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A Reliable New Approach in Solving Multi-attribute Decision Making Problem through Fuzzy Rule-base System and Z-Numbers

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ABSTRACT

The parameters of decision-making in the Multi-Attribute Decision Making (MADM) process are often imprecise, vague, uncertain, or incomplete information. To cope with these problems, the researchers apply fuzzy set theory as the best-developed approach. Among different fuzzy methods, the fuzzy rule-based system (FRBS) due to its flexibility, simplicity, and experts' knowledge modelling is an adequate technique for solving MADM problems. The main objective of this study is to apply experts' opinions by Z-numbers in MADM issues to enhance the accuracy of the decision-making process. The fundamental issue in solving MADM problems is that inadequate information in the experts' opinions leads to some degree of uncertainty in decisions. Indeed, in FRBS research to ranking, the reliability level (Z-numbers) in experts' opinions within the decision-making process has not been taken into account. Whereas, the Z-numbers play a key role in the decision-making process to reach more precise decisions affecting the final ranking results. In the proposed approach (Z-FRBS), by considering experts' opinions in the form of Z-numbers to deal with inadequate information and modelling experts' knowledge through FRBS, the process of making a decision is performed without using conventional techniques which resulted in a more accurate solving MADM problems. The effectiveness and validity of the proposed method was approved with an illustrative example, sensitivity analysis, and comparison with three other validated method.

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1. Introduction

Decision-making depicts the method by which a course of activity is chosen as the way to deal with a particular issue. This is one of the most critical tasks in engineering problems. Indeed, in this process, conflicting multiple criteria are involved and often take place in the complex environment in the field of engineering. Multi-Criteria Decision-Making (MCDM) strategies can be classified into two categories: Multiple Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM) [39]. MADM is considered an approach to select the best alternative among a set of a finite number of attributes by considering different criteria which have conflict with each other with the aid of Decision-Maker(s) (DM).

In the real world, MADM problems are hard for individual DMs with different knowledge and practical experience of structures to consider all the aspects of decision problems. Thus, Group Decision-Making (GDM) plays an important role in assuring the correctness and rationality of decisions [39,50]. In the GDM Model, members of the group attempt to satisfy each group member with the decision concerning his/her interest.

Those experts who solve decision-making problems express their opinions about alternatives in real-world situations by using linguistic variables [12,45]. In general, the executed linguistic variables are considered to be vague, uncertain, imprecise, or incomplete data. To deal with these ambiguous data, Zadeh proposed fuzzy set theory as a strong and useful technique. This theory is commonly used to describe uncertainty in decision-making owing to a lack of complete knowledge. There are several successful applications and executions of the fuzzy set theory in MADM. To that end, several research also combined MADM approaches with fuzzy set theory, resulting in Fuzzy MADM (FMADM) [9]. Some of them are the "fuzzy technique for order of preference by similarity simulation to ideal solution" known as TOPSIS [20,11], fuzzy analytical hierarchy process (AHP)[21,13], fuzzy Viekriterijumsko kompromisno rangiranje (VIKOR)[11,8], fuzzy elimination et Choice translating reality (ELECTRE)[1], fuzzy analytic network process (ANP) [40], and fuzzy preference ranking organization method for enrichment evaluations (PROMETHEE) [31].

Estimating the utility function of DM is among the complex issues in the decision-making process. DMs usually express their knowledge, preferences, and utility in the form of IF-THEN rules. The fuzzy rule-based system (also known as the fuzzy inference system) is built on IF-THEN rules and has been widely used to develop intelligent systems for simulating experts' knowledge and utility. It can ease the computing process to resolve sophisticated issues in MADM that is emphasized by the researchers[42];[33];[23]. For this reason, systems based on fuzzy rules are prominent techniques to overcome these problems due to simplicity and flexibility in modeling the experts' knowledge[22];[5]. In the last decade, several types of research have been accomplished to solve problems related to MADM using fuzzy rule-based systems. A modular fuzzy inference system has been proposed by Carrera and Mayorga [10] to overcome impreciseness and uncertainty in selecting a suitable supplier. In their proposed methods, the alternatives were evaluated numerically (crisp) and only the opinions of an expert were used. Amindoust *et al*.[5] presented a fuzzy system for the selection of a sustainable supplier. The experts' opinions were collected in the shape of linguistic variables, ranked a set of suppliers, and gave weight to each criterion. A fuzzy system has been built [22] by taking into account an extra stage that included qualification stages with a non-compensatory rule. Furthermore, they utilized the Design of Experiments (DOE) to test the rules and obtain criterion weights from the rules. Meanwhile, Kumar *et al.* [26]proposed a fuzzy inference system that is similar to [10] by taking into account a mechanism for rule designing. Rezaei and Ortt [35] suggested a method for segmenting providers based on a fuzzy rule-based framework. As well, Yaakob et al. [43] also tried to improve type 2 TOPSIS to deal with ambiguity and proposed a novel approach that by considering the expert empirical knowledge by using FRBS could solve the MADM problem. They considered one expert and crisp value for ranking. Mahmoudi *et al.* [27] proposed a hybrid method of PROMETHEE and fuzzy systems to solve group decision-making problems which claimed their method is reliable. Ghadimi *et al* [15] an audition checklist linguistically based fuzzy inference system approach suggested for performance scoring problems by one expert. A new decision-making model by Zanon *et al.* [48]was presented to forecast the influence of supply chain performance aspects on customer perceived value to assist decision-makers. Yuan et al., [44] developed an agent-based artificial general intelligence system for the process of their system converted numerical data into fuzzy symbolic and reasoning rules to support cognitive decision-making. Also, Hatamzad et al., [17]designed a reliable and accurate prediction model for the road surface friction coefficient to make a decision support system using fuzzy logic networks. Almadi et al. [4] used fuzzy logic to evaluate the emotions of drivers' Correlation between weather/road conditions and speed limits. As well, Zahran et al. [47] by using fuzzy logic presented a model to forecast seasonal rainfall in Jordan. In summary, the range of papers relevant to FRBS and some papers that have been published about the combination of FRBS and conventional techniques to solve MADM problems issues are shown in Table 1.

Table 1. Summary of Reviewed Papers about FRBS & Combinated

Author(s)	Year	Method	Single/ Group Expert	Crisp/Linguistic	Objective
[10]	2008	FRBS	One Expert	Crisp	Proposed a modular fuzzy inference system to select suitable alternatives in supplier selection issues (four rules for three main criteria)
[5]	2012	FRBS	Grouping	Linguistic	Presented a fuzzy system for Sustainable supplier selection.
[26]	2013	FRBS	-	-	Proposed a new model, based on fuzzy logic to handle the various attributes, associated with supplier evaluation problems.
[22]	2013	FRBS+DOE	Grouping	Crisp & Linguistic	Developed a fuzzy system by considering an addition stage including qualification stages with a non-compensatory rule. Also, they used DOE (design of experiment) to test the rules and extract weights of criteria from the rules.
[35]	2013	FRBS	One Expert	Crisp	Proposed an approach based on the fuzzy rule-based system for the segmentation of suppliers.
[43]	2015	FRBS+TOPSIS	Grouping	Linguistic	A novel version of the TOPSIS method using interval type 2 FRBS approach proposed to improve the type 2 TOPSIS ability to deal with ambiguity through the combination of the mathematical process involved in the type 2 TOPSIS with expert empirical knowledge.
[27]	2016	FRBS+ PROMETHEEF RBS	Grouping	Linguistic	Proposed a hybrid method of PROMETHEE and fuzzy systems to solve group decision-making problems.
[15]	2017	FRBS	One Expert	Crisp	Proposed an audition checklist-based fuzzy inference system approach for performance scoring problems.
[48]	2017	FRBS	Grouping	Linguistic	Designed an expert system for determining the best suitable type of supply chain collaboration (SCCOL) for a given enterprise
[25]	2019	FRBS	One Expert	Linguistic	Presented a new decision-making model to aid decision-makers in predicting the impact of supply chain dimensions of performance on customer perceived value.
[44]	2022	FRBS	-	Linguistic	Develop an agent-based artificial general intelligence system to support cognitive decision-making.
[17]	2022	FRBS+ANFIS	-	Linguistic	Design a reliable and accurate prediction model for the road surface friction coefficient to make a decision support system using fuzzy logic networks
[4]	2022	FRBS	One Expert	Linguistic	Proposing a method to predict outputs of drivers' emotions by using fuzzy logic algorithm
[29]	2023	FRBS	One Expert	Linguistic	Presenting a more accurate model to forecast seasonal rainfall in Jordan

Logically, reliable information will give reliable and accurate decisionsp[36]. Because of different knowledge, expertise, and experience, the reliability of experts' opinions and judgments has become an essential property of information. Therefore, in real problem situations, in the decision-making process, the reliability of information is very important to achieve suitable decisions. Zadeh [46] introduced the Z-number concept as a more appropriate idea for describing real-world information. This idea has the potential for important applications in the decision-making process which is used in many areas, especially in the field of engineering and other problems.

In everyday decision-making, most decisions are in the form of Z-numbers. The Z-numbers have more potential to explain human knowledge compared to the classical fuzzy numbers. Thus, some researchers have investigated the concept of Z-number in decision-making problems. [24] presented a method for multi-criteria decision-making using Z-numbers. Criteria weights and criteria values of alternatives are provided as Z-numbers in the proposed methodology. To simplify and diminish computational complications, they converted Z-numbers to crisp values and then ranked the alternatives[25]. Aliev and Zeinalova [2] suggested a method for making decisions under uncertainty in which decision-relevant information is characterized by Z-numbers. In the proposed decision analysis, Z-numbers were first transformed to fuzzy numbers using the methodology given in [24], then they computed the fuzzy utility function values for alternatives, and the best was chosen based on the fuzzy utility value. Azadeh *et al.* [7] propounded an application of the AHP approach beneath the Z-number. Zeinalova [49] developed an approach founded on the concept of expected utility to solve the MADM problem under Z-information. Yaakob and Gegov [43] introduced Z-TOPSIS, a unique variation of the TOPSIS approach to ease MCDM issues based on the notion of Z-numbers. They used a fuzzy rule basis technique to modify the Z-TOPSIS technique proposed in[38]. In [43] the authors feed data into TOPSIS and utilize the results to create rules to improve their capacity to cope with ambiguity and reflect judgments more accurately. The main objective of this rectification was to demonstrate the high ability of the fuzzy rule-based system. Indeed, the proposed technique demonstrated that the experts' empirical knowledge may be taken into account in the decision-making process using a fuzzy rule. Peng and Wang [30] introduced a reliable tool to solve hesitant uncertain linguistic Z-numbers problems which integrated linguistic variables using VIKOR and ERP. As well, Azadeh and Kokabi [6] proposed a Z-number DEA model to transform into possible linear programming by applying an alternative α-cut approach. Qiao *et al.* in [32] proposed a simple computational method to rank Z-numbers for solving MADM problems by extending the PROMETHEE technique. In addition, Shen *et al.*[37] combined the classical Multi-Attribute Border Approximation area Comparison (MABAC) approach with a Z-number environment to answer choice issues using the directed distance and regret theory. In Peng et al., [30], a simple but efficient technique for modeling Z-number with low computational complexity has been used to solve game problems. In [38], strategies from different players were compared systematically. Mahammad Nuriyev [29] proposed the Z-numbers hybrid method with the aid of TOPSIS, PROMETHEE, and AHP techniques for ranking and selection of energy resources. Dong et al. [14] proposed a unique technique for solving evaluation alternatives when attribute values are extreme by integrating the best-worst method (BWM), the maximizing deviation method (MDM), and the reference ideal method (RIM) in a Z-number environment. Rao et al. [34] developed a MAGDM method to solve a decision problem that which attributes were fuzzy numbers. To validate the method has been provided sustainable supplier selection case. Hoseinzada [19] suggested a new method to solve the IT engineer selection process under fuzzy by using Z-numbers using the distance between Z-vectors. As well, in this study to validate the proposed method has been used a numerical example. The validity of all the proposed above-mentioned methods has been demonstrated by applying a numerical example. As well likewise, their applicability and superiority have been presented with sensitivity and comparative analyses along with other existing methods. Also, in almost all the introduced methods, such as [27] , [43], and [14], the experts' opinions process were first aggregated in a matrix and then computational operations have been performed on them. In summary, the range of papers relevant to Z-numbers combined with conventional techniques to solve MADM problems which have been published from 2011 to 2023 are shown in Table 2.

Table 2. Summary of Reviewed Papers about Z-numbers (2011-2023)

Authors	Year	Objective, Tools, and approaches	Methods & Techniques
[46]	2011	Introducing the concept of a Z-number and the way to computer Z-numbers.	CWW with Z-number
[25]	2012	Using fuzzy to develop a method to transform Z-number to classical fuzzy number.	Fuzzy Set
[7]	2013	Propose a new AHP method for the first time based on Z-number.	AHP & Fuzzy set
[49]	2014	The problem of making decisions represented and suggested a new approach to solving a real economic problem in various areas.	Z-information in MCDM
[3]	2014	Using Z-numbers and Choquet integral to solve decision-making issues.	Z-number and Choquet Integral
[43]	2015	The enhanced fuzzy rule-based approach in MCDM by using Z-number on TOPSIS	Z-number and TOPSIS
[38]	2015	Introduce a new DEA model with compound to Z-number to measure the efficiency of decision-making for handling real problems	Z-number with DEA model
[30]	2019	Introduced a reliable tool to solve hesitant uncertain linguistic Z-numbers problems which integrated linguistic variables using VIKOR and ERP	VIKOR, Z-number and ERP
[6]	2016	The proposed Z-number DEA model is transformed into possible linear programming.	DEA with Z-numbers
[32]	2018	General-difference of continuous Z-numbers was introduced and also, a new partial order to sort the Z-numbers is defined using generalized centroids	Z-number with g-difference
[41]	2019	Develop a novel approach based on the TOPSIS method and the power aggregation operators for solving the MCGDM problem where the weight information for decision makers (DMs) and criteria was incomplete.	MGDM with Z-numbers and TOPSIS
[29]	2020	Proposed Z-numbers hybrid method with the aid of TOPSIS, PROMETHEE, and AHP techniques for ranking and selection of energy resource	TOPSIS & PROMETHEE & AHP with Z-numbers
[14]	2020	Developed a novel method by combining the best-worst method (BWM), maximizing deviation method (MDM), and RIM under a Z-number environment to solve one issue in evaluating alternatives between the values of extreme values	BMW+MDM+RIM+Z -numbers
[34]	2022	Developed a MAGDM method to solve a decision problem that attributes values were fuzzy numbers.	MAGDM with Z-numbers
[19]	2023	Suggested a new method to solve the IT engineer selection process under fuzzy by using Z-numbers by using distance between Z-vectors.	Z-numbers with Distance between Z-vectors

Based on literature review for both fields of fuzzy inference systems based MADM methods and decision-making under Z-numbers (summary in Table 1, 2) reveals a gap. Indeed, it means that the level of reliability as a critical property of information has been disregarded in FRBS-reviewed papers. Now the basic question that arises is how to fill this gap. By considering experts' opinions under Z-numbers in the form of linguistics, it is possible to be able to increase the level of reliability in make-decisions. Indeed, in this study, designing and presenting a reliable approach through a fuzzy rule-based system under Z-numbers will be able to make more precise decisions in solving multi-attribute group decision-making (MAGDM).

The main motivational reasons for combining the Z-number concept and fuzzy rule-based system are: enhancing accuracy in decision-making by paying attention to the Z-number concept, modeling utility function and preferences of DM by applying fuzzy rule-based systems using IF-THEN rules, and removing computational complexity utilized in papers related to Z-number and decision making by combining fuzzy rule-based system and Z-number.

As a summary, we present our main contributions as, first, we propose an approach to consider Z-rule in MADM problem for ranking Z-numbers with emphasis on ease the computational process; second, in this paper, experts' opinions are considered by Z-numbers, including two components, performance rating, and its reliability; third, experts' opinions are collected in form of fuzzy linguistic variables, whereas studies by crisp value are vagueness and unreliable; forth, we consider MADM problem under GDM and propose an approach to solve MAGDM; fifth, we estimate experts' weight in decision making based on the reliability of their opinion and will indicate that experts with high reliability carry more weight in decision making.

The rest of this paper is organized as follows: Preliminary concepts including Z-numbers and fuzzy inference systems are defined in Section 2; In Section 3, we propose our novel approach to solve the MADM problem; In Section 4, the applicability of the proposed method by using an illustrative example in the area of supplier section process is presented, and the validity of our proposed approach is determined by sensitivity analysis and as well as having strong validity is investigated with comparing to three other validated methods; finally, the conclusion is given in Section 5.

2. Preliminaries

In this section, we will discuss the primary notion of the Z-number, as well as some basic ideas and terminology related to fuzzy systems and fuzzy computing, which were used in the selection process.

2.1. Concept of Z-number

Decision-makers cannot avoid dealing with uncertain aspects in real life. In most real decision-making cases, experts are not able to demonstrate their knowledge and experience as exactly as numerical values. In such cases, it is preferred to use linguistic variables instead of numerical values to express preference [18] [2][28]. The theory of fuzzy set is among the well-known frameworks to encounter uncertainty, vagueness, and ambiguity and to model linguistic variables. It is important to mention that reliable information significantly affects the reliability of decision-making. Because of this, Zadeh [46] proposed the idea of a Z-number as a more appropriate term for the representation of real-world information. A Z-number is an ordered pair of fuzzy numbers denoted as $Z = (\tilde{A}, \tilde{B})$. Its first component namely $\tilde{A}$ is a constraint on the values that a real-valued uncertain variable, X, is allowed to take. Its second component namely $\tilde{B}$ is a measure of reliability (certainty) of the first component. Typically, $\tilde{A}$ and $\tilde{B}$ are described in a natural language and are also fuzzy numbers. Consider a company that wants to assess its potential suppliers. DM is asked to evaluate the suppliers for a given criterion. Assume that DM gives his/her opinion as "very perfect". Then, DM is asked about the reliability (certainty) of his/her opinion. Let's consider his answer as "likely". This evaluation can be formalized as a Z-number "X is $Z = (\tilde{A}, \tilde{B})$", where X is the evaluation score of criterion, $\tilde{A}$ is very perfect as a fuzzy set is used to describe X, and $\tilde{B}$ is likely as a fuzzy number is used to describe the reliability of $\tilde{A}$. The example mentioned above is a simple form of Z-number and is demonstrated in Figure 1. Other simple examples of Z-number are as follows [46].

- (Anticipated budget deficit, close to 2 million dollars, very likely)
- (Population of Spain, is about 45 million, quite sure)
- (The price of oil shortly, significantly over 100 dollars/barrel, very likely)

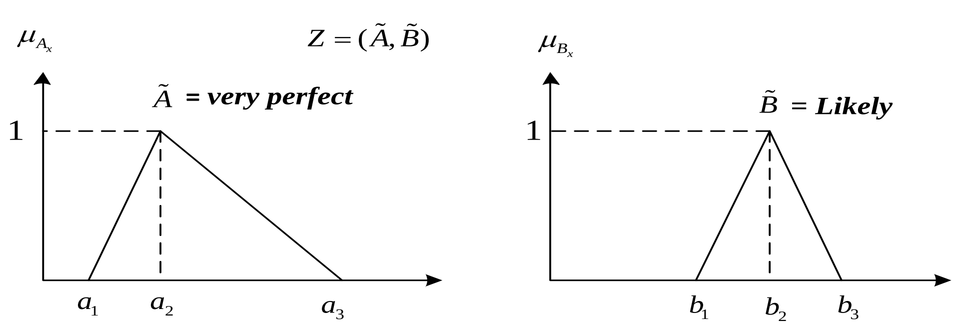

Figure 1. A simple Z-number

The ability to model experts' knowledge in the form of fuzzy IF-THEN rules is one of the robustness of fuzzy logic in many applications. Generally, a simple fuzzy rule can be expressed as: if X is A Then Y is B, where A and B are fuzzy numbers. The meaning of a rule like this is defined as (1):

if X is A then Y is B→ (X,Y) is A × B (1)

Where, $A \times B$ are the Cartesian products of A and B. Zadeh [46] introduced the concept of Z-rules – IF-THEN rules in which the antecedents and/or consequents involve Z-numbers. A Z-rule can be shown as in (2):

$$\text{if } (X, A_X, B_X) \text{ then } (Y, A_Y, B_Y) \quad (2)$$

Where, A_X and A_Y are restrictions on X and Y, respectively. Also, B_X and B_Y are reliabilities on A_X and A_Y, respectively. Let's consider a company that wants to evaluate the performance rating of suppliers in terms of the suppliers' relationship. A rule may be applied as "if (relationship, High, very likely) then (performance rating, moderate, very likely)". A Z-rule- IF-THEN rule may be expanded as in (3):

$$\text{If } (X_1, A_{X1}, B_{X1} \text{ and/or } X_2, A_{X2}, B_{X2} \text{ and/or } \ldots \text{ and/or } X_n, A_{Xn}, B_{Xn}) \text{ then } (Y, A_Y, B_Y) \quad (3)$$

Where, $X_1, X_2, \ldots, X_n$ are input variables and Y is the output variable.

2.2. Basis of Fuzzy Expert Systems

In many instances, decision-making and optimization occur in an uncertain environment, and no deterministic mathematical function exists. In this situation, human knowledge and experience have been utilized for making decisions and optimizing. In many circumstances, experts can express their knowledge in linguistics terms in real-world settings. Zadeh pioneered the notion of fuzzy logic as "word computing"[45]. Human knowledge can be expressed using fuzzy logic. The quantity of data that has to be examined is unimportant in fuzzy logic systems[10]. The heart of a fuzzy system is called the knowledge-based which the rules inside, are expressed by IF-THEN commands.

Mamdani method [28] is among the most common types of fuzzy rule-based systems and has widely been applied in various fields due to its high capacity to express knowledge. The first step in creating a fuzzy system is to collect a set of fuzzy rules from human experts and undertake a fuzzy rule-based analysis. A fuzzy rule is a concept expressed as an IF-THEN rule that is separated into two parts:

"If" section depicts the premise portion of the fuzzy rule "Then" section depicts the conclusion portion of the fuzzy rule. A general type of fuzzy rule [28] is as follows:

$$R: \text{if } x_1 \text{ is } \mu_R^{(1)} \text{ and } \ldots \text{ and } x_n \text{ is } \mu_R^{(n)} \text{ Then } y \text{ is } \mu_R \quad (4)$$

Where $x_1, x_2, \ldots, x_n$ are input variables and y is an output variable. Moreover, $\mu_R^{(i)}$ or μ_R are linguistic variables; that is, fuzzy sets are represented to express imprecise ideas such as “about small” or “very high”. The basic Mamdani fuzzy machine is shown in Figure 2.

Let x_1 and x_2 be two input variables and also, x_1^0 and x_2^0 be their input fuzzy singletons. The following stages demonstrate the fuzzy inference technique[18]:

Step 1: Incorporate fuzzy singletons into their universal sets as inputs.

Step 2: The active rules are obtained by combining input fuzzy sets made up of fuzzy singletons.

Step 3: Calculate $\mu_{X_1}(x_1^0)$ and $\mu_{X_2}(x_2^0)$ as membership functions.

Step4: Determine the degree of matching for each rule as follows in (5):

$$= \min(\mu_{X_1}(x_1^0), \mu_{X_2}(x_2^0)) \quad (5)$$

Step 5: To find the output area, use the Max-Min operator.

Step 6: Repeat steps 3-5 for all active rules.

Step 7: By using centroid defuzzification, all outputs are aggregated and obtain a crisp value of output in (6). Let y^{-j} be the centre of output for j^{th} rule.

$$y^* = \frac{\sum_{j=1}^{J} y^{-j} \times \alpha_j}{\sum_{j=1}^{J} \alpha_j} \quad (6)$$

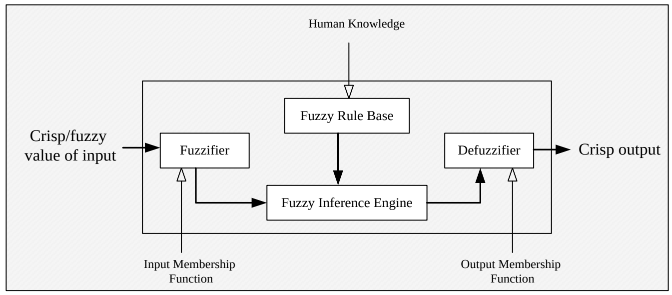

Figure 2. The basic configuration of Mamdani fuzzy systems [28]

As an example, Figure 3 illustrates the fuzzy inference procedure by considering three rules.

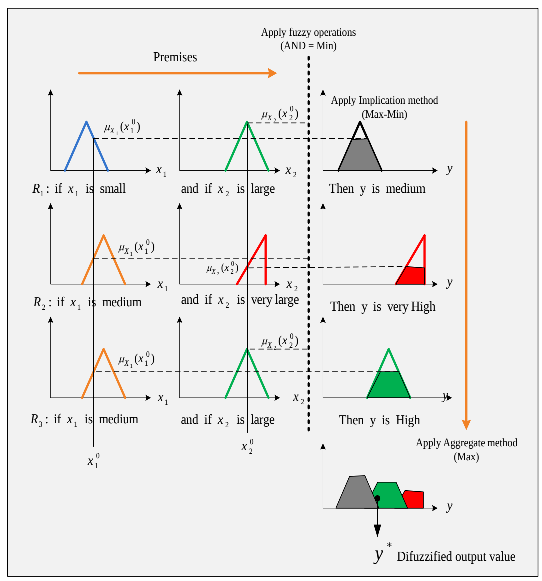

Figure 3. Illustration of the Mamdani fuzzy inference system scheme

3. Proposed methodology

In this work, we are using a fuzzy rule-based technique and considering experts' opinions in the form of Z-numbers as a critical property of information to increase the level of reliability in the make-decision process. This method is presented to deal with uncertainty and incomplete information in decisions more effectively. Reference to the literature review illustrated that the empirical knowledge of experts is capable of facilitating the solution of MADM problems by using the FRBS technique. Another advantage of this method is that it can solve decision-making problems independently without using conventional techniques. Indeed, the main goal of researchers is that by the aid of experts' knowledge with maximum influence degree (means using only fuzzy rules) and considering reliability in their opinions' (by Z-numbers) to provide a method with at least a computational process to be able to solve MADM problems with accepted accuracy.

To introduce this method, let us consider a general structure to define a decision-making problem as in Figure 4 with n main criteria where each criterion has m sub-criteria. It is perceptible that the final evaluation score of each alternative is dependent on the main criteria and the score of each criterion is dependent on their sub-criteria. More concretely, $y = f(x_1, x_2, \ldots, x_n)$ where $x_1 = f(x_{1.1}, x_{1.2}, \ldots, x_{1.m})$, $x_2 = f(x_{2.1}, x_{2.2}, \ldots, x_{2.m})$ and finally $x_n = f(x_{n.1}, x_{n.2}, \ldots, x_{n.m})$ where y is the final score of alternatives and also $x_1, x_2, \ldots, x_n$ are the evaluation scores of the main criteria.

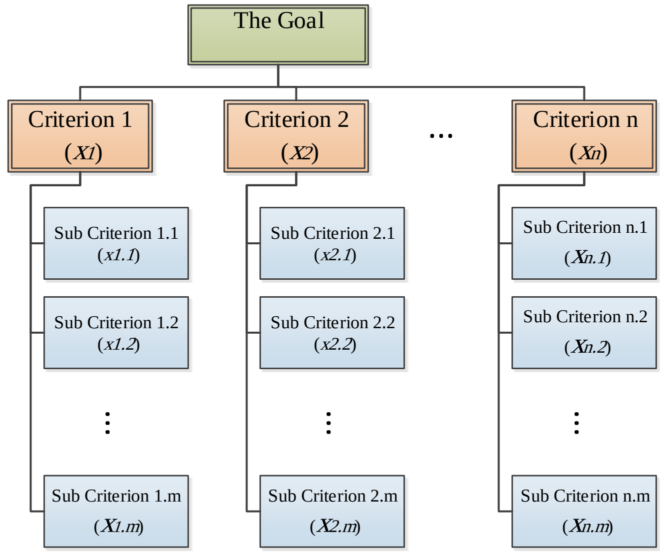

Figure 4. Structure of decision-making problem

To enhance the accuracy of decision-making, experts evaluate alternatives concerning the sub-criteria by Z-numbers. Thus, the rating of each alternative for sub-criteria is an ordered pair of fuzzy numbers, $Z=(\tilde{A},\tilde{B})$, where $\tilde{A}$ is performance rating of alternatives for the sub-criteria is in the form of linguistic variables and $\tilde{B}$ is a linguistic variable that exhibits the reliability of experts in their opinion. Since the input values have two components (performance rating and reliability), it is reasonable that the final score of alternatives contains two components (score of performance and reliability).

Estimating the utility function of DM is an important issue in the decision-making process. Normally, a decision-maker will present their knowledge as linguistic terms in the form of IF-THEN rules. As the main contribution, we propose an approach to establish a fuzzy rule-based system under Z-number. As we mentioned earlier, a fuzzy IF-THEN rule under Z-number may be exhibited as in (7):

If X_1 is (A_{X1}, B_{X1}) and X_2 is (A_{X2}, B_{X2}) then Y is (A_Y, B_Y) (7)

Where, A_{X1}, A_{X2} and A_Y impose restrictions on variables of X_1, X_2 and Y, respectively. Also, B_{X1}, $B_{X2,}$ and B_Y ensure reliabilities on A_{X1}, A_{X2} and A_Y, respectively. It is rational and acceptable that the measure of reliability in antecedents influences the amount of reliability in consequents. It means that if an expert expresses his opinion with high certainty as an input of a rule, the output of a rule will have more strength and reliability. In other words, the reliability of output is dependent on the reliability of inputs. Therefore, we conclude that the aforementioned rule can be divided into two rules as in (8):

(a) If X_1 is (A_{X1}, B_{X1}) and X_2 is (A_{X2}, B_{X2}) then Y is (A_Y, B_Y)

(b) If X_1 is A_{X1} and X_2 is A_{X2} then Y is A_Y (8)

If X_1 is B_{X1} and X_2 is B_{X2} then Y is B_Y

where rule (a) is defined in the domain of variables and rule (b) is defined in the domain of reliabilities. The range of output variable in rule (a) depends on its nature but in rule (b) it is between 0 and 1. Therefore, in our approach, we consider two fuzzy rule-based systems as follows: (a) to determine the alternatives performance score, and (b) to determine the value of reliability.

Before describing the computational stages of our proposed approach, we assume that DM has determined suitable criteria, potential alternatives, and a committee of experts. Assume that experts have been asked about the performance rating of the alternative for sub-criteria and their opinions have been collected in the form of Z-numbers.

Figure 5 depicts a graphical design of the suggested methodology. Indeed, to ease the computational process, the scores of performance and reliability are segregated at first and then finally they will emerge together. Another advantage of the provided model is that it is comprehensive and it can solve any issue defined in the area of MADM with any criteria and sub-criteria but only for simplification is considered a problem with 3 main criteria ($j'=1,2,3$) and 3 sub-criteria (*j* = *1, 2, 3*). Also, let us consider *K* experts (k = 1, 2, …, K) and *S* alternatives (i = 1, 2, …, *S*). The executive procedure of the proposed method is designed into three stages as follows in Figure 5.

Stage1. In this stage, collected opinions in the form of Z-numbers are converted into appropriate fuzzy numbers. Next, fuzzy numbers of each sub-criterion are multiplied by the weight of the sub-criterion. Then, weighted fuzzy numbers are converted into crisp values $\left(x^{i.k}_{j'.j.p}, x^{i.k}_{j'.j.r}\right)$ as inputs of fuzzy rule-based systems where $x^{i.k}_{j'.j.p}$ the performance score is and $x^{i.k}_{j'.j.r}$ is the score of reliability for main criterion j' and sub-criterion j provided by k^{th} expert for i^{th} alternative.

Stage2. The score of performance and reliability of each main criterion for each alternative and each expert is obtained from the fuzzy rule-based system. As indicated in fuzzy expert systems, in our approach, DM defines the fuzzy IF-THEN rules based on their experience and utility. After designing fuzzy rule-based for the performance and reliability separately, a score for the performance and reliability is obtained by utilizing the Mamdani method as shown in Figure 2. Outputs of this stage are denoted as $(x^{i.k}_{j'.p}, x^{i.k}_{j'.r})$ where $x^{i.k}_{j'.p}$ is the performance score for alternative i for the main criterion j' provided by expert k and also $x^{i.k}_{j'.r}$ is the score of reliability for $x^{i.k}_{j'.p}$.

Stage3. The final score of performance and its reliability are calculated for each alternative for each expert. Here, two fuzzy systems are utilized to estimate the score of performance and its reliability. Outputs of stage 2 are considered as input values of fuzzy systems for stage 3. During the decision-making process, the importance of criteria must be considered. As a result, DM specifies the relevant criteria in the form of linguistic variables. In stage 2, the fuzzy weight of a criterion is defuzzified to a crisp value and multiplied by its related output value to determine its relevance. Outputs of this stage may be denoted as $y^{i.k}_p, y^{i.k}_r$) where $y^{i.k}_p$ is the final performance score for i^{th} alternative determined by k^{th} expert and also $y^{i.k}_r$ is the score of reliability for $y^{i.k}_p$. At the end of this stage, we summarize the results of fuzzy systems in Table 3.

Table 3. Outputs of fuzzy systems

Alternatives	Experts			
	1	*2*	…	*K*
1	$(y^{1.1}_p, y^{1.1}_r)$	$(y^{1.2}_p, y^{1.2}_r)$	…	$(y^{1.k}_p, y^{1.k}_r)$
2	$(y^{2.1}_p, y^{2.1}_r)$	$(y^{2.2}_p, y^{2.2}_r)$	…	$(y^{2.k}_p, y^{2.k}_r)$
…	…	…	…	…
S	$(y^{s.1}_p, y^{s.1}_r)$	$(y^{s.2}_p, y^{s.2}_r)$	…	$(y^{s.k}_p, y^{s.k}_r)$

Stage4. In this stage, the final ranking of alternatives is performed using an expected value concept, where for each alternative we compute the expected value as in (9).

$$E_i = \sum_{k=1}^{k} y^{i.k}_p . y^{i.k}_r \qquad ; \quad i = 1,2,\dots,s \tag{9}$$

Based on the calculated expected values, we rank alternatives in descending order.

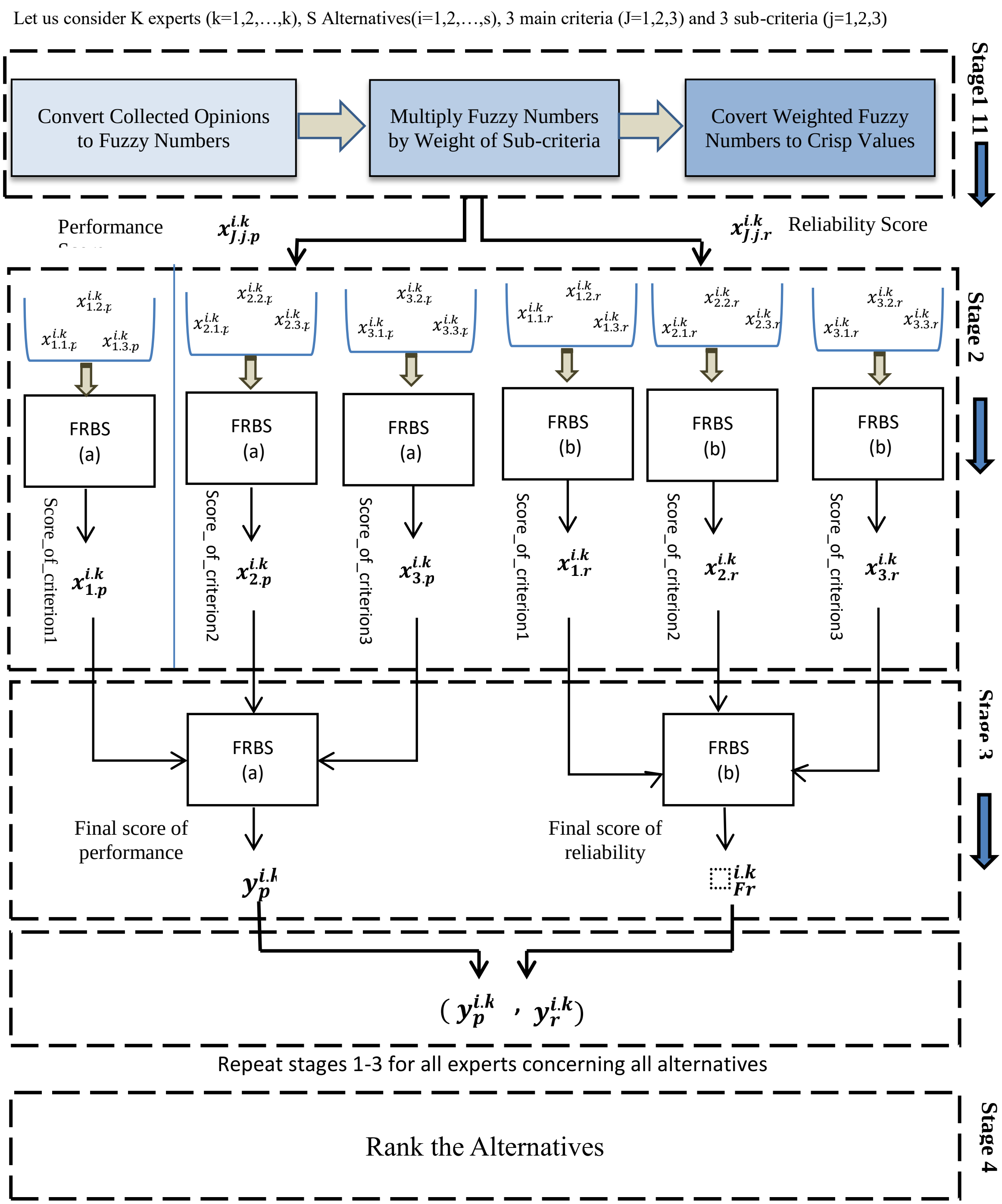

Figure 5. Conceptual framework of the proposed approach

4. Validity analysis

First, the effectiveness of the proposed model is examined by applying and defining an illustrative example for an issue in the supplier selection area like many other research in this area. Thereafter, using the sensitivity analysis will then show that this method works properly. Furthermore, to demonstrate that the proposed method is highly effective and has the beneficial potential to solve the MADM problem with high reliability (more accuracy), it is compared with data sets of three other methods that have been before evaluated.

4.1. Illustrative Example

The performance of the suggested model is demonstrated in this part by using a numerical example in the area of supplier selection. Meanwhile, the proposed method is implemented with software computing through MATLAB. Also, the numerical example data of an assumptive firm are applied inside written code. Assume that a corporation needs to choose a suitable supplier for a key commodity that has a significant impact on the manufacturing process. In this study, 10 specialists were formed by DM as a committee. After starting screening, four candidate suppliers (A_1, A_2, A_3 and A_4) stay for advance assessment. DM provides a hierarchical structure of the criteria to select a suitable supplier based on his knowledge as in Figure 6. Besides that, DM determines the weight of criteria and sub-criteria in the form of fuzzy linguistic variables.

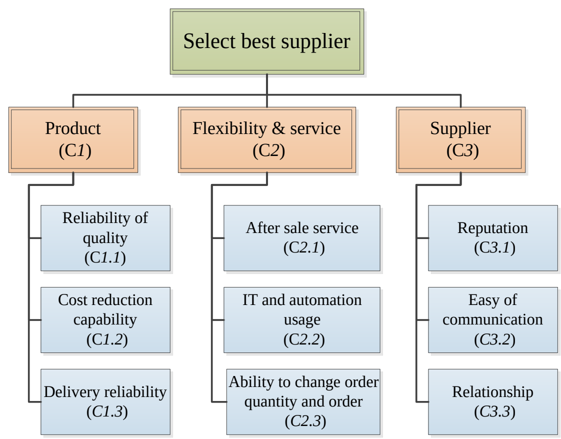

Figure 6. The hierarchical structure of supplier selection

All experts are asked about their judgment on all suppliers for all sub-criteria. Experts' judgments are provided as Z-number, including two components: rate of performance and reliability. Therefore five linguistic variables are considered for the performance rating and reliability of the alternatives. Table 4 demonstrates these linguistic variables with their fuzzy numbers.

The judgments of the suppliers concerning the sub-criteria are collected using experts' knowledge. As an instance, the Supplier's rating provided by expert1 is presented in Table 5 under Z-number. The important weight of the criteria and sub-criteria is necessary before the proposed approach can be applied. Table 6 presents the weight of each criterion and sub-criteria provided by DM.

Table 4. Defining linguistic variables with their fuzzy numbers

Performance rating		**Reliability**		**Importance of (sub) criteria**	
Labels	Triangular Fuzzy numbers	Labels	Triangular Fuzzy numbers	Labels	Triangular Fuzzy numbers
Weakly perfect	(0,0,2.5)	Very low likely	(0,0,0.25)	Very low important	(0,0,0.25)
Low perfect	(0,2.5,5)	low likely	(0,0.25,0.5)	low important	(0,0.25,0.5)
Moderate Perfect	(2.5,5,5)	Likely	(0.25,0.5,0.75)	Moderate important	(0.25,0.5,0.75)
Very perfect	(5,7.5,10)	Very likely	(0.5,0.75,1)	Very important	(0.5,0.75,1)
Extremely perfect	(7.5,10,10)	Extremely likely	(0.75,1,1)	Extremely important	(0.75,1,1)

Table 5. Judgment of expert 1

Criteria & Sub-Criteria		A1	A2	A3	A4
C1	C1.1	Moderate perfect Very likely	Moderate perfect Very likely	Very perfect Very likely	Extremely perfect Likely
	C1.2	Moderate perfect Extremely likely	Very perfect likely	Moderate perfect likely	Very perfect Extremely likely
	C1.3	Very perfect Extremely likely	Very perfect Very likely	Moderate perfect Very likely	Moderate perfect Extremely likely
C2	C2.1	Extremely perfect Likely	Moderate perfect likely	Extremely perfect likely	Very perfect Very likely
	C2.2	Very perfect Extremely likely	Extremely perfect Likely	Very perfect likely	Very perfect Likely
	C2.3	Moderate perfect Very likely	Moderate perfect likely	Extremely perfect likely	Very perfect Extremely likely
C3	C3.1	Moderate perfect Likely	Moderate perfect Very likely	Moderate perfect Very likely	Very perfect Extremely likely
	C3.2	Very perfect Very likely	Extremely perfect Extremely likely	Moderate perfect Very likely	Extremely perfect Very likely
	C3.3	Extremely perfect Likely	Moderate perfect Extremely likely	Very perfect Extremely likely	Low perfect Very likely

Table 6. Importance weight of criteria & sub-criteria

Criteria & Sub-criteria	Weight	Criteria & Sub-criteria	Weight	Criteria & Sub-criteria	Weight
Product (C1)	Extremely important	Flexibility & Service (C2)	Very important	Supplier (C3)	Very important
Reliability of quality (C1.1)	Very important	After-sale service (C2.1)	Extremely important	Reputation (C3.1)	Moderate important
Cost reduction capability (C1.2)	Extremely important	IT & Automation usage (C2.2)	Moderate important	Easy of communication (C3.2)	Very important
Delivery reliability(C1.3)	Extremely important	Ability to change order quantity & time (C2.2)	Very important	Relationship (C3.3)	Very important

After an initial condition was satisfied and the necessary information provided, the proposed approach is tried in the following stages:

Stage1. At the outset, linguistic variables of performance ratings and their reliability are converted into the related fuzzy numbers, and then fuzzy numbers of each sub-criterion are multiplied by the weight of the sub-criterion and weighted fuzzy numbers are obtained. At the end of this stage, weighted fuzzy numbers are converted into crisp values. Table 7 indicates the weighted crisp value of expert1 including an ordered pair of values, where the first part is the performance score and the second part is the score of reliability.

Table 7. Weighted crisp value of expert 1

Criteria & sub-criteria		Suppliers			
		A1	A2	A3	A4
C1	C1.1	(3.96,0.58)	(3.96,0.58)	(5.83,0.58)	(7.29,0.40)
	C1.2	(4.90,0.93)	(7.29,0.49)	(4.9,0.49)	(7.29,0.93)
	C1.3	(7.29,0.93)	(7.29,0.73)	(4.9,0.73)	(4.90,0.93)
C2	C2.1	(9.27,0.49)	(4.9,0.49)	(9.27,0.49)	(7.29,0.73)
	C2.2	(3.96,0.49)	(4.9,0.27)	(3.96,0.27)	(3.96,0.27)
	C2.3	(3.96,0.58)	(3.96,0.4)	(7.29,0.4)	(5.83,0.73)
C3	C3.1	(2.71,0.27)	(2.71,0.40)	(2.71,0.40)	(3.96,0.49)
	C3.2	(5.83,0.58)	(7.29,0.73)	(3.96,0.58)	(7.29,0.58)
	C3.3	(3.96,0.58)	(3.96,0.58)	(5.83,0.58)	(7.29,0.40)

Stage2. The supplier selection process is continued by computing the score of the main criteria (*C1, C2, and C3*) based on two components, score of performance and reliability. At this point, two fuzzy rule-based systems based on DM's knowledge and utility are developed, which are to compute the performance score and score of reliability. Rules of fuzzy systems are defined by DM for a score of performance and reliability are indicated in Table 8 and Table 9, respectively.

Table 8. DM's rules for computing performance scores (first fuzzy system)

Rule Number	If Part			Then Part
	Input1	Input2	Input3	Output
1	Weakly perfect	Extremely perfect	Extremely perfect	Very perfect
2	Low perfect	Extremely perfect	Extremely perfect	Very perfect
3	Weakly perfect	Moderate perfect	Extremely perfect	Moderate perfect
…	…	…	…	…
124	Extremely perfect	Low perfect	Weakly perfect	Low perfect
125	Extremely perfect	Extremely perfect	Extremely perfect	Extremely perfect

Table 9. DM's rules for computing score of reliability (second fuzzy system)

Rule Number	If Part			Then Part
	Input1	Input2	Input3	Output
1	Very low likely	Very low likely	Very low likely	Very low likely
2	Very low likely	Very low likely	low likely	Very low likely
3	low likely	Very likely	Very likely	low likely
…	…	…	…	…
124	Extremely likely	Extremely likely	Likely	Very likely
125	Extremely likely	Extremely likely	Extremely likely	Extremely likely

Table 10 shows the score of performance and its reliability concerning the main criteria for expert1. Here is an explanation to calculate the score. For example, to calculate the performance score of supplier *A1* for the criterion of the product (*C1*) by expert1, the first components of *C1.1*, *C1.2* and *C1.3* (3.96, 4.90 and 7.29) in

Table 7 are applied to the fuzzy system and its output was 3.9 (in Table 8). Similarly, to obtain the score of reliability for supplier *A1* for the criterion of the product (*C1*) of expert1, the second component of *C1.1*, *C1.2*, and *C1.3* (0.58, 0.93, and 0.93) in Table 7 are applied to the second fuzzy system and its output is 0.57 (in Table 10). The aforementioned example is highlighted cell in Table 8.

To show the structure of rule viewers getting from MATLAB Software for the modular FIS system which presents the roadmaps of FIS systems, one of the FIS systems was chosen as an example. As can be seen in Figure 7, each row of plots implies a rule and each column suggests a variable (input1, input2, and input 3). The input values can be varied by moving the red line while the FIS system gives the output value (output1). As the system contains five membership functions and three variables are considered for inputs, the number of rules will be 125 (5^3) to have the output value [16].

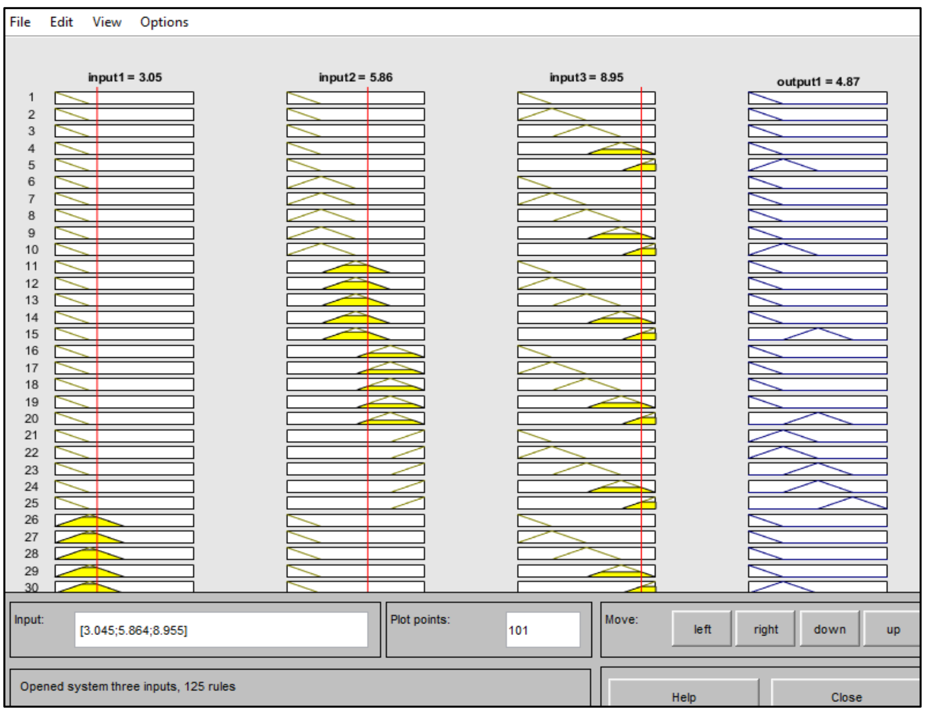

Figure 7. The rule viewer for one of the suppliers in the proposed system

After verifying the rules, it was quite clear that the output value (output1) increased similar to the results obtained from the input values (input1 and input2 & input3). What is more, when we ran MATLAB Software to obtain the output surface produced for the aforementioned FIS system, it was revealed that any increase in the level of output 1 is accompanied by an increase in the amount of input1, input2, and input3 (See Figure 7).

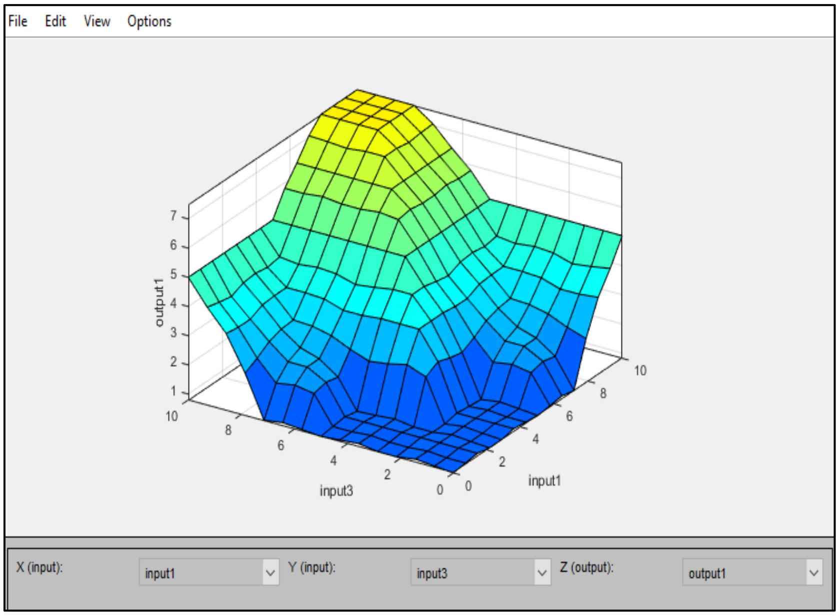

Figure 8. The output surface for one of the suppliers in the proposed method

Table 10. Score of performance and its reliability for each main criterion provided by expert1

Main criteria	Suppliers			
	A1	A2	A3	A4
C1	(3.90,0.57)	(6.08,0.19)	(4.82,0.19)	(6.98,0.55)
C2	(3.92,0.09)	(3.83,0.09)	(6.25,0.09)	(4.84,0.25)
C3	(3.73,0.14)	(2.82,0.25)	(2.39,0.20)	(2.45,0.19)

Stage 3. Given the outputs of stage 2, the final score of performance and its reliability are computed in the stage. In so doing, two fuzzy systems implemented in stage 2 are utilized again. At the outset of stage 3, the fuzzy weight of the main criteria is defuzzified to crisp and multiplied by its corresponding output value in stage 2. Then, weighted values are considered as inputs of two fuzzy systems. Similar to stage 2, the first components of the criteria (from Table 10) are considered as inputs of the first fuzzy system for estimating the performance of suppliers. Also, the second component of criteria (from Table 8) is considered as inputs of the second fuzzy system for estimating the score of reliability. It is worthwhile to mention that to derive the outputs of this stage, the two fuzzy systems indicated in Table 8 and Table 9 are utilized to derive the output at this stage. At the end of this stage, the final score of performance and its reliability are calculated and indicated in Table 11.

Table 11. Final score of performance and its reliability

Experts	Suppliers			
	A1	A2	A3	A4
1	(3.85,0.0918)	(3.44,0.0925)	(3.96,0.0925)	(4.73,0.0935)
2	(5.05,0.0915)	(1.38,0.0906)	(4.75,0.0925)	(3.12,0.0925)
3	(1.56,0.0925)	(3.43,0.0906)	(3.75,0.0900)	(3.95,0.0925)
4	(2.96,0.0925)	(1.42,0.0926)	(4.16,0.0925)	(4.06,0.0876)
5	(0.87,0.0915)	(4.67,0.0925)	(2.33,0.0839)	(4.11,0.0925)
6	(0.9,0.09150)	(4.18,0.0925)	(3.03,0.0900)	(6.62,0.0925)
7	(3.42,0.0871)	(4.18,0.0925)	(3.03,0.0925)	(5.31,0.0935)
8	(3.42,0.0871)	(4.18,0.0925)	(4.02,0.0900)	(4.71,0.0925)
9	(3.42,0.0871)	(4.02,0.0925)	(3.03,0.0925)	(3.95,0.0925)
10	(3.85,0.0918)	(3.44,0.0925)	(3.96,0.0925)	(4.73,0.0935)

Stage4. In this stage, the expected value of suppliers is computed using equation (9). At the end of this stage, the final ranking of the supplier is performed using the supplier's expected value. The computed expected value and final ranking are presented in Table 12. As can be seen in this table, the ranking of the suppliers is $A4 \succ A2 \succ A3 \succ A1$ based on the expected value in ascending order. By using the final scores of reliabilities, we can learn about the weight of each expert in the decision-making process, where the summations of the score of reliabilities were calculated. The values in the table show that experts with high reliability have more weight in decision-making (see Table 13). The diagram based on the values in this table is shown in Figure 9.

Table 12. Supplier's expected value and final ranking

Supplier	1	2	3	4
Expected value	2.66	3.29	3.19	4.11
Ranking index	4	2	3	1

Table 12. Weight of experts in the decision-making process

Experts	1	2	3	4	5	6	7	8	9	10
Sum of reliability	0.3702	0.3671	0.3656	0.3652	0.3604	0.3665	0.3657	0.3621	0.3647	0.3685
Ranking index	1	3	6	7	10	4	5	9	8	2

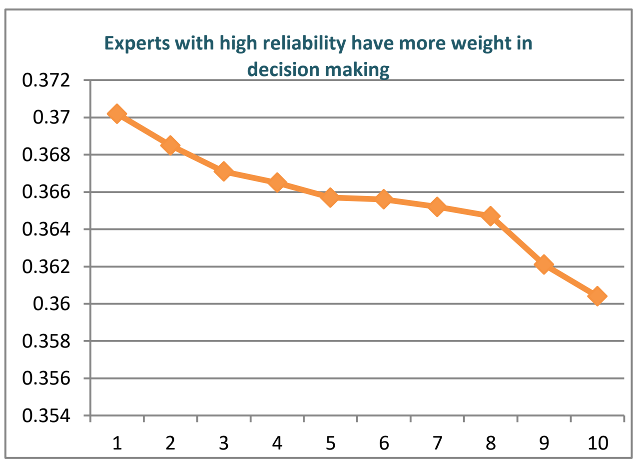

Figure 9. Relation between reliability and weight in the decision-making process based on Z-numbers

4.2. Sensitivity analysis

Sensitivity analysis is used as a demonstration to validate the proposed approach and also to test the robustness of the model. In this respect, it is continued to indicate how considered reliability in experts' opinions is effective in the decision output. In this sensitivity analysis, the reliability of some experts for a given supplier(s) is changed to investigate the influence on expected values and ranking. Consider the following cases with the given changes.

Case1: Reliability of expert 1 is changed to "Extremely likely" for A2 according to all sub-criteria.

Case 2: Reliability of experts 1 and 2 is changed to "Extremely likely" for A2 according to all sub-criteria.

***Case* 3:** The reliability of experts 1, 2, and 3 is changed to "Extremely likely" for A2 according to all sub-criteria.

Case 4: Reliability of Expert 1 is changed to "Extremely likely" for A3 according to all sub-criteria.

Case 5: Reliability of experts 1 and 2 is changed to "Extremely likely" for A3 according to all sub-criteria.

***Case* 6:** The reliability of experts 1, 2, and 3 is changed to "Extremely likely" for A3 according to all sub-criteria.

Case 7: Reliability of all experts for all suppliers is equal to “Extremely likely”.
Note that the remaining components are kept as an original numerical example except for the changed component. The results of the analysis are shown in Table 12.

It is intelligible from cases 1-3 in Table 14 that supplier A2 overcame supplier A4 by increasing the reliability of experts 1-3 simultaneously for supplier A2. And also, as seen from cases 4-6, supplier A3 is overcome by other suppliers if the reliability of three experts is enhanced to "extremely likely". The aim of case 7 is to investigate the influence of considering the reliability of the experts' opinions as a result of the mentioned numerical example. For this purpose, the same reliability is considered “extremely likely” for all experts. As can be seen from case 7 in Table 14, the ranking of suppliers has changed toward the original example. Thus, supplier A2 is the best choice under the same reliabilities. In other words, if the component of reliability is not considered in experts’ opinions, then supplier A2 will be the selected choice. However, in the original numerical example, by considering reliability in experts’ opinions, supplier A4 has been the suggested choice.

Table 14. Ranking orders of suppliers under different reliabilities

Case Number	Expected value				Ranking
	A1	A2	A3	A4	
Case1	2.65	**3.65**	3.17	4.11	$A4 \succ A2 \succ A3 \succ A1$
Case2	2.65	**3.79**	3.17	4.11	$A4 \succ A2 \succ A3 \succ A1$
Case3	2.65	**4.15**	3.17	4.11	$A2 \succ A4 \succ A3 \succ A1$
Case4	2.65	3.28	**3.59**	4.11	$A4 \succ A3 \succ A2 \succ A1$
Case5	3.65	3.28	**4.09**	4.11	$A4 \succ A3 \succ A2 \succ A1$
Case6	2.65	3.28	**4.5**	4.11	$A3 \succ A4 \succ A2 \succ A1$
Case7	**5.81**	**8.04**	**6.94**	**7.91**	$A2 \succ A4 \succ A3 \succ A1$

The ranking order related to the types of cases from 1 to 7 based on different reliability scores is depicted in Figure 10. For example, in case 2, this chart shows that, as the reliability increases for case 3, the alternatives ranked between A2 and A4 are substituted. Likewise, in case 3 when reliability is increasing for case 4, there is a change in alternatives rank between A2, A3, and A4. According to the figure, each alternative that has a higher rank is placed lower.

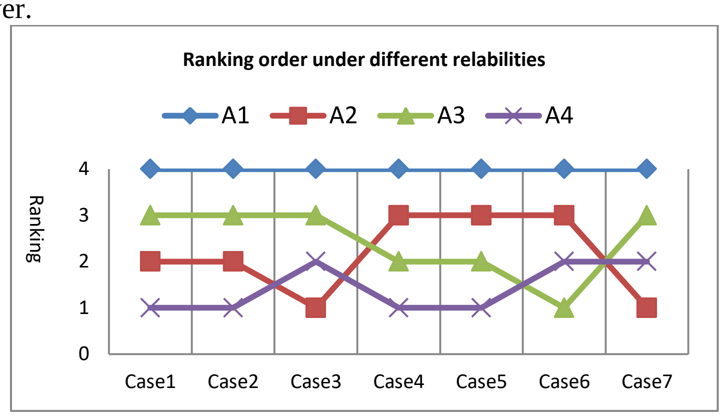

Figure 10. Ranking order under different reliabilities

4.3. Results of Comparisons and Discussion

In this section, the proposed method is verified by comparing it with three other methods. As these methods are the benchmark methods they provide the opportunity to validate the performance of the proposed method. For comparison, the data set used in the numerical example into methods of Yaakob *et al.*[43], Mahmoudi *et al.* [27], and Dong *et al.* [14] have been applied into the proposed method. Since the proposed method is being proposed for the first time, it is necessary to choose a method that has used FRBS, combined with conventional technology and also, the Z-number method for comparison, to demonstrate that the proposed method is highly effective. Indeed, the comparison with each of the methods is established separately, and the results of each one will be discussed at the end of each sub-section as follows.

4.3.1. Comparison with Yaakob's method

Yaakob[43], integrated FRBS with the TOPSIS technique by considering the Z-number concept. Yaakob has modified the Z-TOPSIS method that was introduced in [38] and has combined it with FRBS. They have chosen a stock selection problem as a case study with three DM and 25 stocks to compare with [43]. It should be noted that, in evaluating experiments relevant to make-decisions research, quantity in the sample data is not important [10]. What is important is to show by comparison that the proposed method makes a reliable and more accurate decision. Table 15 shows the comparison performance between the proposed method and with Yaakob method in [43] based on Eq. (10).

$$\rho = 1 - \frac{6 \sum \partial_i^2}{n^3 - n} \quad (10)$$

Table 15. Measurement performance Yaakob method and the proposed method

	Yaakob Method[43]				The Proposed Method			
	Ranking Z-numbers				Ranking Z-numbers			
STOCK	Actual	Z-FRBS	∂_i	∂_i^2	Actual	Z-FRBS	∂_i	∂_i^2
S1	2	5	-3	9	**2**	**2**	**0**	**0**
S2	4	10	-6	36	4	3	1	1
S3	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**0**
S4	21	20	1	1	21	20	1	1
S5	19	24	-5	25	19	22	-3	9
S6	11	9	2	4	11	8	3	9
S7	17	16	1	1	17	11	6	36
S8	24	18	6	36	24	13	11	121
S9	23	25	-2	4	**23**	**23**	**0**	**0**
S10	22	14	8	64	22	21	1	1
S11	8	13	-5	25	8	15	-7	49
S12	13	15	-2	4	13	12	1	1
S13	25	22	3	9	**25**	**25**	**0**	**0**
S14	9	8	1	1	**9**	**9**	**0**	**0**
S15	**3**	**3**	**0**	**0**	3	7	-4	16
S16	5	2	3	9	**5**	4	1	1
S17	18	21	-3	9	18	24	-6	36
S18	12	19	-7	49	12	14	-2	4
S19	15	11	4	16	15	16	-1	1
S20	16	17	-1	1	16	19	-3	9
S21	7	4	3	9	7	6	1	1
S22	20	23	-3	9	20	18	2	4
S23	6	7	-1	1	6	5	1	1
S24	14	6	8	64	14	10	4	16
S25	10	12	-2	4	10	17	-7	49
Spearman Rho Coefficient				**0.850**	Spearman Rho Coefficient			**0.859**

4.3.1.1. Comparison with Yaakob's method

A comparison of the performance and properties of both methods is summarized in Table 16. The findings according to values in this table show that the amount of Spearman Rho Coefficient (SRC) in the proposed method is higher than Yaakob method. The value of SRC in Yaakob method was 0.850 and in the proposed method is 0.859. Meanwhile, in this example, the number of matches shows that decision-making accuracy is important. Indeed, in the rows in this table that $\partial=0$ and have been highlighted, it indicates the Z-numbers ranked that are matched with the actual values. And also the values in this table show that the number of matches and the strength of association in the proposed method are more than Yaakob method. This indicates that the proposed method has a higher accuracy in solving MADM problems.

Table 16. Compare of two methods in terms of performance and properties

Comparative Properties	Yaakob Method	The Proposed Method
FRBS	✓	✓
Z-number	✓	✓
TOPICS	✓	-
Number of matches with actual value	**2**	**5**
The strength of the association	**0.850**	**0.862**
Rank method	Z-FRBS+ TOPSIS	Z-FRBS

4.3.1.2. Comparison with Mahmoudi's Method

In the second comparison, the proposed method is compared with Mahmoudi et al.[27]. method, they have used experts' opinions in group decision-making by linguistics, FRBS, and also a conventional technique called PROMETHEE for ranking. For comparison, Mahmoudi's method data [27] are located in the implemented code, and also, the linguistics variables are defined in the method equally to Mahmoudi's method. The output results of the implemented method with Mahmoudi's method for ranking four suppliers are shown in Table 17.

Table 17. The proposed method preferences comparisons with Mahmoudi's method

Proposers	Techniques used	Ranking
Mahmoudi *et al.* (Mahmoudi et al., 2016)	FRBS and PROMETHEE	*A1 > A4 > A3 > A2*
Proposed approach	FRBS based on Z-numbers	*A4 > A1 > A3 > A2*

Based on Table 17, both methods show that the priority of A2 and A3 is at the same rank, however, the other two alternatives (A1 and A4) have been ranked discordantly. To investigate the reason for this inconsistency between the two methods, more details need to be considered. Employing a theoretical analysis similar to [27], will be shown that the ranking by the proposed method is more accurate. To draw this comparison, the opinions of all experts for each supplier and criteria, which are fuzzy, are converted to crisp value. As an example, Table 18 shows the crisp values for expert1 (E1) (based on the performance rating of suppliers in [27] from Table 4.

Table 18. Crisp decision matrix of expert1 (d1)

Suppliers	Criteria				
	C_1	C_2	C_3	C_4	*Avg.*
A1	0.65	0.65	0.80	0.92	**0.755**
A2	0.65	0.80	0.80	0.65	0.725
A3	0.80	0.50	0.65	0.92	0.717

A4	0.92	0.80	0.65	0.80	**0.790**

Based on Table 18, the priority for each alternative can be simply evaluated by comparing the differences between the alternatives (A1 to A4) for each criterion, which is the average of their criteria. For instance, the average weight for A4 is higher than A1 for Expert. Thus, it can be concluded that A4 has a higher priority than A1 (A4 > A1). After we transform all the performance ratings of suppliers into crisp values that are shown in [27], we found that supplier A4 has higher priority than A1 except for the opinions of E2, E6, and E8 (which means 7 out of 10 suppliers). This analysis shows that the output of the proposed method (A4 having a higher priority than A1) seems to be more accurate than the output of the Mahmoudi method [27]. One of the reasons for the accuracy of the results of the proposed method compared to the Mahmoudi method can be mentioned in the use of the definition of fuzzy numbers in the form of reliability.

4.3.1.3. Comparison with Dong's Method

To perform the third comparison of the proposed method, the data set of Dong's method [14] is applied to our proposed method. Also, the linguistic variables for the data have been implemented in our method equally to the Dong method. In Dong study, 5 criteria for 5 alternatives were evaluated as Z-numbers. After executing the implemented code, Table 19 shows the comparative performance between Dong *et al.* [14] and our proposed method.

Based on Table 19, the values in column two show the final ranking of the proposed method (namely the Z-FRBS method). Indeed, it is quite clear that the final ranking in this column for the proposed method is *A4 > A5 > A2 > A3 >A1*. As well, the values of the third column namely the Z-RIM method are derived from [14].

Table 19. The final performance comparison for the z-frbs method with the z-rim method

Alternatives	Final ranking (the Z-FRBS method)	Final Ranking (the Z-RIM method in (Dong et al., 2020))
A_1	0.336	0.725
A_2	0.401	0.799
A_3	0.383	0.874
A_4	0.477	0.676
A_5	0.460	0.980

Table 20 shows the ranking of our proposed method and Dong *et al.*[14]. The main difference in the ranking is on alternative A4, which is ranked as the first option in the proposed method, while it is ranked as the last option [14].

Table 20. Ranking comparison between the proposed method and Dong's method

Research	Techniques used	Ranking
Dong *et al.*	BMW+MDM+Z-RIM	*A5 > A3 > A2 > A1 > A4*
Proposed approach	FRBS based on Z-numbers	*A4 > A5 > A2 > A3 >A1*

From Table 20, it is clear that the A4 alternative is the least valuable since it is the last option in the ranking [14]. Also, by referring to the values of the decision matrix in [14], it is obvious that the alternative A4 has the highest (in 3 criteria) degree of repetition of VH, which is the maximum amount of linguistics variables among all alternatives. This number is in the next alternative for A5, the number of which is twice repeated among the criteria. On the other hand, by comparing the reliability values relevant to the VH variable in two alternatives A4 and A5, is found that the reliability value of the A4 alternative (two H and one VH) is higher than A5 (two M). A simple comparison between the A4 and A5 alternatives simply reveals that the A4 has a better rating than the A5, and again it shows the high-ranking power and accuracy of the proposed method. The theoretical analysis presented along with a numerical example, further confirms the high potential of the proposed method and its high capability in solving MADM-type problems. In addition, the comparison of the proposed approach with three other methods showed that has high accuracy and reliability for ranking alternatives. The limitations of the proposed method are that it works only with linguistics variables and the variables for performance and reliability could be defined only in the form of 5 variables. Although the limitations mentioned in the other three methods also exist.

5. Conclusion and future work

In this study, a reliable approach known as the Z-FRBS was introduced for solving any issue that can be propounded in the area of group MADM. Due to the nature of its issues and different conditions, a lot of approaches have been developed. However, the proposed Z-FRBS method has the following advantages.

i. The Z-FRBS method because of the consideration of experts' opinion by Z-number as an innovative model and with possessing a strong validity can be a robust and reliable model to solve group MADM problems.

ii. In the proposed method, the variables are by linguistics. Because employing information by linguistic reinforces the flexibility and reliability of decision-making models, and is more feasible in the realistic decision-making process. Also, the vagueness of the original information can be fully retained and employed to address practical problems with precision.

iii. The method output in determining experts' weight in decision-making based on the reliability of their opinions, indicated that experts with high reliability have more weight in decisions making.

iv. This method is comprehensive because it can be used with any criteria and sub-criteria for solving any issue in MADM.

v. The experts' opinion in form of a group is used in the proposed method to ensure the accuracy and rationality of decisions.

vi. The proposed method using FRBS can solve any MADM issues by modelling experts' knowledge and utility function through at least a computational process.

Based on the sensitivity analysis, especially in case 7 indicated the experts' opinions even with reliability may change ranking results. Also, by measuring the performance of the method with another method using statistical analysis was proved that this method has strong robustness. As a result, ranking with FRBS by considering the Z-number may be more efficient in real-world decision-making.

Although this research is very new in its approach generally the limitations, are that the solving MADM problems are performed only on linguistics variables and the number of variables for definition can be only 5 variables. A further effort is needed to be performed using the proposed Z-FRBS method in a real case as a future work; for example in an organization, industry, or business. As well, implementation of a decision support system is suggested to solve MADM problems for all stakeholders as another future work.

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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