E^{\alpha R}_{n١}\right]$	$E^{\alpha}_{n٢}=$	$E^{\alpha}_{n٣}=$	$E^{\alpha}_{n۴}=$	...	$E^{\alpha}_{nn}=$
	$\frac{١}{n}\sum_{p=١}^{n} E_{p١}$	$\frac{١}{n}\sum_{p=٢}^{n} E_{p٢}$	$\frac{١}{n}\sum_{p=٣}^{n} E_{p٣}$	$\frac{١}{n}\sum_{p=۴}^{n} E_{p۴}$	...	$\frac{١}{n}\sum_{p=n}^{n} E_{pn}$

$$
\begin{aligned}
& Max\ \sum_{r=١}^{s} u_{rp}\left[\sum_{\substack{j=١\\ j\neq p}}^{n} (١-\alpha)\, y^{L}_{rj}+\alpha\ \overset{M}{y}_{rj}\right]\\
& s.t.\ \sum_{i=1}^{m} v_{ip}\left[\sum_{\substack{j=1\\ j\neq p}}^{n} (1-\alpha)\overset{R}{x}_{ij}+\alpha\ \overset{M}{x}_{ij}\right]=1\\
& \sum_{r=1}^{s} u_{rp}\left[(1-\alpha)\overset{R}{y}_{rp}+\alpha\ \overset{M}{y}_{rp}\right]-E^{R\alpha}_{pp}\sum_{i=1}^{m} v_{ip}\left[(1-\alpha)\overset{L}{x}_{ip}+\alpha\,\overset{M}{x}_{ip}\right]=0\\
& \sum_{r=1}^{s} u_{rp}\left[(1-\alpha)\overset{L}{y}_{rj}+\alpha\ \overset{M}{y}_{rp}\right]-\sum_{i=1}^{m} v_{ip}\left[(1-\alpha)\overset{R}{x}_{ij}+\alpha\,\overset{M}{x}_{ij}\right]\leq 0 \qquad j=1,\dots,n,\ j\neq p\\
& \qquad u_{rp}\geq 0 \qquad r=1,\dots,s\\
& \qquad v_{ip}\geq 0 \qquad i=1,\dots,m
\end{aligned}
$$

And

$$
Max\ \sum_{r=1}^{s} u_{rp}\left[\sum_{\substack{j=1\\ j\neq p}}^{n} (1-\alpha)\overset{R}{y}_{rj}+\overset{M}{y}_{rj}\right]
$$

$$s.t.\ \sum_{i=1}^{m} v_{ip}\left[\sum_{\substack{j=1\\ j\neq p}}^{n} (1-\alpha)\, x_{ij}^{L} + \alpha\, x_{ij}^{M}\right] = 1$$

$$\sum_{r=1}^{s} u_{rp}\left[(1-\alpha)\, y_{rp}^{L} + \alpha\ y_{rp}^{M}\right] - E_{pp}^{L\alpha}\sum_{i=1}^{m} v_{ip}\left[(1-\alpha)x_{ip}^{R} + \alpha\, x_{ip}^{M}\right] = 0$$

$$\sum_{r=1}^{s} u_{rp}\left[(1-\alpha)\, y_{rj}^{R} + \alpha\ y_{rj}^{M}\right] - \sum_{i=1}^{m} v_{ip}\left[(1-\alpha)x_{ij}^{L} + \alpha\, x_{ij}^{M}\right] \le 0 \qquad j = 1,\dots,n,\ j\neq p$$

$$u_{rp} \ge 0 \qquad r = 1,\dots,s$$

$$v_{ip} \ge 0 \qquad i = 1,\dots,m$$

4. Numerical Example

Using an example in which there are one input and one output with fuzzy triangular values, first provided by (Guo and Tanaka, 2000), we investigate fuzzy cross efficiency.

Table 2 contains five fuzzy DMUs with one input and one output, each with fuzzy triangular values.

Table 2: DMUs with fuzzy triangular values

DMU_S	Input (Loft, mean, right)	Output (left, mean, right)
DMU_1	(1.5, 2, 2.5)	(0.7, 1, 1.3)
DMU_2	(2.5, 3, 3.5)	(2.3, 3, 3.7)
DMU_3	(2.4, 3, 3.6)	(1.6, 2, 2.4)
DMU_4	(4, 5, 6)	(3, 4, 5)
DMU_5	(4.5, 5, 5.5)	(1.8, 2, 2.2)

We explain this example with $\alpha = 0.1$ and $\alpha = 0.5$.

First, for $\alpha = 0.1$, we present the optimal solutions obtained by Models (5) and (6) in Tables 3 and 4, respectively

Table 3: Results of model (5) for $\alpha = 0.1$

DMU_S	$\left(u^{R\alpha *}, v^{R\alpha *}\right)$
DMU_1	(0.4532, 0.64521)
DMU_2	(0.2755, 0.3922)
DMU_3	(0.2856, 0.4065)
DMU_4	(0.1713, 0.2439)
DMU_5	(0.1544, 0.2198)

Table 4: Results of model (6) for $\alpha = 0.1$

DMU_S	$\left(u^{L\alpha *}, v^{L\alpha *}\right)$
DMU_1	(0.5942, 0.4082)
DMU_2	(0.4219, 0.2899)
DMU_3	(0.4112, 0.2825)
DMU_4	(0.2267, 0.1695)
DMU_5	(0.2671, 0.1835)

Table 5: Fuzzy cross-efficiency scores

DMU_S	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5
DMU_1	$\theta_{11}^{\alpha} = [0.2092, 1.1926]$	$\theta_{12} = [0.4825, 2.0721]$	$\theta_{13} = [0.3253, 1.3965]$	$\theta_{14} = [0.3690, 1.7396]$	$\theta_{15} = [0.2345, 0.6974]$
DMU_2	$\theta_{21} = [0.2093, 1.1925]$	$\theta_{22} = [0.4825, 2.0716]$	$\theta_{23} = [0.3254, 1.3961]$	$\theta_{24} = [0.3690, 1.7394]$	$\theta_{25} = [0.2345, 0.6972]$
DMU_3	$\theta_{31} = [0.2092, 1.1927]$	$\theta_{32} = [0.4826, 2.0721]$	$\theta_{33} = [0.3254, 1.3964]$	$\theta_{34} = [0.3691, 1.7395]$	$\theta_{35} = [0.2345, 0.6974]$

DMU_4	$\theta_{41}=[0.2092,1.1926]$	$\theta_{42}=[0.4824,2.0719]$	$\theta_{43}=[0.3253,1.3964]$	$\theta_{44}=[0.3690,1.7395]$	$\theta_{45}=[0.2345,0.6973]$
DMU_5	$\theta_{51}=[0.2092,1.1926]$	$\theta_{52}=[0.4825,2.0720]$	$\theta_{53}=[0.3254,1.3963]$	$\theta_{54}=[0.3690,1.7395]$	$\theta_{55}=[0.2345,0.6973]$
$\bar{E}_j^{0.1}$	$\frac{1}{5}[0.2092,1.1927]$	$\frac{1}{5}[0.4824,2.0721]$	$\frac{1}{5}[0.3253,1.3965]$	$\frac{1}{5}[0.3690,1.7396]$	$\frac{1}{5}[0.2345,0.6974]$
$R_j^{0.1}$	0.9835	1.5897	1.0712	1.3706	0.4625

Ranking of DMUs at $\alpha=0.1$

Finally, the fuzzy cross-efficiency scores for five fuzzy DMUs with one input and one output, each having fuzzy triangular values, are given in

It is evident from the last row of Table 5 that DMU_2 is the most efficient one at $\alpha=0.1$.

Now, for $\alpha=0.5$, we present the optimal solutions of Model (5) in Table 6 and those of Model (6) in Table 7.

Table 6: Optimal solutions of Model (5)

DMU_S	$\left(\begin{matrix} R\alpha* \\ u \end{matrix}, \begin{matrix} R\alpha* \\ v \end{matrix}\right)$
DMU_1	(0.4742, 0.5714)
DMU_2	(0.2985, 0.3597)
DMU_3	(0.3074, 0.3704)
DMU_4	(0.1844, 0.2222)
DMU_5	(0.1747, 0.2105)

Table 7: Optimal solutions of Model (6)

DMU_S	$\left(\begin{matrix} L\alpha* \\ u \end{matrix}, \begin{matrix} L\alpha* \\ v \end{matrix}\right)$
DMU_1	(0.5451, 0.4444)
DMU_2	(0.3774, 0.3077)
DMU_3	(0.3716, 0.3030)
DMU_4	(0.2230, 0.1818)
DMU_5	(0.2336, 0.1905)

Finally, the fuzzy cross efficiency obtained for the fuzzy DMUs is presented in Table 4.7. It is obvious that DMU_2 is the most efficient amongst all DMUs for $\alpha=0.5$, as well. Similarly, the fuzzy cross efficiency can be calculated for all $\alpha\in[0,1]$.

We here briefly illustrate the concept of cross efficiency by adopting the cross-efficiency matrix from Doyle and Green (1994). we have six DMUs. is the (cross) efficiency of DMU_j based upon a set of DEA weights calculated for DMU_p. 'This set of DMU weights gives the best efficiency score for DMU_p under evaluation by a DEA model, and End (in the leading diagonal) is the DEA efficiency for DMU_k. The cross efficiency for a given DMU_j is defined as the arithmetic average down column j, given by (we point out that in Doyle and Green (1994), the efficiency score for DMU k is not included as part of the average.)

Table 8: Ranking of DMUs at $\alpha=\frac{1}{2}$

DMU_S	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5
DMU_1	$\theta_{11}=[0.3134,0.8059]$	$\theta_{12}=[0.6766,1.4780]$	$\theta_{13}=[0.4526,0.9994]$	$\theta_{14}=[0.5281,1.2265]$	$\theta_{15}=[0.3003,0.5422]$
DMU_2	$\theta_{21}=[0.3134,0.8060]$	$\theta_{22}=[0.6766,1.4779]$	$\theta_{23}=[0.4526,9993]$	$\theta_{24}=[0.5280,1.2265]$	$\theta_{25}=[0.3003,05422]$
DMU_3	$\theta_{31}=[0.3134,0.8059]$	$\theta_{32}=[0.6766,1.4778]$	$\theta_{33}=[0.4526,9992]$	$\theta_{34}=[0.5281,1.2264]$	$\theta_{35}=[0.3003,05421]$

DMU_4	$\theta_{41}=[0.3134,0.8060]$	$\theta_{42}=[0.6766,1.4780]$	$\theta_{43}=[0.4526,9995]$	$\theta_{44}=[0.5281,1.2266]$	$\theta_{45}=[0.3003,05423]$
DMU_5	$\theta_{51}=[0.3133,0.8058]$	$\theta_{52}=[0.6766,1.4778]$	$\theta_{53}=[0.4526,9992]$	$\theta_{54}=[0.5281,1.2263]$	$\theta_{55}=[0.3003,05421]$
$\overline{E}_j^{0.5}$	$\frac{1}{5}[0.3133,0.8060]$	$\frac{1}{5}[0.6766,1.4780]$	$\frac{1}{5}[0.4526,0.9995]$	$\frac{1}{5}[0.5281,1.2266]$	$\frac{1}{5}[0.3003,0.5423]$
$R_j^{0.5}$	0.4927	0.8014	0.5469	0.6985	0.242

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