

## Optimization of reinforced concrete cantilever retaining walls by the use of intelligent water drops algorithm (IWDA)

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### Abstract

In this paper, cost and weight optimization for reinforcement cantilever concrete retaining wall is investigated using intelligent water drops algorithm (IWDA). The algorithm capability was compared with that of others in the literature. A computer program has been developed to analyze reinforced concrete cantilever retaining walls using the IWDA algorithm. The results show that the IWDA algorithm is able to optimize retaining walls from viewpoints of cost and weight. The effect of the wall geometry has been investigated by considering four types of T-shape walls in order to discover the influence of contributing variables on objective functions. It has been found that backfill slope, unit weight and friction angle are important parameters affecting retaining wall cost and weight. Also, the Coulomb method in computing lateral earth pressure on retaining walls leads to lower cost and weight for retaining walls than the Rankine method. In general, optimizing retaining walls may lead to approximately 44 percent saving.

**Keywords:** Concrete retaining walls, Lateral earth pressures, Optimization, Intelligent water drop algorithm

### 1. Introduction

The optimization of retaining walls has been investigated using various methods. These methods involve nonlinear programming (Sribas and Erbatur, 1996) [1], simulated annealing algorithm (Ceranic and et al, 2001) [2], target reliability approach (Sivakumar and Munwar, 2008) [3], simulated annealing algorithm (Yepes et al., 2008) [4], ant colony algorithm (Ghazavi and Bazazzian, 2011) [5], foraging bacterial algorithm (Ghazavi and Salavati, 2011) [6], charged system search algorithm (Kaveh and Behnam, 2013) [7], dolphin echolocation echolocation optimization (Kaveh and Farhoudi, 2016) [8], gases brownian motion optimization algorithm (Shalchi et al., 2021) [9], firefly algorithm (Laali and Shalchi, 2018) [10], artificial bee colony algorithm (Shalchi and Laali, 2018) [11], hybrid firefly algorithm (Sheikholeslami et al., 2016) [12] and evolutionary algorithms (Gandomi and Kashani, 2018) [13]. Sribas and Erbatur (1996) [1] used specially prepared computer program (RETOPT) and

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performed sensitivity analyses stem height, backfill slope, and surcharge load. They concluded that with increasing the stem height and surcharge load, the cost and weight objective functions increase. Moreover, with increasing the backfill slope values from 0 to 20 degree, the wall cost and weight decrease and then increase.

Ghazavi and Salavati (2011) [6] used foraging bacterial algorithm (BFOA) and found that this algorithm is capable of finding the optimal cost and weight for retaining walls. Ghazavi and Bazazzian (2011) [5] applied ant colony optimization algorithm (ACO) and found that the mentioned algorithm is able to find quickly the minimum weight and cost of walls.

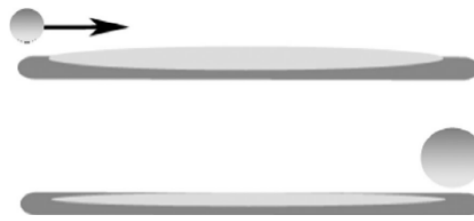
In this paper, a new method for wall optimization is applied and the cost and weight of wall will be minimized. The intelligent water drops algorithm (IWDA) is a new swarm-based optimization algorithm that inspired from observing water drops in rivers. A natural river can choose the best path among lots of possible paths from the source to destination. The IWDA was initially presented by Shah-Hosseini (2009) [14].

In order to investigation the capability of the IWDA predictions, its predictions are compared with those reported by Saribas and Erbatur (1996) [1]. Moreover, the IWDA optimization data will be compared with those calculated by conventional method normally used in practice by design engineers. Parametric studies will then be performed to optimize four wall geometries. In addition, the influence of backfill slope, backfill unit weight and method of

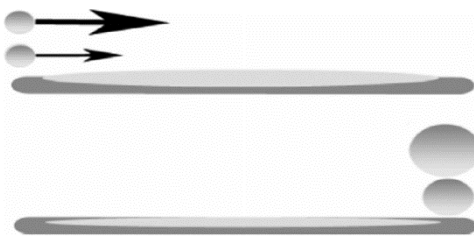
computing lateral earth pressures on objective functions are investigated by performing sensitivity analyses.

## 2. Introduction of Intelligent Water Drops Algorithm [14, 17]

The velocity and soil are two main factors for each water drop optimization method. Each water drop flows from a source to destination with an initial velocity and zero soil. Water drops carry soil particles when they move between two locations of river bed and add the transported soil to the in-situ available soil in water drop (Fig. 1). It should be noted that the size of circle in Fig. 1 shows the amount of soil that the water drop can carry. In addition, water drop with greater velocity gathers more soil at the end of its path (Fig. 2). As seen, water drop with bigger narrow shows a higher velocity. As also observed in right side of Fig. 2, water drop with greater velocity can carry more soil in the right side. Thus, soil properties depend on the water drop velocity. In contrast, the velocity of water drop is changed by the soil of the path. In other words, water drop in the path with little soil has greater velocity and can carry more soil particles (Fig. 3). This means that the path with larger amount of the soil has greater resistance against flowing water drop. Both velocity and soil may be altered during the movement from source to destination. Each IWD needs a path selection mechanism to determine the next location. The selection of next path is proportional to the inverse of the available soil in paths. Therefore, there is higher chance that paths with lower soil are selected by the IWD.



1- Fig. 1. Schematic of IWD carrying soil with moving from left to right [14]



2- Fig. 2. IWD with higher velocity in left side, gathering more soil in the right side [14]



3- Fig. 3. Performance comparison of two paths based on different kinds of available soil [14]

Each IWD algorithm consists of (N, E) graph that its role is an extensive memory. In this graph, N is the number of water drops that can move on edge (E). Each IWD begins from one node of graph and moves along the edges until the IWD finally completes its solution. In this algorithm, one iteration is complete when all water drops have completed their solution [14].

The iteration-best solution  $T^{IB}$  is obtained after each iteration. This parameter is used for updating the total-best solution  $T^{TB}$ . Based on the quality of the soil, the amount of soil on the edges of the iteration-best solution  $T^{TB}$  decreases. After that, the algorithm begins another iteration with new IWDs. It is important to note that this

$$q(T^{TB}) = +\infty$$

The maximum number of iterations,  $iter_{max}$ , is determined by users based on the solution

iteration is performed with the same soils on the paths of the graph and the mentioned process is repeated. The algorithm ends up if the number of iteration reaches  $iter_{max}$  or the total-best solution  $T^{TB}$  reaches the expected quality. There are two groups of parameters. The first group are constant during the lifetime of algorithm which is called "static parameters". The second group are "dynamic parameters" that are reinitialized after each iteration. In the following section, the steps of IWDA are presented:

- 1- Initialization of static parameters. The problem is defined with (N, E) graph for algorithm. At the first step, the quality of the total-best solution ( $q(T^{TB})$ ) is set to the worst value

$$(1)$$

accuracy and required time for algorithm run. In this study, this parameter is 1000. In

addition, the iteration count,  $iter_{count}$ , is set to zero. The number of water drops,  $N_{IWD}$ , is a positive integer value which is usually selected to the number of graph nodes determined by user. The value of this parameter is affected by accuracy and run time of algorithm. In other words, the greater value causes to increase the accuracy and run time. In this study, this parameter is 80 based on the problem dimension. According to Shah-Hosseini (2009), the required static parameters are: For velocity updating  $a_v = 1$ ,  $b_v = 0.01$ , and  $c_v = 1$ . For soil updating,  $a_s = 1$ ,  $b_s = 0.01$ , and  $c_s = 1$ . For local soil updating  $\rho_n = 0.9$ . For global soil updating  $\rho_{IWD} = 0.9$ . The initial soil value on path,  $Initsoil = 10000$ . The initial velocity for each IWD,  $Initvel = 200$ . It is important to note that the "Initsoil" and "Initvel" parameters

$$p_i^{IWD}(j) = \frac{f(soil(i,j))}{\sum_{k \in V_c(IWD)} f(soil(i,k))} \tag{2}$$

$$f(soil(i,j)) = \frac{1}{\epsilon_s + g(soil(i,j))} \tag{3}$$

$$g(soil(i,j)) = \begin{cases} soil(i,j) & \text{if } \min_{l \in V_c(IWD)} (soil(i,l)) \geq 0 \\ soil(i,j) - \min_{l \in V_c(IWD)} (soil(i,l)) & \text{else} \end{cases} \tag{4}$$

This selection is in a way that does not violate any constraints of problems and is not in the visited node list,  $V_c(IWD)$ . After that, the new node is added to the visited node list. It is important to note that the value of  $\epsilon_s$  is  $vel^{IWD}(t+1) = vel^{IWD}(t) + \frac{a_v}{b_v + c_v \cdot soil^2(i,j)}$

where  $vel^{IWD}(t+1)$  is the updated velocity of the IWD in new node.

5-3- The amount of soil that can be moved from node  $i$  to node  $j$  ( $\Delta soil(i,j)$ ) are calculated by:

$$\Delta soil(i,j) = \frac{a_s}{b_s + c_s \cdot time^2(i,j; vel^{IWD}(t+1))} \tag{6}$$

$$time(i,j; vel^{IWD}(t+1)) = \frac{HUD(j)}{vel^{IWD}(t+1)} \tag{7}$$

where the heuristic undesirability,  $HUD(j)$  is defined appropriately with problem condition. This parameter is calculated from (Shah-Hosseini) [18] as:

$$HUD(j) = ||c(i) - c(j)|| \tag{8}$$

The function  $c(i)$  and  $c(j)$  represents the two dimensional positional vector for water drop.

5-4- The available soil value in traversed path by IWD ( $soil(i,j)$ ) and the soil that the IWD carries ( $soil^{IWD}$ ) are updated by:

$$soil(i,j) = (1 - \rho_n) \cdot soil(i,j) - \rho_n \cdot \Delta soil(i,j) \tag{9}$$

$$soil^{IWD} = soil^{IWD} + \Delta soil(i,j) \tag{10}$$

7- The iteration-best solution,  $T^{IB}$  from all the solution of IWD,  $T^{IWD}$  is calculated from:

$$T^{IB} = \arg \max_{T^{IWD}} q(T^{IWD}) \tag{11}$$

where function  $q(T^{IWD})$  shows the quality of the solution.

can be selected by users based on the problem conditions.

3- Initialization of dynamic parameters.

These parameters are changed in each run of the algorithm. The list of the visited nodes is defined by  $V_c(IWD)$  which is empty at first step,  $V_c(IWD) = \{\}$ . After each iteration of the algorithm, the visited node is added to it.

4- IWDs spread on graph nodes randomly as the first visited nodes.

5- The list of visited nodes is updated.

6- The steps 5-1 to 5-5 are repeated for those IWDs with partial solution.

5-1- For each IWD that exists in node  $i$ , next node  $j$  is selected by using the following probability:

0.01 based on research carried out by Shah-Hosseini [18].

5-2- For each movement of IWD from node  $i$  to node  $j$ , the velocity is updated by using the following function:

$$\tag{5}$$

8- The existing soils on iteration-best solution paths  $T^{IB}$  are updated by:

$$\text{soil}(i, j) = (1 + \rho_{IWD}) \cdot \text{soil}(i, j) \tag{12}$$

$$-\rho_{IWD} \cdot \frac{1}{(N_{IB}-1)} \cdot \text{soil}_{IB}^{IWD} \quad \forall (i, j) \in T^{IB} \tag{13}$$

The number of nodes in the solution  $T^{IB}$  is  $N_{IB}$ .

9- The total best solution,  $T^{TB}$  is updated by iteration-best solution.

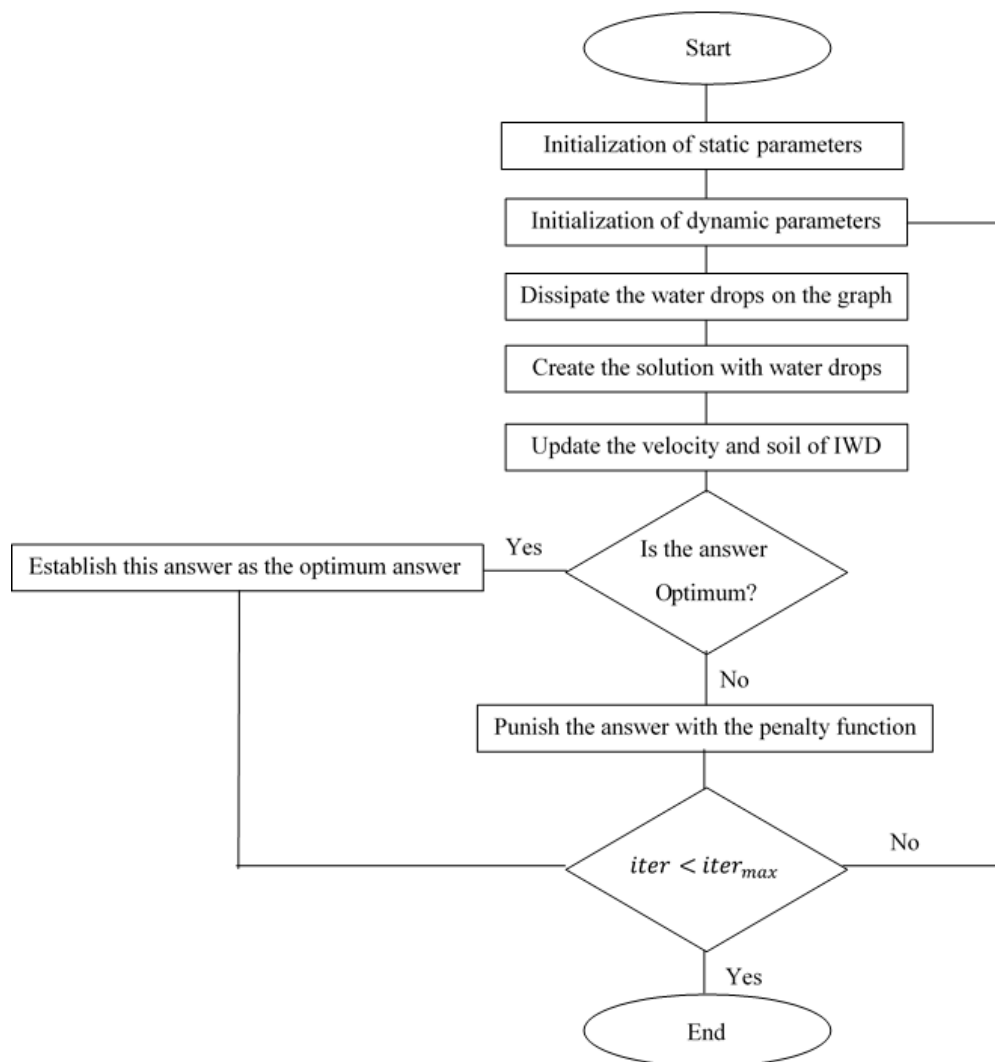
$$T^{TB} = \begin{cases} T^{LB} & \text{if } q(T^{TB}) > q(T^{IB}) \\ T^{TB} & \text{otherwise} \end{cases} \tag{14}$$

It is important to note that the obtained values with bad quality in each step are scrambled from the algorithm process by using penalty function.

10- The number of iterations increases as:

$$\text{Iter}_{\text{count}} = \text{Iter}_{\text{count}} + 1 \tag{15}$$

If the bottom condition be established, all processes from step 2 will be repeated. The IWDA flowchart is presented in Fig. 4.



4- Fig. 4. Flowchart of Intelligent Water Drops Algorithm (IWDA)

### 3. Parameters and Requirement for Retaining Wall Optimization

In order to optimize a retaining wall based on IWDA from cost and weight viewpoints, the wall geometry shown in Fig. 5 is considered and corresponding variables, objective functions, and constraints will be presented subsequently.

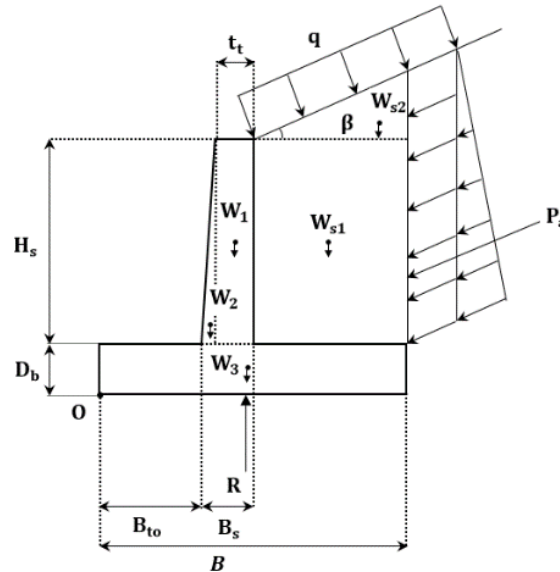


Fig. 5. Wall geometry for optimization

**3-1- Design variables**

All parameters considered for design procedure such as wall dimension, the required steel value, the concrete compressive strength, yield strength of steel and the diameter of bar are defined as variables shown in Table 1. The required number of steel bars for the stem, toe and

$F_y=350, 400, 500 \text{ MPa}$   
 $F_c=21, 24, 28, 35, 42, 45 \text{ MPa}$   
 $d_b = 10, 12, 14, 16, 18, 20, 22, 24, 26 \text{ mm}$

The upper and lower bounds of continuous variables are shown in Table 2. The height of the wall stem is represented by  $H_s$  which is an initial parameter. The maximum and minimum values in Table 2 are presented based on Bowles (1982) [16]. In addition, values of required steel are controlled by

heel is obtained from the software output. Also, these values are controlled with maximum and minimum values based on the American Concrete Institute Code (ACI-2008) [15]. The discrete parameters are selected as follow:

$$(16)$$

$$(17)$$

$$(18)$$

ACI 2008 [15] in constraints and the numbers that appear in Table 2 are initial values. In programming, if the tension steel is sufficient for the applied moment, the compression steel will be obtained zero. For this reason, the lower bound value of required steel is selected as zero.

Table 1 Problem variables and software output for retaining wall design

Groups	Name	Unit	Symbol	Type
Variables of wall geometry	Total base width		B	
	Toe width		$B_{to}$	
	Stem thickness at bottom	m	$B_s$	Continuous
	Thickness of base		$D_b$	
	Stem thickness at top		$t_t$	
Variables for specification of steel	Stem tensile steel area	$(\text{cm}^2/\text{m})$	AstS	Continuous
	Toe tensile steel area		AstT	

	Heel tensile steel area		AstH	
	Stem compressive steel area		AscS	
	Toe compressive steel area		AscT	
	Heel compressive steel area		AscH	
	Yield strength of stem tensile steel		$F_{y1}$	
	Yield strength of stem compressive steel		$F_{y2}$	
	Yield strength of toe tensile steel	(MPa)	$F_{y3}$	
	Yield strength of toe compressive steel		$F_{y4}$	Discrete
	Yield strength of heel tensile steel		$F_{y5}$	
	Yield strength of heel compressive steel		$F_{y6}$	
	Diameter of steel bar	mm	$d_b$	
Variables of specification of concrete	Compressive strength of stem concrete		$F_{cs}$	Discrete
	Compressive strength of base concrete	(MPa)	$F_{cf}$	
Software output	Number of stem tensile steel		$n_1$	Discrete
	Number of toe tensile steel		$n_2$	
	Number of heel tensile steel		$n_3$	
	Number of stem compressive steel	–	$n_4$	
	Number of toe compressive steel		$n_5$	
	Number of heel compressive steel		$n_6$	

Table 2 Upper and lower bounds of continuous variables

Variables name	Lower bound	Upper bound
Total base width (m)	$(24 \times H_s)/55$	$(7 \times H_s)/9$
Toe width (m)	$(8 \times H_s)/55$	$(7 \times H_s)/27$
Stem thickness at bottom (m)	0.2	$H_s/9$
Thickness of base (m)	$H_s/11$	$H_s/9$
Stem thickness at top (m)	0.2	0.3
Area of tensile and compressive steel ( $\text{cm}^2/\text{m}$ )	0	80

**3-2- Objective functions**

In this paper, the wall optimization is considered by using two objective functions representing wall cost and weight. In the optimization procedure, dimensions and specification of the wall are obtained from structural and geotechnical constrains with minimizing the wall cost and weight. It

should be noted that the required development length of bars ( $l_{dh}$ ,  $l_{dc}$ ) are obtained according to the ACI code. The cost and weight objective functions are respectively defined as:

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$$f(C) = C_s W_s + C_c V_c \tag{19}$$

$$f(W) = W_s + 100 V_c \gamma_c \tag{20}$$

where  $C_s$  is the cost of steel unit (\$/kg),  $C_c$  is the cost of concrete unit (the selected value is considered for forming, concretion, vibration and work force cost) (\$/m<sup>3</sup>),  $W_{st}$  is the steel weight in the wall length unit (kg),  $V_c$  is the concrete volume in the wall

length unit (m<sup>3</sup>), and  $\gamma_c$  is the weight of concrete unit (kN/m<sup>3</sup>).

It should be noted that the unit of cost and weight objective functions are \$ and kg in per unit length of the wall, respectively.

**3-3- Design constraints**

In optimization scheme, both structural and geotechnical constraints are considered as:

$$g_i(x) \leq 0, \quad i = 1, 2, \dots, m \tag{21}$$

where  $m$  is the number of constraints presented in Table 3.

and bearing capacity calculations, respectively.

It is noted that the Rankine and Hansen methods are used for lateral earth pressure

Table 3 Design constraints

Names of constraints	Unit	Names of constraints	Unit
Overturning stability	kN.m	Yielding of tensile steel	–
Sliding stability	kN	Yielding of compressive steel	–
No tension condition in foundation	m	Minimum footing depth	m
Bearing capacity	kPa	Stem slope control	–
Shear control	kN	Minimum distance of tensile steel	m
Moment control	kN.m	Minimum distance of compressive steel	m
Minimum of tensile steel	–	Maximum distance of tensile steel	m
Maximum of tensile steel	–	Maximum distance of compressive steel	m

**4- Verification**

In this section, the capability of the IWDA prediction is evaluated using optimized data for retaining walls given by Saribas and Erbatur (1996) [1] with initial parameters given in Table 4. The optimization is performed for T-shape wall with variables shown in Fig. 5. The considered variables and constraints of this verification are given

in Table 5. The method used by Saribas and Erbatur [1] is a nonlinear programming by a specially prepared computer program called RETOPT [1]. Since the design in this research was done with a differential manner, results are in lowest values and the other methods like metaheuristic algorithms try to reach these results.

Table 4 Initial parameters for verification with data reported by Saribas and Erbatur (1996) [1]

Parameter	Symbol	Example	Example
		1	2
Height of stem (m)	$H_s$	3	4.5
Stem thickness at the top (m)	$t_t$	0.2	0.25



Yield strength of reinforcing steel (MPa)	$F_y$	400	400
Compressive strength of concrete (MPa)	$f'_c$	21	21
Concrete cover (cm)	$d_{co}$	7	7
Maximum steel percentage	$\rho_{max}$	0.016	0.016
Minimum steel percentage	$\rho_{min}$	0.00333	0.00333
Shrinkage and temporary reinforcement percent	$\rho_{st}$	0.002	0.002
Diameter of bar (cm)	$\phi_{bar}$	1.2	1.4
Surcharge load (kPa)	$q$	20	30
Backfill slope (Degree)	$\beta$	10	15
Internal friction angle of retained soil (Degree)	$\phi$	36	36
Internal friction angle of base soil (Degree)	$\phi$	0	34
Unit weight of retained soil (kN/m <sup>3</sup> )	$\gamma_s$	17.5	17.5
Unit weight of base soil (kN/m <sup>3</sup> )	$\gamma'_s$	18.5	18.5
Unit weight of concrete (kN/m <sup>3</sup> )	$\gamma_c$	23.5	23.5
Cohesion of base soil (kPa)	$c$	125	100
Design load factor	LF	1.7	1.7
Depth of soil in front of wall (m)	$D_f$	0.5	0.75
Cost of steel (\$/kg)	$C_s$	0.4	0.4
Cost of concrete (\$/m <sup>3</sup> )	$C_c$	40	40
Factor of safety against sliding	$SF_s$	1.5	1.5
Factor of safety for overturning stability	$SF_o$	1.5	1.5
Factor of safety for bearing capacity	$SF_b$	3	3

Table 5 List of the variables and constraints for the first case of verification [1]

Variables	constraints
Total base width	Shear at bottom of stem
Toe width	Moment at bottom of stem
Stem thickness at the bottom	Overturning stability
Thickness of base	Sliding stability
Area of stem tensile steel	No tension condition in foundation
Area of toe tensile steel	Bearing capacity
Area of heel tensile steel	Toe shear
	Toe moment
	Heel shear
	Heel moment

Table 6 compares results of optimization for a retaining wall using four methods including current IWDA, RETOPT (Saribas and Erbatur, 1996) [1], BFOA (Ghazavi and Salavati, 2011) [6], and ACO (Ghazavi and Bazzazian, 2011) [5]. As seen, there is satisfaction between all optimization algorithms, stating the capability of the

current IWDA predictions. The small differences between results of IWDA and RETOPT [1] indicate that the IWDA is also suitable for retaining wall optimization. In addition, the IWDA optimization results are close to those given by BFOA and ACO algorithms [5, 6].

Table 6 Comparison of results of present IWDA with three optimization algorithms: RETOPT (Saribas and Erbatur, 1996) [1], BFOA (Ghazavi and Salavati, 2011) [6], and ACO (Ghazavi and Bazzazian, 2011) [5]

Objective function	RETOPT minimum value [1]	IWDA minimum value	BFOA minimum value[6]	ACO minimum value [5]	Difference between RETOPT and IWDA minimum values	Difference between RETOPT and BFOA minimum values	Difference between RETOPT and ACO minimum values
Example 1							
Cost (\$/m)	82.474	82.5877	-	-	%0.137	-	-
Weight (Kg/m)	2498.7	2499.5	-	-	%0.032	-	-
Example 2							
Cost (\$/m)	189.546	189.6473	190.574	201.185	%0.053	%0.542	%6.140
Weight (Kg/m)	5280	5289.5	5343.221	5540.3	%0.179	%1.197	%4.929

Table 7 summarizes the values of variables determined from the current IWDA and those predicted by Saribas and Erbatur (1996) [1]. As seen, optimized values of X1 to X7 computed from two methods for two examples are close, indicating reasonable predictions of IWDA. As mentioned above, the method used by Saribas and Erbatur (1996) [1] is differential manner. This method for big problem with more dimension is so difficult and needs a lot of time. In addition, there is a risk of mistake with more variables and constraints in this method. The IWDA can solve the problem with more accuracy and less time. Moreover, the IWDA has some advantages in comparison with ACO [5] and BFOA [6]. It is able to local search besides global search, less catches in local optimum points and also can solve the problem with high dimension.

For this reasons, the IWDA is suitable and capable in optimization. The geometric model considered for the wall in this section is shown in Fig. 6. As another versification, a retaining wall is selected from Bowles (1982) [16] who used conventional engineering design procedure. The aim of this comparison is to clarify the effectiveness of using IWDA. Table 8 shows the required initial parameters for this verification. The results show that the IWDA reduces %43.22 of the wall cost and %44.92 of the wall weight. The optimum values of the objective functions and variables for this verification are presented in Tables 9 and 10, respectively. The above verifications show that the IWDA is capable of designing optimized retaining walls.

Table 7 Values of variables in the optimum points of the first case of verification

Design variables	Optimum values for minimum cost		Optimum values for minimum weight	
	RETOPT	IWDA	RETOPT	IWDA

Example 1					
$X_1$	Total base width (m)	1.578	1.5807	1.574	1.5754
$X_2$	Toe width (m)	0.436	0.4364	0.441	0.4752
$X_3$	Stem thickness at the bottom (m)	0.258	0.2545	0.200	0.200
$X_4$	Thickness of base (m)	0.273	0.2727	0.273	0.2727
$X_5$	Area of stem tensile steel (cm <sup>2</sup> /m)	12.574	12.940 4	21.072	21.138 3
$X_6$	Area of toe tensile steel (cm <sup>2</sup> /m)	6.551	6.551	6.551	6.551
$X_7$	Area of heel tensile steel (cm <sup>2</sup> /m)	6.551	6.551	6.681	6.551
Example 2					
$X_1$	Total base width (m)	2.254	2.2572	2.238	2.239
$X_2$	Toe width (m)	0.655	0.6454	0.655	0.6545
$X_3$	Stem thickness at the bottom (m)	0.417	0.4191	0.300	0.300
$X_4$	Thickness of base (m)	0.409	0.4091	0.409	0.409
$X_5$	Area of stem tensile steel (cm <sup>2</sup> /m)	23.475	23.304 2	41.626	42.410
$X_6$	Area of toe tensile steel (cm <sup>2</sup> /m)	11.059	11.059	11.059	11.058
$X_7$	Area of heel tensile steel (cm <sup>2</sup> /m)	11.059	11.058	11.059	11.058

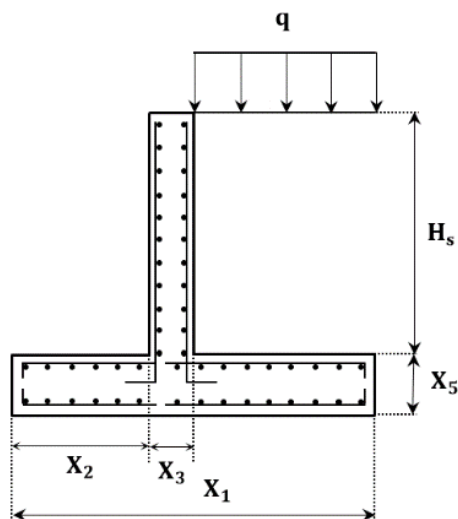


Fig. 6. Wall model in second case of verification

Table 8 Design parameter for second case of verification

Parameter	symbol	value
Height of stem (m)	$H_s$	2.44
Concrete cover (cm)	$d_{co}$	5
Shrinkage and temporary reinforcement percent	$\rho_{st}$	0.0018
Diameter of bars (cm)	$\phi_{bar}$	2
Surcharge load (kPa)	$q$	12
Backfill slope (degree)	$\beta$	0
Internal friction angle of retained soil (degree)	$\phi$	36
Internal friction angle of base soil (degree)	$\phi'$	0
Unit weight of retained soil (kN/m <sup>3</sup> )	$\gamma_s$	18.86

Unit weight of base concrete (kN/m <sup>3</sup> )	$\gamma_c$	23.6
Unit weight of soil (kN/m <sup>3</sup> )	$\gamma_s$	17.3
Cohesion of base soil (kPa)	c	120
Depth of soil in front of wall (m)	$D_f$	1.22
Factor of safety for bearing capacity	$SF_b$	3
Factor of safety against sliding	$SF_s$	1.5
Factor of safety against overturning	$SF_o$	1.5

Table 9 Optimum values of cost and weight objective functions for second case of verification

Method	Objective function	value
Bowles (manual design) [10]	Cost (\$/m)	86.7692
	Weight (Kg/m)	3543.6
IWDA	Cost (\$/m)	49.2664
	Weight (Kg/m)	1951.8

Table 10 Optimum variable values for cost and weight for verification with Bowles (1982) [16]

		Optimum	
value		Cost	Weight
<b>Design parameters</b>			
$X_1$	Total base width (m)	1.4614	1.4114
$X_2$	Toe width (m)	0.5278	0.5094
$X_3$	Stem thickness at the bottom (m)	0.201	0.2001
$X_4$	Thickness of base (m)	0.232	0.2272
$X_5$	Stem tensile steel area (cm <sup>2</sup> /m)	6.7328	7.389
$X_6$	Toe tensile steel area (cm <sup>2</sup> /m)	12.2739	11.3343
$X_7$	Heel tensile steel area (cm <sup>2</sup> /m)	6.5926	7.5967
$X_8$	Stem compressive steel area (cm <sup>2</sup> /m)	0	0
$X_9$	Toe compressive steel area (cm <sup>2</sup> /m)	0	0
$X_{10}$	Heel compressive steel area (cm <sup>2</sup> /m)	0	0
$X_{11}$	Yield strength of stem tensile steel (MPa)	500	400
$X_{12}$	Yield strength of stem compressive steel (MPa)	400	350
$X_{13}$	Yield strength of toe tensile steel (MPa)	500	400
$X_{14}$	Yield strength of toe compressive steel (MPa)	400	400
$X_{15}$	Yield strength of heel tensile steel (MPa)	350	400
$X_{16}$	Yield strength of heel compressive steel (MPa)	500	500
$X_{17}$	Compressive strength of Stem concrete (MPa)	35	45
$X_{18}$	Compressive strength of base concrete (MPa)	28	28
$X_{19}$	Diameter of bar (m)	14	20
$X_{20}$	Number of stem tensile steel	5	3
$X_{21}$	Number of toe tensile steel	8	4
$X_{22}$	Number of heel tensile steel	5	3
$X_{23}$	Number of stem compressive steel	0	0

$X_{24}$	Number of toe compressive steel	0	0
$X_{25}$	Number of heel compressive steel	0	0

**5- Parametric Studies**

In this section, the influence of various wall geometries is investigated from cost and weight viewpoints. Fig. 7 shows various geometries for retaining walls. For all walls, required initial parameters are shown in Table 4 for Example 2. The influence of discrete variables such as compressive strength of concrete and yield strength of

steel on objective functions is significant. For this reason, these variables are eliminated in order to increase the accuracy and consistent comparison. It is important to note that all presented constraints in Table 3 are examined for all types of walls. The values of cost and weight objective functions are presented in Table 11

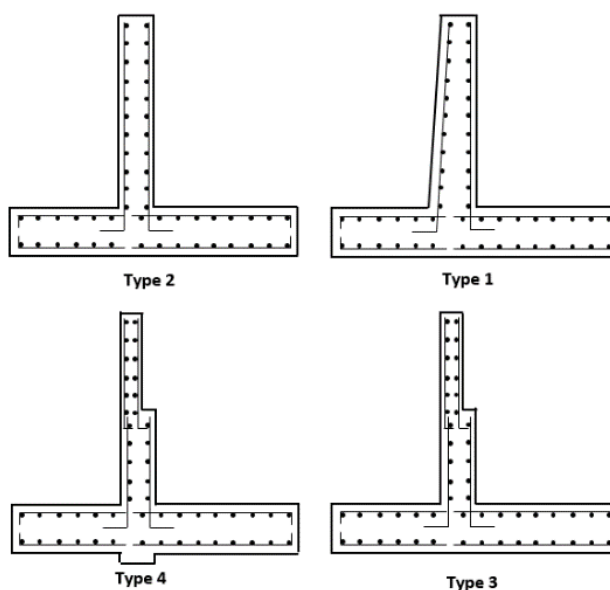


Fig. 7. Wall geometries considered for parametric studies

Table 11 Objective functions values considered for parametric studies

Type of wall	Objective function	Cost (\$/m)	Weight (kg/m)
Type1		146.911	5089.81
Type2		163.946	5514.25
Type3		139.04	4863.51
Type4		143.084	4991.62

A comparison between four walls types shows that the minimum cost causes to minimize the wall weight. This means that among four walls in Fig. 7, wall type 3 with two thicknesses has the lowest cost and weight. Moreover, normal T-shape wall is the worst one from cost and weight viewpoints. As seen, the use of wall type 3 reduces about %15.2 and %11.8 the wall cost

and weight, respectively. Also, the cost and weight of wall with varying stem thickness (wall type 1) are respectively %10.4 and %7.7 less than those of wall type 2. It is important to note that the shear key causes to slightly increase the wall cost and weight for this case.

**6- Sensitivity Analyses**

To find out the influence amount of contributing parameters on optimized walls, sensitivity analyses are performed in this section. For this purpose, effects of backfill unit weight, friction angle, and backfill slope are investigated on cost and weight objective functions. The required initial parameters are similar to those considered in section 5 (Table 4). It is noted that the backfill unit weight is computed in terms of linear

variation with the backfill internal friction angle (Table 12). It is noted that the effect of the wall stem height and surcharge were also investigated and found that with increasing the wall height and surcharge increases both objective functions, as obviously expected. These results are not presented here, however, they show that the IWDA optimizes retaining walls properly.

Table 12 Parameters values of sensitivity analyses

Parameter	Symbol	Value
Backfill slope (degree)	$\beta$	0°-10°-20°-30°
Unit weight of retained soil (KN/m <sup>3</sup> )	$\gamma_s$	15-16-17-18
Internal friction angle of soil (degree)	$\phi$	30-33.33-36.66-40

### 6-1- Backfill slope

Fig. 8 shows the effect of backfill slope,  $\beta$ , on objective functions. As seen, with increasing  $\beta$  from zero to 20°, the cost and weight objective functions initially decrease and then increase. The maximum values of objective functions are obtained for  $\beta=0^\circ$ . Also, the maximum value of weight objective function for the second type of wall is obtained for  $\beta=30^\circ$ . A increase of  $\beta$  affects significantly shear control constraint of the wall toe. This means that, for the applied shear, in toe is obtained in similar process as same as objective functions. For this reason, the parameter variations are investigated with increasing  $\beta$ . Variations of  $\beta$  can change the applied force to the wall toe, the minimum and maximum of force ( $q_{\max}$ ,  $q_{\min}$ ). Increasing this parameter causes to decrease  $q_{\max}$  and increase  $q_{\min}$ . Moreover, the variations of  $q_{\max}$  and  $q_{\min}$

are effected on tension control constraint in foundation. The values of constraints for two optimum points corresponding to  $\beta=10^\circ$  and  $\beta=20^\circ$  are compared. The results show that the variables of optimum point for  $\beta=20^\circ$  cannot gratify the constraints of angle of  $\beta=10^\circ$ . Therefore, the variation of backfill slopes is effected on tension control constraints, too.

### 6-2- Unit weight and internal friction of soil

According to Fig. 9, with increasing the backfill unit weight and internal friction angle, both objective functions rocket for all wall types. In addition, the variation of cost and weight objective functions is different for all wall types. Moreover, the reduction rate of functions is in the range of %15 to %20 for all wall types.

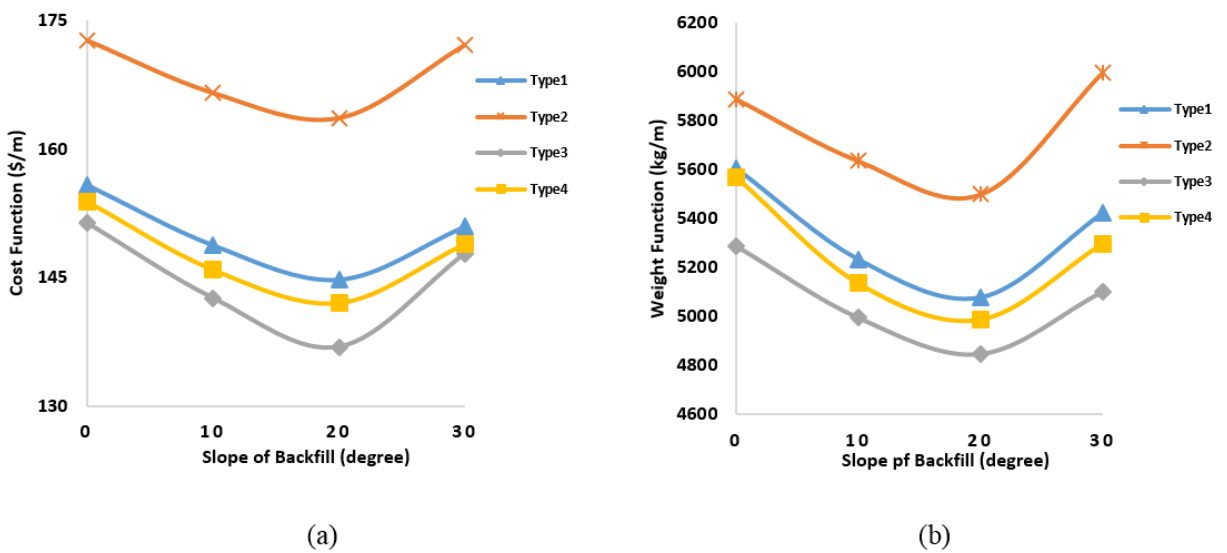


Fig. 8. Effect of backfill slope angle ( $\square$ ) on objective function values; (a) cost; (b) weight

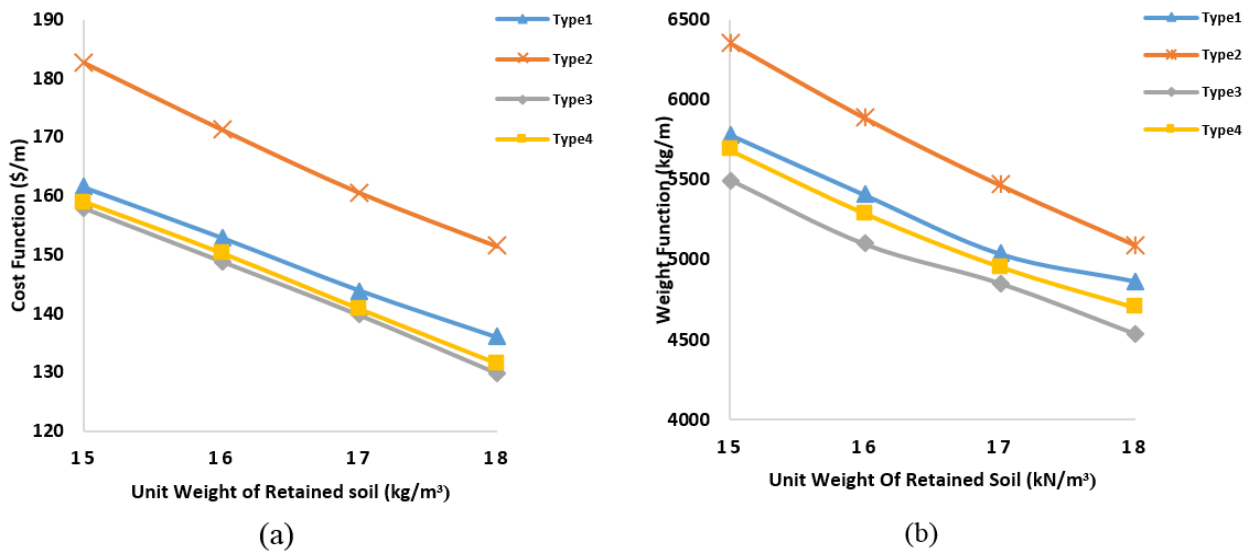


Fig. 9. Effect of variation of backfill unit weight on objective function values: (a) cost; (b) weight

**7- Effect of Rankine and Coulomb Method**

The influence of Rankine and Coulomb methods for calculation of lateral earth pressures is presented for normal T-shape wall and T-shape wall with two stem thicknesses. The initial parameters for this section are presented in Table 4. Moreover, the results given by Rankine method is given

in section 5. The results show that the Coulomb method decreases cost and weight objective functions more than the Rankine method. As seen in Table 13, the reduction range is 4 to 6 percent for cost and weight objective function.

Table 13 Comparison of objective functions values determined from Rankine and Coulomb methods

Wall type	Method	Objective function	
		Cost (\$/m)	Weight (kg/m)
Type1	Rankine	146.911	5089.81

	Coulomb	140.63	4867.39
	Percentage decrease	%4.27	%4.36
Type2	Rankine	163.946	5514.25
	Coulomb	155.12	5188.65
	Percentage decrease	%5.38	%5.9

**8- Design Chart**

In this section, a design chart for the T-shape retaining wall with varying stem thickness (wall type 1) is presented. The initial and fixed parameters are similar to those considered in section 5 (Table 4, Example 2) except the internal friction angle of retained

soil ( $\phi$ ) and surcharge load ( $q$ ). For this section,  $\phi$  and  $q$  are  $35^\circ$  and 0, respectively. The geometric wall with variables is shown in Fig. 10. The values of variables for walls with varying stem height (design chart) are presented in Table 15.

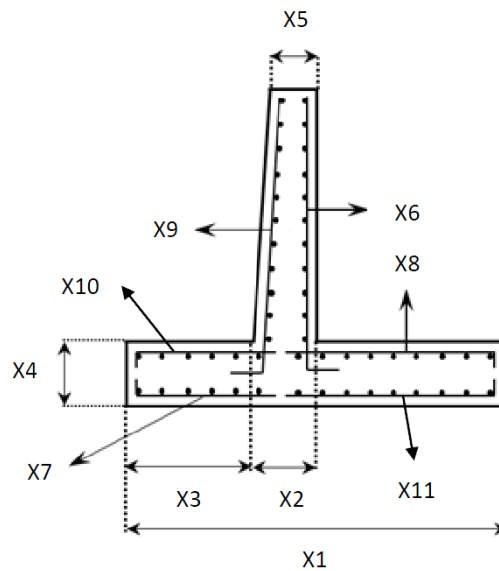


Fig. 10. Variables and wall geometry for design chart preparation

Table 14 Design chart for T-retaining wall

		Stem height							
		(m)	2.5	3	3.5	4	4.5	5	5.5
$X_1$	Total base width (m)	1.28	1.52	1.67	1.91	2.19	2.46	3.00	3.12
$X_2$	Stem thickness at the bottom (m)	0.64	0.43	0.51	0.58	0.65	0.99	1.26	1.44
$X_3$	Toe width (m)	0.20	0.20	0.27	0.30	0.34	0.37	0.41	0.47
$X_4$	Thickness of base (m)	0.23	0.27	0.31	0.36	0.41	0.46	0.50	0.54
$X_5$	Stem thickness at the top (m)	0.23	0.20	0.20	0.20	0.20	0.21	0.20	0.20
$X_6$	Stem tensile steel area (cm <sup>2</sup> /m)	5.28	8.26	7.61	9.88	12.4	15.2	21.4	21.8
$X_7$	Toe tensile steel area (cm <sup>2</sup> /m)	12.5	7.02	8.46	10.0	11.6	24.5	21.3	28.6



$X_8$	Heel tensile steel area (cm <sup>2</sup> /m)	5.93 6	6.96 7	8.69 8	10.1 5	11.7 1	14.1 6	14.9 9	16.9 3
$X_9$	Stem compressive steel area (cm <sup>2</sup> /m)	0	0	0	0	0	0	0	0
$X_{10}$	Toe compressive steel area (cm <sup>2</sup> /m)	0	0	0	0	0	0	0	0
$X_{11}$	Heel compressive steel area (cm <sup>2</sup> /m)	0	0	0	0	0	0	0	0

## 9- Discussion

In this paper, the optimization of retaining wall has been performed by using intelligent water drops algorithm (IWDA). The influence of wall geometries was also been investigated. The main concluding remarks may be mentioned as:

- ♦ The minimum and maximum values of objective functions are obtained for T-shape with two stem thicknesses and normal T-shape, respectively.

- ♦ Sensitivity analyses show that with increasing the backfill slope, cost and weight objective functions initially decrease and then increase. The minimum values of objective functions are obtained for  $\alpha=20^\circ$ . In addition, with increasing the backfill unit weight and internal friction angle, both objective functions decrease.

- ♦ The Coulomb method for lateral earth pressure calculation gives lower wall cost and weight than the Rankine method.

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