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Original Research

Determining the Investment Portfolio Selection Model based on Investor Information using Multi-Criteria Decision Making in the Presence of Uncertainty

Heshmatollah Shokrian[a], Mohammad Soleimani Vareki[b, *], Reza Shahverdi[c], Mohsen Rabbani[a]

[a]*Department of Mathematics, Sari Branch, Islamic Azad University, Sari, Iran*
[b]*Department of Mathematics, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran*
[c]*Department of Mathematics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran*

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ABSTRACT

The goal of investors in forming a stock portfolio is to obtain the highest return for bearing the lowest risk and portfolio optimization is one of the most complicated problems in the field of finance and investment. It is an NP-hard problem, and in general there is no definite method in polynomial time to find an exact solution for it. In this research, to solve the problem of choosing the optimal stock portfolio, the multi-criteria decision-making method has been used under conditions of uncertainty. In order to implement the algorithm and evaluate it, the monthly returns of the Tehran Stock Exchange indices were used between 2018 and 2013. The results can be examined from two different perspectives. From an analytical and technical point of view, the results can be discussed. From a technical point of view, presenting a new technique for doing things can give the capital market participants the confidence that they can choose a stock portfolio using a new tool. From an analytical point of view, the existence of decision-making algorithms in providing the optimal portfolio is a new step that can be used in the combination of fundamental analysis and the use of dynamic stock portfolio.

1 Introduction

The activity and prosperity of the capital market in each country is one of the signs of the development of the countries, which is known internationally. In such countries, most of the investments are made through the financial markets, and the active participation of the people in the stock market will guarantee the life of the capital market and the sustainable development of the country [1]. Today, the formation of the optimal stock portfolio requires a lot of expertise and experience and is required to

* Corresponding author. Tel.: +989113544116
E-mail address: *m.soleimanivareki@sutech.ac.ir*

study and research carefully. Considering the growth, diversity and complexity of the investment environment, investors have a wide range of choices and the need to use mathematical models, software and new methods to optimize the stock portfolio is evident [2]. The activity and prosperity of the capital market in each country, one of the signs of the development of countries, is known internationally. In such countries, most of the investments are made through the financial markets, and the active participation of the people in the stock market guarantees the life of the capital market. It will be a sustainable development of the country. In the active participation of people in the society in the capital markets, the main problem that each investor faces are deciding to choose the appropriate securities and assets for investment and forming an optimal portfolio [3]. Optimization means choosing the best combination between financial assets in a way that maximizes the return of the investment portfolio and minimizes the portfolio risk. The general idea is that if investments are made in assets that are not completely correlated, the risk of those assets will neutralize the other and make it possible to get a return with less risk [4]. Despite the advantages that each of the techniques of selecting and optimizing the stock portfolio has, when the types of these methods and investment theories are examined in real world conditions, it is easy to realize that many of these methods are facing two basic problems in the operation stage. The first problem that can be noticed in relation to this topic is the assumptions that these theories are based on. If these assumptions do not represent the real conditions, they will lead to completely different results [5].

There are different types of investors, risk taking group and risk avoiding group. Risk-taking investors seek to obtain higher returns by accepting high investment risk, and risk-averse investors obtain lower returns by accepting low risk. Generally, investors seek to increase their profit and reduce risk. Logically, it is not possible for one to decrease and the other to increase. For this reason, investors accept a certain risk to earn a certain percentage of profit, or by accepting a certain rate of risk, they expect to earn a certain percentage of profit. Today, it is possible to reduce investment risk by doing some methods, or in other words, manage it and bring it to an acceptable level; In fact, by means of these methods, at a certain level of risk, it is possible to obtain the highest return or bear the minimum risk at a certain rate of profit [6]. If the investor makes a logical decision in choosing stocks, he can achieve a favorable return. What is defined as the main problem today is facing and dealing with environmental changes. Meanwhile, information agencies play a high role in order to clarify the market as much as possible. These institutions help decision makers in making their decisions by providing timely, correct, appropriate and relevant information [7].

Company rating institutes are also of this type of institutes. These institutes, by introducing the best companies in the industry, determine their position in the competitive environment based on various indicators or variables. This causes, on the one hand, the weaker companies to recognize their distance from the best and develop appropriate strategies to reach them. On the other hand, top companies strengthen their superiority by defining appropriate programs and strategies. Since users do not properly identify investment opportunities and threats without benefiting from sufficient information, providing a ranking list of companies present in the stock exchange helps the diversity and adequacy of information in the direction of market efficiency [8]. The current research was carried out with the aim of determining the investment portfolio selection model based on the investor's information using multi-criteria decision making in the presence of uncertainty. In this study, we present a novel approach for determining an investment portfolio selection model that leverages investor information through multi-criteria decision making (MCDM) techniques while addressing the inherent uncertainty in investment decisions. We develop a comprehensive framework that integrates various criteria relevant to investors' preferences, risk tolerance, and investment objectives.

2 Theoretical Foundations and Research Background

All of us make different decisions consciously and unconsciously in our personal and organizational lives. The problem of choosing the optimal stock portfolio is a problem that all investors, both real and legal, always face. The issue of stock selection involves creating a stock portfolio that maximizes the investor's utility [9]. One of the reasons for the success of individuals and organizations is making appropriate and logical decisions. It is obvious that the use of scientific methods helps people in this field. Decision making is not a linear and one-dimensional process, but a successful decision maker is the one who examines the issue of decision from different aspects and from several criteria jointly and simultaneously. use and then choose the best option according to priority while examining the various factors affecting it, if the problem to be decided is only trying to optimize one goal or criterion, it is called single criterion, but if there is more than one criterion, it is called the target of decision can be called a multi-criteria MCDM decision making problem [10]. Optimization models have always been the focus of mathematicians and industry practitioners since the era of the industrial movement in the world, especially since the Second World War. The main emphasis in classic optimization models is to have a measurement criterion (or an objective function) [11].

Table 1: An overview of previous work done

Writers	Year	Title	Results
Nazari et al [12]	2018	Investigating multi-objective portfolio optimization using colonial competition algorithm in Tehran Stock Exchange	The results of the research showed that the modified Kai model has a portfolio with a higher return than the Sharp and Kano models, and the Kano model shows less risk in its portfolio. According to the findings of the research, the application of the colonial competition algorithm in the selection and optimization of dividends is confirmed and recommended.
Abdul Rahimian and Qutb al-Dini [13]	2018	Investigating investment profit optimization using linear models and Sharpe model (Tehran Stock Exchange)	The results of the research showed that the modified Kai model has a portfolio with a higher return than the Sharp and Kano models, and the Kano model shows less risk in its portfolio.
Amin Lo et al [14]	2017	Portfolio selection model design in Tehran stock exchange using fuzzy MCADM techniques	The results obtained from the analysis using fuzzy topography method using Expert Choice software show that the company's perspective factor has the highest priority. Liquidity ratio is the second priority. The growth criterion is the third priority and finally the profitability ratio has the lowest priority
Nikkinen and Plavmäki [15]	2019	An optimization study based on artificial algorithm for predicting the performance of the stock market index	The results showed that this algorithm is more accurate than the multi-variable multi-algorithm method.

Dongmei et al, discussed comprehensive modeling for the optimal selection of stock portfolios using multi-criteria decision-making methods in companies listed on the Tehran Stock Exchange. A sample of 79 companies listed on the Tehran Stock Exchange was used to conduct this research. After simulating the data and programming them with MATLAB software, the cumulative data analysis model was performed, and 24 companies were selected. This research data was collected from the financial statements of companies listed on the Tehran Stock Exchange in 2020. The index in the Tehran Stock Exchange can be used to provide a comprehensive and optimal model for the stock portfolio; different multi-index decision-making methods (TOPSIS method), the taxonomy method (Taxonomy), ARAS method, VIKOR method, The COPRAS method and the WASPAS method can all identify the optimal stock portfolio and the best stock portfolio for the highest return [16].

Doaei et al, is developed a two-phase approach based on multi-criteria decision making and multi-objective optimization to solve the problem of optimal portfolio selection. According to the obtained results, it is observed that cash adequacy ratio with score 0.1604 is the most important criterion and operating profit with score 0.004 is the least important one. In the alternative prioritization section, it is concluded that Shraz, Shavan, Shenft and Vanft companies have a high priority for investment. In solving the mathematical model under certain conditions, it is observed that the Pareto members 152, 154 and 193 have the smallest distance from the ideal solution (0.0121) and therefore each of them can be used as the final solution. In solving the problem under uncertain conditions, numerical scenarios resulting from changes in the prioritization of companies based on the coefficient v is used in the VIKOR model. After solving the model, it is observed that the impact of different scenarios on corporate investment is not negligible and consequently investors need to pay attention to this fact [17]. Brito expected utility, entropy and variance (EU-EV) model is applied for stock selection, which can be used as preselection model for mean–variance portfolio optimization problems. Based on the EU-EV risk, stocks are ranked and the best ranked stocks with lower risk are selected in order to form subsets of stocks, which are then used to construct portfolios.

The EU-EV model is applied to the PSI 20 index, to the Euro Stoxx 50 index and to the Nasdaq 100 index. Subsets of selected stocks are analysed and their portfolios' efficiencies are compared with those of the portfolios obtained from the whole set of stocks using the mean–variance model. The results reveal that the EU-EV model is an adequate stock selection model for building up efficient portfolios with a lower number of stocks [18]. Other findings suggest that cost stickiness has a positive impact on the relationship between institutional investors and passive institutional investors with conservatism [20]. In another study provide a useful and effective tool to assist professionals and researchers in portfolio selection theory. This study, while comprehensively reviewing the literature on the subject and the developments and expansions made in the area of portfolio selection and optimization, reviews the types of problems and optimization methods [21].

Other findings research, through a regular and logical process based on the judgment method in a survey of 14 experts in the field of capital market investment and a quantitative and multivariate model of fuzzy network analysis, to assess the level of importance, ranking and refining the effective factors. Portfolio optimization was undertaken. Based on the analysis, the variables of profit volatility, return on capital, company value, market risk, stock profitability, financial structure, liquidity and survival index can be introduced as the most important factors affecting the optimization of the stock portfolio [22].

3 Research Methodology

This research is considered as applied research from the point of view of the objective and as optimization and modeling research from the nature and method point of view. In terms of gathering research information, it is a retrospective event because it used past information.

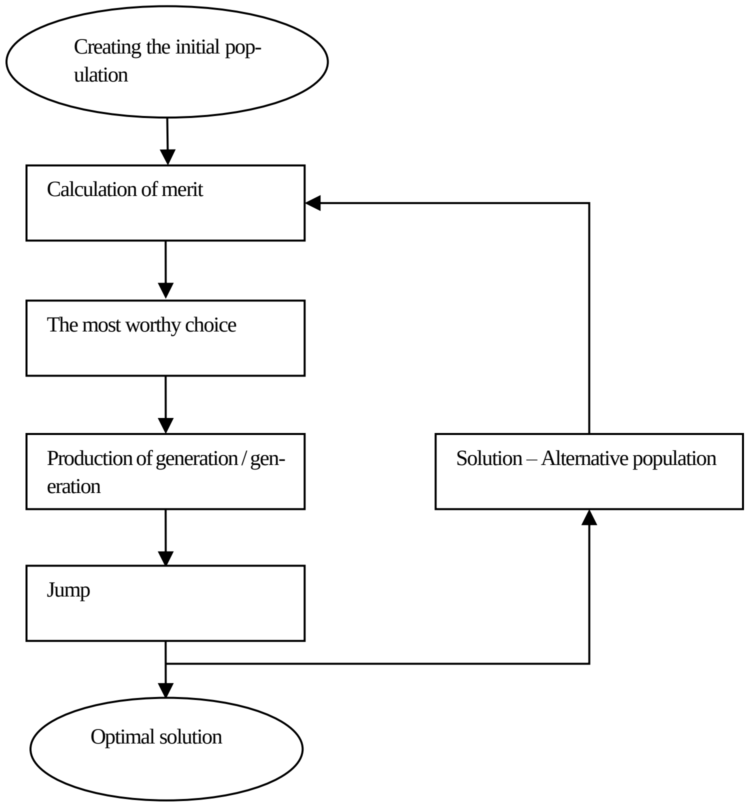

Fig. 1: Graphic Representation of the Algorithm Outline

The method of collecting information and data in this research is divided into two categories: library and field. The first category of information is related to the literature and background of the research, which is obtained by studying articles, theses, publications, magazines, books, and research done in this field. The second category includes the data obtained through the reports of the Stock Exchange Organization. The statistical population of the research includes companies active in the Tehran Stock Exchange, and a sample of 215 companies was randomly selected during the financial period of 1401 and their information was used. In this research, after the qualitative method, the quantitative method and decision-making techniques are used to rank the identified factors and criteria. Hierarchical decision-making process has been used to determine the weight and importance of each of the indicators of the optimal portfolio. In this regard, to facilitate the acquisition of knowledge and the judgment of experts and to save time, two questionnaires in the form of a 9-point Likert scale for the hierarchical decision-making process were designed using the AHP method and were asked from the experts. The data of this research includes all the stocks and their returns related to the studied companies during a financial period which was arranged first in Excel software and then using techniques and methods that include multi-criteria decision-making algorithm, particle swarm and colonial competition and analyzed by Matlab2012 software. Then they were analyzed by Sharpe test and t-test. Each of the variables is calculated as follows. The return of the portfolio is made and the monthly return of the portfolio is also obtained from the calculation of the weighted average of the monthly returns of investment companies as relation (1):

$$r_{pt} = \sum_{i=1}^{n} r_i w_1 \quad (1)$$

r_i Return of investable companies in month t, $w_1$weight of investable companies in portfolio t. In addition, due to the fact that the returns of investing companies and investors are calculated continuously, arithmetic average is used to calculate the average return (2):

$$\overline{r_p} = \frac{\sum i = 1\ r_p}{n} \tag{2}$$

Monthly returns of the investment company $\overline{r_p}$ market return: To calculate the market return (r_m) in the research, the price index and cash return have been used as follows (3):

$$r_m = lnp_{m,t} - lnp_{mt-1} \tag{3}$$

p_{mt}The price index and cash yield at the end of the current month, $p_{mt-1}$the price index and cash yield at the end of the previous month, $r_m$are obtained from the average market yield.

Here, r_m the average market return is obtained by calculating the arithmetic mean of monthly market returns.

Total risk: Total risk is obtained from the deviation of portfolio returns from the average portfolio return [19].

Market risk is obtained from the deviation of market returns from the average market return (4).

$$\sigma_p^2 = \frac{\sum_{t=1}^{n}(r_{pt-\overline{r_p}})}{n-1} \tag{4}$$

$\mathrm{r_{pt}}$ Monthly returns of portfolio P in period t, $\mathrm{\bar{r}_p}$average monthly returns and market risk $\sigma_m$is calculated similarly to total risk.

Sharp index: Professor Sharp conducted a research on 34 mutual investment funds during the years 1954 to 1966 to evaluate the performance and achieved a standard known as the reward-to-variability ratio (RVAR).

This measure shows the excess return of the portfolio against each unit of the total risk. In this indicator, a higher RVAR ratio indicates a better performance and is obtained from the following relationship (5):

$$RVAR = \frac{\overline{TR_p - \overline{R_f}}}{SD_p} = \frac{Excess\ Return}{Risk} \tag{5}$$

where TR_p the average return of the total portfolio during a period of time $\overline{R_f}$ is the average rate of return without risk during a period of time SD_p, the standard deviation of the return of the portfolio during a period of time is the excess return [17-19].

3.1 Statistical model

If the assets of the portfolio have average and variance returns, then the return of this portfolio will be equal to (6):

$$\mu = E(R) = E\left[\sum_{i=1}^{n} w_i R_i\right] = \sum_{i=1}^{n} w_i E(R_i) = \sum_{i=1}^{n} w_i \mu_i \tag{6}$$

And the return variance of this portfolio will be equal to (7):

$$\sigma^2(R) = E[R-\mu]^2 = E\left[\left(\sum_{i=1}^{n} w_i (R_i - \mu_i)\right)\right] \tag{7}$$

$$= E\left[\sum_{i=1}^{n}\sum_{i=1}^{n} w_i w_j (R_i - \mu_i)(R_i - \mu_i)\right]$$

$$= \sum_{i=1}^{n}\sum_{i=1}^{n} E[w_i w_j (R_i - \mu_i)(R_i - \mu_i)]$$

$$= \sum_{i=1}^{n}\sum_{i=1}^{n} w_i w_j Cov(R_i R_j)$$

$i = 1\ j = 1$

If the weight matrix and the covariance matrix are as follows (8):

$$w = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}, \sum = [\sigma_{ij}] = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix} \tag{8}$$

Then we will have the return and variance of the portfolio as (9):

$$E(R) = w^t \mu \tag{9}$$

$$E(R) = R^2\mu = w^t \sum w$$

Since according to Markowitz's theory, people seek to maximize their utility at the end of the period, and considering that investors, considering the two factors of return and risk, seek to maximize their return with the least risk or minimize their risk with are the maximum returns, so the optimal portfolios can be obtained in three ways that all have the same answers. These three ways of solving the problem are as follows (10):

$$\begin{cases} \min \quad w^t \sum w \\ subject\ to:\ w^t\ \ \mu\ \geq\ \mu^* \end{cases}$$

(10)

$$\begin{cases} \max \quad w^t\ \ \mu \\ subject\ to:\ w^t\ \ \sum w \leq \sigma^{2*} \end{cases}$$

$$Max \quad w^t\mu - w^t \sum w$$

μ^*: The expected return of the investor, which can be the return of the index portfolio.

σ^{2*} : The most expected risk of the investor that can be considered as the index portfolio risk.

3.2 Particle population algorithm model (PSO)

In this section, the components of the PSO model are introduced as (11):

$$f(x_1, x_2, x_3, \ldots\ldots, x_n) = f(x)$$

$$x_i^{t+1} = x_i^t + \vartheta_i^{t+1} with\ x_i^o \sim U(x_{min}, x_{max})$$

$$P_{best,1}^{t+1} = \begin{cases} P_{best,i}^t \\ x_i^{t+1} \end{cases} \quad \begin{cases} if\ f(x_i^{t+1}) < P_{best,i}^t \\ if\ f(x_i^{t+1}) \geq P_{best,i}^t \end{cases}$$

(11)

$$G_{best} = min\{P_{best,i}^t\}, where\ i\ \in [1, \ldots\ldots n] and\ \ n > 1$$

$$\vartheta_{ij}^{t+1} = \vartheta_{ij}^t + c_1 r_{1j}^t [P_{best,i}^t - x_{ij}^t] + c_1 r_{2j}^t [G_{best} - x_{ij}^t]$$

3.3 Colonial Competition Algorithm (CCA)

After introducing the components of the PSO method in 3.2, in this section, the CCA approach introduced as follows relations (12):

$$\begin{cases} f_1(x_1, x_2, x_3, \ldots\ldots, x_n) = 0 \\ f_2(x_1, x_2, x_3, \ldots\ldots, x_n) = 0 \\ f_3(x_1, x_2, x_3, \ldots\ldots, x_n) = 0 \end{cases}$$

(12)

$$cos = f(country) = f(x_1, x_2, x_3, \ldots\ldots, x_n$$

$$C_n = c_n - max\{c_i\}$$

$$p_n = \frac{C_n}{\sum_{i=1}^{Nimp} C_i}$$

$$N0.C_n = round(p_{n.N_{col}})$$

$$x \sim u(0, \beta \times d), \qquad \beta > 1$$

$$N.T.C_n = T.C_n - \max\{T.C_i\}$$

$$p_{pn} = \left[\frac{N.T.C_n}{\sum_{i=1}^{n_{imp}} N.T.C_i}\right]$$

$$p = [p_{p1} p_{p2} p_{ps} \ldots . p_{pnimp}]$$

4 Finding and Results

4.1 Portfolio Risk

According to the formation of different portfolios, the risk of the portfolios is as follows. Using each of the algorithms used in this research, 50 stock portfolios have been formed. These portfolios are created according to the efficient frontier at different risk levels. The risk of the portfolios is calculated from the following formula for each portfolio.

$$\sum_{i=1}^{n}\sum_{i=1}^{n} w_i w_j \sigma_{ij}) \tag{13}$$

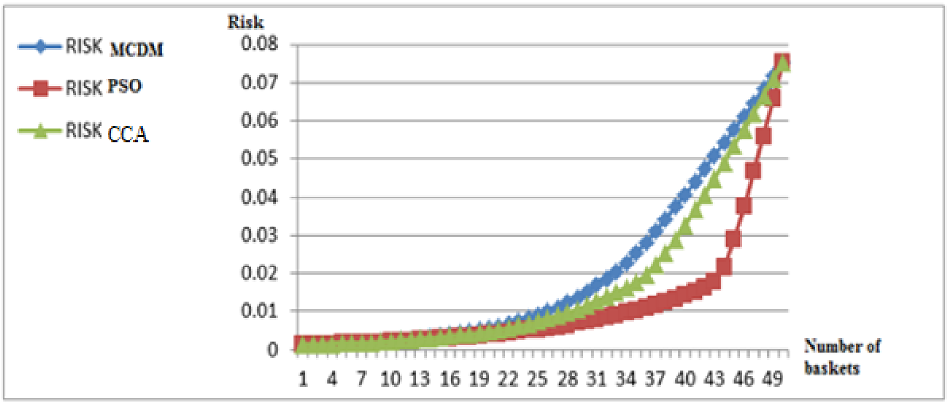

Fig. 2: Risk level

Table 1: Risk Calculation of Portfolios

RISK			RISK		
CCA	PSO	MCDM	CCA	PSO	MCDM
0.008188	0.005762	0.010076	0.001555	0.001555	0.001555
0.008952	0.006141	0.011128	0.001564	0.001561	0.001564
0.009792	0.006548	0.012335	0.001587	0.001578	0.001589
0.010705	0.006985	0.013746	0.001626	0.001607	0.00163
0.011696	0.007456	0.015308	0.001684	0.001649	0.001694
0.012752	0.007962	0.016985	0.001762	0.001703	0.001781
0.013871	0.008508	0.018749	0.001861	0.001771	0.001889
0.015074	0.009088	0.020653	0.00198	0.001855	0.002019
0.016388	0.0097	0.022903	0.002116	0.001956	0.002167
0.017903	0.010347	0.025445	0.002271	0.002074	0.002335
0.019824	0.011036	0.028214	0.002446	0.002205	0.002527
0.02236	0.011778	0.031165	0.002643	0.00235	0.00275
0.025463	0.012565	0.034252	0.00286	0.002509	0.003009
0.028955	0.013399	0.03744	0.003101	0.002683	0.003301
0.032711	0.014275	0.040706	0.003365	0.002872	0.00362
0.036651	0.015189	0.044033	0.003648	0.003075	0.00396
0.040721	0.016198	0.047407	0.003948	0.003289	0.004318
0.044885	0.017708	0.05082	0.004262	0.003513	0.00469
0.049121	0.021797	0.054264	0.004586	0.003746	0.005098
0.05341	0.02895	0.057733	0.004933	0.003987	0.005565
0.057741	0.037452	0.061224	0.005331	0.004238	0.006112
0.062105	0.046569	0.064732	0.005784	0.004506	0.006734
0.066496	0.056002	0.068255	0.006302	0.004791	0.007441
0.070908	0.065614	0.071791	0.006875	0.005093	0.008234
0.075338	0.075338	0.075338	0.007499	0.005412	0.009118

The risk created in the corresponding baskets in the multi-criteria decision-making method and the particle swarm algorithm and the colonial competition algorithm in baskets 1 to 22 have the same risk, in the same baskets from 25 to 50, the multi-criteria decision-making method has the highest risk, and the particle swarm algorithm has the lowest risk. The risk and algorithm of colonial competition has a medium risk. It is expected that the yield behavior will be the same.

Return of the portfolios: The return of the portfolio is made and the monthly return of the portfolio is also obtained from the calculation of the weighted average of the monthly returns of investment companies as relation (14):

$$r_{pt} \sum_{i=1}^{n} r_i w_1 \tag{14}$$

r_i, Return of investable companies in month t, w_1 weight of investable companies in portfolio t. In addition, due to the fact that the returns of investing companies and investors are calculated continuously, arithmetic average is used to calculate the average return: $\overline{r_p} = \frac{\sum_{i=1}^{n} r_p}{n}$

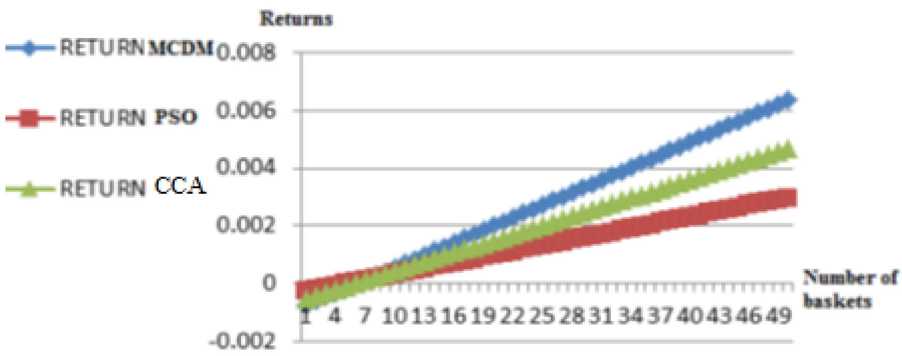

Fig. 3: Efficiency level

The efficiency of the baskets is linear in all cases, and the multi-criteria decision-making method has the highest efficiency. On the other hand, the colonial competition algorithm has had the second position in the creation. The last place is given to the particle swarm algorithm. As mentioned earlier, the risk behavior is repeated here as well.

The efficient frontier of each algorithm using their stock portfolio is given below.

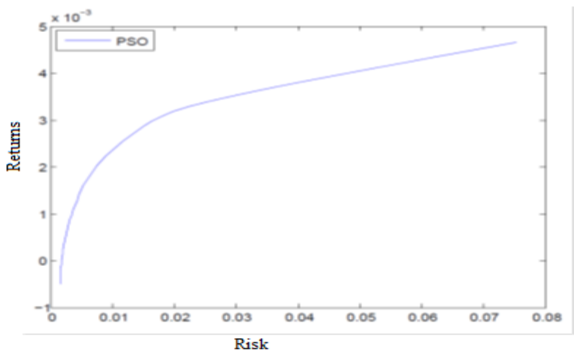

Fig. 4: Efficiency Limit of Particle Swarm Algorithm

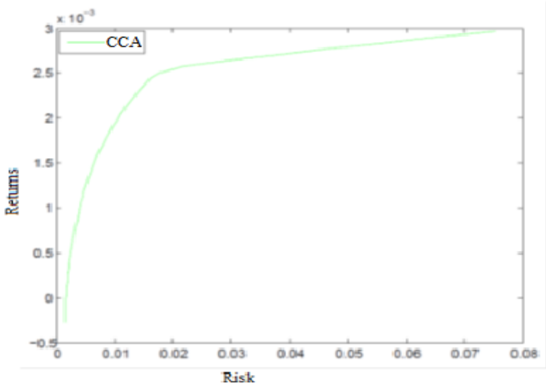

Fig. 5: Efficiency Frontier of Algorithm a of Colonial Competition

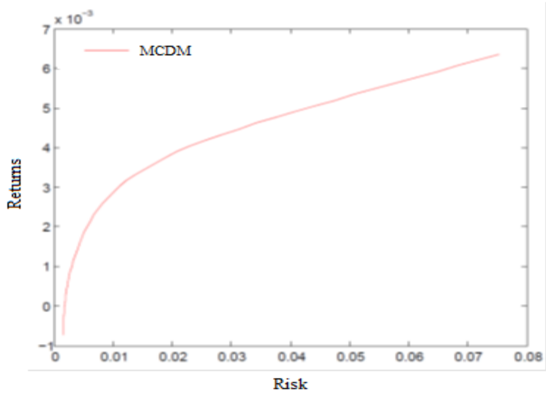

Fig. 6: Efficiency Limit of Multi-Criteria Decision-Making Method (MCDM)

Drawing the efficient frontier for each of the different methods means that the corresponding algorithm can be used in the presentation and optimization of the portfolio. Providing the efficient frontier by any of the used models can be used for a group of investors. To clarify the status of each of the stock portfolios, a pairwise comparison has been used. To compare two efficient frontiers, it is compared by drawing two efficient frontiers.

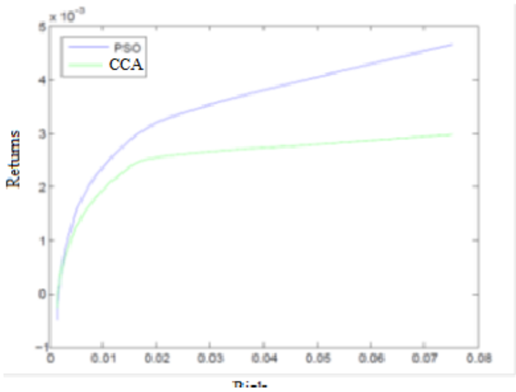

Fig. 7: Comparison of Particle Swarm Algorithm Performance and Colonial Competition

Table 2: Calculation of Efficiency of Portfolio 5 Efficient Frontiers

RETURN			RETURN		
CCA	PSO	MCDM	CCA	PSO	MCDM
0.002129	0.001379	0.00288	-0.00051	-0.00028	-0.00074
0.002235	0.001445	0.003024	-0.0004	-0.00021	-0.0006
0.00234	0.001511	0.003169	-0.0003	-0.00014	-0.00045
0.002446	0.001577	0.003314	-0.00019	-7.80E-05	-0.00031
0.002551	0.001643	0.003459	-8.63E-05	-1.17E-05	-0.00016
0.002657	0.00171	0.003604	1.92E-05	5.45E-05	-1.60E-05
0.002762	0.001776	0.003748	0.000125	0.000121	0.000129
0.002868	0.001842	0.003893	0.00023	0.000187	0.000274
0.002973	0.001908	0.004038	0.000336	0.000253	0.000418
0.003079	0.001974	0.004183	0.000441	0.000319	0.000563
0.003184	0.002041	0.004328	0.000547	0.000385	0.000708
0.00329	0.002107	0.004472	0.000652	0.000452	0.000853
0.003395	0.002173	0.004617	0.000758	0.000518	0.000997
0.003501	0.002239	0.004762	0.000863	0.000584	0.001142
0.003606	0.002305	0.004907	0.000969	0.00065	0.001287
0.003712	0.002372	0.005051	0.001074	0.000717	0.001432
0.003817	0.002438	0.005196	0.00118	0.000783	0.001577
0.003923	0.002504	0.005341	0.001285	0.000849	0.001721
0.004028	0.00257	0.005486	0.001391	0.000915	0.001866
0.004134	0.002636	0.005631	0.001496	0.000981	0.002011
0.004239	0.002703	0.005775	0.001602	0.001048	0.002156
0.004345	0.002769	0.00592	0.001707	0.001114	0.002301
0.00445	0.002835	0.006065	0.001813	0.00118	0.002445
0.004556	0.002901	0.00621	0.001918	0.001246	0.00259
0.004661	0.002968	0.006355	0.002024	0.001312	0.002735

Comparing the particle swarm algorithm and the colonial competition algorithm, it can be easily seen that the model based on the particle swarm algorithm has been able to create a higher performance and efficiency limit than the colonial competition algorithm.

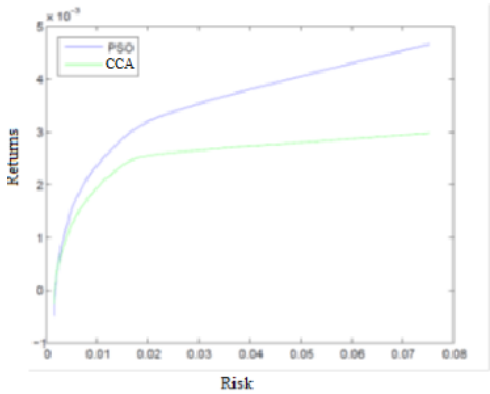

Fig. 8: Comparison of Efficiency of Multi-Criteria Decision-Making Method and Particle Swarm

Comparing the particle swarm algorithm and the multi-criteria decision-making method, it can be easily seen that the model based on the multi-criteria decision-making method has been able to create a higher performance and efficiency limit than the particle swarm algorithm.

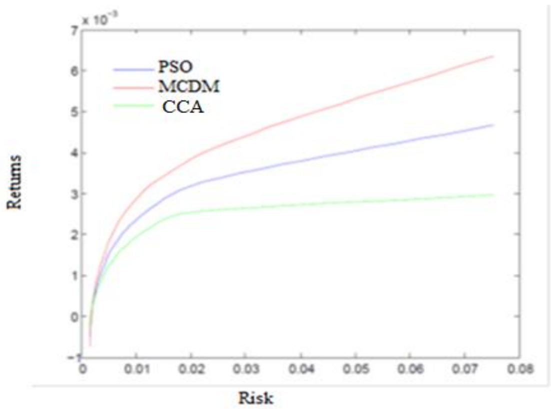

Fig. 9: Comparison of Efficiency of MCDM, Colonial Competition Algorithm and Particle Swarm

In another view, it is possible to compare all three efficient frontiers, among which the multi-criteria decision-making method has the best possible performance.

Test of the First Hypothesis: Choosing the stock portfolio using the multi-criteria decision-making method can lead to the formation of an optimal portfolio. Based on the drawing of the efficient frontier, considering the fact that the efficient frontier of the portfolios formed by the multi-criteria decision-making method is higher than the colonial competition and particle swarm algorithm, it is possible to accept the first hypothesis of the research based on this issue that the selection of the stock portfolio using the decision-making method Using multiple criteria can lead to the formation of an optimal portfolio at the same risk levels. This claim has been made on the basis that the higher the efficient frontier is, the better the performance of the model in choosing the stock portfolio. This can be clearly seen in Figure 8.

Test of the Second Hypothesis: Choosing a stock portfolio using the colonial competition algorithm can lead to the formation of an optimal portfolio. Based on the drawing of the efficient frontier, considering the fact that the efficient frontier of the portfolios formed by the multi-criteria decision making method is higher than the colonial competition algorithm and the particle swarm algorithm, it is possible to accept the second hypothesis of the research based on this issue that the selection of the stock portfolio using the algorithm Colonial competition can lead to the formation of an optimal portfolio at the same levels of risk rejection. Like the first hypothesis, this claim has been made on the basis that the higher the efficient frontier is, the better the performance of the model in choosing the stock portfolio. This can be clearly seen in Figure 8. And the efficient frontier due to the multi-criteria decision-making method is higher than other models.

Test of the Third Hypothesis: Choosing a stock portfolio using the particle swarm algorithm can lead to the formation of an optimal portfolio. In this hypothesis, as well as other hypotheses based on the drawing efficient frontier, considering the fact that the efficient frontier of the baskets formed by the multi-criteria decision-making method is higher than the particle swarm algorithm and the colonial competition algorithm, it is possible to accept the third hypothesis of the research based on this issue that selection The stock portfolio using the particle swarm algorithm can lead to the formation of an optimal portfolio at the same levels of risk rejection.

The fourth Hypothesis Test: There is a significant difference between the returns of the stock portfolio based on each of the multi-criteria decision-making methods, the colonial competition algorithm, and the particle swarm.

H0: There is no significant difference between the portfolio returns of the two algorithms.

H1: There is a significant difference between the returns of the stock portfolio of the two algorithms.

Table 3: Turns of the Stock Portfolio Based on Each of the Multi-Criteria Decision-Making Methods

	Pairwise difference					T	Significance level
	Average	Standard deviation	Standard deviation of the mean error	Difference			
				Low level	High level		
MCDM – PSO	0.0014618	0.0011455	0.000162	0.0011363	0.0017874	9.024	0
MCDM – CCA	0.0007309	0.0005727	0.000081	0.0005682	0.0008937	9.024	0
PSO – CCA	-0.0007309	0.0005727	0.000081	-0.0008937	-0.0005681	-9.024	0

Considering that the significance level of sig is less than 0.5 for all three, then the hypothesis H1 that there is a significant difference between the portfolio returns of the two algorithms is accepted and the opposite hypothesis H0 is rejected.

H0: There is no significant difference between the portfolio risks of two algorithms.

H1: There is a significant difference between the stock portfolio risks of the two algorithms.

Table 4: Portfolio Returns of the Two Algorithms

Paired Samples Test									
		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	MCDM - PSO	.0087091	.0105448	.0014913	.0057123	.0117059	5.840	49	.000
Pair 2	MCDM - CCA	.0026361	.0029755	.0004208	.0017904	.0034817	6.264	49	.000
Pair 3	PSO - CCA	-.0060731	.0081963	.0011591	-.0084024	-.0037437	-5.239	49	.000

Considering that the significance level of sig is less than 0.5 for all three, then the hypothesis H1 that there is a significant difference between the stock portfolio risks of the two algorithms is accepted, and the opposite hypothesis H0 is rejected.

4.6 Sharpe Index

After forming the baskets, the baskets can be compared with each other using the Sharpe index.
As mentioned in the efficient frontier review, it can be seen that in all portfolios with positive returns, the value of the Sharpe index for the portfolio formed by the multi-criteria decision-making method is higher than other algorithms. The above Table contains three columns, each of these columns examines one of the methods of selecting the stock portfolio. The output results due to risk and return have been calculated using the Sharpe formula for each of the baskets separately. It is important that even if the value of the Sharpe index is higher for a portfolio, the said portfolio has a better performance. The above Table has shown that this index has not been able to perform properly in separating the models from each other, although this issue can be considered as one of the limitations of the research.

Table 5: Sharpe Index Results for Three Algorithms

portfolios	MCDM	PSO	CCA	portfolios	MCDM	PSO	CCA
1	-0.4758	-0.17784	-0.32682	26	643.0248	643.0248	643.0248
2	-0.38048	-0.13474	-0.25758	27	639.3002	640.5191	639.5255
3	-0.28341	-0.09134	-0.18731	28	629.2876	633.6194	630.123
4	-0.18742	-0.0485	-0.11794	29	613.3109	622.1728	614.9927
5	-0.09493	-0.00713	-0.05125	30	590.38	606.4965	593.9986
6	-0.00899	0.031977	0.010912	31	561.5951	587.1809	567.6535
7	0.068155	0.068133	0.06702	32	529.2642	564.653	537.3716
8	0.135524	0.10075	0.11628	33	495.4104	539.146	505.0933
9	0.193053	0.129368	0.158629	34	461.4669	511.185	472.5185
10	0.241132	0.15395	0.19425	35	428.1981	482.1764	440.2713
11	0.280097	0.174805	0.223483	36	395.6635	453.4648	408.7834
12	0.310066	0.192222	0.246791	37	363.6275	425.5591	378.3993
13	0.331528	0.206457	0.264898	38	332.3619	398.6441	349.6104
14	0.346068	0.217705	0.278343	39	302.9638	372.7155	322.4581
15	0.355581	0.226411	0.287867	40	276.2739	348.1584	297.1723
16	0.361562	0.233037	0.294419	41	252.5144	325.237	274.087
17	0.365126	0.237998	0.298767	42	231.586	304.0647	253.2616
18	0.367057	0.241658	0.301539	43	213.2292	284.6627	234.6292
19	0.366079	0.244294	0.303214	44	196.1623	266.9497	218.0346
20	0.361349	0.246143	0.303287	45	179.6868	250.8238	202.7095
21	0.352735	0.247192	0.30046	46	163.6232	235.9724	187.593
22	0.341617	0.24719	0.295146	47	148.4927	221.9443	172.8873
23	0.328626	0.246277	0.287649	48	134.3882	208.717	158.6898
24	0.31455	0.244681	0.279018	49	121.4414	196.348	145.4621
25	0.299961	0.242473	0.269873	50	109.6779	184.7603	133.3597

5 Discussion and Conclusions

The selection of stock portfolios using decision-making algorithms are at a high level of confidence in terms of accuracy, which can be used to examine a large volume of stocks and finally select the optimal stock portfolios with the lowest risk and the highest return. The difference that this research has with similar researches that have been done in this field is that the efficiency limit of the portfolios has been measured and at the same risk levels, the performance of the portfolios of the algorithms has been compared. It was observed that at the same risk level, multi-criteria decision-making algorithm, particle swarm, colonial competition has more efficiency and are more efficient. According to the comparison chart of three algorithms, we come to the conclusion that the choice of algorithm does not differ much at low levels of risk, but at high levels of risk, first multi-criteria decision making and then particle swarm algorithm and colonial competition give better answers. The results can be examined from two different perspectives. From an analytical and technical point of view, the results can be discussed. From a technical point of view, presenting a new technique for doing things can give the capital market participants the confidence that they can choose a stock portfolio using a new tool. From an analytical point of view, the existence of decision-making algorithms in providing the optimal portfolio is a new step that can be used in the combination of fundamental analysis and the use of dynamic stock portfolio.
According to the obtained results, the following suggestions can be made: 1- Due to the high importance of country risk in choosing an investment portfolio, investors should be able to control the stock portfolio according to the risk and uncertainties imposed on them by the market; therefore, in order to reduce the effects of the country's problems, such as sanctions and exchange rates, it is necessary for companies to choose stocks that

have the least impact on the country's problems. 2- Due to the high importance of market profit in choosing an investment portfolio, investors should be aware that some products do not earn the expected return in the whole market in a certain period or under special conditions; therefore, it is necessary for investors to consider this issue and choose products for their stock portfolio that have not suffered losses in the past few periods and have given the necessary amount of profit to their shareholders by referring to the entire market and evaluating its profitability. 3- When a product has experienced a decrease in added value in previous periods, its production has decreased, or the possibility of investing in that product has decreased, it may face losses in future periods.

Also, according to the results obtained in this research, suggestions for future research can be made:

1- It is suggested that in future research, other criteria such as social, economic, etc. criteria should be investigated in the selection of the investment portfolio under conditions of uncertainty.

2- It is suggested that in future researches, this research should be done by structural equation modeling to present the model.

3- It is also suggested that in future researches, this research should be carried out by the ant algorithm due to its high accuracy in choosing the investment portfolio.

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