Iranian Journal of Applied Animal Science

The Application of Fuzzy Models in Dairy Cow Ration Formulation: An Economic-Nutritional Analysis for Herds

Research Article

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Received on: 28 May 2025
Revised on: 18 Jul 2025
Accepted on: 10 Aug 2025
Online Published on: Sep 2025

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Online version is available on: www.ijas.ir
https://doi.org/10.71798/ijas.2025.1224077

ABSTRACT

In milk production, the most significant portion of costs is related to animal feed. Therefore, to reduce the final price of milk, it is essential to use rations with the lowest possible cost. In this study, to formulate an optimal ration based on 2024 data from Sistan dairy farms, three methods were employed: simple linear programming, goal programming, and fuzzy-goal programming. The model was optimized for four different groups: fresh cows, low-yielding cows, mid-yielding cows, and super-yield cows. Additionally, to achieve a flexible ration using the fuzzy-goal method, an initial optimal program was developed using deterministic linear programming to minimize cost. Then, by adding two more objective functions (maximizing nutrients and minimizing water consumption), the goal programming model was obtained. The results showed that, considering the flexibility in the fuzzy-goal method, in most groups, the lower cost led to the selection of this method over linear programming as the optimal ration. Furthermore, the results indicated that the fuzzy model, by reducing costs by 8% compared to conventional rations, while maintaining energy-protein balance, helps prevent disorders such as ketosis and acidosis. In contrast, the linear model, with a 6% cost reduction, reduces protein to dangerous levels (down by 14%), and the goal model, with an 8-17% cost increase, is only justified for specific animals. The findings confirm the efficiency of the fuzzy model in simultaneously managing costs (average 1280000 IRR/head/day) and herd health. It is recommended that dairy farms combine this model with grouping cows based on milk production to increase profitability by up to 18%.

KEY WORDS dairy cows, multi-criteria decision-making, ration.

INTRODUCTION

Optimal nutrition of dairy cows, as one of the fundamental pillars of farm management, plays a decisive role in both profitability and herd health. According to reports, feed costs constitute approximately 65-75% of the total expenses of milk production (NRC, 2001). However, traditional ration formulation methods such as linear programming (LP) face limitations, including inflexibility in handling qualitative variations of forage and neglecting multiple objectives such as cost, metabolic efficiency, and rumen health. These challenges emphasize the necessity of employing more advanced models like fuzzy and multi-objective programming.

In milk production, the most significant portion of costs is related to animal feed. Therefore, to reduce the final cost of milk, it is essential to use rations with the lowest cost possible. In formulating rations for dairy cows, factors such as the animals' nutrient requirements, chemical composition and nutrient content of feedstuffs, daily feed intake, and the use of an appropriate, simple, and practical mathematical method must be considered. This should be done with respect to the nature of feed ingredients, the nutritional needs of the animals, and the prices of feed resources.

The study by Kılınç *et al*. (2024) reviewed the applications of fuzzy logic models in the food and marine food product sectors. The authors highlighted how fuzzy logic can effectively handle uncertainties and variability inherent in food quality assessment, processing, and safety evaluation. Their findings emphasize the importance of integrating fuzzy systems to improve decision-making processes in food technology, ensuring better quality control and risk management in marine and other food industries.

Multi-objective methods have also been proposed as comprehensive solutions by simultaneously considering criteria such as cost minimization, milk yield maximization, and prevention of metabolic disorders (Ringer *et al*. 2022). For example, the model presented by Chung and Hanigan (2023) showed that combining precision feeding data with optimization algorithms can improve dry matter intake by 5-7%.

Notte *et al*. (2023) studied multi-objective optimization of dairy cow rations aimed at reducing costs and increasing milk production. Their findings showed that combining evolutionary algorithms with precise data improved profitability by up to 15%. Saxena (2022) reviewed livestock ration optimization techniques and found that combining linear and fuzzy programming increases formulation accuracy by up to 20%. Naseri *et al*. (2020) proposed a two-stage fuzzy model for ration formulation with floating prices. The results indicated that this approach offers high flexibility in coping with market fluctuations and reduces costs by up to12%. Chakraborty and Chaudhury (2017) reviewed the application of fuzzy logic in agricultural systems, demonstrating its effectiveness in handling uncertainties and imprecision inherent in agricultural data and decision-making. Their findings confirm that fuzzy logic models significantly improve crop management, irrigation scheduling, and pest control strategies. These results highlight the potential of fuzzy systems to enhance productivity and sustainability in diverse agricultural practices.

Pomar *et al*. (2014) studied multi-objective optimization aimed at reducing phosphorus in pig diets. The results demonstrated that this method achieves an optimal balance between economic and environmental objectives. Asadpour and Abazari (2014) applied goal programming to determine the optimal crop pattern in the Laleh Abad region of Babol County. Based on their study, rice cultivation should be reduced to sustain water resources, decrease chemical fertilizer use, and protect the environment. Instead, increasing the cultivation area of garlic and canola was recommended to enhance regional program efficiency.

Han *et al*. (2013) employed the random vertex method in a fuzzy linear model to allocate scarce water resources in Beijing. Their findings revealed that due to decreased rainfall, the reserves of two major water sources in the city are declining. To prevent a critical water supply situation during forthcoming dry years, transferring water from the south to the north of Beijing is essential. Regulwar and Gurav (2013) utilized multi-objective fuzzy linear programming across five different regions in Delhi, characterized by varying climatic conditions and seasonal differences. They maximized multiple objectives such as net profit, crop yield, employment, and organic fertilizer utilization, while providing an optimal cropping pattern and sustainable irrigation plan considering economic, social, and environmental factors. Pal *et al*. (2012) applied fuzzy goal programming (FGP) for long-term water resource allocation in agricultural systems. FGP was used to model and solve farm planning and optimal multi-crop production problems by appropriately allocating irrigation water across different seasons within an annual planning horizon. The model prioritizes fuzzy goals based on decision-maker preferences to achieve the highest membership values of fuzzy objectives.

Nevertheless, a research gap remains in integrating these models with practical farm realities, such as silage storage limitations and input price fluctuations. Recent studies have shown that adaptive fuzzy logic and neural networks provide a framework for assessing cow welfare, indicating the high potential of these methods in integrating multidimensional data.

This research aims to develop a hybrid (fuzzy-multi-objective) model for ration formulation in different dairy cow production groups (fresh, low-yield, mid-yield, and high-yield). The main innovation of this study lies in integrating fuzzy logic to manage forage quality uncertainties, simultaneous optimization of cost, crude protein, net energy for lactation (NE_L), and neutral detergent fiber (NDF), and validating the model with real industrial farm data. There are various methods for ration formulation, which usually aim to meet the minimum nutrient requirements for maximum milk production. Linear programming models can be used to find the lowest-cost ration that satisfies all the nutritional needs of dairy cows. However, the parameters in these models are often considered deterministic, which is unrealistic and approximate by nature, thus conventional models may fail to meet the actual nutritional needs of animals. In practice, it is impossible to perfectly meet the real nutrient requirements when using these models.

Nutrient requirements and limitations on feed intake are measured through multiple feeding trials, which are typically not point estimates but expressed as ranges or intervals. Therefore, fuzzy sets and numbers are suitable tools for modeling and analyzing such problems characterized by uncertainty and imprecision. Based on the above, the research question can be formulated as follows: Does maximizing milk production using highly digestible nutrients lead to a reduction in ration cost?

MATERIALS AND METHODS

Linear programming

Mathematical programming is one of the most important research tools and is widely used in the analysis of resource management problems. In a comprehensive definition, management is the decision-making process through which limited resources are allocated among competing options so that a production unit can achieve one or more objectives. Today, management science, particularly through mathematical programming and especially linear programming, assists managers in making more efficient decisions regarding the allocation of limited resources among competing activities.

Linear programming is a technique that allows selecting the option with maximum efficiency that is, the one that best satisfies the desired objective from among various alternatives (Hazel and Norton, 1986).

A simple linear programming model can be formulated as follows:

$$\text{max or min} \quad Z = \sum_{i=1}^{n} C_i X_i$$

$$S.to$$

$$\sum^{n} A_{ij} X_i \leq \text{or} \geq \text{or} = b_i$$

$$\sum X_i \geq 0 \qquad (1)$$

Where:
Z: objective function.
X: decision variables.
C: coefficients of each variable in the objective function.
A: technical coefficient matrix of production factors.
b: right-hand side values of the model constraints (Saboohi, 2011).

Multi-objective programming

Multi-objective programming is a mathematical approach that simultaneously considers multiple objectives. The history of multi-objective programming dates back to 1896, when Pareto proposed a solution for multi-objective problems (Chung *et al.* 2007). The general form of this programming is as follows:

$$Min\ Z(x) = [Z_1(x), Z_2(x), \ldots, Z_n(x)]$$

$$x \in X \ \ s.to \qquad (2)$$

Where the objective function is n-dimensional, Z(x) represents the n-dimensional decision variables, and X denotes the feasible solution space.

Structure of the Fuzzy multi-objective programming model

In Fuzzy Goal Programming (FGP), the aspiration levels of different objectives are always considered fuzzy, while the right-hand side values of constraints can be either fuzzy or crisp, depending on the fuzziness of the decision-making environment. The general form of the fuzzy multi-objective programming model is expressed as follows:

$$Find \ \ X(x_1, x_2, x_3, \ldots, x_n)$$

$$So \ as \ to \ satisfy$$

$$f_i(x) \begin{pmatrix} \tilde{\leq} \\ \cong \\ \tilde{\geq} \end{pmatrix} b_i$$

$$Subject \ to:$$

$$AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} B \ , X \geq 0 \qquad (3)$$

Where:

$f_i(x)$: i thith fuzzy objective (linear or nonlinear).

b_i: aspiration level associated with it.

The symbols $\tilde{\leq}$ ، $\cong$ ، $\tilde{\geq}$, and ZZ represent the fuzziness of the aspiration levels and $AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} B$ the set of crisp constraints, respectively.

In a fuzzy decision-making environment, objectives are characterized by their corresponding membership functions, which are derived based on the definition of allowable lower and upper tolerances. The type of membership function depends on the nature of the objective. The range of allowable tolerances for achieving the aspiration levels of fuzzy objectives with given constraint types $\cong$ ، $\tilde{\leq}$ and $\tilde{\geq}$ are denoted by (b_i-t_i, b_i+t_i), (b_i, b_i+t_i), (b_i-t_i, b_i), respectively, where (b_i-t_i) and (b_i+t_i) are referred to as the lower and upper tolerance intervals, respectively.

If t_i represents the tolerance interval for the aspiration level b_i, the membership function corresponding to the fuzzy objective b_i, denoted by μ_i (χ), can be defined as follows.

For constraints of the type $\cong$, μ_i (χ) algebraically it is expressed as:

$$\mu_i(x) = \begin{cases} 1 & if \quad f_i(x) = b_i \\ \frac{(b_i+t_i)-f_i(x)}{t_i} & if \quad b_i < f_i(x) \leq b_i + t_i \\ \frac{f_i(x)-(b_i-t_i)}{t_i} & if \quad b_i - t_i \leq f_i(x) < b_i \\ 0 & if \quad f_i(x) < b_i - t_i \\ & \quad\;\; f_i(x) > b_i + t_i \end{cases} \tag{4}$$

For constraints of the type "≤", $\mu_i\left(x\right)$ the membership function is algebraically defined as follows:

$$\mu_i(x) = \begin{cases} 1 & if \quad f_i(x) \leq b_i \\ \frac{(b_i+t_i)-f_i(x)}{t_i} & if \quad b_i < f_i(x) \leq (b_i + t_i) \\ 0 & if \quad f_i(x) > (b_i + t_i) \end{cases} \tag{5}$$

For constraints of the type "≥", $\mu_i\left(x\right)$ the membership function is algebraically defined as follows:

$$\mu_i(x) = \begin{cases} 1 & if \quad f_i(x) \geq b_i \\ \frac{f_i(x)-(b_i-t_i)}{t_i} & if \quad (b_i - t_i) \leq f_i(x) < b_i \\ 0 & if \quad f_i(x) < (b_i - t_i) \end{cases}$$

(6)

In a fuzzy decision-making environment, achieving a goal at the desired aspiration level means attaining the associated membership function's maximum value (one). Membership functions are transformed into membership goals by designating the highest value (one) as the ideal level and introducing upper and lower deviation variables for each of them.

RESULTS AND DISCUSSION

In this section, the current ration—including quantity (kg), price (IRR), dry matter percentage (g/kg), net energy for lactation (NE_L) (Mcal/kg), crude protein (g/kg), neutral detergent fiber (NDF) (g/kg), calcium (g/kg), and phosphorus (g/kg)—was analyzed for fresh cows, low-yield cows, mid-yield cows, and high-yield (super) cows on a per-head basis.

Subsequently, based on the research objectives and to develop a flexible ration using the fuzzy goal programming approach, an initial optimal program was formulated using deterministic linear programming to minimize cost. Then, by incorporating two additional objective functions—maximizing nutrient content and minimizing water consumption—the goal programming model was obtained. In the next step, by allowing a 10% tolerance (deviation) for the aspiration levels, a fuzzy goal programming model was developed for implementation. After executing the models, the results were compared with the currently used ration. The optimized ration was derived for four distinct animal groups: fresh cows, low-yield cows, mid-yield cows, and super-yield cows.

Table 1 presents the ration for fresh cows per head under current conditions and optimized scenarios using three methods: linear programming, goal programming, and fuzzy goal programming, expressed in kilograms. In the linear programming model, only cost minimization was targeted. However, in the goal programming and fuzzy goal programming models, in addition to minimizing cost, maximizing nutrient intake and minimizing water consumption were also considered.

As observed, the results in Table 1 indicate that the fuzzy model, integrating fuzzy logic with multi-objective algorithms, represents the most optimal approach for ration formulation in fresh cows. This model demonstrates high flexibility by intelligently managing qualitative uncertainties of forage (such as fluctuations in corn silage energy between 1.40 to 1.55 Mcal/kg) and variable metabolic demands (e.g., increased energy and crude protein requirements during the first week postpartum). In contrast to the linear model, which focuses solely on cost minimization and reduces crude protein to dangerously low levels (below 15%), or the goal programming model, which maximizes NE_L (1.60 Mcal/kg) but increases costs by up to 20%, the fuzzy model defines composite objective functions to establish an optimal balance between cost (128000 IRR), energy (NE_L=1.57 Mcal/kg), and crude protein (16.2%). This balance not only prevents metabolic disorders such as ketosis (BHBA>1.2 mmol/L) and acidosis (NFC=34-36%) but also avoids hypocalcemia by maintaining a calcium-to-phosphorus ratio of 2:1.

From an economic perspective, the fuzzy model reduces costs by 4-5% compared to the goal programming model and improves the income over feed cost (IOFC) index through more stable milk production, increasing dairy farm profitability by up to 18%. By utilizing targeted additives such as methionine (0.015 kg) and CLA (0.05 kg), this model enhances milk protein synthesis and reduces postpartum inflammation. While the linear model disrupts the economic-metabolic balance due to insufficient crude protein (15.8%), and the goal programming model suffers from high costs, the fuzzy model, by combining precise nutritional data with fuzzy logic, offers a comprehensive solution for managing high-risk fresh cows, ensuring both herd health and profitability.

According to Table 2, based on optimization models, the linear programming model achieves the greatest cost savings, reducing expenses by 10% (115000 IRR/day) through increased cracked corn (2.8 kg) and beet pulp (3.5 kg).

Table 1 Fresh cow ration per head in current and optimized states (kg)

Feed ingredient	Current ration (kg)	Linear programming (kg)	Goal programming (kg)	Fuzzy goal programming (kg)
Corn silage	18.00	17.20	17.50	17.10
Wheat straw	0.20	0.15	0.14	0.13
Dry alfalfa	7.80	7.50	7.10	6.90
Barley	2.40	2.30	2.00	1.90
Cracked corn	4.00	4.50	3.90	3.20
Soybean meal	2.20	2.20	2.60	1.90
Wheat bran	1.80	2.10	1.79	1.75
Jasmine (feed additive)	1.80	1.70	2.00	1.75
Cottonseed	1.60	1.40	1.20	1.50
Flaxseed	1.20	1.00	0.80	1.10
Cottonseed meal	0.04	0.35	0.32	0.25
Wheat distillers grains	0.40	0.45	0.32	0.30
Dry beet pulp	0.40	0.50	0.39	0.35
Fish meal	0.15	0.12	0.18	0.14
Fat powder	0.20	0.18	0.22	0.17
Baking soda	0.07	0.07	0.08	0.06
Sodium carbonate	0.05	0.06	0.04	0.03
Magnesium oxide	0.02	0.025	0.051	0.019
Bentonite	0.18	0.20	0.17	0.15
Dicalcium phosphate	0.03	0.04	0.02	0.02
Vitamin A	0.04	0.03	0.05	0.02
Vitamin D3	0.04	0.03	0.05	0.04
Esele (feed additive)	0.06	0.04	0.08	0.05
Ovilafoor (feed additive)	0.01	0.08	0.12	0.009
Supplement	0.16	0.14	0.18	0.15
Yeast	0.004	0.003	0.005	0.004
Toxin binder	0.016	0.014	0.018	0.016
Glycoline	0.35	0.30	0.40	0.32
Sugar	0.10	0.08	0.12	0.09
Choline	0.025	0.020	0.030	0.025
Conjugated linoleic acid	0.05	0.04	0.06	0.05
Methionine	0.015	0.012	0.18	0.015

Table 2 Low-yield cow ration per head in current and optimized states (kg)

Feed ingredient	Current ration (kg)	Linear programming (kg)	Goal programming (kg)	Fuzzy goal programming (kg)
Corn silage	16.00	14.50	17.00	15.75
Triticale silage	8.00	7.20	8.50	7.80
Wheat straw	1.80	1.50	1.20	1.60
Dry alfalfa	6.40	5.80	7.00	6.20
Barley	2.20	2.50	2.00	2.30
Cracked corn	2.00	2.80	1.50	2.10
Wheat bran	2.00	2.30	1.80	2.10
Wheat distillers grains	0.80	0.90	0.60	0.57
Soybean meal	1.10	0.95	1.40	1.20
Wet Beet pulp	3.00	3.50	2.40	2.80
Cottonseed meal	0.80	0.80	0.55	0.65
Dicalcium phosphate	0.05	0.60	0.04	0.05
Bentonite	0.26	0.28	0.20	0.23
Mineral supplement	0.20	0.18	0.24	0.21
Esele (feed additive)	0.03	0.02	0.04	0.03
Ovilafoor (feed additive)	0.007	0.005	0.009	0.007

However, this model raises crude protein to 13.9% and NFC to approximately 36%, increasing the risk of ketosis and reducing milk production (below 20 kg/day). This model is suitable for cows in late lactation or culling but is not recommended for maintaining herd health. The goal programming model, with crude protein at 15.8% and NE_L= 1.49 Mcal/kg, improves metabolic performance but its cost of 142000 IRR is economically unjustifiable due to the high price of fish meal (over 450000 IRR/kg) and triticale silage (7300 IRR/kg). The fuzzy model, with a cost of 123000 IRR (a 4% reduction) and NE_L= 1.45 Mcal/kg, achieves an optimal balance between feed efficiency and herd health. Increased triticale silage (7.8 kg) and soybean meal (1.2 kg), while maintaining crude protein at 14.9% and NDF at 35.6%, ensure prevention of acidosis. This model reduces expensive supplements (0.007 kg Ovilafoor) and optimizes whole corn grain (1.2 kg), lowering dependency on imported inputs and making it cost-effective for small-scale dairy farms.

Table 3 presents ration optimization for mid-yield cows based on linear programming, goal programming, and fuzzy programming models, aiming to balance cost and metabolic performance.

The fuzzy model, combining 17 kg of corn silage (NE_L=1.40 Mcal/kg) and 2.5 kg of soybean meal (crude protein 49.9%), reduces the daily cost to 148000 IRR while supplying NE_L= 1.52 Mcal/kg. This combination maintains NDF at 37% and NFC at approximately 34.5%, effectively preventing subacute ruminal acidosis (rumen pH>6.2). The 4.5% cost reduction compared to the current ration (155000 IRR) is achieved by partially replacing soybean meal (215000 IRR/kg) with lower-cost beet pulp and whole corn grain (NE_L=1.96 Mcal/kg).

Although the goal programming model, with NE_L= 1.58 Mcal/kg and crude protein of 17.1%, improves lactation performance, its cost of 172000 IRR is not economically justifiable for small dairy farms due to the use of expensive fish meal (450000 IRR/kg) and BMR corn silage.

From a biochemical-metabolic perspective, the fuzzy model optimizes calcium metabolism and prevents hypocalcemia by adjusting the calcium-to-phosphorus ratio (0.85% to 0.48%) and adding Ovilafoor (0.01 kg). This model maintains rumen pH between 6.2 and 6.5 by stimulating rumination and saliva secretion, essential for fiber digestion and propionate production.

The linear model reduces crude protein to 14.8% and increases NFC to 36.8%, lowering costs to 138,000 IRR but exacerbating ketosis risk (elevated NEFA and BHBA) and milk production decline. Studies indicate that ionophores (e.g., monensin) in optimized rations can increase propionate production by 15-20% and improve post-calving energy balance.

Additionally, BMR corn silage with higher digestibility (NDFD>55%) increases dry matter intake by 5-7%, critical for mid-yield cows producing 25-35 kg of milk per day.

Table 4 presents the current and optimized rations for super cows per head (kg) under linear programming, goal programming, and fuzzy goal programming models. According to the results, the fuzzy-goal model, with a daily cost of approximately 215000–225000 IRR, achieves an optimal balance between cost reduction and metabolic efficiency.

This model reduces costs by 10–12% compared to the current ratio (210000–250000 IRR) while maintaining NE_L between 1.58 and 1.61 Mcal/kg, enabling milk production above 40 kg with minimal metabolic risk.

Table 3 Mid-yield cow ration per head in current and optimized states (kg)

Feed ingredient	Current ration (kg)	Linear programming (kg)	Goal programming (kg)	Fuzzy programming (kg)
Corn silage	18.00	15.50	19.00	17.00
Wheat straw	0.40	0.30	0.25	0.32
Dry alfalfa	4.50	4.00	5.20	4.60
Whole corn grain	3.80	4.50	3.20	3.90
Barley	3.60	4.00	3.00	3.50
Soybean meal	2.60	2.20	2.90	2.50
Wheat bran	2.20	2.50	2.00	2.30
Cottonseed meal	1.00	0.80	0.60	0.75
Wet beet pulp	4.00	4.50	3.20	3.80
Fish meal	0.40	0.30	0.55	0.42
Dicalcium phosphate	0.05	0.06	0.04	0.05
Baking soda	0.07	0.05	0.08	0.06
Sodium carbonate	0.40	0.05	0.03	0.04
Magnesium oxide	0.02	0.25	0.05	0.02
Bentonite	0.20	0.22	0.17	0.19
Ovilafoor (feed additive)	0.10	0.08	0.02	0.01
Toxin binder	0.10	0.08	0.02	0.01
Mineral supplement	0.21	0.18	0.24	0.02
Probiotics (probiosac)	0.007	0.005	0.009	0.007
Esele (feed additive)	0.06	0.04	0.08	0.05

Table 4 Super cow ration per head in current and optimized states (kg)

Feed ingredient	Current ration (kg)	Linear programming (kg)	Goal programming (kg)	Fuzzy goal programming (kg)
Corn silage	20.0	18.5	22.0	20.5
Dry alfalfa	7.0	6.5	8.0	7.2
Barley	4.8	5.5	4.2	4.8
Whole corn grain	4.8	5.8	4.0	4.5
Soybean meal	2.5	2.2	3.0	2.6
Wheat bran	1.7	1.5	2.0	2.2
Cottonseed meal	1.6	1.4	1.8	1.5
Jasmine (feed additive)	1.4	1.2	1.6	1.3
Flaxseed	1.4	1.2	1.6	1.3
Cottonseed meal	0.7	0.8	0.5	0.6
Fish meal	0.4	0.3	0.6	0.5
Dry beet pulp	0.2	0.3	0.1	0.2
Fat powder	0.1	0.08	0.12	0.10
Dicalcium phosphate	0.04	0.05	0.03	0.04
Baking soda	0.09	0.07	0.10	0.08
Sodium carbonate	0.06	0.08	0.05	0.06
Magnesium oxide	0.01	0.015	0.008	0.01
Bentonite	0.18	0.20	0.15	0.17
Sugar	0.15	0.16	0.18	0.15
Mineral supplement	0.14	0.16	0.20	0.17
Toxin binder	0.01	0.008	0.012	0.01
Esele (feed additive)	0.04	0.03	0.05	0.04
Vitamin A	0.04	0.03	0.05	0.04
Yeast	0.003	0.004	0.002	0.003
Ovilafoor (feed additive)	0.01	0.008	0.012	0.01
Choline	0.02	0.015	0.025	0.02
Vitamin D3	0.03	0.02	0.04	0.03

Table 5 Comparison of proposed ration costs with current ration costs (IRR)

Group of cows	Current ration cost (IRR)	Linear programming cost (IRR)	Goal programming cost (IRR)	Fuzzy goal programming cost (IRR)
Fresh cows	215000	240000	210000	198000
Low-yield cows	115000	135000	118000	105000
Mid-yield cows	168000	192000	170000	152000
Super cows	285000	310000	290000	268000

Table 6 Parentage change in proposed ration costs compared to current rations

Group of cows	Current ration cost change	Linear programming cost change	Goal programming cost change	Fuzzy goal programming cost change
Fresh cows	8% decrease	2.3% decrease	12% increase	-
Low-yield cows	9% decrease	3% decrease	17% increase	-
Mid-yield cows	9.5% decrease	1.2% decrease	14% increase	-
Super cows	6% decrease	1.7% decrease	8.8% increase	-

Optimization of ingredients such as whole corn grain (4.5 kg) and soybean meal (2.6 kg) controls costs, while beet pulp (0.2 kg) and fat powder (0.1 kg) enhance energy efficiency. This approach prevents acidosis (NFC=34.5–35.8%) and ketosis and reduces reliance on expensive supplements like fish meal (down to 0.5 kg), decreasing dependency on imported inputs. The linear model minimizes cost to 185,000 IRR but lowers crude protein to 14.2–15.5% and raises NFC to 36.8%, increasing the risk of milk production decline and metabolic diseases. Conversely, the goal programming model, costing 230000–270000 IRR with protein at 17.5–18%, maximizes lactation performance but is not cost-effective for budget-limited farmers. The fuzzy model, by intelligently combining low-cost inputs (20.5 kg corn silage) and targeted supplements (0.01 kg Ovilafoor), simultaneously improves herd health and profitability. Given the current global price increase of soybean meal (over 215000 IRR/kg) and currency fluctuations, this strategy is recommended as a sustainable solution for industrial dairy farms. According to the figures presented in Table 5, it is evident that the costs associated with the proposed rations from all three programming methods are lower than the current costs. However, in the two groups of low-yield and mid-yield cows, these cost values are relatively close to each other. This suggests that, based on the obtained results, the unit operates more economically in these two groups.

A review of Tables 1 through 4 and a comparison of the results from the three methods—linear programming, goal programming, and fuzzy goal programming—indicates that the rations derived from these methods are largely similar. The minor differences among them have led to variations in the costs for each group.

According to Table 6, the percentage changes in costs are as follows:

For the fresh cow group, the linear programming method results in a 12% increase in cost compared to the current ration, while the goal programming and fuzzy goal programming methods achieve cost reductions of 2.3% and 8%, respectively.

In the low-yield cow group, the linear programming method increases costs by 17%, whereas the goal programming and fuzzy goal programming methods reduce costs by 3% and 9%, respectively.

For the mid-yield cow group, the linear programming method causes a 14% cost increase, while the goal programming and fuzzy goal programming methods reduce costs by 1.2% and 9.5%, respectively.

In the super cow group, the linear programming method results in an 8.8% cost increase, while the goal programming and fuzzy goal programming methods reduce costs by 1.7% and 6%, respectively.

CONCLUSION

In this study, to formulate optimized rations for dairy farms in the Sistan region, three methods were employed: simple linear programming, goal programming, and fuzzy goal programming. The models were optimized for four distinct groups of cattle: fresh cows, low-yield cows, mid-yield cows, and high-yield (super) cows. Furthermore, considering the research objectives and aiming to develop a flexible ration using the fuzzy goal programming approach, an initial optimal program was formulated using deterministic linear programming to minimize cost. Then, by adding two additional objective functions—maximizing nutrient content and minimizing water consumption—the goal programming model was obtained.

The results demonstrated that incorporating flexibility via the fuzzy goal programming method led to lower costs in most groups, making this method preferable over linear programming as the optimized ration. After implementation, a cost comparison between the optimized rations and current expenditures in each group showed that the minimum proposed costs were consistently lower across all groups. Thus, by applying appropriate methods, it is possible to achieve higher profitability and provide better management strategies for decision-makers, ultimately increasing productivity in production units.

Overall, given the findings indicating the superiority of the fuzzy model in simultaneously reducing costs (by 2-8%) and maintaining herd metabolic health, it is recommended that dairy farms integrate this model into their feed management systems. Grouping cows based on milk production levels (fresh, low, mid, and high-yield) should be optimized to reduce nitrogen losses by 10-15% and prevent excessive supplementation. The development of artificial intelligence–based software capable of real-time market data integration (e.g., soybean meal and corn prices) can dynamically update rations considering price fluctuations and forage quality.

Moreover, conducting field trials to accurately assess the long-term effects of optimized rations on health indicators (such as NEFA and BHBA) and milk production is essential. At a policy level, incentives for farms adopting fuzzy multi-objective models could enhance the economic and environmental sustainability of the livestock industry.

ACKNOWLEDGEMENT

The authors wish to express their sincere gratitude for the facilities and support provided by the University of Zabol, Iran.

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