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Fuzzy Goal Programming Model to Rolling Performance Based Budgeting by Productivity Approach (Case Study: Gas Refineries in Iran)

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ABSTRACT

This research presents a mathematical model for performance-based budgeting and combines it with rolling budget for increased flexibility. The model has been designed by Chebyshev's goal programming technique with fuzzy approach. The parameters or coefficients of the model are derived by measuring the productivity of the organizations considering eight criteria. Data for calculating productivity indicators were collected from gas refineries of Iran in 2011–2015 and analysed by Excel and GAMS software. Then, the model was tested for determining the 2016 budget of those refineries. The model was solved by LINGO software by linking it to Excel. The solution of the model reduced 0.68% of the total refinery's budget compared with the actual budgets for 2016, which is higher than the annual budget of some of the companies in this group.

1 Introduction

The budge method used by Iran's executive agencies is the line-item or traditional budgeting that has many weaknesses; therefore, one of the macro policies of the state is to transform the country's traditional budgeting system into a performance-based budgeting (PBB) system". PBB shows better performance outcomes than other budgeting systems and is considered one of the successful budgeting systems for optimal allocation of resources and managing them. Targeted allocation of the budget to the activities can clarify the distribution of resources, allow operational monitoring and meet expectation of the costs. The use of PBB will be an effective step in increasing the efficiency and effectiveness of the budget. The PBB system is a set of processes that show the relation between the allocated budget and its outputs or outcomes. The use of this efficient budgeting system calls for a need to design appropriate PBB models. The main obstacle in PBB implementation is the performance meas-

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urement standards of organizations. Researchers have found that the main reasons for failure to measure performance are inappropriate design of performance measurement systems and failure to execute these systems (Safari et al. [24]). Among the indicators of performance measurement, productivity is one of the most important aspects that cover both financial and non-financial issues of the organization. During execution, all government companies are required to measure their productivity to improve it. Therefore, the use of productivity measures to evaluate the performance of an organization is one of the most appropriate indicators of performance evaluation for optimal budget allocations. Considering the importance of the issue, various models were designed for PB, but there has been no model for PBB that allocates budget based on organizational productivity.

By studying the functioning of the Iran's gas refineries in the preceding years, no link was found between resources (revenues) and expenditures. In most cases, expenditures are higher than revenue. This is due to the inadequacy of the budgeting system and it needs to be reformed. The existence of joint gas reservoirs with neighbouring countries increased the sensitivity of budget allocation for gauging performance of these companies. This is because more of the resources were used by the neighbouring countries. Considering the importance of gas refineries in the Iran's economy, the use of accountable and PBB systems is necessary to increase financial discipline and optimize the use of limited resources. The use of PBB leads to transparency, accountability, and more investment in this industry. The main issue of this applied research is using mathematical modelling to establish a relationship between productivity (performance) and budget allocation in Iran's gas refineries within the framework of the rolling budgeting (RB) model (which is a flexible budget). So, appropriate indexes of productivity are identified and measured, and then the mathematical model of budgeting is designed based on the organization's performance in these indexes. The importance of mathematical approaches becomes apparent when the number of decision variables, activities, and goals increases (Azar and Najafi, [3]). Research Innovation: In previous researches, productivity-based budget allocations and the combination of RB and PBB budgeting models have not been observed. For this subject, the research has designed crisp and fuzzy mathematical models.

2 Literature Review

Performance-based budgeting (PBB): The PBB system is a form of budgeting that relates to the allocation of funds to measure results from outputs and outcomes (Curristine, [6]). The organization for economic co-operation and development (OECD) defines PBB as a form of budgeting that relates to fund allocated for measurable results (Schick, [25]). The government accountability office (GAO) in US (1999) defines PBB as the concept of linking performance information with the budget. Across the globe, the sensitivity and importance of promoting budgeting efficiency is through PBB, which is an initiative in new public management (Mkasiwa and Gaspar, [13]). The PBB model is a diagnostic tool for assessing government program performance. This is a basis for program funding decisions, aiming for more transparent, robust and systematic links between performance and resource allocations, and focusing on results and outcomes rather than inputs or outputs (Momeni, [17]). The PBB process is a technique where the administrators can apply to manage more cost-efficient and effective budgeting programs (Mohammadipour [15]).

Performance evaluation: Today, various indicators have been proposed as performance indicators of organizations where efficiency, effectiveness, and productivity are among the most important criteria (Khadivar, et al. [12]). The performance of an organization comes from the input of resources and is seen in its outputs or outcomes; therefore, the productivity index that measures the ratio of outputs to inputs is a good criterion for measuring the organization's performance. So, we have to focus on designing a comprehensive productivity measurement system. By choosing productivity as a measure of performance, time and cost of performance evaluation need not be done. It is not needed by the new assessment team in the organization because the productivity indicators are calculated and reported by the organizations. In the field of productivity studies, technical efficiency (T) has been used as an indicator for measuring productivity. One of the goals of a stock company is to increase revenue and create wealth for shareholders; measuring revenue efficiency (R) of these companies can measure the effectiveness. The data envelopment analysis approach (DEA) used to measure performance is like a black box, and it describes only the output to the input without describing the operation of internal units. Using the other productivity indicators will help clarify the performance of this black box. To measure general productivity, labor productivity (L), capital productivity (K), and total factor productivity (TFP) are usually used. Also, in the gas refineries, the specific energy consumption (SEC) index for the gas refinery industry is a suitable indicator of evaluation energy efficiency. These indicators cover the performance of an organization in terms of productivity, but we should use of the financial indicators for performance evaluation, too. For this purpose, "the percentage of revenue achieved" (for measurement of the organization's income obligations) and "total budget decrease" (the aim of all budgeting models) are appropriate indicators. Therefore, the criteria for a comprehensive system of performance measurement by productivity are: 1. Technical efficiency (T) 2. Revenue efficiency (R) 3. Productivity of labor (L) 4. Capital productivity (K) 5. Total factor productivity (TFP) 6. Specific energy consumption (SEC) 7. Revenue achieved (re) and 8. Total budget reduction (b).

Rolling budget (RB): RB is a budget that is always available for a specific period in the future. This course can be a month, three months (seasonal) or one year (Bhimani et al. [5]). In the sequential implementation of RB, risks and opportunities are identified. RB is also capable of creating scenarios and managing different scenarios for prediction (Garlapti et al.[22]). In RB, budget is constantly updated by considering the latest changes. With each period, a new period is added so that the management can undertake annual budget revision. The RB system is to create a seasonal budget allocation mechanism and with seasonal replication, warranty efficiency and effectiveness. Duplicate costs can be fully standardized, thus helping in standardization. With combined RB and PBB, the next season's budget is the result of performance of the past season. Therefore, the results of performance appraisal are understandable to its personnel. The receipt of the budget in each season of the organization reflects the performance of the organization in the previous season. So, it can improve performance of the organization. The process of creating the RB in this study is shown in Fig. 1 .In the first step, the indexes of productivity (eight performance measurement indicators introduced in this research) are calculated for five periods and their averages are extracted. In the second step, according to the experts, target levels for the vision of the goals in the future season are determined. In the third step, the mathematical model with the parameters obtained from the previous stages is solved and the budget coefficient of the next season is determined based on it. These steps are always followed as a rolling process for allocating funds .In this research, productivity indicators were computed for five periods yearly and their average was used to solve the mathematical model and allocate next year's budget. After calculating productivity indicators of the organization in the next period, new data was replaced with the first period's data and the average of the five new periods was updated. The mathematical model was solved again. Finally, with the new coefficient, the budget for the next period was determined. In this flexible budgeting (PBB+RB), budget allocations are changed continuously with the performance information.

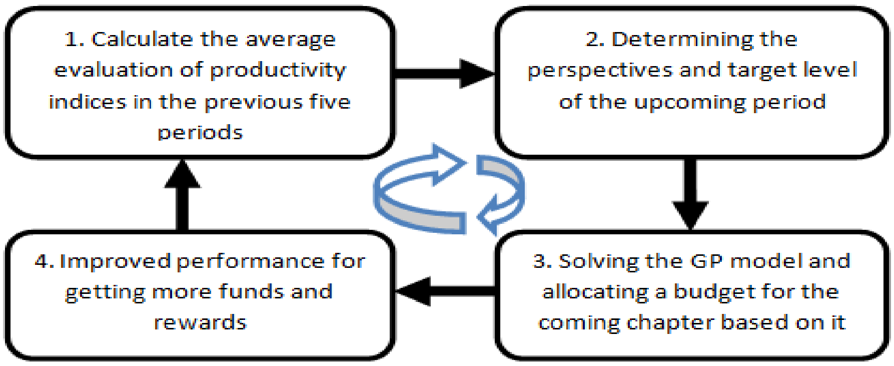

Fig. 1: Schematic of RB system

Goal programming (GP): The GP approach is a linear programming that is presented as a multi-objective decision-making model for analyzing problems with different and sometimes conflicting objectives. GP is an optimization technique that defines the goal deviation from the target levels and sets its objective function to minimize the sum of the unwanted. GP is of several types. They are: lexicographic (priority) GP, weighted GP, and Chebyshev GP.

Use of the GP approach model in budgeting has increased in recent years; It helps in various ways: To allocate budget by the Fars province to its cities (Namazi, [18]), allocate budget to government agencies (Azar and Najafi, [3]), in designing the optimal budget allocation model for the ministry of health to the provinces (Rajabi [21]), and to design the budgeting model of Tarbiat Modares University (Azar, et al. [2]). It is also helpful for construction of a budgeting of the Isfahan municipality (Akbari and Hoseini, [2]), and for designing a budgeting model for hospitals affiliated to Tehran University of Medical Sciences (Valipor et al. [26]) .It is also helpful for the Nigerian economic modeling (Habeeb [9]), optimal allocation of funds to the US public sector (Greenberg and Nunamaker, [8]), allocating funds to supplementary education in England (Ho et al.,[10]), and for allocating funds to various departments in Qom province (Mohammadi, et al. [14]). More of its applications are: to allocate funds to the Graduate Institute of the University of Nigeria (Dan Dan ,[7]), for budget allocation to the ministry of Health of Romania (Zamfirescu and Zamfirescu, [27]), and to design a type of budget allocation model for India's research & development organizations (Mohan and Sasikumar, [16]).

3 Methodology and Make Model

In this section, the GP's mathematical model was designed to allocate budget to gas refineries based on the criteria set in productivity. Similar criteria will be combined as an indicator and other criteria are considered as non-additive goals; the GP’s target is to achieve a satisfactory level for all of them. Some of the goals are in one direction and the goal is to reach the highest or lowest level of the target. But they cannot be combined because the scale of these criteria is different and also according to the research objectives, special inputs and outputs are chosen to identify the non-productive sector. In the Chebyshev GP, the decision maker tries to create a balance while achieving the goals and offers the most appropriate answer at the various levels of goals. There is a certain imbalance in lexicographic GP and weighted GP, which means some goals were on the verge of being achieved but other were a long way from being achieved. This property is called "ruthless optimization" (Jones & Tamiz,

[19]). Therefore, Chebyshev's technique was chosen for the research model, and normalization scheme had not been achieved because the units of the amounts of the goals are the same

Table 1: Goals of The PBB Model Based on Productivity

Goals	**Target**	**Undesirable deviation**
Combined productivity criteria (pr)	max	n_1
Revenue achieved (re)	max	n_2
Total budget (b)	min	p_3
Specific energy consumption (se)	min	p_4

If λ is the maximum deviation from the set of goals, then the ideal formula for the Chebyshev goal programming (CGP) is the following ([11]):

$$\min\ a = \lambda$$

Subject to:

$$f_q(\underline{x}) + n_q - p_q = b_q \quad , q = 1, \dots, Q$$
$$\frac{u_q n_q}{k_q} + \frac{v_q p_q}{k_q} \le \lambda \quad , q = 1, \dots, Q$$
$$\underline{x} \in F$$
$$n_q \, , p_q \ge 0 \, , q = 1, \dots, Q$$

Therefore, the Chebyshev goal programming rolling performance based budgeting (CGP-RPBB) based on productivity approach is as follows:

$$\min\ a = \lambda \tag{1}$$

Subject to:

$$w_1 n_1 \le \lambda \tag{2}$$
$$w_2 n_2 \le \lambda \tag{3}$$
$$w_3 p_3 \le \lambda \tag{4}$$
$$w_4 p_4 \le \lambda \tag{5}$$
$$\sum_{i=1}^{m} pr_i X_i + n_1 - p1 = G_1 \quad ; \ i = 1, 2, \dots, 7 \ ; \tag{6}$$
$$\sum_{i=1}^{m} re_i X_i + n_2 - p2 = G_2 \ ; \ i = 1, 2, \dots, 7 \ ; \tag{7}$$
$$\sum_{i=1}^{m} b_i X_i + n_3 - p_3 = G_3 \quad ; \ i = 1, 2, \dots, 7; \tag{8}$$
$$\sum_{i=1}^{m} se_i X_i + n_4 - p_4 = G_4 \quad ; \ i = 1, 2, \dots, 7; \tag{9}$$
$$\sum_{i=1}^{m} X_i < 1 \quad ; \ i = 1, 2, \dots, 7; \tag{10}$$
$$\sum_{i=1}^{5} w_j = 1 \quad ; \tag{11}$$
$$Lo_i \le X_i \le up_i \quad ; \ i = 1, 2, \dots, 7; \tag{12}$$

$up_i \leq rev_i \quad ; \ i = 1, 2, \dots, 7;$ (13)

$n_j \, , p_j \geq 0 \ ; \ X_i \in [0,1];$ (14)

Model indexes: i: gas refinery, j: goals;

Model decision variables: X_i: budget allocation coefficient, p_j: overachievement, n_j: underachievement;

Model parameters: pr_i: general refinery productivity, re_i: revenue achieved, b_i: total allocated budget, se_i: Specific energy consumption, lo_i: lower limit of refinery budget, up_i: upper limit of refinery budget, Rev_i: refinery's revenue and G_1 to G_4: target levels for first goal to fourth goal.

Constraints: Equation (2) to (5) are the Chebyshev model's constraints, eq. (6) to (9) are goal constraints, eq. (10) controls the sum of allocated budget, which is less than the total available budget, eq. (11) represents the sum of the weight of the goals, and eq. (12) is the lower limit and upper limit budget to be allocated to refineries. Moreover, eq. (13) controls the budget's upper limit, which must be less than the revenue and eq. (14) shows that overachievement and underachievement are positive and the budget allocation coefficient for each refinery is a closed distance between zero and one. Lower/upper limit of the budget has been extracted from the performance of previous years in Iran's gas refineries. So, they have an uncertainty for future forecasting. For fixing ambiguity, the eq. (12) is defined as fuzzy. For this purpose, the initial linear programming model follows:

$$\max f(x) = C^T X$$

Subject to:

$$AX \leq b$$
$$X \geq 0$$

The following style converts it to new model with crisp objective function and fuzzy constraint (Jones and Tamiz [11]):

$$max\, Z = \lambda$$
$$C^T X - (f_1 - f_0)\lambda \leq f_0$$

Subject to:

$$Ax + \lambda p \leq b + p$$
$$\lambda \, , x \geq 0$$

The final model of this research is called fuzzy Chebyshev goal programming roll performance-based budgeting model (FCGP-RPBB) with fuzzy restrictions and crisp objective function as follows:

$\max \quad \gamma$ (15)

Subject to:

$\lambda + \gamma(a^{max} - a^{min}) \leq a^{max}$ (16)

(Repeat equations (2) to (11) of the crisp model)

$X_i + \gamma q_i^{up} \leq up_i + q_i^{up}$ (17)

$LO_i - q_i^{lo} \leq X_i - \gamma q_i^{lo}$ (18)

(Repeat equation (13) of the crisp model)

$n_i \, , p_i \, , \gamma \, , q_i^{up} \, , q_i^{lo} \ \geq 0 \ ; \ X_i \in \{0,1\};$ (19)

Where γ is the degree of satisfaction of the objective function and the fuzzy constraints; a^{max}, a^{min}: the minimum and maximum values of the crisp model; q_i^{up}: the amount of permissible violation (fuzzy changes) of the upper limit of the allocated budget to the refineries; q_i^{lo}: The rate of permissible violation (fuzzy changes), which is the lower limit of the allocated budget to the refineries. By surveying the financial statements and performance reports of Iran's gas refineries in 2011–2015, the data required for this research was collected. Target levels were determined by interview with budget experts of these companies. The data collected was analyzed for determining performance evaluation criteria separately for each year. Excel and GAMS software were used and the average of five years was calculated as the parameters of the mathematical model. The mathematical model was also solved by the LINGO software linking to Excel.

4 Research Findings

In this section, the parameters of the mathematical model of expression are calculated, and then the results are compared with the gas refineries' actual budget for 2016.

(A) Relative importance of each goal (w_j): By utilizing the opinion of the society's experts, the goals are compared. The relative importance (weights) of the goals extracted is shown in the Table 2.

Table 2: Weight of Goals in The Objective Function

Goal number	Normalization matrix				w_j
1	0.48	0.54	0.47	0.31	0.450
2	0.24	0.27	0.32	0.38	0.303
3	0.16	0.14	0.16	0.23	0.171
4	0.12	0.05	0.05	0.08	0.076

For coupling comparisons, the rate of inconsistency (IR) is 0.038 and it is less than 10%. So, there is a reasonable balance between consistency and stability and the weights are reliable.

(B) The target levels of goals: Target level for the first goal was calculated with the single-objective optimization process by the LINGO software. But using this process determines all target levels of goals, which affects the philosophy of satisfaction and leads to optimization rather than GP (Romero, et al. [23]). The target levels of other goals are defined via field surveys and interviews with experts. The values obtained were: (G1 = 0.838), (G2 = 0.98), (G3 = 0.98) and (G4 = 0.95).

(C) The first goal: Maximizing budget allocation to refineries based on combined productivity criteria (pr) :The indicators; technical efficiency (T), revenue efficiency (R), productivity of labor (L), capital productivity (K), and total factor productivity (TFP) were selected as a combination of productivity indicators. These indices are calculated annually for five years and the average of each index is taken separately. After matching the units, we use the MADM methods to rank the refineries. MADM has several models and decision makers do not restrict themselves to just one solution (Momeni [17]), so with most popular MADM methods including SAW, TOPSIS and ELECTERE we calculated and ranked each company from the total of these indicators. The result of these methods did not give the same rank to refineries. So, the average rating method was used in their prioritization strategy and the final score in the ranking was considered as a combined productivity parameter (Pr).

Technical efficiency (E) and revenue efficiency (R): Efficiency is defined as how well an organization uses its resources to produce outputs relative to the best practice at a point of time (Pierce, [19]). This research is a comparison between units within an industry so the structural efficiency for refineries is not calculated; it is also assumed that gas refineries operate on an optimal scale. So, calculating the scale efficiency here is meaningless. In this research, DEA is used to calculate efficiency. The DEA approach has the ability to calculate efficiency, and it calculates cost efficiency, revenue efficiency, profitability and relative efficiency (profit/revenue) models(Ho et al., [10]). Here, according to research goals, measuring the technical efficiency (T) to specify the units that use the least inputs for output production and revenue efficiency (R) is done in order to measure the refiners' privileges in generating more revenue using the amount specific costs (the current budget). For the technical efficiency (E) input (raw gas) and its outputs (gas delivery to the gas transmission line (u_1), total utility products (u_2) and fuel gas (u_3)) were selected. The data with BCC and CCR models based on input-oriented and output-oriented was calculated by GAMS software and results were compared. The CCR model, with the fixed returns to scale in the calculations, was selected as the technical efficiency (E) measurement model. If the number of DMUs is less than the sum of the inputs and outputs, the CCR model will show efficiently a large number of DMUs and it is difficult to distinguish between them (Ho et al., [10]). The results of the calculations of this research showed that most refiners' score in the (E) are numerically similar and efficient. Therefore, the use of this indicator alone has a poor accuracy for refinery ratings and the use of revenue efficiency (R) is necessary to overcome it. In R, the efficiency unit is a unit that uses a certain cost for generating more revenue (Bader et al. [4]). In R, the DMU scores are calculated by the distance between each one and the efficient boundary. Like other DEA models, the efficient boundary is formed by units that have the best performance. The realized revenue is selected as the output of the revenue model (R) and the wages & salary cost (which represents the cost of manpower for realization of revenue) and the cost of depreciation (which represents the assets used to realize the revenue) are selected as inputs of the model. The collected data was analyzed by the GAMS software. Results show R has more separation between refineries and the problem of non-separation in similar cases, E, is eliminated. Productivity of labor (L), capital productivity (K), and total factor productivity (TFP): These indicators show the internal strengths and weaknesses of the DMUs and they are comparable with national targets and other industries. So, they are suitable for research purposes. The calculation of these indicators has been in accordance with the national guidelines and the oil ministry's norms.

(D) Second goal: Maximizing budget allocation to the refineries based on the revenue achieved: Revenue is one of the performance indicators. Improving productivity will mean revenue growth. Therefore, the "revenue achieved" was selected as a performance index and its average in years 2011–2015 was expressed for use in the mathematical model.

(E) The third goal: Reduce the total budget of gas refineries (b): Reducing cost is the primary aim for any budgeting model and it is one of the main goals of productivity. Therefore, "reduction of the total budget of each refinery" was chosen as the goal. To calculate the coefficients of this goal, the budget of several years was extracted from the financial statements of refineries and their average was used as parameters of this goal.

(F) Fourth goal: Reduction in specific energy consumption (SEC) in gas refinery (se): To calculate energy efficiency in gas refineries, a reliable indicator called the SEC was used.

The SEC in the refineries is calculated monthly in Gigajoules per ton (GJ/T). The information for the study period was collected from the gas refineries and the SEC average of each refinery was used for the parameters of this goal.

Table 3: Final Score of Gas Refineries among Productivity Indicators

Refinery	B ↓	Re ↑	Pr ↑	Se ↓
SPGC	0.5257	1.1623	0.0714	0.1766
FGTC	0.1366	0.8048	0.0475	0.0850
PGTC	0.0941	0.8101	0.1546	0.0331
HNGTC	0.1180	1.021	0.0954	0.1653
SGHGTC	0.0342	0.8859	0.2025	0.0787
BBGTC	0.0576	0.4277	0.2382	0.0992
IGTC	0.0338	0.5869	0.1904	0.3621

(H) The upper and lower limits $\mathbf{up_i}$ ، $\mathbf{lo_i}$ and $\mathbf{Rev_i}$: These coefficients were defined by referring to the budget documents and the actual performance of previous years in the Iranian gas refineries and use of experts' views. Budget transferred a refinery from a unit with lower productivity to a unit that can be more productive. The upper limit of the refinery budget must be less than or equal to the amount of revenue generated. There is an accumulated loss in financial statements of some refinery. It indicates that the funds allocated to it in the previous years were more than the income it generated and this is not consistent with the PBB philosophy.

5 Model Solving Results

The **FCGP-RPBB** model has crisp parameters and a single period that is obtained from the average of five periods. Its variable's number is 47 (24 main, eight goal, 15 fuzzy) and its limitation's number is 41 (22 main, four goal, 15 fuzzy).

Fuzzy model: By solving the fuzzy model for γ = 1, γ = 0, the minimum and maximum values of the initial (crisp) model were determined $a^{max} = 1, a^{min} = 0$. After interviewing the experts, permissible violations for upper/lower limit of the budget were determined at 6% of the nominal budget for each refinery ($q_i^{lo} = q_i^{up} = 0.06$). After solving the model, the degree of satisfaction of the objective function was γ = 0.96. Based on the performance in 2011–2015, refineries with the crisp and fuzzy models solved that the results of solving them are shown in Table 4. It should be noted that because of the confidentiality of information, the actual budget is not shown and the budget of the Iran's gas refineries is expressed as percentages.

Table 4: Compare Actual Budget of 2016 with New Budget

Refinery	Actual Budget	Budget in new model		Improved value	
		Crisp	Fuzzy	Crisp	Fuzzy
SPGC	0.7674	0.7353	0.7500	0.0320	0.0174
FGTC	0.0690	0.0700	0.0700	(0.0010)	(0.0010)
PGTC	0.0364	0.0400	0.0332	(0.0036)	0.0032
HNGTC	0.0617	0.0700	0.0700	(0.0083)	(0.0083)
SGHGTC	0.0156	0.0212	0.0212	(0.0056)	(0.0056)
BBGTC	0.0330	0.0315	0.0315	0.0015	0.0015
IGTC	0.0170	0.0173	0.0173	(0.0003)	(0.0003)
Sum	1.0000	0.9853	0.9932	0.0147	0.0068

As can be seen in Table 4, the mathematical model of the 2016 budget was reduced by a total of 0.68% which is more than the annual budget of some gas refineries. The optimal allocation of budget in refineries will increase financial discipline and optimal utilization of funds so that their budget goes towards standardization. In the mathematical model, the refineries received the minimum budget (equivalent to the lower limit) except for HNGTC. The budget allocated to HNGTC in the proposed model is more than the actual budget allocated to it. This is due to HNGTC's superiority in the productivity indicators compared to other refineries The re index of this company, unlike most refineries, was higher than the expected target. Therefore, in the proposed model, it has been allocated more funds than the actual budget (see equation (13)). It should be noted that despite an increase in the budget of HNGTC, the decrease in total budget was 0.68%. SPGC saw the highest decline in the PPB model. In the current method, the allocation coefficient to most refineries is more than the ratio of income to the cost. This was well controlled by the new model (for example, in the budget document of 2016, SPGC generated 62% of the revenues of the refinery but receive 77% of its budget allocation). Although the SPGC's re index is more than all refineries, its pr index is low and its negative criterion se is much lower; therefore, the mathematical model reduced the budget. Decrease of SPGC's budget is more intense than others because it is the largest (b) refinery. The rate of achievement of the expected goals is shown in Table 5.

Table 5: Rate of Achievement of Expected Goals

Goal number	**Target**	**Target level**	**Crisp**		**Fuzzy**	
			Result	**Achieved value**	**Result**	**Achieved value**
1-pr	max	0.0838	0.0832	99.34	0.0832	99.20
2-re	max	0.98	0.8518	86.91	0.8513	86.87
3-b	min	0.98	1.0366	(5.77)	1.0355	(5.66)
4-se	min	0.95	0.9656	(1.64)	0.9652	(1.60)

Achieving the goals is desired. The third (b) and fourth (se) goals are fulfilled. The best level of achievement is related to the third goal or the total budget reduction (b). The first goal (pr) is almost fully achieved. The achieved value of the second goal (re) shows a significant distance from the target level but in comparison with the actual performance it is desirable. In the financial statements of previous the years, in the gas refineries that this model has calculated based on their data, the allocation of budget to some refineries is higher than the revenue generated by them (Table 3). For example, the actual amount realization of revenue in BBGTC and IGTC is 57% and 47%, respectively and in large refineries such as FJGTC and PGTC, it is about 80%. Therefore, the percentage of realized revenue in Iranian gas refineries isn't a good situation .The actual performance of the year 2016, in SPGC, Iran's largest gas refinery, shows it is number two in achieving goals at 82% and for IGTC refinery it is 46%; therefore, the achieved value of this goal at 98% of the revenues is desired. The proposed model provides optimal answers for the expected goals. So, choosing Chebyshev's GP method for allocating budget to gas refineries is a good choice. The solution the model shows is that the target level of goals in some cases is rigorous but in some cases easy. So, it is necessary to revise the target level of goals in subsequent periods.

6 Conclusion

This research was conducted to design a model for optimal budget allocation based on productivity for Iranian gas refineries. Productivity indicators were selected for the gas refineries and the refiners' privileges in these indices were the basis for designing a mathematical model for budget allocation .This research will provide a control tool to managers by evaluating productivity in different parts of the company; it will identify lower productivity units and leverage of the budget to improve them.

In this study, the data of five seasons or time period required for the rolling budget was five consecutive years and budget suggested was for the year after. The users of this model can use five seasons (quarterly) data to allocate budget for the coming season. It is important that for applying RB, the future (seasonally or annual) budget must be allocated using performance data of the previous period. So, performance appraisal is understandable for the personnel and leads to increased organizational dynamism. The results of this study showed that the presented model provides significant improvement in the level of achievement of goals and the objective function .Saving costs and collecting funds from low-productivity units and directing them to high productivity is a strategic and important policy. Reduction of financial resources from some company due to low productivity leads to their amendment. In other words, in order to maximize productivity and attract higher funding, a healthy competition must be created among gas refineries. Therefore, budget allocation by this model in gas refineries and similar organizations will improve performance. The model presented in this study, like other models (see (Azar et al. [2]; Azar and Najafi, [3]; Mohan and Sasikumar [16]; Rahmani and Arabmazar [20]; Valipor et al., [26]; Zamfirescu and Zamfirescu [27])), improves the budget allocation and leads to better results for the organization.

The special advantages of this model include: enhancing the flexibility of budget allocation with combined rolling budget and PBB, allocation of budget-basis productivity, and measuring productivity including the comprehensive criteria that has not been seen in previous research on productivity. For design of the model, we didn't use complex mathematical formulas. So, the proposed model is easier to apply. For example, instead of using direct multi-period data in a model whose data was also collected and calculated, average of the periods was used. So, the multi-cycle model becomes a cycle model. Therefore, understanding and using the model becomes easier and less time is allocated to solve it .The results showed no improvement from the development of the crisp model to the fuzzy model. This indicates that the upper and lower limits of the budget in this study are highly accurate. However, in other studies, it may not be similar. So, always using a fuzzy model is recommended in order to reduce uncertainty.

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