Option Pricing Accumulated with Operational Risk

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ABSTRACT

In this paper we distinguish between operational risks depending on whether the operational risk naturally arises in the context of model risk. As the pricing model exposes itself to operational errors whenever it updates and improves its investment model and other related parameters. In this case, it is no longer optimal to implement the best model. Generally, an option is exercised in a jump-diffusion model, if the stock price either exactly hits the early exercise boundary or the price jumps into the exercise price region. However paths of the diffusion process are continuous. In this paper the impact of operational risk on the option pricing through the implementation of Mitra's model with jump diffusion model is presented. A partial integral differential equation is derived and the impact of parameters of Merton's model on operational risk and option value by operational value at risk measure is employed. The option values in the presence of operational risk on data set are computed and some of the results are presented.

1 Introduction

Since Black and Scholes [5] has presented the model on option pricing, there are huge number of theoretical and empirical papers on the subject. In the other hand, operational risk has always been present, but in the last 20 years, with rapid changes in the financial industry leading to larger and more complex financial institutions, a widespread concern has grown significantly. Our main aim in this paper is to undertake a comprehensive analysis of the decision making in the presence of operational risk. More detailed, we study the impacts of operational risk for an optimal investment within a standard asset allocation framework.

This study would be one of the attempts to directly include operational risk into such a pricing model and framework. The feature of our work is the presence of implementation operational risk and jumps that gives volatile pricing process. In particular, within our paper, an investor makes its decisions based on an investment model, but it has incomplete information on what the true model for investment decisions should be. This induces model risk or some parameters to improve its investment model with the employ of new information over time. Thus, there has been an increasing interest in option pricing research in financial mathematics and in particular, on derivatives pricing. Also, many academics work on option pricing research and have presented alternative formulas to the original Black Scholes pricing formula with each making different assumptions about the various factors

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that affect the price of an option, Bahiraie et al. [3]. Most researches on option pricing studies assume that there is no risk. Risk management concerns the investigation of four significant risks of a loss to a firm or portfolio: market risk, credit risk, liquidity risk, and operational risk. Here, one of the issues in option pricing is the presence of risks such as market, liquidity and credit. There are many researches on credit risk such as Hull and White [10], Klein [16], Su and Wang [19], and Feng et al., [11], Acharya and Pedersen [1] in liquidity risk. However, all the above option pricing studies on risk management are widely researched. The other type of risk that is less studied is Operational Risk. Operational risk has become a great topic of interest in the financial markets, yet there exists relatively little research on it, [14].

This risk is defined as losses resulted from inadequate or failed internal processes, people and systems, or from external events, Bahiraie et al., [4]. Hence, in operational risk area for option pricing, Mitra [18] introduced a model for measuring operational risk in option pricing. He shows that option pricing and hedging contains significant operational risk, for instance, due to high volume of activity involving operational risks, (for example,) accounting reconciliation, data entry, and failed reporting, which increases the probability of making errors in the process of rebalancing. The Mitra's model is based on Black Scholes's model$(B.S.m)$, where impact of some parameters in option price such as expiry time (T), interest rate (r) by an Operational Value at Risk is studied, Bahiraie and Alipour [2]. Mitra shows that operational risk increases when T decreases, and that this risk with increasing$r$, increases for $K > S$ (where K is the strike price) and decreases for$K < S$. In general, he shows that operational risk increases with $|K - S|$ decreases. In this paper we add jumps to Mitra's Model to indicate sudden changes in stock price due to, for instance, managerial changes and evolutions, employees strike and governmental decisions which may have effect on option price. Our aim is: considering market as incomplete and observe the impact of jumps on operational risk.

The organization of the paper is as we review the prerequisites in three subsections. We introduce operational risk and firstly review the current risk measurement methods, followed by discussions on the fundamental causes in option pricing. There are review of Black Scholes and Merton's jump diffusion models. We show our extension in Mitra's model by Merton's jump diffusion model. We conduct numerical experiments with the use of the parameters that calculated from collected data set and we show some estimation of the operational Value-at-Risk for a range of parameters values on option prices with different figures and we conclude in last section.

2 Methodological Implementation

Generally, an option is exercised in a jump-diffusion model, if the stock price hits the early exercise boundary or the price jumps into the exercise region. Paths of the diffusion process are almost surely continuous, hence an early exercise at the boundary is due to the pure diffusion. However, conditional on stopping of the process, continuously distributed jumps will almost surely overshoot the critical price and trigger the early exercise inside of the stopping region. In this section we illustrate the prerequisites in our work with dividend about operational risk. Operational risk has become an important risk component in the financial world. This type of risk is closely associated with human errors, system failure, fraud, and inadequate procedures and controls. The Basel II Capital Accord, introduced a capital requirement for operational risk (in addition to credit and mar-

ket risk). This fact has further fostered the focus on operational risk management. The first step to understand this risk is by knowing about its definition. There are many different definitions of operational risk and many institutions have adopted their own definition which better reflects their area of business, and hence it is difficult to present a standard definition for operational risk; nonetheless, we consider the most common definition for operational risk that is presented by Basel II as**, **"*the risk of losses resulting from inadequate or failed internal processes, people and systems, or from external events*".

The Basel committee enhanced operational risk assessment efforts by encouraging the industry to develop methodologies and collect data related to managing operational risk. There are three main operational risk measurement methods for calculation, presented by Basel II: Basic Indicator Approach (BIA), Standardized Approach (SA) and Advanced Measurement Approach (AMA).

3 Economic and Market Setting

The most basic approach allocates operational risk capital using a single indicator as a proxy for an institution's overall operational risk exposure. The basic indicator approach's capital κ_{BIA} is:

$$\kappa_{BIA} = \alpha Ex \tag{1}$$

where α is a constant and Ex is the exposure indicator of the entire institution.

3.1 The Conventional Approach Accumulated with Operational Risk

The standardized approach represents a further refinement along the evolutionary spectrum of approaches for operational risk capital. Operational risk arises from the inadequate information of the models that investors adopt to perform their pricing process. According to an inadequate implementation, which can be caused by different types of errors (e.g., bugs in programming codes, mistakes in data collection and processing), is more likely to occur at times when investor makes changes to its model. In our framework, as introduced in the previous section, model risk creates the updates of model with the arrival of new information. In the presence of operational risk, however, operational errors introduce a wedge between the estimated model and the one that is ultimately implemented. In particular, if the investor decides to change its model by $d\kappa_{SA}$, it will end up implementing a change given by $d\kappa_{SA} + \sigma dwt$, where σdwt captures the operational error. Here, *wt* is another standard Brownian motion adapted to the institution's filtration, and represents operational uncertainty. The volatility parameter σ is a constant and is our measure of operational risk. This approach differs from the basic indicator approach in which a bank's activities are divided into a number of standardized business units and business lines. Thus, the standardized approach is able to better reflect the different risk profiles across banks as reflected by their broad business activities. In the standardized approach's capital κ_{SA} is:

$$\kappa_{SA} = \sum_{i=1}^{8} \beta_i E_i \tag{2}$$

3.2 Advanced Measurement Approach

The Basel committee define AMA generally to reduce capital requirement for operational risk i.e. capital requirement in AMA will be lower under simpler approaches. The moment models used for AMA are: VaR approach and loss distribution approach (LD) and scorecard and Bayesian approaches, Bahiraie et al. [2]. One of the most commonly used measures of risk is the Value-at-Risk(VaR). The interpretation for this measure is: "What is the maximum amount that I can expect to lose with a certain probability over a given horizon?" In the context of operational risk, VaR is, informally speaking, the total one-year amount of capital that would be sufficient to cover all unexpected losses with a high level of confidence. Operational value at risk model determines the worst possible loss that may occur with a given confidence level and for a given timeframe. The $100\,(1-\xi)\%$ VaR is defined as the $(1-\xi)$th percentile of the loss distribution over a target time horizon, T.

The mathematical definition of Value at Risk at a confidence level$\xi \in (0,1)$,

$$VaR^T_{1-\xi} = F_L^{-1}(1-\xi) \tag{3}$$

where F_L is a continuous and strictly increasing loss distribution function and F_L^{-1} is inverse of this function.

3.3 Review of Black Scholes and Merton Jump Diffusion Models

The option pricing theory was introduced by Black and Scholes [5]. It is well known that asset prices can be modelled by the following Stochastic Differential Equation (SDE):

$$dS_t = \alpha S_t \delta t + S_t \sigma dW_t \tag{4}$$

where α and σ^2 are expected return and volatility of the asset return respectively, dW_t is a standard Brownian motion process. The Black-Scholes partial differential equation (PDE) for option price C =C (t; S) is given by,

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} + r\frac{\partial C}{\partial S_t} S_t - rC = 0 \tag{5}$$

where r is interest rate.

While the assumption of an exponentially distributed maturity leads to simple approximations, they generate large numerical errors. To improve the approximation, some papers instead assume that the time to maturity may be subdivided into N periods, Cont [8]. As $N \to \infty$, the distribution converges to a point mass concentrated at the mean. Hence, for large N, the value of an option with random maturity approximates the value of the option with the original maturity. Merton [17] studied option pricing in the case where changes in the asset price consist of a combination of geometric Brownian motion and discontinuities (or jump). This can be described by,

$$dS_t = (\alpha - \lambda\beta)S_t\delta t + S_t\sigma dW_t + (J-1)dN_t \tag{6}$$

where α, σ^2 and dW_t are as in equation (4), dN_t is a Poisson process with intensity λ, J_t is the size of jump and $\beta = E[J_t - 1] = e^{\mu_j + \frac{1}{2}\delta^2}$ (μ_j and δ are the mean and standard deviation of the jumps respectively). In addition, Merton also assumes that the absolute price jump size J_t is a lognormal random variable, that is,

$$J_t \sim \text{i:i:d lognormal } (e^{\mu_j + \frac{1}{2}\delta^2}; \ e^{e^{2\mu_j + \delta^2}}(e^{\delta^2} - 1)) \tag{7}$$

Merton derived a pricing formula for European option on asset S under Merton's jump diffusion model. The Merton partial-integro differential equation (PIDE) for option price $C = C(t; S)$ is:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S_t^2} + r\frac{\partial C}{\partial S_t} S_t - rC + \lambda E[C(J_t S_t; t) - C(S_t; t) - \Delta(J - 1)] = 0 \tag{8}$$

It's clear in above equation, if jump is not occur (i.e $\lambda = 0$), the PIDe's Merton reduce to Black-Scholes PDE. Closed form solution of PIDe's Merton is function price of European options under jump-diffusion. This function is:

$$C(S, t) = \sum_{n=0}^{\infty} \frac{(\hat{\lambda}T)^n}{n!} e^{-\hat{\lambda}T} BS(S_0, \sigma_n, r_n, T, K) \tag{9}$$

4 Optimal Behaviour with Operational Risk

Generally, in option hedging, operational tasks are required such as recording stock price, calculating the number of shares and bonds required for hedging, trading the number of stocks and bonds on the market and settle the trades, data entry, checking new market data, accounting reconciliation and other operations associated with transacting assets. We consider a hedging portfolio to hedge out a European call option $C(t)$ and portfolio with value $V(t)$ with:

$$\delta V(t) = \Delta(t)\delta S(t) + \Phi(t)rB(t)t \tag{10}$$

All these operational activities are associated with operational risks and activities must also be done at each time interval (interval hedging), δt. By using the following lemma, we will able to illustrate the impact of δt on the operational risk in option price. The operational risk in option hedging increases as the hedging interval δt decreases and vice versa, Sulaiman and Andera [20]. Since option hedging intervals can be chosen for different of time periods such as daily or weekly, it is clear that when this interval decreases, then operational risk associated with hedging increases. In this section, we explain the model for operational risk, for option pricing and hedging.

4.1 Model Sophistication and Risk Exposures

The first step to measure the operational risk is to determine exposure indicator. For this, gross income was commonly chosen. As for option hedging, there is no gross income, so, as in Mitra's paper, we choose the operational costs in rebalancing the replicating portfolio as exposure indicator for option hedging. We can express exposure mathematically by:

$$Ex = lQ(t)S(t) \tag{11}$$

where $Q(t)$ is the quantity of shares held, and l is some scaling constant, Thus, operational risk exposure can be expressed as:

$$\kappa_{BIA} = \alpha lQ(t)S(t) = l'|\delta\Delta|S(t) \tag{12}$$

where in last term we take magnitude from $Q(t)$, due to the importance in the number of shares traded (and that no shares that are held). This form indicates the necessity of distribution for $OpRisk$ in option hedging. Here we must first derive the distribution related to $OpRisk$ in option hedging with jump. Since we choose the operational costs as exposure indicator, we set $\delta\Delta$ as number of stock trade from t to $t + \delta t$, i.e. we have:

$$\delta\Delta = \frac{\partial C}{\partial S}(t + \partial t, S + \partial S) - \frac{\partial C}{\partial S}(t,S) \tag{13}$$

Using Taylor series, expansion for $\frac{\partial C}{\partial S}(t + \partial t, S + \partial S)$ we obtain:

$$\delta\Delta = \partial S \frac{\partial^2 C}{\partial S^2}(t,S) + \partial t \frac{\partial^2 C}{\partial S \partial t} + O(\delta t) \tag{14}$$

where $O(\delta t)$ denotes terms of order of (δt), and by using Eq (6), we have:

$$\delta\Delta = \left((r - \lambda\beta)S_t\delta t + S_t\sigma dW_t + (J-1)dN_t \frac{\partial^2 C}{\partial S^2}(t,S) + \partial t \frac{\partial^2 C}{\partial S \partial t} + O(\delta t) \right) \tag{15}$$

since $E[dN_t] = \lambda\delta t$, then from (16) we have:

$$\delta\Delta = (S_t\sigma dW_t)\frac{\partial^2 C}{\partial S^2}(t,S) + O(\delta t) \tag{16}$$

Thus, the distribution of operational risk for option pricing with jump coincide with that derived by Mitra. On the other hand:

$$D(t) \sim \left| l'\sigma \frac{\partial^2 C}{\partial S^2}\delta W_t \right| \tag{17}$$

Note that this distribution is a half normal distribution, because $dW_t \sim N(0, \sqrt{\delta t})$ and this clarifies if X is $N(0, \sqrt{\delta t})$ then $|X|$ have half normal distribution. So, the quantile function for half normal distribution can be used and the VaR for some confidence level and by a quantile function can be derived:

$$OpVaR_\xi = \Phi_{0,\#}^{-1}\left(\frac{\xi}{2} + 0.5\right) \tag{18}$$

where $\Phi_{0,\#}^{-1}$ is the cumulative distribution function for $X \sim N(0, \#)$ and $\Phi_{0,\#}^{-1}$ is the inverse cumulative distribution function.

4.2 Option Pricing in Presence Operational Risk with Jump

In this subsection we present extended Mitra's model in option pricing with jump. If operational costs in hedging portfolio is applied then from Eq (10) we have:

$$\delta V(t) = \Delta(t)\delta S(t) + \Phi(t)rB(t)t - l'|\delta\Delta|S(t) \tag{19}$$

while δt is the expected change in the value of the portfolio. Thus we have:

$$E[\delta V(t)] = E[\delta C(t)], \ \ t \le T \tag{20}$$

In other words, the expected hedging error over, should equal zero,

$$E[\delta V(t) - \delta C(t)] = 0, \ \ t \le T \tag{21}$$

Now we must determine the option price in presence of operational risk. For it, we firstly, write Ito's formula with jump C given by Elandt [10] and Cont [8]:

$$dC = \frac{\partial C}{\partial t}dt + (r - \lambda\kappa)S_t\frac{\partial C}{\partial S_t}dt + \frac{\sigma^2 S_t}{2}\frac{\partial^2 C}{\partial S_t{}^2}dt + \sigma S_t\frac{\partial C}{\partial S_t}dW_t \tag{22}$$
$$+ [C(J_t S_t, t) - C(S_t, t)]dN_t$$

from equations (19), (21) and (22) we can write:

$$E\left[\frac{\partial C}{\partial t}dt + (\alpha - \lambda\kappa)S_t\frac{\partial C}{\partial S_t}dt + \frac{\sigma^2 S_t}{2}\frac{\partial^2 C}{\partial S_t{}^2}dt + \sigma S_t\frac{\partial C}{\partial S_t}dW_t + [C(J_t S_t, t) - C(S_t, t)]dN_t - \right. \tag{23}$$
$$\left. \Delta\delta S(t) - r\Phi(t)B(t)\delta t + l'|\delta\Delta|S(t) \right] = 0$$

now from Eq. (10) we have,

$$\Phi(t)B(t) = C(t) - \Delta(t)S(t) \tag{24}$$

By substituting equations (6) and (24) into Eq. (23) we obtain,

$$E\left[\frac{\partial C}{\partial t}dt + \frac{\sigma^2 S_t{}^2}{2}t\frac{\partial^2 C}{\partial S_t{}^2}dt + [C(J_t S_t, t) - C(S_t, t)]dN_t - \Delta(J - 1)dN_t - r\big(C(t) + \right. \tag{25}$$
$$\left. \Delta\delta S(t)\big)\delta t + \sigma\frac{\partial^2 C}{\partial S_t{}^2}l'|\delta W_t|S_t{}^2 \right] = 0$$

It can be shown that $|\delta W_t|$ can be approximated by its expectation when t approaches small value (for instance see [6]), so we must obtain expectation $\delta|W_t|$. Since $dW_t \sim N\big(0; \sqrt{\delta t}\big)$, we can write $\delta W_t = \sqrt{\delta t}z$ where $z \sim N(0; 1)$.

Thus, from Eq. (26) and $E[dN_t] = \lambda\delta t$, Eq. (25) becomes:

$$\frac{\partial C}{\partial t} + \frac{\hat{\partial}^2}{2}S_t{}^2\frac{\partial^2 C}{\partial S_t{}^2} + r\frac{\partial C}{\partial S_t}S_t - rC + \lambda E[C(J_t S_t, t) - C(S_t, t) - \Delta(J - 1)] = 0 \tag{26}$$

Where $\hat{\partial}^2 = (1 + \chi)\sigma^2$. It's clear that the above equation is Merton's *PIDe* for option pricing with jump but with volatility corrected from σ to $\hat{\sigma}$. with this equation we can use from closed form solution *PIDe*'s Merton with volatility corrected $(\hat{\sigma})$ to determination of option value in presence operational risk and we are able to observe the impact of this risk on option price. In the next section we present numerical experiment.

5 Numerical Experiments and Empirical Results

In this section we conduct numerical experiments to determine the value at risk with option price in the presence of operational risk. We use parameter values that are estimated by Hanson and Westman [12] on the $S\&P$ 500 index. The $Matlab\ R2012$ software for calculations is used. The impact of jumps on option prices has been recently considered in Sulaiman and Andera [20]. They examine jump effects by early exercise boundary with and without jumps. In contrast, we consider the disentanglement of jumps directly by analytically the exercise premium into the contributions. Furthermore, our approach does not require the use of different models, due to model misspecification. Therefore, idea for employed parameters has been previously studied in the literature by Sulaiman and Andera [20] and Bahiraie and Alipour [2]. We use the parameter values for Merton's jump-diffusion as given are:

$$\lambda = 55.46 \qquad \mu = 0.007624 \qquad \delta = 0.0192 \qquad \sigma = 0.0742$$

Also, we set the interest $r = 0.06$ and time $T = 0.5$. Although the option hedging intervals δt has different type (e.g. daily, weekly and monthly), we set it daily (i.e. $\delta t = 1/252$). Similarly we use $\hat{\sigma}^2 = (1 + \chi)\sigma$ and equate $\hat{\sigma}$ to implied volatility and hence estimate.

Using $\chi = \frac{2}{\sigma} l' \sqrt{\frac{2}{\pi \delta t}}$, we can finally obtain l'. Using these estimates, iterate $\hat{\sigma}$ until convergence.

The parameters T, λ, μ, δ and r to analyze the impact operational risk and option price shows the same results as presented at Sulaiman and Andera [20]. We increase these parameters while keeping all other parameters constant. The results are presented in the next section.
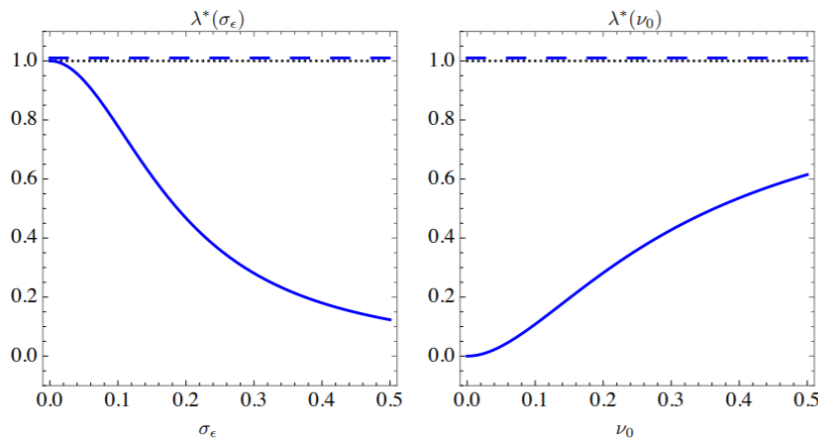

Fig. 1: The Changes of Volatility Corrected σ in Different Range from Stock Price with K=800
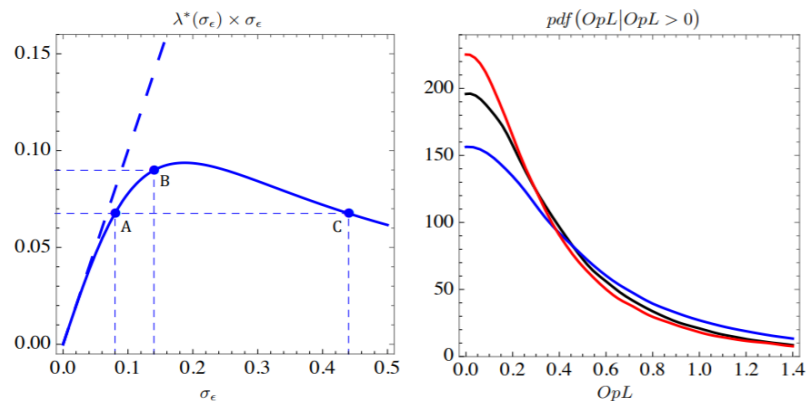
Fig. 2: Option Optimal Values with Merton and Operational Risk Models with K=800

The first step in our calculations is to compute $\hat{\sigma}$. Figure 1 displays the changes of volatility corrected $\hat{\sigma}$ to stock price. This figure shows that $\hat{\sigma}$ decreases as $|K - S|$ decreases. In other words, $\hat{\sigma}$ have the least amount around at-the-money options ($K = S$). We calculate the option values with *tow* models which includes jump diffusion Merton model and jump diffusion Merton model for $K = 800, T = 0.5$ and different S with no parameters altered. We plot the option values in Figure 2 with payoff of call option (that is: $\max\{S_t - K, 0\}$). Figure 2 shows the impact of operational risk on option price and it shows this risk create the value of options is more than value of options that is computed by Merton model in particular in around at the money options. Option prices are computed via the jump-diffusion models.
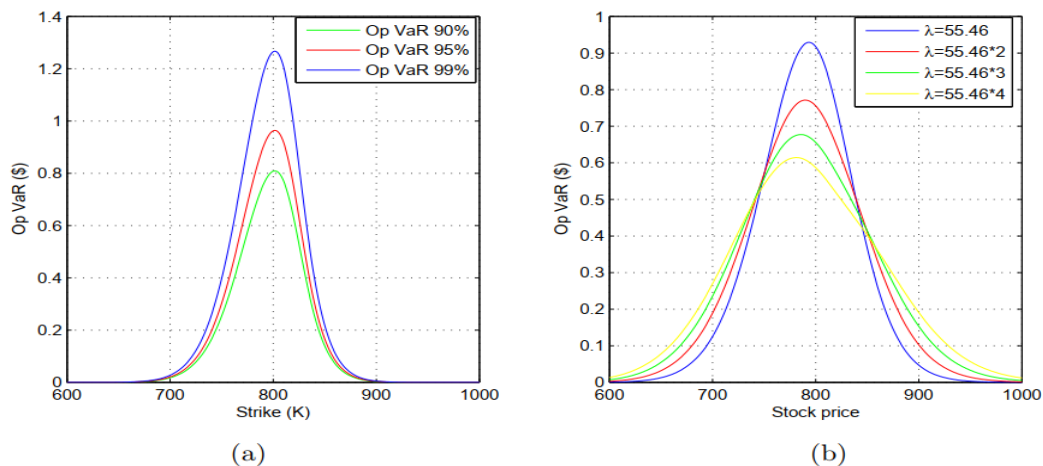

Fig. 3: (a) $Op\,VaR$ at Different Quantile Levels with $K = 800$. (b) Effect of λ at Quantile

The underlying asset price is set to $S_t = 100$, the risk-free rate is $r = 0.04$, the dividend yield is fixed at $\delta = 0.02$, and the conditional probability of a negative jump is $q = 0.7$. Variable input parameters are strike, volatility σ, jump intensity λ, positive jump parameter η, and negative jump parameter θ. We compare the results of our approach to the benchmark and report the relative pricing errors in percentages.
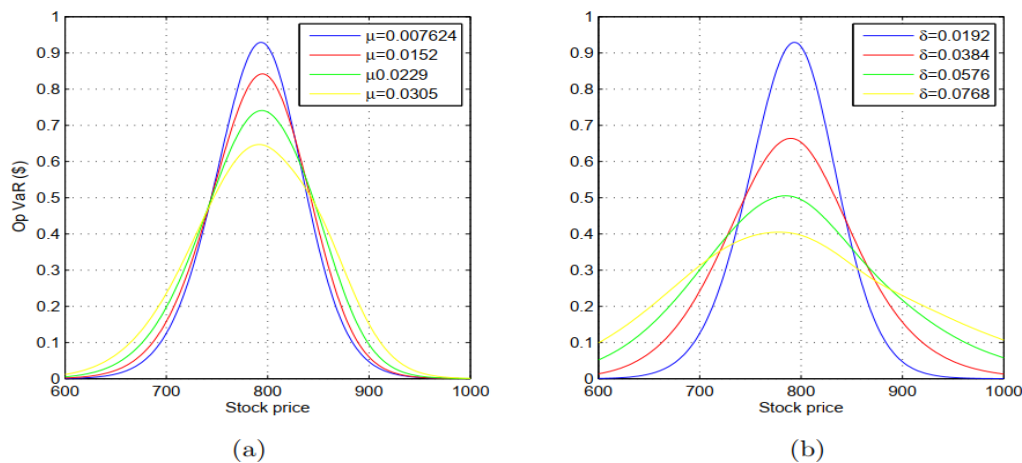
Fig 4: (a) Effect of μ on $Op\ VaR$ at Quantile Level 99% with K=800.(b) Effect of δ on $Op\ VaR$ at Quantile Level 99% with k=800.

Notice that the option price calculated with Merton model is smaller than the option price with operational risk model. This results shows that the operational value at risk in quantile levels increases with decreasing strike prices. On the other hand, we have most operational value at risk around at the money options. For some more data sets, we employed a summary of option price calculated from the Merton model (we denote it with C) and option price calculated from the operational risk model and VaR at three different quantile levels ($90\%, 95\%\ and\ 99\%$) for a range of strike prices, $T = 0.5$. We fixed the stock price $S = 800\$$.
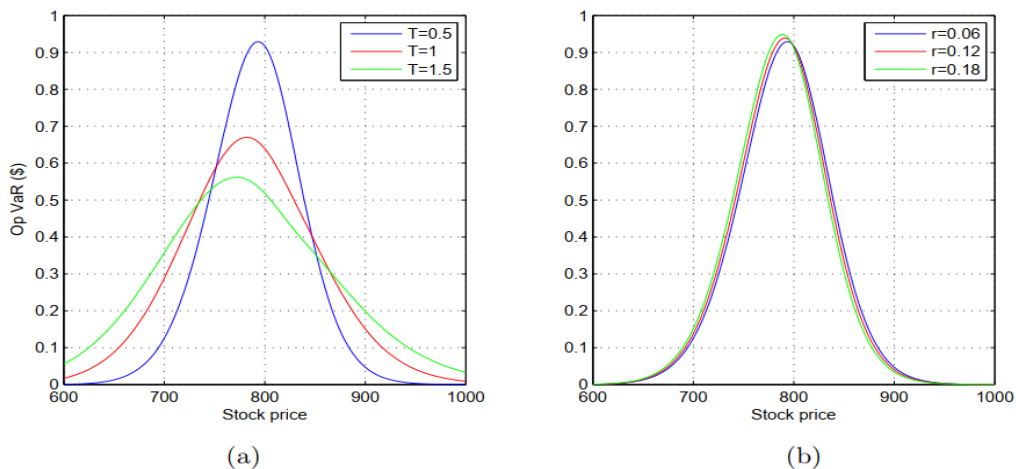

Fig 5: (a) Effect of T on $OpVaR$ at Quantile Level 99 % with K=800.(b) Effect of Interest Rate on $OpVar$ at Quantile Level 99% with k=800.

The optimal model in the presence of financial constraints cannot be obtained in the non-linear nature of the problem. Nonetheless with comparison the results we can confirm this subject. On the other hand, increases in T, decreases operational risk at options, and for more difference of S and K, this result is vice versa. We also compute and analyze the effect of other tersλ, μ, δ, as illustrated in Figures 3 and 4. In Figure 3(a), we plot operational risk at different quantile levels, and notice that as λ increases, the operational risk decreases around at-the-money options but it increases at other points. To investigate the impact on operational risk, the

values of μ and δ are increased. Fig. 4(a) shows the impact of μ and Figure 4(b) shows the impact of δ. Similarly, we increase T and r to investigate the impact on operational risk. The impact of T interest rate r is shown in Figure 5. Similar analysis for μ, δ and T, with different intensity can be given. Figure 5(a) shows the effect of r on operational risk: for $K < S$, increasing r decreases operational risk, but for $K > S$, increasing r increases operational risk.

6 Conclusion

In this paper we extended Mitra's model with Merton's jump diffusion model. We show that the operational risk distribution with jumps has the same distribution that is derived by Mitra, hence we used the same quantile function. We derived the option price formula in the presence of operational risk. We compare the option values in Merton model with the proposed formula, with reference to some parameters in option pricing with jump diffusion Merton's model on operational risk. The result shows T, λ, μ and δ have the similar impact on operational risk at different aspects of option pricing which the investor will process.

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