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Mutiobjective Optimization Strategy for Solving Interval-valued Fuzzy Constrained Shortest Path Problems

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ABSTRACT

The constrained shortest path (CSP) problem is one of the most used and tangible applications of network flow problems that aside from its straightforward application is arised as auxiliary problems in flight planning, tail assignment problem in aircraft scheduling and crew rostering problems, among others. The objective of the CSP problem is to determine a minimum cost path between two specified nodes that the traversal time of the path does not exceed from a specified time. Conventional CSP problem generally assumes that the weights of arc costs and times are defined by real variables, though these values are unpredictable due to some uncontrollable factors. The present study formulates a CSP problem when values of arc costs and times are interval-valued triangular fuzzy numbers and proposes a multi-objective optimization strategy to obtain the efficient solution of the resulting problem. The applicability of the proposed approach is illustrated through an example dealing with wireless sensor networks.

1. Introduction

As a particular type of network optimization problem, shortest path (SP) problems are used in a wide range of applications, from transportation to routing and communication networks. The main goal of SP problems in this general sense is to determine the path between two distinct nodes with the least weight (cost, time, or distance). For example, this paper considers a network where the arc weights could represent transportation costs or travel time. In SP problems, it is often assumed that the weights are precise, which does not necessarily correspond to the real-life feeling. In this particular case, the uncertainty of both traffic and weather conditions are represented by generalized fuzzy numbers, such as intuitionistic or interval-valued fuzzy numbers.

There has been a number of approaches to solution of different SP problems under uncertainty which have been proposed in the literature [3, 4, 6, 7, 19, 24]. Nevertheless, two main lines of research may be segmented with regard to solution techniques for fuzzy SP (FSP). These are direct and heuristic approaches. Direct

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approaches include a variety of methods, and involve the use of the extension principle to identify membership boundaries along the SP [20], or use of α-cuts to define distances among fuzzy weights [25]. The evaluation structure in principle related to defining the SP is quite complex, and considers multiple objective functions. In this context, a combination of methods should be employed.

In recent years, the importance of numerical heuristic approaches has increased noticeably. These can involve simply using established procedures, for example, genetic algorithms [14], and ant colony optimization [15], to develop hybrid models that bring together a number of heuristics [11]. In this regards, Lin et al. [18] introduced an innovative algorithmic method based on genetic algorithms for determining the shortest path between two nodes in a fuzzy graph with fuzzy arc lengths in the SP problem.

Keshavarz and Khorra [16] simplified the FSP problem by establishing a bi-level programming model. They advanced and implemented an algorithm based on this model, taking advantage of the relationship between SP and programming problems. Further development of the FSP issue in multi-constraint networks was accomplished by Dou et al. [8] with the use of vague multi-criteria decision-making approaches based on similarity measures. Deng et al. [5] generalized the Dijkstra algorithm to solve FSP problems applying the graded mean integration representation of fuzzy numbers. Kung et al. [17] adopted a dynamic programming technique to analyze FSP problems in a network with discrete fuzzy arc weights. Yang et al. [25] introduced a new algorithm for solving reliable FSPs in a mixed network, taking into account various fuzzy arc weights. Parimala et al. [22] employed the Bellman algorithm for a network with trapezoidal picture fuzzy numbers (TPFNs) and introduced a new algorithm to identify the shortest picture fuzzy path between each pair of nodes.

The analysis context has also been extended to include interval valued fuzzy arc weights in accordance with more recent developments. In this case, the membership functions are defined in intervals, rather than crisp values [10]. Two more examples of direct and heuristic solutions have emerged from this extension in order to identify the related SP model for this context. Dey et al. [6] utilized a genetic algorithm to investigate the interval-valued fuzzy problem.Conversely, Enayattabr et al. [12] employed dynamic programming techniques to address an all-pairs SP problem. In particular, these authors adapted the Floyd-Warshall algorithm to identify SPs within interval-valued fuzzy environments. Ebrahimnejad et al. [10] formulated the SP problem in a directed interval-valued triangular fuzzy network and converted it into a multi objective linear programming (MOLP) problem. They then employed a lexicographic optimization structure to obtain an efficient solution to the resulting MOLP problem.

Despite the extensive research conducted on the SP problem in a fuzzy environment, there has been a paucity of studies exploring the concept of a constrained shortest path (CSP) with fuzzy data. The CSP problem is one of the most used and tangible applications of network flow peoblems that aside from its straightforward application is arised as auxiliary problems in flight planning, tail assignment problem in aircraft scheduling and crew rostering problems, among others. Abbaszadeh Sori et al. [1] employed an elite artificial bees' colony algorithm for the resolution of fuzzy CSP (FCSP). Furthermore, the application of the fuzzy constrained shortest path has been demonstrated on a location-based online service, Snap [2]. In a recent study, Peng et al. [23] proposed a recursive relation for a FCSP problem, which was then used to develop a dynamic algorithm employing the distance function for comparing fuzzy constrained paths. In a recent contribution to the field, Dudeja [9] put forth the use of the particle swarm optimization (PSO) algorithm as a means of attaining the optimal global solution within the confines of a CSP problem, particularly within the context of an undirected network with intuitionistic fuzzy arc weights, which are known to present a considerable challenge in terms of search space complexity.

In the present paper, we formulate the CSP problem within a directed interval-valued triangular fuzzy network and propose an efficient solution technique for determining the optimum interval-valued fuzzy path weight and the corresponding interval-valued fuzzy optimal path. Moreover, we illustrate the applicability of the proposed approach through an illustrative example pertaining to wireless sensor networks (WSNs) [13]. Indeed, an interval-valued fuzzy constrained shortest path (IVFCSP) problem is formulated in order to take account of the fact that energy consumption and the quality of service (QoS) cannot be measured with precision due to the influence of environmental conditions. Subsequently, the proposed approach will identify an optimal path in WSN that will minimise energy consumption while maintaining a minimum level of quality of service (QoS).

To sum up with all the above aspects, the main contributions of the present study are sumarized as below:

- To formulate the CSP problem within a directed network with interval-valued triangular fuzzy arc costs and interval-valued triangular fuzzy arc times.
- To converte the interval-valued triangular fuzzy CSP problem into a model with crisp constraints and interval-valued triangular fuzzy objective function.
- To converte the interval-valued triangular fuzzy CSP problem into a crisp multi objective linear programming (MOLP) problem.
- To employ a multi-objective optimization strategy to obtain the efficient solution of the resulting problem.
- To illustrate the applicability of the proposed approach through an applicable example pertaining to wireless sensor networks.

The paper proceeds as follows. The next section introduces the main definitions required to build the interval-valued fuzzy constrained shortest path (IVFCSP) problem presented in Section 3. Section 4 illustrates numerically its applicability. Section 5 summarizes the main results obtained and suggests potential extensions.

2. Interval-valued fuzzy numbers

In this section, we review the concepts of interval-valued fuzzy numbers [10, 12].

Definition 1. A level λ triangular fuzzy number $\tilde{H}$ on $\mathbb{R}$, denoted by $\tilde{H}=(h_1,h_2,h_3;\lambda)$, $0<\lambda\leq 1$, is a fuzzy set with the following membership function:

$$\mu_{\tilde{H}}(x) = \begin{cases} \lambda\left(\dfrac{x-h_1}{h_2-h_1}\right), & h_1 \leq x \leq h_2, \\ \lambda, & x=h_2, \\ \lambda\left(\dfrac{h_3-x}{h_3-h_2}\right), & h_2 \leq x \leq h_3, \\ 0, & otherwise. \end{cases} \tag{1}$$

Denote $F_{TN}(\lambda)= \ (h_1,h_2,h_3;\lambda), h_1\leq h_2\leq h_3 \ \ , 0<\lambda\leq 1.$

Definition 2. Let $\underline{\tilde{H}}\in F_{TrN}(\underline{\lambda})$ and $\bar{\tilde{H}}\in F_{TrN}(\bar{\lambda})$. A level $(\underline{\lambda},\bar{\lambda})$–interval–valued triangular fuzzy number $\underline{\bar{\tilde{H}}}$, denoted by $\underline{\bar{\tilde{H}}}=\left[\underline{\tilde{H}},\bar{\tilde{H}}\right]=\left\langle(\underline{h}_1,h_2,\underline{h}_3;\underline{\lambda}),(\bar{h}_1,h_2,\bar{h}_3;\bar{\lambda})\right\rangle$ is an interval–valued fuzzy set on $\mathbb{R}$ where the lower triangular fuzzy number $\underline{\tilde{H}}$ and the upper triangular fuzzy number $\bar{\tilde{H}}$ are expressed as follows, respectively:

$$\mu_{\underline{\tilde{H}}}(x) = \begin{cases} \underline{\lambda}\left(\dfrac{x-\underline{h}_1}{h_2-\underline{h}_1}\right), & \underline{h}_1 \leq x \leq h_2, \\ \underline{\lambda}, & x=h_2, \\ \underline{\lambda}\left(\dfrac{\underline{h}_3-x}{\underline{h}_3-h_2}\right), & h_2 \leq x \leq \underline{h}_3, \\ 0, & otherwise. \end{cases} \tag{2}$$

$$\mu_{\tilde{\tilde{H}}}(x)=\begin{cases}\bar{\lambda}\left(\dfrac{x-\bar{h}_1}{h_2-\bar{h}_1}\right), & \bar{h}_1 \le x \le h_2,\\ \bar{\lambda}, & x=h_2,\\ \bar{\lambda}\left(\dfrac{\bar{h}_3-x}{\bar{h}_3-h_2}\right), & h_2 \le x \le \bar{h}_3,\\ 0, & otherwise.\end{cases} \tag{3}$$

where $\bar{h}_1 \le \underline{h}_1 \le h_2 \le \underline{h}_3 \le \bar{h}_3$, $0<\underline{\lambda} \le \bar{\lambda} \le 1$. The family of all level $(\underline{\lambda},\bar{\lambda})$–interval–valued triangular fuzzy numbers is denoted by $F_{IVTN}(\underline{\lambda},\bar{\lambda})$.

Definition 3. Let $\tilde{\tilde{\underline{H}}}=\left[\tilde{\underline{H}},\tilde{\tilde{H}}\right]=\left\langle(\underline{h}_1,h_2,\underline{h}_3;\underline{\lambda}),(\bar{h}_1,h_2,\bar{h}_3;\bar{\lambda})\right\rangle$ and $\tilde{\tilde{\underline{G}}}=\left[\tilde{\underline{G}},\tilde{\tilde{G}}\right]=\left\langle(\underline{g}_1,g_2,\underline{g}_3;\underline{\lambda}),(\bar{g}_1,g_2,\bar{g}_3;\bar{\lambda})\right\rangle$ belong to $F_{IVTN}(\underline{\lambda},\bar{\lambda})$ and k be a non–negative real number. Then, interval-valued fuzzy arithmetic operations are defined as follows:

$$\tilde{\tilde{\underline{H}}}+\tilde{\tilde{\underline{G}}}=\left\langle(\underline{h}_1+\underline{g}_1,h_2+g_2,\underline{h}_3+\underline{g}_3;\underline{\lambda}),(\bar{h}_1+\bar{g}_1,h_2+g_2,\bar{h}_3+\bar{g}_3;\bar{\lambda})\right\rangle,$$

$$k\tilde{\tilde{\underline{H}}}=\begin{cases}\left\langle(k\underline{h}_1,kh_2,k\underline{h}_3;\underline{\lambda}),(k\bar{h}_1,kh_2,k\bar{h}_3;\bar{\lambda})\right\rangle, & k>0,\\ \left\langle(k\underline{h}_3,kh_2,k\underline{h}_1;\underline{\lambda}),(k\bar{h}_3,kh_2,k\bar{h}_1;\bar{\lambda})\right\rangle, & k<0,\\ \left\langle(0,0,0;\underline{\lambda}),(0,0,0;\bar{\lambda})\right\rangle=\tilde{\tilde{\underline{0}}}, & k=0.\end{cases} \tag{4}$$

Definition 4: Two interval–valued trianular fuzzy numbers $\tilde{\tilde{\underline{H}}}=\left[\tilde{\underline{H}},\tilde{\tilde{H}}\right]=\left\langle(\underline{h}_1,h_2,\underline{h}_3;\underline{\lambda}),(\bar{h}_1,h_2,\bar{h}_3;\bar{\lambda})\right\rangle$ and $\tilde{\tilde{\underline{G}}}=\left[\tilde{\underline{G}},\tilde{\tilde{G}}\right]=\left\langle(\underline{g}_1,g_2,\underline{g}_3;\underline{\lambda}),(\bar{g}_1,g_2,\bar{g}_3;\bar{\lambda})\right\rangle$ are said to be equal, i.e., $\tilde{\tilde{\underline{H}}}=\tilde{\tilde{\underline{G}}}$ if and only if

$$\underline{h}_1=\underline{g}_1,h_2=g_2,\underline{h}_3=\underline{g}_3,\bar{h}_1=\bar{g}_1,\bar{h}_3=\bar{g}_3 \tag{5}$$

Definition 5: Let $\tilde{\tilde{\underline{H}}}=\left[\tilde{\underline{H}},\tilde{\tilde{H}}\right]=\left\langle(\underline{h}_1,h_2,\underline{h}_3;\underline{\lambda}),(\bar{h}_1,h_2,\bar{h}_3;\bar{\lambda})\right\rangle$ and $\tilde{\tilde{\underline{G}}}=\left[\tilde{\underline{G}},\tilde{\tilde{G}}\right]=\left\langle(\underline{g}_1,g_2,\underline{g}_3;\underline{\lambda}),(\bar{g}_1,g_2,\bar{g}_3;\bar{\lambda})\right\rangle$ be two interval–valued trapezoidal fuzzy numbers. Then $\tilde{\tilde{\underline{H}}} \precsim \tilde{\tilde{\underline{G}}}$ if $\underline{h}_1 \le \underline{g}_1,h_2 \le g_2,\underline{h}_3 \le \underline{g}_3,\bar{h}_1 \le \bar{g}_1,\bar{h}_3 \le \bar{g}_3$.

3. Interval-valued fuzzy constrainted shortest path problem

In this section, we incorporate inetrval-valued fuzzy costs and times within a standard constrained shortest path problem and suggest a multi-objective optimization strategy.

Let $G=(V,E)$ be a directed network with $V=\{1,2,\ldots,m\}$ and $E=\{(i,j):i,j\in V,i\ne j\}$ as the set of nodes and the set of arcs, respectively. Assume nodes s and t are the source node and the destination node of the network under consideration, respectively. In such network there exists a unique directed arc (i,j) from node i to node j. A path p_{ij} from node i to node j is defined as a sequence of arcs $p_{ij}=\{(i,i_1),(i_1,i_2),\ldots,(i_k,j)\}$ in which the initial node of each arc is same as the end node of preceding arc in the sequence.

The objective of CSP problem is to find the shortest path from initiation point to destination point so that the traversal time of the path does not exceed from a specified time. Standard CSP problems assume that the arc weights are defined by crisp values. However, inetrval-valued fuzzy parameters must be considered in many real-life situations dealing with imprecise evaluations of the arc weights.

Assume two non–negative inetrval-valued fuzzy weights $\tilde{\underline{\tilde{c}}}_{ij}$ and $\tilde{\underline{\tilde{t}}}_{ij}$ for each arc (i,j) are associated with the inetrval-valued fuzzy cost and the inetrval-valued fuzzy the traversal time, respectively. Moreover, let $\tilde{T}$ denotes the predefined amount of the traversal time. Given these notations, the interval-valued fuzzy constrained shortest path problem is formulated as follows:

$$
\begin{aligned}
&\min \ \ \tilde{\underline{\tilde{Z}}} = \sum_{i=1}^{m}\sum_{j=1}^{m} \tilde{\underline{\tilde{c}}}_{ij} x_{ij} \\
&s.t. \\
&\sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{m} x_{ki} = \begin{cases} 1, & i = s, \\ 0, & i \neq s, t, \\ -1, & i = t, \end{cases} \\
&\sum_{i=1}^{m}\sum_{j=1}^{m} \tilde{\underline{\tilde{t}}}_{ij} x_{ij} \preceq \tilde{\underline{\tilde{T}}}, \\
&x_{ij} \geq 0, \ i,j = 1,2,\ldots,m.
\end{aligned} \tag{6}
$$

where x_{ij} are binary variables associated with each arc (i,j). If arc (i,j) belongs to the set of arcs included in the constrained optimal path, then $x_{ij} = 1$; otherwise $x_{ij} = 0$.

Let P_{st} denotes the set of all paths from node s to node t. Define $\tilde{\underline{\tilde{C}}}(p) = \sum_{(i,j)\in p} \tilde{\underline{\tilde{c}}}_{ij}$ and $\tilde{\underline{\tilde{D}}}(p) = \sum_{(i,j)\in p} \tilde{\underline{\tilde{t}}}_{ij}$. Given $\tilde{\underline{\tilde{T}}} \succsim \tilde{\underline{\tilde{0}}}$, let $P_{st}(\tilde{\underline{\tilde{T}}})$ be the set of all paths p_{st} from node s to node t such that $\tilde{\underline{\tilde{D}}}(p_{st}) \preceq \tilde{\underline{\tilde{T}}}$, i.e. $P_{st}(\tilde{\underline{\tilde{T}}}) = \ p_{st} : \tilde{\underline{\tilde{D}}}(p_{st}) \preceq \tilde{\underline{\tilde{T}}}$. Each path belonging to the set $P_{st}(\tilde{\underline{\tilde{T}}})$ is called an interval-valued fuzzy feasible path. In this case, the interval-valued fuzzy CSP problem is to find a minimum cost interval-valued fuzzy feasible path.

Assume that interval-valued fuzzy parameters of interval-valued CSP problem (6) are all triangular. Therefore, $\tilde{\underline{\tilde{c}}}_{ij}$, $\tilde{\underline{\tilde{t}}}_{ij}$ and $\tilde{\underline{\tilde{T}}}$ are all represented by interval-valued triangular fuzzy numbers $\left\langle (\underline{c}_{ij,1}, c_{ij,2}, \underline{c}_{ij,3}; \underline{\lambda}), (\overline{c}_{ij,1}, c_{ij,2}, \overline{c}_{ij,3}; \overline{\lambda}) \right\rangle$, $\left\langle (\underline{t}_{ij,1}, t_{ij,2}, \underline{t}_{ij,3}; \underline{\lambda}), (\overline{t}_{ij,1}, t_{ij,2}, \overline{t}_{ij,3}; \overline{\lambda}) \right\rangle$ and $\left\langle (\underline{T}_1, T_2, \underline{T}_3; \underline{\lambda}), (\overline{T}_1, T_2, \overline{T}_1; \overline{\lambda}) \right\rangle$, respectively. Thus, the interval-valued fuzzy CSP problem (6) can be rewritten as follows:

$$\begin{aligned}
&\min\ \tilde{\underline{\tilde{Z}}} = \sum_{i=1}^{m}\sum_{j=1}^{m}\left\langle(\underline{c}_{ij,1},c_{ij,2},\underline{c}_{ij,3};\underline{\lambda}),(\overline{c}_{ij,1},c_{ij,2},\overline{c}_{ij,3};\overline{\lambda})\right\rangle x_{ij}\\
&\ s.t.\\
&\qquad \sum_{j=1}^{n}x_{ij}-\sum_{k=1}^{m}x_{ki}=\begin{cases}1, & i=s,\\ 0, & i\neq s,t,\\ -1, & i=t,\end{cases}\\
&\qquad\quad \sum_{i=1}^{m}\sum_{j=1}^{m}\left\langle(\underline{t}_{ij,1},t_{ij,2},\underline{t}_{ij,3};\underline{\lambda}),(\overline{t}_{ij,1},t_{ij,2},\overline{t}_{ij,3};\overline{\lambda})\right\rangle x_{ij}\preceq\left\langle(\underline{T}_1,T_2,\underline{T}_3;\underline{\lambda}),(\overline{T}_1,T_2,\overline{T}_1;\overline{\lambda})\right\rangle,\\
&\qquad\qquad\qquad x_{ij}\geq 0,\ i,j=1,2,\dots,m.
\end{aligned} \tag{7}$$

Definition 6: A feasible path p of the Model (7) is called an optimal inetrval-valued fuzzy pah if there is no feasible solution $\hat{p}$ such that $\tilde{\underline{\tilde{C}}}(\hat{p})=\sum_{(i,j)\in\hat{p}}\tilde{\underline{\tilde{c}}}_{ij}\preceq\tilde{\underline{\tilde{C}}}(p)=\sum_{(i,j)\in p}\tilde{\underline{\tilde{c}}}_{ij}$.

Regarding Definition 3, the interval-valued triangular fuzzy objective function of Model (7) is simplified as follows:

$$\tilde{\underline{\tilde{Z}}}=\left\langle\left(\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{c}_{ij,1}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij,2}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{c}_{ij,3}x_{ij};\underline{\lambda}\right),\left(\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{c}_{ij,1}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij,2}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{c}_{ij,3}x_{ij};\overline{\lambda}\right)\right\rangle \tag{8}$$

In a similar way, by Definitions 3 and 5, the interval-valued fuzzy triangular constraint of Model (7) is simplified as follows:

$$\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{t}_{ij,1}x_{ij}\leq\underline{T}_1,\ \sum_{i=1}^{m}\sum_{j=1}^{m}t_{ij,2}x_{ij}\leq T_2,\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{t}_{ij,3}x_{ij}\leq\underline{T}_3,\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{t}_{ij,1}x_{ij}\leq\overline{T}_1,\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{t}_{ij,3}x_{ij}\leq\overline{T}_3 \tag{9}$$

Hence, the interval-valued fuzzy CSP problem (6) can be reformulated as the following model:

$$\begin{aligned}
&\min\tilde{\underline{\tilde{Z}}}=\left\langle\left(\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{c}_{ij,1}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij,2}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{c}_{ij,3}x_{ij};\underline{\lambda}\right),\left(\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{c}_{ij,1}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij,2}x_{ij},\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{c}_{ij,3}x_{ij};\overline{\lambda}\right)\right\rangle\\
&\ s.t.\\
&\qquad \sum_{j=1}^{n}x_{ij}-\sum_{k=1}^{m}x_{ki}=\begin{cases}1, & i=s,\\ 0, & i\neq s,t,\\ -1, & i=t,\end{cases}\\
&\qquad\quad \sum_{i=1}^{m}\sum_{j=1}^{m}\underline{t}_{ij,1}x_{ij}\leq\underline{T}_1,\ \sum_{i=1}^{m}\sum_{j=1}^{m}t_{ij,2}x_{ij}\leq T_2,\\
&\qquad\quad \sum_{i=1}^{m}\sum_{j=1}^{m}\underline{t}_{ij,3}x_{ij}\leq\underline{T}_3,\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{t}_{ij,1}x_{ij}\leq\overline{T}_1,\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{t}_{ij,3}x_{ij}\leq\overline{T}_3\\
&\qquad\qquad\qquad x_{ij}\geq 0,\ i,j=1,2,\dots,m.
\end{aligned} \tag{10}$$

Clearly, the objective function of Model (10) is given by an interval-valued fuzzy triangular variable, but all fuzziness has been eliminated from its constraints. Therefore, Model (10) can be interpreted as a multi-objective linear programming problem (MOLP). To do this assume that

$$\begin{aligned}
&\underline{Z}_1\ x_{ij}\ =\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{c}_{ij,1}x_{ij}, Z_2\ x_{ij}\ =\sum_{i=1}^{m}\sum_{j=1}^{m}c_{ij,2}x_{ij}, \underline{Z}_3\ x_{ij}\ =\sum_{i=1}^{m}\sum_{j=1}^{m}\underline{c}_{ij,3}x_{ij},\\
&\overline{Z}_1\ x_{ij}\ =\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{c}_{ij,1}x_{ij}, \overline{Z}_3\ x_{ij}\ =\sum_{i=1}^{m}\sum_{j=1}^{m}\overline{c}_{ij,3}x_{ij}
\end{aligned} \tag{11}$$

Hence, Model (10) can be reformualted as follows:

$$\begin{aligned}
&\min \underline{\tilde{\tilde{Z}}}\ x_{ij}\ =\left\langle\ \underline{Z}_1\ x_{ij}\ , Z_2\ x_{ij}\ , \underline{Z}_3\ x_{ij}\ ; \underline{\lambda}\ ,\ \overline{Z}_1\ x_{ij}\ , Z_2\ x_{ij}\ , \overline{Z}_3\ x_{ij}\ ; \overline{\lambda}\ \right\rangle\\
&s.t.\\
&\ \text{Constraints of Model (10).}
\end{aligned} \tag{12}$$

Definition 7: A feasible solution x_{ij} of the Model (12) is called an efficient solution if there is no feasible solution $\hat{x}_{ij}$ such that $\underline{\tilde{\tilde{Z}}}\ x_{ij}\ \preceq \underline{\tilde{\tilde{Z}}}\ \hat{x}_{ij}$ and $\underline{\tilde{\tilde{Z}}}\ x_{ij}\ \not\preceq \underline{\tilde{\tilde{Z}}}\ \hat{x}_{ij}$, i.e., $\underline{Z}_1\ x_{ij}\ \leq \underline{Z}_1\ \hat{x}_{ij}$, $Z_2\ x_{ij}\ \leq Z_2\ \hat{x}_{ij}$, $\underline{Z}_3\ x_{ij}\ \leq \underline{Z}_3\ \hat{x}_{ij}$, $\overline{Z}_1\ x_{ij}\ \leq \overline{Z}_1\ \hat{x}_{ij}$, $\overline{Z}_3\ x_{ij}\ \leq \overline{Z}_3\ \hat{x}_{ij}$ and with strict inequality holding for at least one index.

Remark 1: Every efficient solution to the problem (12) is associated with an optimal interval-valued fuzzy path to the problem (7).

According to Remark 1, Definitions 6 and 7, we need to adopt an approach for solving the MOLP (12) that provides us with an efficient solution. Here, we solve this model using the weighted sum method which is formulated as a weighted sum of all of the objective functions in the original MOLP problem.

The weighted sum model for the Model (12) is given by:

$$\begin{aligned}
&\min\ \ w_1\underline{Z}_1(x_{ij})+w_2Z_2(x_{ij})+w_3\underline{Z}_3(x_{ij})+w_4\overline{Z}_1(x_{ij})+w_5\overline{Z}_3(x_{ij})\\
&s.t.\\
&\ w_1+w_2+w_3+w_4+w_5=1, w_t\geq 0, t=1,2,3,4,5,\\
&\ \text{Constraints of Model (10).}
\end{aligned} \tag{13}$$

The relationships between the optimal solution of the weighting problem (13) and the efficient solution concept of the MOLP model (12) can be characterized by the following theorems.

Theorem 1: If (x_{ij}^{*}) is an optimal solution of the weighted sum model (13) for some $w=\ \ w_1,w_2,w_3,w_4,w_5\ \ >0$, then (x_{ij}^{*}) is an efficient solution of the of the MOLP model (12).

Theorem 2: If (x_{ij}^{*}) is an unique optimal solution of the weighted sum model (13) for some $w=\ \ w_1,w_2,w_3,w_4,w_5\ \ \geq 0, w\neq 0$, then (x_{ij}^{*}) is an efficient solution of the of the MOLP model (12).

Theorem 3: If (x_{ij}^{*}) is an efficient solution of the of the MOLP model (12), then (x_{ij}^{*}) is an optimal solution of the weighted sum model (13) for some $w=\ \ w_1,w_2,w_3,w_4,w_5\ \ \geq 0, w\neq 0$.

One of the main advantages of the proposed approach relies on its computational simplicity, allowing for its implementation within complex network structures. In other words, a single linear minimization problem provides an intuitive foundation for constructing the weighted sum structure designed to address the MOLP problem defined by the interval-valued arc weights of the network.

In conclusion, the principal benefits of the aforementioned methodology can be enumerated as follows:

- The computational simplicity of the proposed approach permits its implementation within complex network structures.
- The solution to the linear minimization problem is sufficient to obtain an efficient solution to the resulting MOLP problem.
- The classical MOLP algorithm is employed for identifying the interval-valued fuzzy optimal path of the interval-valued fuzzy CSP problem. Consequently, the proposed approach offers a straightforward solution to this problem in real-world applications, when compared with existing methods.
- The proposed approach guarantees that the resulting interval-valued fuzzy optimal cost will retain the form of a non-negative interval-valued fuzzy number.

4. Application to wireless sensor networks

A wireless sensor network (WSN) is constituted of a multitude of sensor nodes that utilise irreplaceable batteries. The nodes are typically randomly distributed throughout a given geographical area. In general, the function of a wireless sensor network is to collect data from its surrounding environment and transfer it to a designated node, known as the “sink” node [23].

Two significant challenges in WSNs are the efficient consumption of energy and the provision of quality of service (QoS). One of the primary challenges in WSN is the efficient consumption of energy, which prolongs the lifetime of the network [23]. When determining the shortest path for data transition in WSN in terms of energy consumption, it is essential to consider the quality of service (QoS) of the path. The aim of this pronlem is to identify an efficient path in WSN that minimises energy consumption while maintaining a minimum level of QoS. Thus, an inetrval-valued fuzzy constrained shortest path problem is formulated as energy consumption and the QoS cannot be measured preciously because of environmental conditions.

Figure 1 provides an example of a WSN with six sensor nodes. The neighbor nodes are connected to each other through ten arcs. Table 1 gives the amount of QoS and needed energy for data transferring between neighbor nodes in terms of interval-valued fuzzy numbers. The maximum allowable QoS represented by interval-valued triangular fuzzy number $\langle (40,50,60;0.5),(35,50,65;1) \rangle$.

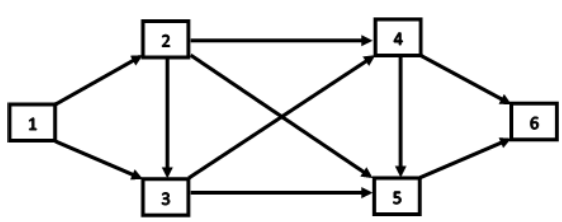

Figure 1. An example of WSN [10]

Table 1. Arc information in terms of interval-valued fuzzy numbers

Arc	Interval-valued fuzzy energy	Interval-valued fuzzy QoS
(1,2)	$\langle(10,12,13;0.5),(9,12,16;1)\rangle$	$\langle(21,25,27;0.5),(19,25,31;1)\rangle$
(1,3)	$\langle(8,10,11;0.5),(7,10,14;1)\rangle$	$\langle(25,31,34;0.5),(22,31,43;1)\rangle$
(2,3)	$\langle(10,12,13;0.5),(9,12,16;1)\rangle$	$\langle(19,23,25;0.5),(17,23,31;1)\rangle$
(2,4)	$\langle(2,3,6;0.5),(1,3,8;1)\rangle$	$\langle(11,16,21;0.5),(6,16,26;1)\rangle$
(2,5)	$\langle(3,5,6;0.5),(2,5,9;1)\rangle$	$\langle(7,11,13;0.5),(5,11,15;1)\rangle$
(3,4)	$\langle(4,8,10;0.5),(3,8,15;1)\rangle$	$\langle(11,19,23;0.5),(9,19,27;1)\rangle$
(3,5)	$\langle(5,7,8;0.5),(4,7,11;1)\rangle$	$\langle(11,15,17;0.5),(9,15,19;1)\rangle$
(4,5)	$\langle(4,6,8;0.5),(1,6,11;1)\rangle$	$\langle(9,13,17;0.5),(3,13,21;1)\rangle$
(4.6)	$\langle(4,7,11;0.5),(3,7,15;1)\rangle$	$\langle(9,15,23;0.5),(7,15,25;1)\rangle$
(5,6)	$\langle(3,5,7;0.5),(2,5,8;1)\rangle$	$\langle(7,11,15;0.5),(5,11,17;1)\rangle$

The IVFCSP problem based on the interval-valued fuzzy energy values as arc cost weights and the interval-valued fuzzy QoS values as arc time weights given in Table 1 is formulated as follows:

$$
\begin{aligned}
\min\ \underset{\sim}{\tilde{Z}} = {} & \langle(10,12,13;0.5),(9,12,15;1)\rangle x_{12} + \langle(8,10,11;0.5),(7,10,14;1)\rangle x_{13} + \langle(10,12,13;0.5),(9,12,16;1)\rangle x_{23} \\
& + \langle(2,3,6;0.5),(1,3,8;1)\rangle x_{24} + \langle(3,5,6;0.5),(2,5,9;1)\rangle x_{25} + \langle(4,8,10;0.5),(3,8,15;1)\rangle x_{34} \\
& + \langle(5,7,8;0.5),(4,7,11;1)\rangle x_{35} + \langle(4,6,8;0.5),(1,6,11;1)\rangle x_{45} + \langle(4,7,11;0.5),(3,7,15;1)\rangle x_{46} \\
& + \langle(3,5,7;0.5),(2,5,8;1)\rangle x_{56} \\
s.t. \quad & \\
& x_{12} + x_{13} = 1, \\
& x_{23} + x_{24} + x_{25} - x_{12} = 0, \\
& x_{34} + x_{35} - x_{13} - x_{23} = 0, \\
& x_{45} + x_{46} - x_{24} - x_{34} = 0, \\
& x_{56} - x_{25} - x_{35} - x_{45} = 0, \\
& -x_{46} - x_{56} = -1, \\
& \langle(21,25,27;0.5),(19,25,31;1)\rangle x_{12} + \langle(25,31,34;0.5),(22,31,43;1)\rangle x_{13} + \langle(19,23,25;0.5),(17,23,31;1)\rangle x_{23} \\
& + \langle(11,16,21;0.5),(6,16,26;1)\rangle x_{24} + \langle(7,11,13;0.5),(5,11,15;1)\rangle x_{25} + \langle(11,19,23;0.5),(9,19,27;1)\rangle x_{34} \\
& + \langle(11,15,17;0.5),(9,15,19;1)\rangle x_{35} + \langle(9,13,17;0.5),(3,13,21;1)\rangle x_{45} + \langle(9,15,23;0.5),(7,15,25;1)\rangle x_{46} \\
& + \langle(7,11,15;0.5),(5,11,17;1)\rangle x_{56} \precsim \langle(40,50,60;0.5),(35,50,65;1)\rangle \\
& x_{12}, x_{13}, x_{23}, x_{23}, x_{25}, x_{34}, x_{35}, x_{45}, x_{46}, x_{56} \geq 0.
\end{aligned} \tag{14}
$$

Regarding Model (10), the interval-valued fuzzy CSP problem (14) can be reformulated as the following model:

$$
\begin{aligned}
&\min\ \tilde{\tilde{\underline{Z}}} = \left\langle \left(\underline{Z}_1, Z_2, \underline{Z}_3; 0.5\right), \left(\bar{Z}_1, Z_2, \bar{Z}_3; 1\right) \right\rangle \\
&\quad s.t. \\
&\qquad x_{12} + x_{13} = 1, \\
&\qquad x_{23} + x_{24} + x_{25} - x_{12} = 0, \\
&\qquad x_{34} + x_{35} - x_{13} - x_{23} = 0, \\
&\qquad x_{45} + x_{46} - x_{24} - x_{34} = 0, \\
&\qquad x_{56} - x_{25} - x_{35} - x_{45} = 0, \\
&\qquad -x_{46} - x_{56} = -1, \\
&21x_{12} + 25x_{13} + 19x_{23} + 11x_{24} + 7x_{25} + 11x_{34} + 11x_{35} + 9x_{45} + 9x_{46} + 7x_{56} \le 40, \\
&25x_{12} + 31x_{13} + 23x_{23} + 16x_{24} + 11x_{25} + 19x_{34} + 15x_{35} + 13x_{45} + 15x_{46} + 11x_{56} \le 50, \\
&27x_{12} + 34x_{13} + 25x_{23} + 21x_{24} + 13x_{25} + 23x_{34} + 17x_{35} + 17x_{45} + 23x_{46} + 15x_{56} \le 60, \\
&19x_{12} + 22x_{13} + 17x_{23} + 6x_{24} + 5x_{25} + 9x_{34} + 9x_{35} + 3x_{45} + 7x_{46} + 5x_{56} \le 35, \\
&31x_{12} + 43x_{13} + 31x_{23} + 26x_{24} + 15x_{25} + 27x_{34} + 19x_{35} + 21x_{45} + 25x_{46} + 17x_{56} \le 65, \\
&x_{12}, x_{13}, x_{23}, x_{23}, x_{25}, x_{34}, x_{35}, x_{45}, x_{46}, x_{56} \in \{0,1\}.
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
\underline{Z}_1 &= 10x_{12} + 8x_{13} + 10x_{23} + 2x_{24} + 3x_{25} + 4x_{34} + 5x_{35} + 4x_{45} + 4x_{46} + 3x_{56} \\
Z_2 &= 12x_{12} + 10x_{13} + 12x_{23} + 3x_{24} + 5x_{25} + 8x_{34} + 7x_{35} + 6x_{45} + 7x_{46} + 5x_{56} \\
\underline{Z}_3 &= 13x_{12} + 11x_{13} + 13x_{23} + 6x_{24} + 6x_{25} + 10x_{34} + 8x_{35} + 8x_{45} + 11x_{46} + 7x_{56} \\
\bar{Z}_1 &= 9x_{12} + 7x_{13} + 9x_{23} + x_{24} + 2x_{25} + 3x_{34} + 4x_{35} + x_{45} + 3x_{46} + 2x_{56} \\
Z_2 &= 12x_{12} + 10x_{13} + 12x_{23} + 3x_{24} + 5x_{25} + 8x_{34} + 7x_{35} + 6x_{45} + 7x_{46} + 5x_{56} \\
\bar{Z}_3 &= 15x_{12} + 14x_{13} + 16x_{23} + 8x_{24} + 7x_{25} + 12x_{34} + 9x_{35} + 10x_{45} + 12x_{46} + 8x_{56}
\end{aligned} \tag{16}
$$

If we choose $w_1 = w_2 = w_3 = w_4 = w_5 = \frac{1}{5}$, regarding the Model (13), the weighted sum model for Model (15) is given as the following model:

$$
\begin{aligned}
&\min\ 12x_{12} + 10x_{13} + 12x_{23} + 4x_{24} + 5x_{25} + 8x_{34} + 7x_{35} + 6x_{45} + 8x_{46} + 25x_{56} \\
&\quad s.t. \\
&\qquad \text{Constraints of Model (15).}
\end{aligned} \tag{17}
$$

The optimal solution of the problem (17) is given by:

$$
x_{12}^* = 1, x_{13}^* = x_{23}^* = x_{24}^* = 0,\ x_{25}^* = 1,\ x_{34}^* = x_{35}^* = x_{45}^* = x_{46}^* = 0, x_{56}^* = 1 \tag{18}
$$

The interval-valued fuzzy shortest path corresponding to this optimal solution is $p_2^*: 1 \to 2 \to 5 \to 6$.

$$
x_{12}^* = 1, x_{13}^* = x_{23}^* = x_{24}^* = 0,\ x_{25}^* = 1,\ x_{34}^* = x_{35}^* = x_{45}^* = x_{46}^* = 0, x_{56}^* = 1
$$

Moreover, the optimal interval-valued fuzzy path weight of this optimal path is given by:

$$\begin{aligned}\tilde{\underline{\tilde{Z}}}^* &= \left\langle \left(\underline{Z}_1^*, Z_2^*, \underline{Z}_3^*; \underline{\lambda}\right), \left(\bar{Z}_1^*, Z_2^*, \bar{Z}_3^*; \bar{\lambda}\right) \right\rangle = \\ &= \tilde{\underline{\tilde{c}}}_{12} + \tilde{\underline{\tilde{c}}}_{25} + \tilde{\underline{\tilde{c}}}_{56} \\ &= \langle (10,12,13;0.5),(9,12,16;1) \rangle + \langle (3,5,6;0.5),(2,5,9;1) \rangle \\ &+ \langle (3,5,7;0.5),(2,5,8;1) \rangle \\ &= \langle (16,22,26;0.5),(13,22,33;1) \rangle \end{aligned} \tag{19}$$

Finally, the interval-valued triangular fuzzy number of QoS of this optimal path is given by:

$$\begin{aligned}\tilde{\underline{\tilde{T}}} &= \left\langle \left(\underline{T}_1, T_2, \underline{T}_3; \underline{\lambda}\right), \left(\bar{T}_1, T_2, \bar{T}_3; \bar{\lambda}\right) \right\rangle = \tilde{\underline{\tilde{T}}}_{12} + \tilde{\underline{\tilde{T}}}_{25} + \tilde{\underline{\tilde{T}}}_{56} \\ &= \langle (21,25,27;0.5),(19,25,31;1) \rangle + \langle (7,11,13;0.5),(5,11,15;1) \rangle \\ &+ \langle (7,11,15;0.5),(5,11,17;1) \rangle \\ &= \langle (35,47,55;0.5),(29,47,63;1) \rangle \end{aligned} \tag{20}$$

which is less or equal that the maximum allowable QoS represented by interval-valued triangular fuzzy number $\langle (40,50,60;0.5),(35,50,65;1) \rangle$.

Figures 1 and 2 show the membership functions of the optimal interval-valued fuzzy path cost weight and the interval-valued triangular fuzzy number of QoS of this optimal path given in (19) and (20), respectively.

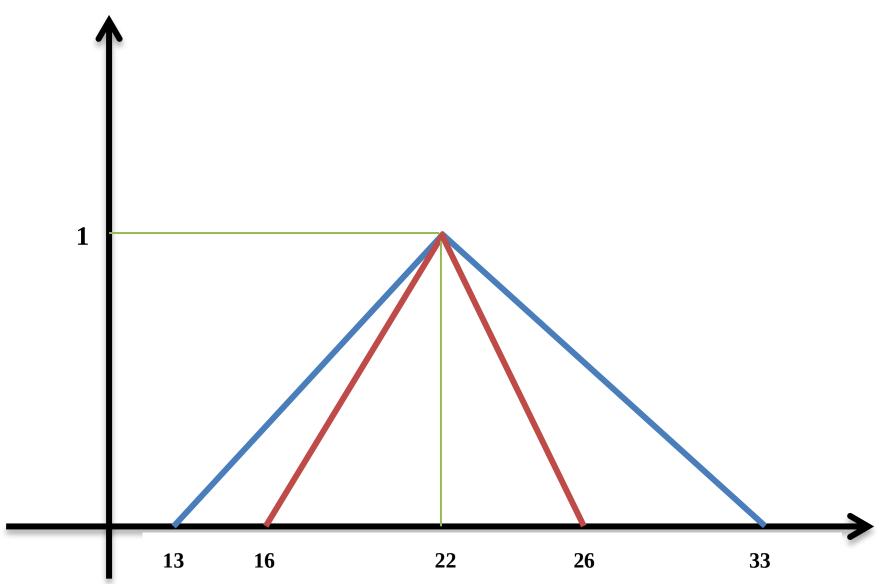

Figure 1. Membeship function of optimal interval-valued fuzzy path cost

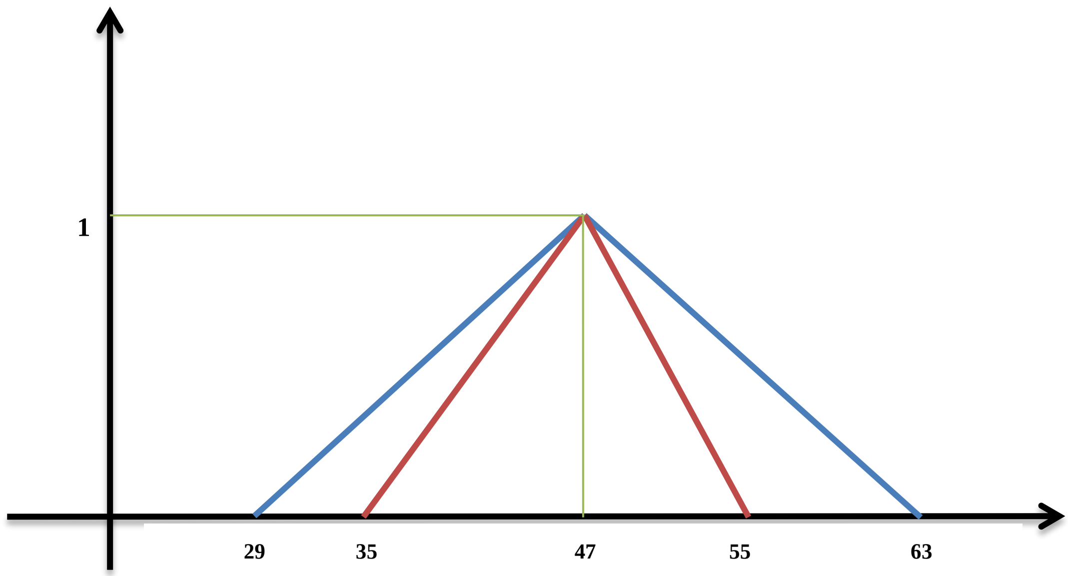

Figure 2. Membeship function of interval-valued fuzzy traversal time

5. Conclusion

Traditional SP problems assume accurate arc weights, which are not always available in real-world scenarios. In this study, we have analyzed a constrained shortest path problem with interval-valued triangular fuzzy arc weights and interval-valued triangular fuzzy arc times and proposed a novel solution approach based on the multi-objective optimization strategy to obtain the efficient solution of the resulting problem. In particular, we have converted the IVFCSP problem into one crisp linear programming problems that can be solved using standard simplex algorithms. The interval-valued fuzzy path derived form the proposed approach satisfied both the set of flow conservation constraints and the set of the traversal time conservation constraints of the network. The proposed multi-objective optimization strategy provides an intuitive framework on which to build the weighted sum structure designed to solve the MOLP problem defined by the fuzzy interval-valued fuzzy arc weights. The primary limitation of the proposed approach is that it increases the number of constraints in contracts in comparison to the primary interval-valued fuzzy CSP problem. In the future, we will attempt to develop new methods to overcome this defect.

There are numerous other areas that require further investigation. Some of these are discussed below.

1) The maximal flow problem represents a fundamental challenge in the field of combinatorial optimization in weighted directed graphs. The objective is to send as much flow as possible between a source node and a sink node in a weighted graph, without exceeding the capacity of any arcs. In real-life situations, there is always uncertainty regarding the capacities and flows. A promising avenue for future research is the generalization of the proposed method for determining the interval-valued fuzzy optimal flow of maximum flow problems with interval-valued fuzzy capacities and flows as interval-valued fuzzy numbers.
2) The proposed approach for solving CSP problems in interval-valued fuzzy environments can be extended to encompass interval-valued intuitionistic fuzzy environments.
3) Accordingly, the formulation of the proposed methodology for deriving the efficient interval-valued fuzzy path of the multi-objective CSP problems when the arc cost and time coefficients are non-negative interval-valued triangular fuzzy numbers is a subject for future investigation.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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