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Optimizing Hub Location for Military Equipment: A Robust Mathematical Model for Uncertainty and Meta-Heuristic Approaches

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ABSTRACT

This research presents a robust mathematical model for optimizing hub location for military equipment, addressing the inherent uncertainties associated with logistical operations in defense contexts. The model aims to minimize transportation costs and enhance the efficiency of equipment distribution while considering various uncertainties, such as demand fluctuations, transportation delays, and operational constraints. To solve this complex optimization problem, we employ advanced meta-heuristic algorithms, including Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), which are designed to navigate the solution space effectively and provide high-quality solutions within reasonable computational time. The performance of the proposed model is evaluated through a series of simulations, demonstrating its effectiveness in identifying optimal hub locations that ensure timely and cost-effective delivery of military equipment. The first objective is to minimize costs, the second objective is to maximize the fulfillment of demands, and the third objective is to minimize congestion on the routes. Taking into account the parameters in the state of uncertainty, the mathematical model is modeled in a robust state and a robust counterpart model of the problem is proposed. In order to solve the problem on a small scale, the exact weighted sum method (WSM) is used in GAMS software. The findings highlight the model's potential to improve logistical decision-making in military operations, ultimately contributing to enhanced operational readiness and resource allocation. This study serves as a foundational framework for future research in military logistics optimization under uncertainty.

1. Introduction

Hubs serve as specialized facilities that function as exchange, transfer, and classification points within various distribution systems. Rather than offering direct services between every origin and destination, hub

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facilities consolidate flows to capitalize on the resulting economic efficiencies. This research focuses on determining hub locations for war equipment under uncertain conditions. Certain locations are identified as hub points, functioning as equipment transfer centers to minimize transportation costs, expedite transfers, and reduce the number of transport vehicles required [5].

War equipment, while often ineffective during peacetime, plays a crucial role in deterring and defending against enemy attacks during wartime [1]. The strategic positioning of this equipment is vital; improper placement can hinder troop readiness during an attack or crisis, potentially leading to significant damage that may be difficult or even impossible to recover from. Accurate and strategic location of war equipment can thwart enemy attacks and ensure maximum responsiveness. Although there is a substantial body of research on hub location across various sectors, the lack of focus on the specific issue of war equipment placement presents a significant gap and an important avenue for future research [10].

The existing literature on hub location problems primarily focuses on deterministic models, often overlooking the complexities introduced by uncertainty in demand, supply, and operational conditions, particularly in the context of war equipment logistics. While some studies have explored stochastic approaches, there remains a significant gap in the application of advanced meta-heuristic algorithms tailored to address these uncertainties effectively. Additionally, most research has not adequately considered the unique constraints and requirements specific to military logistics, such as security concerns and rapid response times. Furthermore, the integration of multi-objective optimization within the hub location framework, particularly under uncertain conditions, is still underexplored. There is also a lack of empirical studies validating the proposed models in real-world military scenarios, which limits the practical applicability of existing theoretical frameworks. This research aims to fill these gaps by developing a comprehensive mathematical model that incorporates uncertainty and employs meta-heuristic algorithms, providing a robust solution for optimizing hub locations for war equipment logistics. Additionally, due to the parallels between wartime and crisis situations, it is essential to incorporate uncertainty into this area of study. In essence, uncertainty should be a fundamental aspect of addressing location and routing challenges, particularly in contexts of conflict and emergencies.

Acknowledging the existing gaps and limitations in hub location for war equipment, this research aims to address these issues by proposing a mathematical programming model specifically for hub location, which will be solved using meta-heuristic algorithms. A key focus of this study is to develop a resilient model that accounts for uncertainty, particularly recognizing that routes to hubs may face disruptions. Consequently, the costs associated with these routes will be treated separately, and the demand on these routes will be represented through multiple arcs, reflecting the critical nature of the problem. While many similar studies have typically considered only one arc per hub, incorporating multiple arcs for each hub enhances the model's alignment with real-world complexities and uncertainties.

Another innovative aspect of this research is the consideration of network congestion. Given the likelihood of congestion during wartime, prioritizing routes with lower congestion levels is crucial. Therefore, the chosen algorithm will factor in the current congestion within the network when selecting hubs. The gray wolf meta-heuristic algorithm will be employed due to its novel approach, which has seen limited application in hub location research. This study aims to assess the effectiveness of this algorithm in addressing the multi-objective nature of the proposed model. In general, the objectives of the research are as follows:

- To create a robust mathematical model for hub location that incorporates uncertainty factors affecting the logistics of war equipment, ensuring that the model addresses various operational constraints and objectives.
- To apply and evaluate the effectiveness of advanced meta-heuristic algorithms in solving the proposed hub location model, aiming to optimize transportation costs, minimize transfer times, and reduce the number of vehicles required for logistics.
- To conduct empirical analyses and case studies that validate the proposed model and algorithms in real-world military logistics scenarios, assessing their practicality and effectiveness in enhancing the strategic positioning of war equipment

Reminder of paper organized follow as: in section 2 presented literature review. In section 3 presented research methodology. For this purpose mathematical modeling processes and research solution approach defined. In section 4, presented findings and in section 5 presented conclusion and further research suggestion.

2. Literature review

The idea of hub networks was proposed by Goldman [7]. Then, O'Kelly [14] proposed the first study of hub network in the field of aerial networks. Although O'Kelly provided the first known mathematical formulation of a hub location problem by studying airline passenger networks. His formulation refers to the single-hub allocation problem [3]. Nevertheless, the first linear integer mathematical model was presented by Campbell [4]. The first generation of hub location research can be seen as the results of the work of Campbell and O'Kelly [13], which made great progress in the understanding of intermediary systems and the development of basic models with a major focus on minimizing the flow cost and fixed cost. Yılmaz et al [18] addressed an optimization problem focused on the location and routing of a homogeneous fleet of unmanned aerial vehicles, incorporating synchronization constraints and proposing a mixed integer linear programming model solved with an ant colony optimization heuristic. Their results showed that the heuristic outperformed a commercial solver, particularly in longer time frames. Shavarani et al. [15] developed a bi-objective mathematical model to optimize the number and location of facilities, aiming to minimize total travel distance, costs, and lost demand. They applied two genetic algorithms to solve this NP-hard problem and analyzed the performance of the algorithms. Li et al. [10] proposed a robust optimization formulation to address uncertainties in flow and hub setup costs, using nonlinear integer programming models for both single and multiple allocation cases. Their findings indicated that the robust strategy resulted in more hubs with a slight cost increase compared to deterministic scenarios. Mokhtarzadeh et al. [11] introduced a p-mobile hub location-allocation problem, allowing hub facilities to be relocated. They developed a multi-objective mixed-integer non-linear programming model and utilized several meta-heuristic algorithms, finding that the KNSGA-II algorithm was the most effective. Soleimani et al. [16] presented a fuzzy multi-objective mathematical model considering cost uncertainties and included backup hubs for disaster response. They proposed a robust possibility method and employed two meta-heuristic algorithms, demonstrating that NSGA-II outperformed MOPSO. Zahiri and Suresh [19] focused on a material transportation network design under uncertainty, aiming to minimize total risk, including response times to hazmat incidents. They developed an interactive approach and two heuristic algorithms, showcasing their methodologies through numerical experiments. Demir et al. [6] evaluated meta-heuristic approaches for multi-allocation hub location with multi-objective capacity, comparing NSGA-II and AMOSA. Their analysis revealed that NSGA-II was more effective for larger networks, while AMOSA excelled in smaller instances. Li et al. [9] designed a multimodal hub-and-spoke transportation network for emergency relief during the COVID-19 pandemic, employing a bi-objective model to minimize transportation time and costs. They customized the grey wolf optimizer, achieving high accuracy and providing valuable insights for transportation management during emergencies. Abolghasemian et al. [1] presented a mathematical modeling is presented to determine efficient locations for deployment of support forces using data envelopment analysis in war conditions. The proposed model has the possibility to first change the manageable inputs in order to improve the outputs according to the principle of managerial accessibility and also if it is not possible to reduce the unmanageable inputs according to the principle of natural accessibility, it keeps them at least at the existing level. Therefore, the most important innovation and contribution of the presented model is the modeling of the positioning of the support forces to support the ground forces in future battles, which is created by using data envelopment analysis and the simultaneous use of natural and managerial accessibility principles. Abolghasemian et al. [2] presented a mathematical modeling approach is to determine efficient locations for deploying support forces using Data Envelopment Analysis (DEA). Additionally, a mixed-integer linear programming model is proposed for routing prioritized support items. The proposed model allows for the adjustment of manageable inputs to improve outputs according to the principle of managerial accessibility, while also maintaining the current levels of unmanageable inputs if they cannot be reduced based on the principle of natural accessibility. Subsequently, routing for the distribution of these prioritized support items is provided using a mixed-integer linear programming model. The proposed model has been used to evaluate 25 potential locations prepared to provide ground support services to assist friendly forces in contested areas, with the aim of ending the conflict in favor of friendly forces. Sixteen viable support locations have been identified. Finally, routing for the distribution of support items to these 16 locations has been presented.

In this research, using a mathematical model, we intend to minimize the transportation cost and determine the best location for war equipment storage in war zones according to the demand of the regions. The demand of the regions is considered dynamically and based on different situations. After the mathematical model is prepared, first we code the model in small dimensions in GAMS software and run the model to make sure the validity of the model. Then, meta-heuristic algorithms are used in MATLAB software to analyze information and data in large dimensions.

3. Problem statement

Locating military equipment requires careful consideration of specific conditions to avoid significant issues in assigning warehouse locations. Meeting demand is essential, and a primary goal is to satisfy maximum demand across various areas. Additionally, selecting routes with minimal congestion is crucial for swift transportation, ensuring vehicles carrying military supplies remain discreet and can deliver quickly. Both demand and route congestion are uncertain factors that can fluctuate, categorized into pessimistic, optimistic, and possible scenarios. This research aims to create a location model for military equipment under uncertainty, focusing on three objectives: minimizing costs, maximizing demand fulfillment, and reducing route congestion.

The assumptions of the problem are as follows:

- It is a single period model.
- The location of the hubs is not known and must be determined.
- The inventory at the beginning of the period is zero.
- There is a capacity limit.
- The model is scenario oriented.
- The number of existing vehicles is considered as congestion.
- The parameters of demand and product transfer cost are uncertain.
- The maintenance cost is considered variable in different periods, but it is the same for all hubs.

Indices

i	Node
j	Hub
k	Location
r	Route
t	Period
l	Demand point
s	Scenario

Parameters

TC_{rs}	Cost of transporting a unit of product on route r under scenario s
DEM_{rst}	Product demand on route r under scenario s in period t
FC_{jk}	Cost of constructing hub j in location k
DIS_{irl}	Distance from node i to demand point l on route r
COG_{rt}	Number of vehicles on route r in period t
CAP_j	Node capacity i
HC_t	Product maintenance cost in time period t
M	A big number

Decision variables

X_{ij}	1 if node i is selected as hub j and zero otherwise
Y_{jk}	1 if hub j is built in location k and zero otherwise
V_{rt}	1 if route r is selected in period t and zero otherwise
Z_{rsjt}	Transfer flow on route r under scenario s to hub j in time period t
U_{jt}	Product inventory in hub j in time period t

3.1. Mathematical model

According to the assumptions of the research, the indices, parameters, and defined variables of the mathematical model of the research are presented as follows:

Objective functions

$$\min z1 = \sum_i \sum_r \sum_l \sum_s \sum_t DIS_{irl} Z_{jrst}\, TC_{rs} + \sum_j \sum_k FC_{jk}\, Y_{jk} + \sum_j \sum_t HC_t\, U_{jt} \quad (1)$$

$$\max z2 = \sum_r \sum_s \sum_t DEM_{rst}\, V_{rt} \quad (2)$$

$$\min z3 = \sum_r \sum_t COG_{rt}\, V_{rt} \quad (3)$$

According above mentioned, equation (1) seeks to minimize the costs, including transfer cost, fixed construction cost, and maintenance cost based on inventory. Equation (2) seeks to maximize the fulfillment of the demand according to the appropriate chosen route. Equation (3) seeks to maximize congestion on the route by choosing the appropriate route.

Constraints

$$\sum_i X_{ij} = 1 \quad (4)$$

$$\sum_k Y_{jk} = 1 \quad (5)$$

$$\sum_k Y_{jk} \le \sum_i X_{ij} \quad (6)$$

$$\sum_r \sum_t V_{rt} = 1 \quad (7)$$

$$Z_{rsj} \le M\, V_{rt} \quad (8)$$

$$U_{jt} = U_{jt-1} + \sum_r Z_{rsjt} \quad (9)$$

$$U_{jt} \le CAP_j \quad (10)$$

$$U_{j1} = 0 \quad (11)$$

$$Z_{rsj} \le DEM_{rst} \quad (12)$$

$$U_{jt} \le M\, Y_{jk} \quad (13)$$

$$X_{ij} \in \{0.1\} \quad (14)$$

$$Y_{jk} \in \{0.1\} \quad (15)$$

$$V_{rt} \in \{0.1\} \quad (16)$$

$$Z_{rsj} \ge 0 \quad (17)$$

$$U_{jt} \ge 0 \quad (18)$$

The Equation (4) shows that each node is assigned to one hub only. The Equation (5) shows that each hub is built in one place. The Equation (6) shows that if a node is assigned to a hub, it is possible to build it. The Equation (7) shows that only one route is selected in all periods. The Equation (8) states that if a route is selected, there is a flow of product transfer from that route. The Equation (9) shows the product inventory in each period. The Equation (10) indicates the limitation of hub capacity. The Equation (11) indicates the assumption that the first period includes zero inventory. The Equation (12) shows the fulfillment of demand by transfer flow. The Equation (13) states that there is an inventory for a hub if that hub has been built. The Equations (14-16) indicate the constraint of the binary variables of the problem. The Equation (17-18) indicate the constraint of integer variables of the problem.

3.2. Providing a robust uncertain model

This section discusses the uncertain problem and the uncertainty approach, contrasting it with the previous deterministic model. Mulvey et al. [12] established a framework for robust optimization, defining a robust solution as one that remains close to optimal across all scenarios, and a robust model as one that is nearly plausible in all situations. A general model of robust optimization has been created for problems with scenario-type data. Mulvey highlights that operations research often deals with fluctuating and uncertain data, making management through sensitivity analysis or probabilistic planning challenging. In optimization models, there are two main components: the fixed structural part and the variable control part influenced by uncertainty.

$$\min C^T X + d^T y \tag{19}$$

$$\text{subject to: } AX = b \tag{20}$$

$$BX + Cy = e \tag{21}$$

$$x, y \geq 0 \tag{22}$$

$$x \in R^{n1}, y \in R^{n2} \tag{23}$$

In the above model:

X: represents the decision variables of the certain parameters,

Y: represents the decision variables of the control part.

Constraints are divided into two parts:

- Structural constraints whose coefficients are fixed and so-called certain.
- Control constraints whose coefficients include the uncertain state.

3.3. Solution approach

In this section, the used solution approach are provided. Since the problem model is presented in an uncertain and robust mode, in order to solve it on a small scale, the exact method is used in GAMS commercial optimization software. For this purpose, multi objective mathematical model convert to single objective using sum weighed method (SWM). Then, meta-heuristic approaches of PSO and NSGA-II are used to solve the model with medium and large scale numerical problems. Also, since the problem model is multi-objective, the epsilon constraint approach is used to solve it, so that it can be solved in GAMS software. Additionally, to solve it, the mentioned meta-heuristic approaches are used in the multi-objective optimization mode, in other words, the multi-objective PSO and the multi-objective NSGA-II are used, which will be explained later.

3.3.1. Particle Swarm Optimization (PSO)

The Particle Swarm Optimization (PSO) algorithm is a computational method inspired by the social behavior of birds and fish. It is used for solving optimization problems by having a group of candidate solutions, called particles, move around in the search space. Each particle adjusts its position based on its own experience and the experience of neighboring particles, aiming to find the optimal solution. Key concept of PSO is as follow:

Particles: Each particle represents a potential solution in the search space. Particles have a velocity that determines how they move through the search space.

Personal Best (pBest): Each particle keeps track of its best position found so far.

Global Best (gBest): The best position found by any particle in the swarm.

PSO Algorithm Steps have following steps:

- Initialize a swarm of particles with random positions and velocities.
- Evaluate the fitness of each particle.
- Update the personal best position (pBest) for each particle.
- Identify the global best position (gBest) among all particles.
- Update the velocity and position of each particle based on its pBest and gBest.
- Repeat steps 2-5 until a stopping criterion is met (e.g., maximum iterations or convergence).

Therefore, PSO main algorithm is follow as:

```
Initialize swarm of particles with random positions and velocities
Set pBest for each particle to its initial position
Set gBest to the best position among all pBest
While stopping criterion not met:
    For each particle in the swarm:
        Evaluate fitness of the particle
        If current fitness is better than pBest:
            Update pBest with current position
        If current fitness is better than gBest:
            Update gBest with current position
    For each particle in the swarm:
        Update velocity:
            velocity = w * velocity + c1 * random() * (pBest - position) + c2 * random() * (gBest - position
        Update position:
            position = position + velocity
```

Main advantages of PSO is as follow:

Simplicity: PSO is easy to understand and implement.

Few Parameters: It requires fewer parameters to tune compared to other optimization algorithms.

Flexible: PSO can be applied to a wide range of optimization problems, including continuous and discrete optimization.

Parallelism: The algorithm can be easily parallelized, allowing for faster computations.

Convergence: PSO is known for its ability to converge quickly to a good solution, especially in complex search spaces.

Overall, PSO is a powerful optimization technique that leverages the collective behavior of particles to efficiently explore the search space and find optimal solutions.

3.3.2. Metaheuristic Non-Dominated Sorting Genetic Algorithm II (NSGA-II)

Multi-objective evolutionary algorithms using non-dominated sorting and sharing have been criticized for three main issues: computational complexity, a non-elitist approach, and the need for a sharing parameter. This section presents NSGA-II, a multi-objective evolutionary algorithm that addresses these challenges. Genetic algorithms, which model animal populations, relate animal traits to objective function values, improving these functions through generational enhancements and interbreeding. The non-dominated sorting genetic algorithm is a multi-objective variant of this approach, with its general procedure described thereafter.

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is a popular choice for optimization, particularly in multi-objective scenarios, for several key reasons:

- NSGA-II uses a fast and efficient non-dominated sorting mechanism, which allows it to categorize solutions based on Pareto dominance quickly. This is crucial for identifying the best trade-offs among competing objectives.
- The algorithm incorporates a crowding distance mechanism that helps maintain diversity among the solutions in the population. This prevents premature convergence to a single solution and ensures a more comprehensive exploration of the solution space.
- NSGA-II is known for its ability to converge towards the true Pareto front effectively. This means it can find high-quality solutions that represent the best trade-offs between multiple objectives.
- The algorithm performs well across various problem sizes and dimensions, making it suitable for large-scale optimization tasks. Its scalability is essential for real-world applications where problem complexity can vary significantly.
- NSGA-II can be adapted to different types of optimization problems, including both constrained and unconstrained scenarios. This versatility allows it to be applied in a wide range of fields.
- The structure of NSGA-II allows for parallel execution, which can significantly enhance computational efficiency, especially for large populations or complex problems.
- The algorithm has been successfully applied in various domains, including engineering design, logistics, environmental management, and finance, demonstrating its robustness and adaptability.
- There is a wealth of literature and case studies that validate the effectiveness of NSGA-II, providing a solid foundation for its application in optimization tasks.
- NSGA-II is relatively straightforward to implement, making it accessible for researchers and practitioners who may not have extensive backgrounds in optimization algorithms.
- The algorithm consistently demonstrates superior performance compared to other multi-objective optimization techniques, establishing it as a leading choice in the field of optimization.

Overall, these characteristics make NSGA-II a powerful and reliable tool for tackling complex multi-objective optimization problems.

3.3.3. Exact solution methods for multi-objective optimization

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objective functions, each of which has its own limitations. In multi-objective optimization, due to conflicting or incomparable objectives, in practice, we cannot reach a solution that simultaneously optimizes all functions and reaches optimal points for all functions.

Solving method of multi-objective problem in this paper is weighted sum method, which tries to optimize the weighted sum of the objective functions by assigning a non-negative weight value to each objective function.

$$\begin{aligned} &\min w_1 f_1 + w_2 f_2 + \ldots + w_n f_n \\ &s.t. \quad g(\mathbf{x}) \le 0, \quad h(\mathbf{x}) = 0 \end{aligned} \qquad (24)$$

By this method, the multi-objective problem becomes a single-objective problem.

Another method used in multi-objective optimization is the normalized weighted sum method, which is the developed state of the weighted sum method, which puts objective functions in the interval [1, 0] by normalizing them.

$$\text{Min}\ \frac{w_1(f_1-z_1^U)}{z_1^N-z_1^U}+\cdots+\frac{w_n(f_n-z_n^U)}{z_n^N-z_n^U}$$
$$\text{s.t.}\quad g(x)\leq 0\ ,\ h(x)=0 \tag{25}$$

4. Findings

4.1. Parameter adjustment

When using meta-heuristic algorithms, adjusting parameters is crucial because poor parameter choices can lead to decreased algorithm efficiency, resulting in solutions that are far from optimal. These parameters are fine-tuned through numerical experiments, and various methods exist for designing these experiments. One of the simplest approaches is to conduct experiments using full factorial designs, but this can become complicated and error-prone when dealing with a large number of factors. An alternative method is the Taguchi method [17], which utilizes a series of fractional factorial experiments, significantly reducing the number of trials while still capturing essential information. According to Taguchi, the factors influencing parameter settings are typically categorized into two groups: controllable factors and uncontrollable factors. The goal of this method is to identify the optimal levels of controllable factors while minimizing the impact of uncontrollable ones. In this method, we need to first measure the qualitative characteristics of the experiments, which are calculated as $\frac{S}{N}$. S denotes the signal value, while N signifies the noise. This ratio reflects the deviations in the solution variable, which in this study corresponds to the objective function. Consequently, each of the algorithms mentioned will be parameterized according to the problem's objective value, and the parameter adjustment process will be carried out twice. Next, we will look at how to adjust the parameters for each algorithm individually.

4.2. Parameter adjustment for the MOPSO algorithm

In the MOPSO algorithm, we will have five parameters ni, np, (w), c_1, c_2. We use a three-level Taguchi design for the operation of adjusting the said parameters, where three different values are considered for each parameter, as shown in Table 1.

Table 1. Display of the level of parameters in the MOPSO algorithm according to Taguchi method

Parameter	*ni*	*np*	*W*	*C1*	*C2*
Three-level values	50, 75, 100	20, 30, 40	0.3, 0.6, 0.9	1,1.5,2	1,1.5,2

To conduct numerical experiments for parameter setting, we will utilize Taguchi tests from the Stat tab and the DOE section in MiniTab software. Each test will be executed five times to minimize randomness effects. The average result from these experiments will represent the solution level in the algorithm. Figurr 1, presented after this section, displays the data analysis results obtained from MiniTab software.

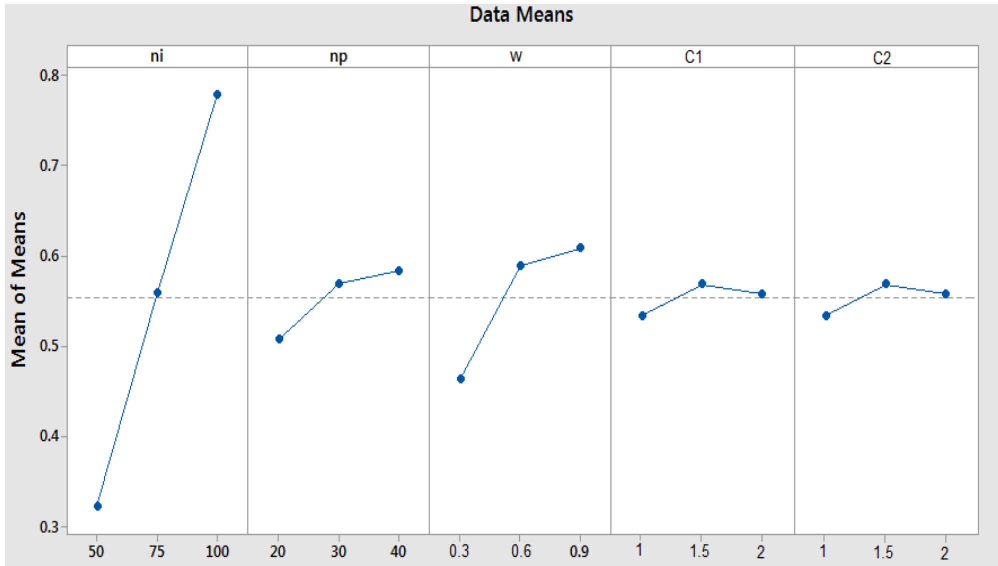

Figure 1. The parameter adjustment analysis diagram for MOPSO in Taguchi method

The best value for each parameter is listed in Table 2. These values are calculated and displayed for each index in the MOGWO algorithm.

Table 2. Adjusted values of MOPSO algorithm parameters

Algorithm	*ni*	*np*	*w*	*C1*	*C2*
GWO	100	40	0.9	1.5	1.5

4.3. Parameter adjustment for the NSGA-11

The NSGA-II has four parameters: the maximum number of iterations (mi), population number (np), mutation (pm), and crossover (pc) that need to be adjusted. We use a three-level Taguchi design for the adjustment of the said parameters. Therefore, three different values for each parameter are considered based on the literature and our expertise, which are shown in Table 3.

Table 3. Level of parameters for Taguchi design

Parameter	Values of each level		
	Level 1	Level 2	Level 3
Percentage of Crossover (Pc)	0.7	0.8	0.9
Percentage of Mutation (Pm)	0.05	0.1	0.15
Number of Solutions in the Population (N-pop)	50	100	150
Maximum iteration(Max-iteration)	100	200	300

Taguchi tests for each algorithm are conducted using MiniTab software, with each test being executed five times to reduce randomness. Consequently, the average objective value from these five runs is taken as the solution level value. Figure 2, which follows this section, presents the results of the data analysis.

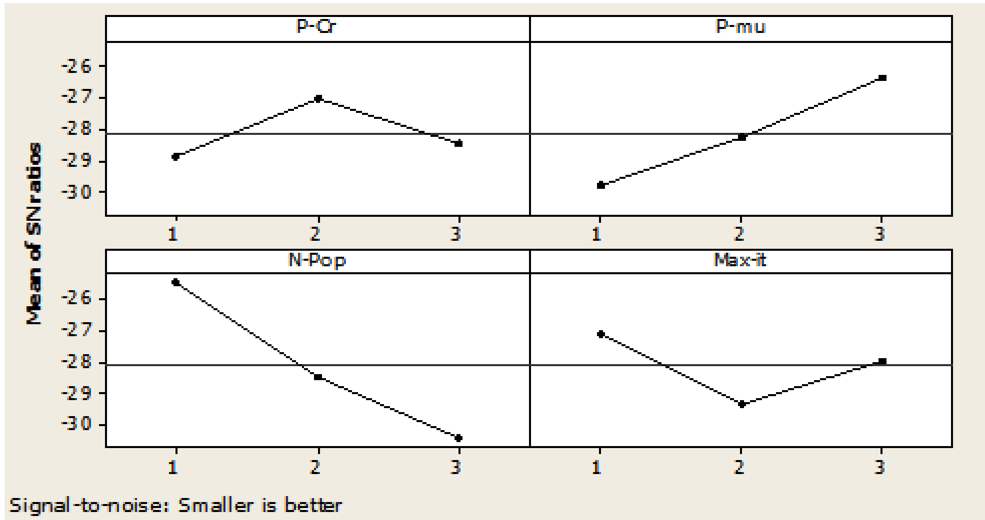

Figure 2. The parameter adjustment analysis diagram for NSGA-II in Taguchi method

The best value for each parameter is listed in Table 4. These values are calculated and displayed for each index in the genetic algorithm.

Table 4. Adjusted values of the NSGA-II parameters

Parameter	Optimal value
Percentage of Crossover (Pc)	0.7
Percentage of Mutation (Pm)	0.05
Number of Solutions in the Population (N-pop)	150
Max iteration	200

4.4. Comparison of the performance of algorithms

In this section, we compare the performance of NSGA-II and MOPSO. There are different methods to do so, we use the method of relative percentage increase in performance, which is defined as the following formula.

$$RPI_s = \frac{f_s - f_b}{f_b} \times 100 \qquad (26)$$

In the above equation, f_s means the value of the objective function, which is calculated by the meta-heuristic method s. s represents the used algorithm. f_b is equal to the most optimal objective value obtained from the algorithms. In the following, each numerical example is solved 10 times in order to ensure more certainty of the efficiency of the algorithms. Table 5 shows the RPI values obtained from the best results of 10 tests.

Table 5. RPI obtained from the most optimal solutions of 10 numerical tests of NSGA-II and MOPSO

Data set	NSGAII	MOPSO
1	5.65	0.6
2	6.85	3.69
3	4.87	3.3
4	6.48	5.48
5	9.14	0.26
6	0.49	0.32
7	7.19	0.98
8	1.66	1.56
9	7.24	11.48
10	5.69	3.65
Mean	5.526	3.132

Table 5 displays the results from the best solutions of 10 numerical tests. The RPI indices for the two proposed algorithms are evaluated in MiniTab software based on the criteria defined earlier, and interval graphs for the optimal value of the objective function with a confidence interval of 0.95% are presented. From this analysis, it can be inferred that the MOPSO algorithm outperforms the other algorithm. However, it's important to note that the reports in Table 6, derived from the best results of 10 algorithm runs, may not be entirely reliable, so this conclusion should be viewed with caution.

As with the numerical tests in Table 6, which were implemented according to the best results, in the following, each numerical test is implemented 10 times based on the mean values of the obtained results. Table 6 reports the RPI values obtained from the mean results obtained from 10 tests.

Table 6. RPI obtained from the mean results of 10 iterations of the numerical test of NSGA-II and MOPSO

Data set	NSGAII	MOPSO
1	3.26	9.82
2	5.62	2.1
3	4.16	6.26
4	10.66	6.56
5	6.5	0.69
6	9.8	3.13
7	6.3	0.68
8	9.6	2.98
9	3.72	7.56
10	3.69	5.91
Mean	6.331	4.569

Next, we compare the solution times of the two algorithms. We designed and executed 10 numerical tests with varying dimensions using MOPSO and NSGA-II. The time taken to solve these numerical tests is recorded and presented below. As shown in Figure 3, the solution time for the MOPSO algorithm, represented by the blue graph, is shorter than that of the other algorithm.

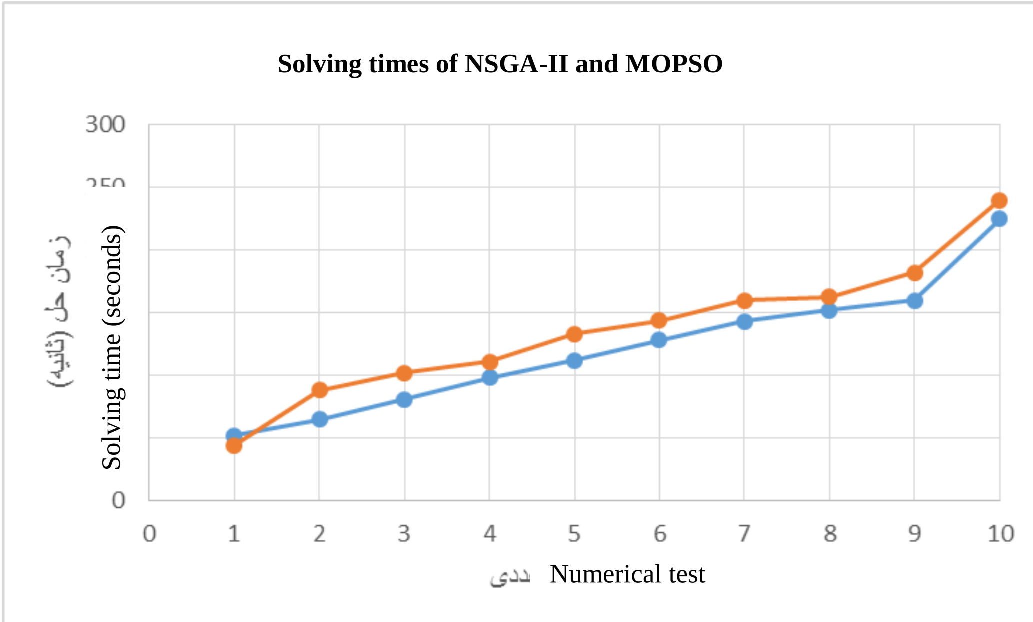

Figure 3. Comparison of the solution times of NSGA-II and MOPSO

4.5 Validation

In this section, the validation of the proposed model is examined. For this purpose, the model is defined in small samples. Then, the feasibility and bounded ness of the model are examined. According to the suggestion of Iraj et al. [8] there are two necessary and sufficient conditions for examining the validity.

First: The feasible space is bounded.

Second: There is X for which a feasible solution to the problem can be determined.

For this purpose, The BARON solver in GAMS 23.5.2 software was used to solve the numerical case due to the nonlinearity of the mathematical model. Additionally, random data following the uniform distribution has been employed due to the absence of real-world data and the mathematical model's ongoing development.

In Table 7, the results of the validation of the model are presented.

Table 7. Validation results

Feasibility of model	Boundry of model	Calculation Time (Seconds)	Objective function
Feasibile	Bounded	0.002	23213.654

According to the results obtained in Table 4, we can conclude the validity of the model.

4.6 Sensitivity analysis

In this section, it shows the effect of the number of route that need hubs in terms of the limitations related to the problem assumption of cost. The calculation results of this analysis based on different percentages of routes that need hub services are shown in Figure 4. As presented in Figure 4, it is evident that the percentage of chosen routes hub services has a significant impact on costs. Also, significantly, increasing the value in this coefficient leads to an increase in total cost, which sometimes may not meet all needs.

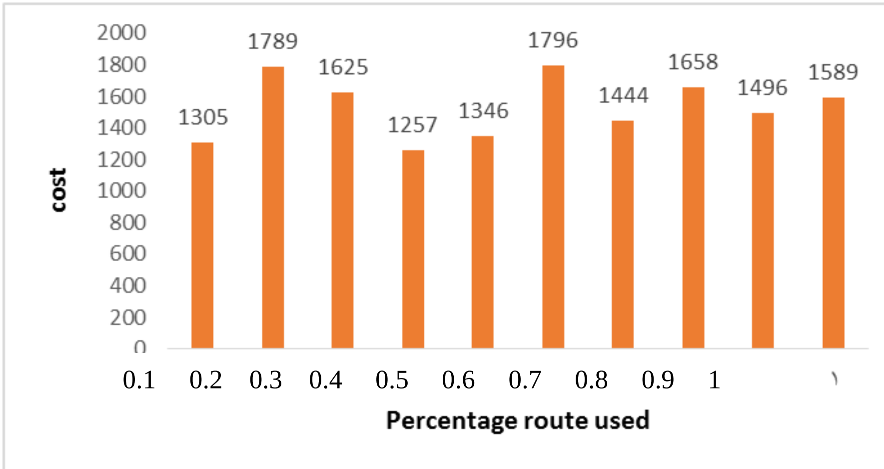

Figure 4. Changing effect of percentage route used on cost

Also, in this research a robust scenario based probabilistic optimization method is used to control the non-deterministic parameters of the model. In this method, to control uncertainty rate to justify the problem and guarantee the maximum demand of damaged points, the robust method is used in different scenarios. Therefore, the change in the most critical parameters related to the robustness of the model will lead to a difference in the total costs and decision variables. Table 8 shows the relief logistics network's total costs in different optimization parameter values.

Table 8. Costs of the entire logistics network under different values of model robustness

Optimistic scenario	Total cost	Pessimistic scenario	Total cost	Possibility scenario	Total cost
1	413098.7	2	413098.7	2	426841.7
10	426841.7	4	418223.8	4	429476.6
20	462846.8	10	429476.6	10	462846.8
100	497842.0	15	433674.8	15	497842.0

4. Conclusion

Locating military equipment involves specific conditions, and neglecting these can lead to significant issues in allocating suitable sites for military equipment warehouses. Beyond security concerns, meeting demand is inherently essential. Thus, a key objective in the hub location of military equipment is to satisfy the maximum demand based on the needs of various areas or routes. Additionally, since military products must be transported swiftly, selecting routes with minimal congestion is crucial. In other words, for both security and transportation speed, it's vital to choose paths with the least traffic, ensuring that vehicles carrying military equipment remain discreet and can quickly reach their intended locations or headquarters.

In this research, a mathematical model was developed to minimize transportation costs and identify optimal storage locations for military equipment in conflict zones based on regional demand. The demand was considered dynamically, adapting to various scenarios. After designing a specific mathematical model and its counterpart under uncertain robust conditions, we first coded and executed the model in small dimensions using GAMS software to validate its accuracy. Subsequently, meta-heuristic algorithms were employed in MATLAB software to analyze data in larger dimensions. For small-scale problem-solving, the exact epsilon constraint method was utilized in GAMS, while meta-heuristic approaches such as PSO and NSGA-II were applied for medium and large dimensions. Finally, appropriate performance indicators were used to compare the effectiveness of the algorithms. After solving several numerical examples and evaluating their performance indicators, it was concluded that the MOPSO algorithm demonstrated superior performance in solving the model.

In conclusion, the development of a mathematical model for hub location of war equipment under uncertainty, utilizing meta-heuristic algorithms, represents a significant advancement in addressing the complex logistical challenges faced in military operations. This research highlights the critical importance of incorporating uncertainty into the hub location and routing processes, ensuring that the model is not only theoretically sound but also practically applicable in real-world scenarios. By leveraging meta-heuristic algorithms, the model effectively navigates the intricate trade-offs between cost, efficiency, and reliability, ultimately enhancing the strategic positioning of war equipment. The findings demonstrate that the proposed model can adapt to various disruptions and uncertainties, providing robust solutions that are essential for maintaining operational readiness in dynamic environments. Furthermore, the insights gained from this study pave the way for future research in military logistics and supply chain management, encouraging the exploration of more sophisticated algorithms and models that can further improve decision-making processes. Overall, this work contributes to the ongoing efforts to optimize military logistics, ensuring that resources are allocated efficiently and effectively in times of crisis.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Abolghasemian, M., Bigdeli, H., & Shamami, N. (2024). Locating Routing Problem (LRP) of distribution of priority support items to ground forces in war conditions. *Engineering Management and Soft Computing*, *10*(1), 262-292.
2. Abolghasemian, M., Bigdeli, H., & Shamami, N. (2024). Modeling the positioning of support forces in future battles using data envelopment analysis and the principles of natural and managerial accessibility. *Defensive Future Studies*, *9*(32), 65-98.
3. Alumur S., Kara B.Y., (2008), Network hub location problems: the state of the art, *European Journal of Operational Research*, 190, 1–21.
4. Campbell J. F. (1994), Integer programming formulation for discrete hub location problems, *European Journal of Operational Research*, 72, 387-405.
5. Chobar, A. P., Adibi, M. A., & Kazemi, A. (2022). Multi-objective hub-spoke network design of perishable tourism products using combination machine learning and meta-heuristic algorithms. *Environment, Development and Sustainability*, 1-28.
6. Demir, İ., Kiraz, B., & Ergin, F. C. (2022). Experimental evaluation of meta-heuristics for multi-objective capacitated multiple allocation hub location problem. *Engineering Science and Technology, an International Journal*, 29, 101032.
7. Goldman, R. L. (1969). Vibration analysis by dynamic partitioning. American Institute of Aeronautics and Astronautics journal, 7(6), 1152-1154.
8. Iraj, M., Chobar, A. P., Peivandizadeh, A., & Abolghasemian, M. (2024). Presenting a two-echelon multi-objective supply chain model considering the expiration date of products and solving it by applying MODM. *Sustainable Manufacturing and Service Economics*, *3*, 100022.
9. Li, C., Han, P., Zhou, M., & Gu, M. (2023). Design of multimodal hub-and-spoke transportation network for emergency relief under COVID-19 pandemic: A meta-heuristic approach. *Applied Soft Computing*, 133, 109925.
10. Li, S., Fang, C., & Wu, Y. (2020). Robust hub location problem with flow-based set-up cost. *IEEE Access*, 8, 66178-66188.
11. Mokhtarzadeh, M., Tavakkoli-Moghaddam, R., Triki, C., & Rahimi, Y. (2021). A hybrid of clustering and meta-heuristic algorithms to solve a p-mobile hub location–allocation problem with the depreciation cost of hub facilities. *Engineering Applications of Artificial Intelligence*, 98, 104121.
12. Mulvey, J.M., Vanderbei, R.J., and Zenios, S.A. (1995). Robust optimization of large-scale systems, *Operations Research*, 43,264-281.

13. O’kelly, M.E., Miller H.J., (1994), The hub network design problem – a review and synthesis, *Journal of Transport Geography*, 2, 31-40.
14. O'kelly M. E., (1987), A quadratic integer program for the location of interacting hub facilities, *European Journal of Operational Research*, 32, 393-404.
15. Shavarani, S. M., Golabi, M., & Izbirak, G. (2021). A capacitated biobjective location problem with uniformly distributed demands in the UAV-supported delivery operation. *International Transactions in Operational Research*, 28(6), 3220-3243.
16. Soleimani, M., Khalilzadeh, M., Bahari, A., & Heidary, A. (2021). NSGA-II algorithm for hub location-allocation problem considering hub disruption and backup hub allocation. *World Journal of Engineering*.
17. Taguchi, G. (1995). Quality engineering (Taguchi methods) for the development of electronic circuit technology. *IEEE Transactions on Reliability*, *44*(2), 225-229.
18. Yılmaz, O., Yakıcı, E., & Karatas, M. (2019). A UAV location and routing problem with spatio-temporal synchronization constraints solved by ant colony optimization. *Journal of Heuristics*, 25(4), 673-701.
19. Zahiri, B., & Suresh, N. C. (2021). Hub network design for hazardous-materials transportation under uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, 152, 102424.