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An inverse DEA approach in the presence of undesirable outputs based on the directional distance function

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Abstract

In many production systems, we can do acquisition and merge operations process to increase productivity. For this purpose, we can use the inverse data envelopment analysis (DEA) approach. In many cases, in addition to producing desirable outputs, we also have the simultaneous production of undesirable outputs. It is important to use a suitable approach in the acquisition and merge operations process. In this paper, we present a new model based on the directed distance function. The new model provides a new unit or a pre-determined target efficiency level by merging two decision-making units (DMUs). Based on this model, level for desirable and undesirable outputs is determined for the newly created unit. In the following, we will show the provided approach with a numerical example and apply it for real world data.

Keywords: Data envelopment analysis; Inverse DEA; Directional distance function, Undesirable outputs.

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1. Introduction

The first goal of DEA was to measure the efficiency of a DMUs with multiple homogeneous inputs and outputs (Charnes et al. [1], Banker et al. [2]). The traditional DEA models measure the efficiency of each DMU by constructing a set called the production possibility set (PPS) based on the inputs and outputs of the observed data. The units located on the frontier of this set are called efficient units, and these units have the best performance among the DMUs; other inefficient units are compared with these units. The inefficient units are projected on the efficiency frontier in order to be efficient. This concept is introduced as technical efficiency in DEA.

The inverse DEA models assume the amount of efficiency as a parameter and the level of efficiency in these models is predetermined, and the levels of inputs or outputs are determined. The inverse DEA idea was proposed by Zhang and Cui [3] and then formally presented by Wei et al. [4]. The inverse DEA articles were further developed from the theoretical and practical perspectives. Amin et al. [5] proposed a general model for firms' restructuring and generalized the concept of M&A. They presented the input and output-oriented model and obtain the minimum efficiency for the post-restructuring units to show the feasibility of the general inverse DEA model. Hadi-Vencheh et al. [6] and Ghiyasi [7] proposed an application of inverse DEA in resource allocation. Ghobadi and Jahangiri [8] proposed inverse DEA models for imprecise data. They proposed a review of inverse DEA and its applications. Gattoufi et al. [9] proposed an inverse DEA model to determining the quantities of inputs that can be saved by a merger when the output ordination model obtains additional outputs that can be produced for a given efficiency target. Jahanshahloo et al. [10] developed inverse DEA models for evaluating systems with inter-temporal dependence. Lim [11] proposed an inverse DEA model by changing the production frontier to set a new production target. Eyni et al. [12] provided an inverse DEA model to the sensitivity analysis of DMUs. Ghiyasi [13] developed the inverse DEA models based on the cost and revenue efficiency concept. Amin et al. [14] proposed a method by combining goal programming and inverse DEA for target setting in mergers. Amin and Oukil [15] developed inverse DEA models for obtaining flexible target settings in mergers. Gerami et al. [16] proposed a generalized inverse DEA model for firm restructuring based on value efficiency. They show that their models obtain potential targets rather than model based on the technical efficiency. Soltanifar et al. [17] developed a new inverse DEA model for merger analysis, they show their models is capable for dealing to negative ratio data. They used their models for evaluation efficiency and merger analysis of 66 branches in Iranian bank. Ghiyasi et al. [18] developed the inverse model based on the DEA-R model. They applied their models for evaluation 130 public hospitals in Iran. Their models developed in both input and output orientations.

The different approaches proposed for dealing with undesirable outputs in DEA. One approach for dealing with undesirable outputs is to consider them as inputs to the model. Seiford and Zhu [19] shown that treating the undesirable outputs as inputs causes the resulting DEA model does not reflect the true production process. The strong disposability of undesirable outputs applies when they are freely disposable. Against the weak disposability is applied when lower production of them depends on lower production of desirable outputs, (see Fare et al.1989). Sueyoshi and Goto [20] shown the two disposability concepts from their conceptual and methodological in their study. They proved that natural disposability implies an environmental strategy that a DMU attempt to decrease an input vector to reduce a vector of undesirable outputs. Halkos and Petrou [21] proposed the treatment of undesirable outputs in DEA. They classified the methods in the context of undesirable outputs in the four groups. (i) Approaches that ignoring undesirable outputs in the model. (ii) Approaches treating them as inputs. (iii) Approaches that treating them as normal outputs. (iv) Approaches that performing necessary transformations to put them into account. Toloo and Hanclova [22] consider multi-valued criteria in the presence of undesirable outputs. Podinovski [23] used non-parametric polyhedral technologies with undesirable outputs. Pishgar-Komleh et al. [24] classified the six approaches assessing the winter wheat cropping system in Poland: (i) Treating undesirables as inputs to the DEA model. (ii) Ignoring undesirable outputs. (iii) Data transformation. (iv) Slack based measurement DEA with undesirable outputs and (iiv) ratio model. Scheel [25] proposed several methods for dealing undesirable outputs and compares the resulting efficient frontiers. He also developed a new radial measure in this regard. As another approach to deal to undesirable outputs in performance evaluation is directional distance functions (DDF) approach in DEA that proposed by Chung et al. [26] and Ball et al. [27], they extended of the idea of Fare et al. [28] and Gerami et al. [29-33].

It can be said that the main contribution of this study is as follows. In this paper, we use the DDF model to perform the acquisition and merge operations process and determine the optimal level of inputs and outputs for the units in the acquisition and merge operations process. In this study, we consider the level of inputs equal to one.

It the following of this paper is organized as follows. The second section presents a new model in DEA for debating to undesirable outputs based on the DDE model in DEA. The third section present a new inverse DEA model in merging process in presence of undesirable outputs. The fourth section show a new model with a numerical example. The fifth section proposed an application in banking and at the end we present the results of the research.

2. DDF model to in DEA by considering a unitary input level

Suppose we have n DMUs with m inputs and s outputs. For $DMU_j = (1, B_j, Y_j)$, $j = 1, \dots, n$, we consider the vectors desirable output as $Y_j = (y_{1j}, \dots, y_{sj})^T \in R_+^s$ and undesirable output $B_j = (b_{1j}, \dots, b_{lj})^T \in R_+^l$. The DDF model of Zanella et al. [34] is as follows.

$$
\begin{aligned}
&\theta_o^* = max\, \theta_o && (1)\\
&s.t.\ \ \textstyle\sum_{j=1}^{n} \lambda_j\, b_{kj} \le b_{ko} - \theta_o g_{b_k}\,, \\
&k = 1, \dots, l, \\
&\textstyle\sum_{j=1}^{n} \lambda_j\, y_{rj} \ge y_{ro} + \theta_o g_{y_r}\,, \ r = 1, \dots, s, \\
&\textstyle\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \ge 0,\ j = 1, \dots, n.
\end{aligned}
$$

The λ_j,, $j = 1, \dots, n$ are the intensity variables. The components of vector $g = (g_{b_k},\ g_{y_r}\colon\ k = 1, \dots, l,\ r = 1, \dots, s\,)$, $g \neq 0_{m+k+s}$ show the direction for change for the desirable and undesirable outputs. The factor θ_o shows the extent of the DMU's inefficiency. We use unitary level of input and setting the directional vector as $g = (g_b, g_y)$ that allows contract undesirable outputs and expand the desirable outputs simultaneously while keeping inputs fixed. Based on the model (1), the efficient frontier avoids downward-sloping segments, with negative trade-offs between desirable and undesirable outputs. This formulation ensures that a DMU will only classified efficient when no further improvements to both desirable and undesirable outputs are possible. Assume $(\theta_o^*, \lambda_j^*,\ j = 1, \dots, n)$ is an optimal solution of model (1). In this case, the optimal value of θ_o measure of technical inefficiency for the evaluated DMU, also the efficiency score can be calculated as $\frac{1}{1+\theta_o^*}$.

3. An inverse DEA approach for dealing with undesirable outputs

In this section of paper considers the case of a consolidation between two DMUs operating in the same market, both merging DMus disappearing to generate a new merged entity. We extended this approach for the case of more than two merging DMUs. This section proposes an alternative formulation for the consolidation between two DMUs operating in the presence of undesirable outputs. This formulation is based on a DDF model thus it allows a proportional interpretation of the improvements required for all outputs as well as avoids the need to change the original measure of the undesirable output. We propose a new model to obtain the optimal level of both desirable and undesirable outputs based on the target efficiency score of the new unit created in the merger process. Let $\bar{\theta}_o$ is the target efficiency score for the new unit created, we show it as the merged entity M in the merger process by selecting DMU_p and DMU_q as units to merge in the inverse DEA process. λ_j is the intensity variable. DMUs p and q are consolidating their activities. Let's M show the merged entity generated by the consolidation and also T is the set of indices of all DMUs except p and q. Suppose ρ_{rk} and ρ_{rh} be the levels of the r-th input from the merging DMU p and DMU q, that is kept by the merged entity M respectively. We proposed the following inverse DDF DEA model.

$$\min \sum_{k=1}^{l} (\rho_{kp} + \rho_{kq}) \qquad (2)$$

$$S.t. \quad \sum_{j \in T} \lambda_j b_{kj} + \lambda_M (b_{kp} + b_{kq}) \leq (\rho_{kp} + \rho_{kq}) - \bar{\theta}_o g_{b_k},\ k = 1, \dots, l,$$

$$\sum_{j \in T} \lambda_j y_{rj} + \lambda_M (y_{rp} + y_{rq}) \geq (y_{rp} + y_{rq}) + \bar{\theta}_o g_{y_r}, r = 1, \dots, s,$$

$$\sum_{j \in T} \lambda_j + \lambda_M = 1, \lambda_j \geq 0, j = 1, \dots, n,$$

$$0 \leq \rho_{kp} \leq b_{kp}, 0 \leq \rho_{kq} \leq b_{kq}, \quad k = 1, \dots, l,$$

$$\lambda_j \geq 0,\ j = 1, \dots, n, \lambda_M \geq 0.$$

The above Inverse DDF DEA model is a nonlinear programming (NLP) problem. In following, we established a linearization of the model (2). The model (2) is the linearized form of model (1) by taking $\lambda_M = 0$. In other words, the model (2) is feasible if and only if the virtual $DMU_M = (2, B_p + B_q, Y_p + Y_q)$ is in the pre consolidation PPS, as an interior point or on the efficiency frontier. Therefore, taking into account the feasibility of model (2) concludes that the virtual DMU_M can be presented in terms of the DMUs indexed in T and vice versa.

4. Numerical Example

In this section, we illustrate the proposed approach in this paper with sample example. In order t, consider 10 DMUs that have one desirable and one undesirable output. These DMUs show in the Table (1).

Table 1: Input–output data and the efficiency scores of models (1).

DMU	desirable	undesirable	Efficiency	DMU	desirable	undesirable	Efficiency
1	7	5	0.7174	6	16	6	0.5932
2	14	25	0.8333	7	5	7	0.9
3	11	3	0.6026	8	7	4	0.6957
4	10	30	1	9	5	3	0.7667
5	13	22	0.7872	10	3	2	1

Table 2: The results of inverse DDF DEA process in the numerical example.

Merge DMU	Target Efficiency	ρ_1	ρ_2	$\rho_1+\rho_2$	DMU_M
$DMU_p = DMU_9$, $DMU_q = DMU_{10}$	0.8	5	2	7	(2,7,8)
$DMU_p = DMU_9$, $DMU_q = DMU_{10}$	0.9	5	0.889	5.889	(2,5.889,8)
$DMU_p = DMU_9$, $DMU_q = DMU_{10}$	1	5	0	5	(2,5,8)
$DMU_p = DMU_5$, $DMU_q = DMU_6$	0.8	13	2	15	(2,15,28)
$DMU_p = DMU_5$, $DMU_q = DMU_6$	0.9	0	11.944	11.944	(2,11.944 ,28)
$DMU_p = DMU_5$, $DMU_q = DMU_6$	1	0	9.5	9.5	(2,9.5,28)

Table (2) show the minimum number of undesirable outputs from each DMUs DMU_k and DMU_h that should be kept in order to reach the predetermined target efficiency. The second column shows the target efficiency scores. Based on the last three columns, show the outputs of merger DMU. These results obtain based on the model (2) in the inverse DEA process. We select the target efficiency scores by attention to the results in the Table (1).

For example, consider

$DMU_p = DMU_5, DMU_q = DMU_6,$

as merging DMUs. At first, we consider target efficiency scores equal to one. By this selection, the merged entity M will be efficient, the optimal level of desirable outputs, and undesirable outputs corresponding to the merged entity M will be as follows.

$$DMU_M = (x_{15} + x_{16}, b_{15} + b_{16}, y_{15} + y_{16}) = (2,9.5,28).$$

5. Case study

In this section, we use of a data in Shirazi and Mohammadi [35] for illustrating the proposed inverse DEA model in this paper in merger process. Airlines in Iran were established in 1967. This organization has 22 airlines. which are 14 service and travel airlines. The names of these airlines are as follows.

Puya Air, Etrak, Meraj, Naft, Kisk Air, Qeshm Air, Taban, Irtor, Caspian, Ata Air, Mahan, Zagros, Aseman and Iran Air.

In this study, we examine these lines and obtain their efficiency using the model presented in this research paper. In this study, the income in terms of passenger transport kilometers is considered as a desirable output and the number of flight delays as an undesirable output, and these data are shown in Table (3). Flight delays are also considered undesirable. Each airline as a DMU has on undesirable output and one desirable output that have been brought in Table 3.

Table 3: Input–output data (Shirazi and Mohammadi [35]).

DMU	desirable	undesirable	Efficiency	DMU	desirable	undesirable	Efficiency
1	5495	60	1	8	1019578	2061	0.5935
2	213833	296	0.6923	9	1662531	2403	0.6266
3	53466	930	0.5263	10	1421762	3650	0.5716
4	504873	1441	0.5718	11	8891611	3118	1
5	1916141	2011	0.6737	12	1952587	3290	0.6075
6	1516563	2293	0.6217	13	2301910	5853	0.5705
7	2279117	1973	0.7092	14	3810524	4465	0.6511

Table 4: The results of inverse DDF DEA process in the case study.

Merge DMU	Target Efficiency	ρ_1	ρ_2	$\rho_1+\rho_2$	DMU_M
$DMU_p = DMU_{13}$, $DMU_q = DMU_{14}$	0.8	4215.224	0	4215.224	(2,4215.224,6112434)
$DMU_p = DMU_{13}$,	0.9	3074.310	0	3074.310	(2,5.3074.310,6112434)
$DMU_q = DMU_{14}$	1	2161.595	0	2161.595	(2,2161.595,6112434)
$DMU_p = DMU_2$, $DMU_q = DMU_3$	0.7	4537.055	0	4537.055	(2,4537.055,267299)
$DMU_p = DMU_2$,	0.85	1949.874	0	1949.874	(2,1949.874,267299)
$DMU_q = DMU_3$	1	150.095	0	150.095	(2,150.095,267299)

As can be seen, we show the merged DMU by M. Based on the model (2), we find the minimum amount of the undesirable outputs of DMU_p and DMU_q in order to reach the desired given efficiency target of airline target in the inverse DDF DEA process.

Table (4) shows the levels of the desirable output and undesirable output from the merging DMU p and DMU q, for predetermined target efficiency score of DMU_M as airline target. We present the results for different selection of DMU_p and DMU_q as units to merge in the inverse DDF DEA process and different target efficiency score in Table (4).

We proposed the results of inverse DDF DEA process in Table (4). We obtain the minimum number of undesirable outputs from each DMUs DMU_k and DMU_h that should be reach in order to reach the predetermined target efficiency. The second column shows the target efficiency scores. Based on the last three columns, show the outputs of merger DMU. These results obtain based on the model (2) in the inverse DEA process. We select the target efficiency scores by attention to the decision maker opinion. For example, consider $DMU_p = DMU_{13}, DMU_q = DMU_{14}$, as merging DMUs. We consider target efficiency scores equal to one.

By this selection, the merged entity M will be efficient, the optimal level of desirable outputs, and undesirable outputs corresponding to the merged entity M will be as follows.

$DMU_M = (x_{113} + x_{114}, b_{113} + b_{114}, y_{113} + y_{114}) = (2,2161.595,6112434).$

6. Conclusion

In this study, we presented a new approach to deal with the presence of undesirable outputs in inverse DEA. This approach provides the optimal level of undesirable outputs to achieve the target efficiency. The model obtains the increase and decrease of desirable and undesirable outputs simultaneously respectively. We obtained the inverse DEA process in the presence of undesirable outputs by choosing different directions in the DDF. If the post-merger efficiency frontier is the same as the pre-merger frontier, the merger is described as a feasible one. Mergers in a market help regulatory authorities to identify those mergers that potentially threaten competitiveness in the market and therefore thoroughly analyze them before any approval. In addition, business intelligence units in a company may use what we suggest to identify potential threats in their business environment. As future work, the models presented in this article can be developed for other data structures in data envelopment analysis, such as fuzzy data. We can also develop models for the presence of inputs.

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