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A New Method for Allocating Fixed Costs with Undesirable Data: Data Envelopment Analysis Approach

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ABSTRACT

Allocating fixed costs with undesirable data has recently been one of the most important issues for managers to discuss. Lack of attention to undesirable data may lead to incorrect cost allocation. Considering and determining undesirable inputs and outputs, the data envelopment analysis (DEA) technique can be significantly helpful in determining the cost allocation strategy. Inputs and outputs are divided into two desirable and undesirable groups. Obviously, desirable inputs and undesirable outputs must be reduced and undesirable inputs and desirable outputs must be increased to improve performance. This manuscript presents two strategies for allocating fixed costs with undesirable data. In the first strategy, each decision-making unit (DMU) first determines the minimum and maximum shares that it can receive from the fixed resources while the efficiency of that DMU and other DMUs remains the same after receiving the fixed resources. Finally, the decision maker chooses the fixed cost for each DMU between the minimum and maximum cost values proposed. In the second strategy, the allocation of fixed costs is done using the CCR multiplicative model with undesirable data. The effectiveness of both methods is examined by an applied study on commercial banks.

1 Introduction

In 1978, with the introduction of the CCR model by Charnes et al. [1], DEA Technique was put under consideration. The CCR model determines the relative efficiency score for each DMU by receiving multiple inputs and outputs. Charens et al. [1] used the constant return to scale technology to present the CCR model. In 1984, Banker et al. [2] developed the CCR model into the BCC model, with a variable return to scale technology. Then other models were introduced by researchers to calculate efficiency [3-6]. There may be undesirable factors in the inputs and outputs in some assessments through DEA. Such as evaluating environmental issues, evaluating the performance of banks, combined cycle

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power plants and so on. Liu et al. [7] have developed envelopment models to evaluate relative performance with undesirable data. Izadikhah et al. [8] have proposed linear models for calculating the upper and lower bounds of efficiency in a two-stage network. Besides, they have proposed a model for calculating the overall performance of DMUs in the network. They then extend these models to stochastic data with undesirable data. Shi et al. [9] have proposed a network data envelopment analysis model based on slack variables (SBM-NDEA) with undesirable outputs to evaluate the performance of production processes that have a complex structure including series and parallel processes. Moghadas et al. [10] have studied the sustainable supply chain with undesirable outputs. Wu et al. [11], Zhao et al. [12], used data envelopment analysis with undesirable data on environmental issues in China. In the last two decades, fixed cost allocation using DEA has been considered.

Fixed cost is shared by all DMUs in shared infrastructure. Cook and Kress [13] first raised the issue of fixed cost allocation in 1999. Cook and Kress [13] have applied the two principles of efficiency invariance and Parato-Minimality of input for fixed cost allocation. In 2004, Jahanshahloo et al. [14], Using only the principle of efficiency invariance, proposed a formula that allocates a fixed cost without having to solve the model. Cook and Zhu [15], Lin [16], did the fixed cost allocation using the principle of efficiency invariance. Most studies have focused on fixed cost allocation while maintaining one of two conditions: efficiency invariance or efficiency improvement. Ghasemi et al. [17] Have introduced a new method to allocate resources and targets based on a common set of weights. Si [18] has used the proportional distribution method to allocate costs. One of the concepts used to allocate fixed costs is cross-efficiency. Du [19], and sharafi et al. [20] have done a fixed cost allocation using the concept of cross-efficiency. Beasley [21] has explored the issue of fixed cost allocation with the aim of maximizing efficiency. Jahanshahloo et al. [22] have performed fixed cost allocation using a set of common weights and efficiency invariance. Feng Li et al. [23] have used the principles presented by Jahanshahlu et al. for fixed resource allocation and target setting. An et al. [24] and Li et al. [25] have allotted fixed costs in the network using game theory. Fixed cost allocation using the principle of network performance reliability has been considered by an et. al. [26], Li et al. [27] have examined the allocation of fixed costs with undesirable data. Hosseinzadeh Lotfi et al. [28] Have introduced a maximum-minimum model for fixed cost allocation. Using the concept of degree of satisfaction, Li et al. [29] have introduced a maximum-minimum model with a fixed cost allocation algorithm. Chu et al. [30] and khodabakhshi et al. [31] have introduced a fixed cost allocation by introducing a maximum-minimum model using the common weight approach. In this manuscript, the issue of fixed cost allocation with two different strategies is proposed.

In the first strategy, each DMU determines the minimum and maximum value that can contribute to the payment of fixed costs with undesirable data, so that the efficiency score of itself and other DMUs after allocating a fixed cost is equal to the efficiency score before allocating the cost. After solving the proposed models, each DMU proposes an interval to participate in the payment of fixed costs. Finally, the decision maker chooses the fixed cost value for each DMU between the minimum and maximum proposed cost value. Li et al. [29], after determining the minimum and maximum fixed cost allocation values, used an algorithm to determine the weights related to each of the costs. In this manuscript, however, we carry out our cost allocation with the assumption of equal weights. In the second strategy, the allocation of fixed costs with undesirable data is done using the CCR multiplicative model. The structure of the paper is as follows: In the second part, the primary concepts of data envelopment analysis are briefly presented. The third section presents some proposed methods with two approaches of fixed cost allocation using the efficiency reliability principle and fixed cost allocation using the CCR multiplicative model. Finally, the proposed models are reviewed using an applied study, and then conclusions and suggestions for future research are presented in section five.

2 Basic concepts

In 1978, Charnes et al. [1] introduced the CCR model to calculate the efficiency of DMUs consistent with multiple inputs and outputs. If n of the DMUs under evaluation with the input vector $x_{ij}(i = 1,\dots,m)$ produce the output vector $y_{rj}(r = 1,\dots,s)$, then the relative efficiency of the unit under evaluation using the CCR fractional model can be calculated as follows:

$$
\begin{aligned}
&\max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\
&s.t: \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad j=1,\dots,n \qquad (1)\\
&u_r, v_i \ge \varepsilon \qquad r=1,\dots,s \qquad i=1,\dots,m
\end{aligned}
$$

u_r and v_i represent the relative importance of input and output vectors, respectively. Using the Charnes-Cooper transformation [28], the fractional model above is transformed into the following linear model, which is known as the multiplier CCR model:

$$
\begin{aligned}
&\delta_o^* = \max \sum_{r=1}^{s} u_r y_{ro} \\
&s.t: \sum_{i=1}^{m} v_i x_{io} = 1 \qquad (2)\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j=1,\dots,n \\
&u_r, v_i \ge \varepsilon \qquad r=1,\dots,s \qquad i=1,\dots,m
\end{aligned}
$$

2.1 Defenition: (CCR-Efficiency)

DMU_o is CCR-efficient if $\delta_o^* = 1$ and there exists at least one optimal (u^*, v^*) with $u^* > 0$ and $v^* > 0$ Otherwise, DMU_o is CCR-inefficient.

If the input vector is divided as $X = (X^D, X^{UD})$ and the output vector as $Y = (Y^D, Y^{UD})$, that X^D and Y^D are desirable input and output vectors, respectively, and X^{UD} and Y^{UD} are undesirable input and output vectors, respectively. Model 1 can be rewritten as follows:

$$
\begin{aligned}
&\max \frac{\sum_{r\in D} u_r y_{ro}^{D} + \sum_{i\in UD} v_i x_{io}^{UD}}{\sum_{i\in D} v_i x_{io}^{D} + \sum_{r\in UD} u_r y_{ro}^{UD}} \\
&s.t: \frac{\sum_{r\in D} u_r y_{rj}^{D} + \sum_{i\in UD} v_i x_{ij}^{UD}}{\sum_{i\in D} v_i x_{ij}^{D} + \sum_{r\in UD} u_r y_{rj}^{UD}} \le 1 \qquad j=1,\dots,n \qquad (3)\\
&u_r, v_i \ge \varepsilon \qquad r=1,\dots,s \qquad i=1,\dots,m
\end{aligned}
$$

Model (3) becomes a linear programming problem as follows by Charnes-Cooper transformations [32] and changing the appropriate variable:

$$
\begin{aligned}
&\theta_o^* = \max\left(\sum_{r\in D} u_r y_{ro}^{D} + \sum_{i\in UD} v_i x_{io}^{UD}\right) \\
&s.t: \sum_{i\in D} v_i x_{io}^{D} + \sum_{r\in UD} u_r y_{ro}^{UD} = 1 \qquad (4)\\
&\sum_{r\in D} u_r y_{rj}^{D} + \sum_{i\in UD} v_i x_{ij}^{UD} - \sum_{i\in D} v_i x_{ij}^{D} - \sum_{r\in UD} u_r y_{rj}^{UD} \le 0 \qquad j=1,\dots,n \\
&u_r, v_i \ge \varepsilon \qquad r=1,\dots,s \quad i=1,\dots,m
\end{aligned}
$$

Solving model (4) n times, the efficiency score of all DMU is determined by undesirable data. The optimal value of the objective function is always greater than zero and less than one.

3 Fixed costs allocation with undesirable data

This section presents a fair allocation of fixed costs with undesirable data through two strategies. First, a model is presented by the efficiency invariance principle, in which each DMU specifies the minimum and maximum values that can contribute to the payment of fixed costs. The decision maker then chooses the fixed cost value of each DMU between the minimum and maximum proposed values. In the second strategy, fixed costs are allocated with undesirable data using a fixed CCR model.

To reduce fixed costs, many organizations, such as banks, work together. Suppose a DMU consumes the input vector x and produces the output vector y. the input vector is divided as $X=(X^{D},X^{UD})$ and the output vector as $Y=(Y^{D},Y^{UD})$. We have used U superscript to show the favorable inputs and outputs and UD superscript to show the undesirable inputs and outputs. Furthermore, assume that an organization has F fixed costs that want to equitably distribute among its subsidiaries. Let f_j ($j = 1,\dots,n$) be the variable corresponding to the allocated fixed costs. It is obvious that

$$\sum_{j=1}^{n} f_j = F$$

3.1 The first strategy

Let θ_j^* be the efficiency score of DMU_j before fixed costs allocation with undesirable data, which is calculated using the model (4). f_j is a new input with a weight of one. We have considered the weight of one for calculation simplicity. We are looking to provide a model in which each DMU can determine the maximum level of participation as well as the minimum level of participation in the payment of fixed costs for itself so that no change in efficiency occurs before and after allocation for itself and other DMUs.

$$\frac{\sum_{r\in D} u_r y_{rj}^{D} + \sum_{i\in UD} v_i x_{ij}^{UD}}{\sum_{i\in D} v_i x_{ij}^{D} + \sum_{r\in UD} u_r y_{rj}^{UD} + f_j} = \theta_j^* \qquad j=1,\dots,n \tag{5}$$

The eq. (5) is a system of linear equations that has n restrictions and $n+s+m$ unknowns. The left section of Equation (5) indicates the relative efficiency score DMU_j after the fixed cost allocation and the right section of Equation (5) indicates the relative efficiency score DMU_j before the fixed cost allocation, which is calculated with the model (4). Given that $\theta_j^* > 0$:

$$f_j = \frac{1}{\theta_j^*}\left(\sum_{r\in D} u_r y_{rj}^{D} + \sum_{i\in UD} v_i x_{ij}^{UD}\right) - \left(\sum_{i\in D} v_i x_{ij}^{D} + \sum_{r\in UD} u_r y_{rj}^{UD}\right) \qquad j=1,\dots,n$$

The desired goals and conditions can be written as model (5):

$$\begin{aligned} & Min \ or \ Max \ \mathrm{f}_o \\ & S.\mathrm{t.} \\ & f_j = \frac{1}{\theta_j^*}\left(\sum_{r\in D} u_r y_{rj}^{D} + \sum_{i\in UD} v_i x_{ij}^{UD}\right) - \left(\sum_{i\in D} v_i x_{ij}^{D} + \sum_{r\in UD} u_r y_{rj}^{UD}\right) \qquad j=1,\dots,n \\ & \sum_{j=1}^{n} f_j = F \\ & f_j \ge 0,\ u_r, v_i \ge \varepsilon \qquad j=1,\dots,n \quad r=1,\dots,s \quad i=1,\dots,m \end{aligned} \tag{6}$$

Model (6) has n+1 constraints and n+s+m variables. The first set of n constraints of the model ensures that the fixed cost value that each DMU receives, is in accordance with the efficiency reliability principle. In the second constraint of the model, the sum of fixed costs allocated to each unit is equal to the total fixed cost. n+m+s of the last constraint indicates that the values of all model variables are non-negative. Model (6) is a linear programming model, which can be solved using the simplex method or Karmarkar's algorithm. GAMS[1] 36.2.0 software was used to solve this model.

Considering the maximum objective function and solving the model (6), the maximum value that each DMU can contribute to the payment of fixed costs is obtained for all DMUs. Model (6) is solved with the objective function min to determine the minimum value of the participation of each DMU in the payment of fixed costs. f_j^{max} and f_j^{min} show the maximum and minimum values of participation in the payment of fixed costs. Solving model (6) 2n times, all f_j^{max} and f_j^{min} values are obtained. Now the fixed cost value for each decision-making unit is considered in the range $[f_j^{min}, f_j^{max}]$. Using the following convex combination, the f_j value is obtained for each decision unit in the proposed range.

$$f_j = \mu_j f_j^{\min} + (1-\mu_j) f_j^{\max} \qquad 0 \le \mu_j \le 1 \qquad j=1,\dots,n \tag{7}$$

Equation (7) contains n equations and 2n unknowns. μ_j and $(1-\mu_j)$ are the weights related to the minimum and maximum values of fixed cost allocation proposed by each DMU. System (7) is a system of non-homogenous linear equations with n equations and $2n$ unknowns, which is written as follows:

$$\begin{cases} f_1 - (f_1^{\min} - f_1^{\max})\mu_1 = f_1^{\max} \\ f_2 - (f_2^{\min} - f_2^{\max})\mu_2 = f_2^{\max} \\ \vdots \\ f_n - (f_n^{\min} - f_n^{\max})\mu_n = f_n^{\max} \end{cases}$$

This system can be rewritten in the following matrix form:

$$F - AX = b$$

Where F and X are unknown matrices, and A and b are known.

$$F = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix},\ X = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}, A = \begin{bmatrix} f_1^{\min} - f_1^{\max} \\ f_2^{\min} - f_2^{\max} \\ \vdots \\ f_n^{\min} - f_n^{\max} \end{bmatrix}\ b = \begin{bmatrix} f_1^{\max} \\ f_2^{\max} \\ \vdots \\ f_n^{\max} \end{bmatrix}$$

One solution for calculating μ_j is to use the algorithm proposed by Li et al. [29]. Here, this algorithm is used from another perspective in order to solve the system of linear equations (7). To make a fair fixed cost allocation we assume that $\mu_1 = \mu_2 = \cdots = \mu_n$. In this case:

$$f_j = \mu f_j^{\min} + (1-\mu) f_j^{\max} \qquad 0 \le \mu \le 1 \qquad j = 1,\dots,n \tag{8}$$

System (8) is a system of non-homogenous linear equations, which can be written as follows:

$$\begin{cases} f_1 - (f_1^{\min} - f_1^{\max})\mu = f_1^{\max} \\ f_2 - (f_2^{\min} - f_2^{\max})\mu = f_2^{\max} \\ \vdots \\ f_n - (f_n^{\min} - f_n^{\max})\mu = f_n^{\max} \end{cases}$$

This system can also be rewritten in the following matrix form:

$$F - AM = b$$

Where F and X are unknown matrices, and A and b are known.

$$F = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix},\ M = \begin{bmatrix} \mu \\ \mu \\ \vdots \\ \mu \end{bmatrix}, A = \begin{bmatrix} f_1^{\min} - f_1^{\max} \\ f_2^{\min} - f_2^{\max} \\ \vdots \\ f_n^{\min} - f_n^{\max} \end{bmatrix}\ b = \begin{bmatrix} f_1^{\max} \\ f_2^{\max} \\ \vdots \\ f_n^{\max} \end{bmatrix}$$

Equation (8), is n equations with n+1 unknowns. placing Equation (8) in the second constraint of Model (6), the value of μ is obtained.

$$\sum_{j=1}^{n} f_j = F \to \sum_{j=1}^{n} (\mu f_j^{\min} + (1-\mu) f_j^{\max}) = F \to \mu \sum_{j=1}^{n} f_j^{\min} + (1-\mu) \sum_{j=1}^{n} f_j^{\max} = F$$

$$\to \mu = \frac{F - \sum_{j=1}^{n} f_j^{\max}}{\sum_{j=1}^{n} f_j^{\min} - \sum_{j=1}^{n} f_j^{\max}} \tag{9}$$

also

$$1-\mu=1-\frac{F-\sum_{j=1}^{n} f_j^{\max}}{\sum_{j=1}^{n} f_j^{\min}-\sum_{j=1}^{n} f_j^{\max}}=\frac{\sum_{j=1}^{n} f_j^{\min}-F}{\sum_{j=1}^{n} f_j^{\min}-\sum_{j=1}^{n} f_j^{\max}} \tag{10}$$

It is clear that $\mu+(1-\mu)=1$. After calculating μ using equation (9), the values of f_j are obtained for all DMUs. μ is the weight relating to f_j^{min} and $(1-\mu)$is the weight relating to f_j^{max} The simplicity of calculations provided is the advantage of calculating the weight with this method. The whole above process can be summarized as the following algorithm:

Fixed cost allocation algorithm

Step 1: Is obtained the efficiency score of each DMU before allocating the costs using model 5.

Step 2: Using Model (6), the minimum amount of participation proposed by each DMU (f_j^{min}) and the maximum amount of participation proposed by each DMU (f_j^{max}) for participation in fixed costs payment is obtained.

Step 3: Using equations (9) and (10), are calculated the weights for $f_j^{min}$and f_j^{max}, respectively.

Step 4: By placing the weights obtained from the third step in relation (8), the fixed cost value for each DMU (f_j) is determined.

The proposed algorithm for allocating fixed costs can be shown in the flowchart of Fig. 1.

3.2 The second strategy

Model (3) is used to evaluate the relative efficiency with input vector $X=(X^D,X^{UD})$ and output vector $Y=(Y^D,Y^{UD})$. Suppose f_j is a fixed cost related to DMU_j $(j=1,\dots,n)$, which is considered as additional input for simplicity by weight 1. Using the model (3), fixed cost allocation with undesirable data can be done as follows:

$$\begin{aligned} & Max\ \frac{\sum_{r\in D} u_r y_{ro}^D+\sum_{i\in UD} v_i x_{io}^{UD}}{\sum_{i\in D} v_i x_{io}^D+\sum_{r\in UD} u_r y_{ro}^{UD}+f_o} \\ & s.t:\ \frac{\sum_{r\in D} u_r y_{rj}^D+\sum_{i\in UD} v_i x_{ij}^{UD}}{\sum_{i\in D} v_i x_{ij}^D+\sum_{r\in UD} u_r y_{rj}^{UD}+f_j}\le 1 \quad j=1,\dots,n \\ & \sum_{j=1}^{n} f_j=F \end{aligned} \tag{11}$$

3.2 The second strategy

Model (3) is used to evaluate the relative efficiency with input vector $X=(X^D,X^{UD})$ and output vector $Y=(Y^D,Y^{UD})$. Suppose f_j is a fixed cost related to DMU_j $(j=1,\dots,n)$, which is considered as additional input for simplicity by weight 1. Using the model (3), fixed cost allocation with undesirable data
can be done as (11).

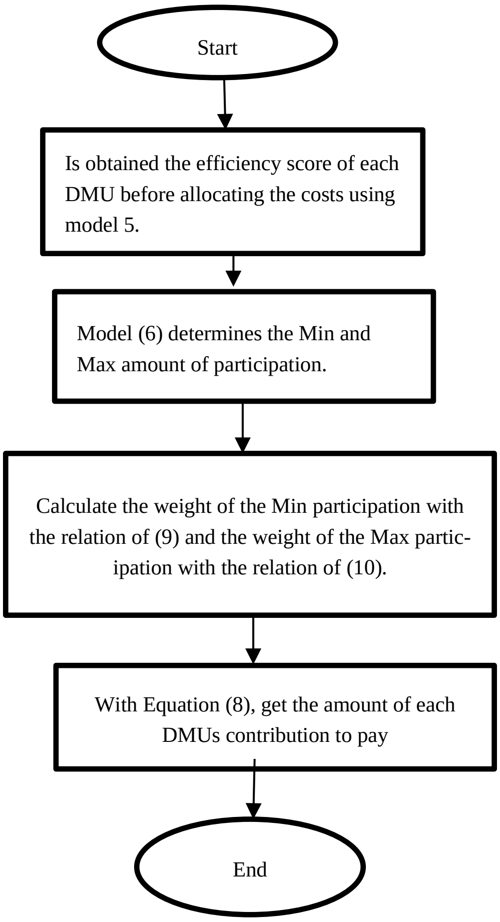

Fig1: Allocation of fixed costs based on the max-min model

$$
\begin{aligned}
& Max \ \frac{\sum_{r\in D} u_r y_{ro}^{D} + \sum_{i\in UD} v_i x_{io}^{UD}}{\sum_{i\in D} v_i x_{io}^{D} + \sum_{r\in UD} u_r y_{ro}^{UD} + f_o} \\
& s.t: \ \frac{\sum_{r\in D} u_r y_{rj}^{D} + \sum_{i\in UD} v_i x_{ij}^{UD}}{\sum_{i\in D} v_i x_{ij}^{D} + \sum_{r\in UD} u_r y_{rj}^{UD} + f_j} \le 1 \qquad j=1,\dots,n \\
& \sum_{j=1}^{n} f_j = F \\
& f_j \ge 0, \ u_r, v_i \ge \varepsilon \qquad j=1,\dots,n \qquad i=1,\dots,m \\
& \qquad\qquad\qquad\qquad\qquad\qquad r=1,\dots,s
\end{aligned} \tag{11}
$$

The first set of constraints of the model (11) shows that f_j is considered as an additional input for simplicity in performing weight-related calculations. The second constraint of the model (11) ensures that the sum of the fixed costs allocated to all units is equal to F. Model (11) is a nonlinear programming problem. The model becomes a form of linear programming problem by Charnes and Cooper transformations [32] and by changing the appropriate variable.

$$\frac{1}{\sum_{i\in D} v_i x_{io}^{D} + \sum_{r\in UD} u_r y_{ro}^{UD} + f_o} = \alpha \rightarrow \sum_{i\in D} \alpha v_i x_{io}^{D} + \sum_{r\in UD} \alpha u_r y_{ro}^{UD} + \alpha f_o = 1$$

Put

$$\alpha v_i = \mu_i \qquad i=1,\dots,m \qquad \alpha u_r = w_r \qquad r=1,\dots,s$$

Using the above variables, model (11) is rewritten as follows:

$$\begin{aligned}
& Max \left(\sum_{r\in D} w_r y_{ro}^{D} + \sum_{i\in UD} \mu_i x_{io}^{UD} \right) \\
& s.t:\ \sum_{i\in D} \mu_i x_{io}^{D} + \sum_{r\in UD} w_r y_{ro}^{UD} + \alpha f_o = 1 \\
& \sum_{r\in D} w_r y_{rj}^{D} + \sum_{i\in UD} \mu_i x_{ij}^{UD} - \sum_{i\in D} \mu_i x_{ij}^{D} - \sum_{r\in UD} w_r y_{rj}^{UD} - \alpha f_j \le 0 \qquad j=1,\dots,n \\
& \sum_{j=1}^{n} f_j = F \\
& f_j \ge 0,\ \mathrm{w}_r, \mu_i \ge \varepsilon \qquad j=1,\dots,n \quad i=1,\dots,m \quad r=1,\dots,s
\end{aligned} \tag{12}$$

The presence of αf_j in the second constraint of the model (12) causes the model to be nonlinear. To linearize the model, the following linear programming model is obtained by changing the variable $\alpha f_j = \tau_j$:

$$\begin{aligned}
& Max \left(\sum_{r\in D} w_r y_{ro}^{D} + \sum_{i\in UD} \mu_i x_{io}^{UD} \right) \\
& s.t:\ \sum_{i\in D} \mu_i x_{io}^{D} + \sum_{r\in UD} w_r y_{ro}^{UD} + \tau_o = 1 \\
& \sum_{r\in D} w_r y_{rj}^{D} + \sum_{i\in UD} \mu_i x_{ij}^{UD} - \sum_{i\in D} \mu_i x_{ij}^{D} - \sum_{r\in UD} w_r y_{rj}^{UD} - \tau_j \le 0 \qquad j=1,\dots,n \\
& \sum_{j=1}^{n} \tau_j = \alpha F \\
& \tau_j, \alpha \ge 0,\ \mathrm{w}_r, \mu_i \ge \varepsilon \qquad j=1,\dots,n \quad i=1,\dots,m \quad r=1,\dots,s
\end{aligned} \tag{13}$$

Model 13 is a linear programming problem and is easily solved. If $(\tau_j^*, \mu_i^*, w_r^*, \alpha^*)$ is the optimal value of the model (13), then the value of cost allocated to DMU_j is equal to:

$$f_j^* = \frac{\tau_j^*}{\alpha^*}$$

4 Applied study

Data from twenty-seven commercial banks in 2015 are used to illustrate the application of the proposed models in the allocation of fixed costs. The goal is to implement a fixed cost allocation model on this data. Data are accessible in the link https://www.thebankerdatabase.com/index.cfm?fuseaction=lite.overview. Information on 27 commercial banks is presented in Table 1. Deposits and staff costs are considered as input. Total loans, total debt and net income on output are considered as the output of the bank. The inputs should be increased and outputs should be reduced to improve relative efficiency. Reducing staff costs is desirable for us, but reducing deposits is not desirable. Therefore, staff costs are a desirable input and deposits are an undesirable input. Total loans are considered an undesirable output because people have difficulty repaying it. Total debt is also an undesirable output and it is better to reduce it because it may be dependent on loans. Increasing net income is desirable, then it is considered a desirable output.

Table 1: Desirable and Undesirable Input and Output Data of 27 Bank Branches in 2015

DMU	Bank Names	Deposits	Staff costs	Total loans	Total debt	Net income
1	ICBC	1228872.04	18307.24	512876.94	3953.42	59096.58
2	China Construction Bank	979258.21	14963.72	510488.81	2022.06	48878.25
3	Bank of China	812886.26	12729.04	421507.76	6656.97	37829.38
4	Agricultural Bank of China	1313220.46	18216.87	384008.17	1183.20	37956.69
5	HSBC Holdings	649313.00	20366.00	412204.00	340669.00	18680.00
6	Credit Agricole	499807.04	13402.91	405582.52	256211.17	9412.62
7	Deutsche Bank	254588.59	15184.47	229252.43	740536.41	3781.55
8	Bank of Communications	221915.67	4034.97	138087.60	1646.35	13879.23
9	RBS	552711.39	8981.28	324141.97	545717.63	895.48
10	Lloyds Banking Group	445397.82	7402.50	530020.28	51773.79	2748.83
11	Groupe BPCE	396480.58	12144.42	341169.90	105668.69	6406.55
12	Societe Generale	177615.29	10981.80	129390.78	274434.47	5309.47
13	BBVA	360364.08	6565.53	190529.13	57503.64	4830.10
14	China Merchants Bank	178628.37	4768.59	93394.35	1674.46	12000.49
15	Credit Mutuel	318000.00	7058.25	229859.22	16887.14	5513.35
16	ING Bank	531286.41	7018.20	383341.02	65990.29	4678.40
17	China Citic Bank	85041.35	3457.43	83939.86	1200.69	8918.78
18	UBS	158166.84	15449.95	166600.61	299770.48	2488.37
19	Industrial Bank	60547.31	2838.54	116413.14	735.09	9903.25
20	Rabobank Group	197264.56	6172.33	265148.06	81990.29	2040.05
21	China Minsheng Bank	89839.68	3665.14	58232.06	418.04	9771.69
22	Royal Bank of Canada	185476.06	9779.26	221315.60	78884.75	10381.21
23	Sberbank	165812.86	5666.75	52580.97	13285.22	6651.50
24	National Australia Bank	395779.53	3965.00	316358.71	51883.64	6959.76
25	ANZ Banking Group	411623.80	4451.44	277368.33	46303.59	9018.37
26	Scotiabank	155286.35	5977.84	202586.88	32303.19	8244.68
27	Toronto Dominion Bank	304290.78	7492.02	213796.99	45014.18	8328.90

Assume that there is a fixed cost of F=1000 that we would like to equitably allocate to each of the DMUs using the proposed method. The numbers are in millions of dollars. Using Model (4), the efficiency of each DMU is calculated before fixed resource allocation, the results of which can be observed in the third column of Table 2.

Table 2: Result for fixed cost allocation.

DMU	Bank Names	θ	f_i^{Min}	f_i^{Max}	$f_i^{model\ 5}$	$f_i^{model\ 12}$
1	ICBC	1	76.092	139.739	105.49	166.66
2	China Construction Bank	1	57.562	112.726	83.04	137.84
3	Bank of China	0.94	52.061	93.349	71.13	106.68
4	Agricultural Bank of China	1	80.863	125.383	101.42	107.04
5	HSBC Holdings	0.46	81.727	117.131	98.08	52.68
6	Credit Agricole	0.49	39.996	68.039	52.95	26.55
7	Deutsche Bank	0.32	0	54.682	25.25	10.67
8	Bank of Communications	1	12.105	31.067	20.86	39.14
9	RBS	0.77	0	48.748	22.51	2.53
10	Lloyds Banking Group	0.69	0	51.571	23.82	7.75
11	Groupe BPCE	0.43	30.503	67.82	47.74	18.07
12	Societe Generale	0.4	2.362	35.764	17.79	14.97
13	BBVA	0.72	13.409	36.751	24.19	13.62
14	China Merchants Bank	0.92	8.523	28.654	17.82	33.84
15	Credit Mutuel	0.59	19.21	42.869	30.14	15.55
16	ING Bank	0.85	7.307	49.146	26.63	13.19
17	China Citic Bank	0.87	1.893	22.44	11.38	25.15
18	UBS	0.28	0	38.784	17.91	7.02
19	Industrial Bank	1	0	21.826	10.08	27.93
20	Rabobank Group	0.39	8.911	36.518	21.66	5.75
21	China Minsheng Bank	1	0.142	21.561	10.03	27.56
22	Royal Bank of Canada	0.37	27.114	61.469	42.98	29.28
23	Sberbank	1	0	15.92	7.352	18.76
24	National Australia Bank	1	3.822	32.182	16.91	19.63
25	ANZ Banking Group	1	12.266	33.269	21.97	25.43
26	Scotiabank	0.42	21.32	44.738	32.14	23.25
27	Toronto Dominion Bank	0.54	32.859	45.54	38.72	23.49

Banks 1, 2, 4, 8, 19, 23, 24, and 25 are efficient. DMU 18 has the lowest efficiency score. By executing Model (6), each DMU proposes the minimum and maximum amounts of fixed costs it can receive while maintaining the same level of efficiency. The results of running Model (6) using GAMS software are given in the fourth and fifth columns of Table 2. The minimum value of participation is related to banks 7, 9, 10, 18, 19, and 23. DMU 5 has the highest value proposed for the minimum participation, which is an inefficient DMU. The maximum value proposed for maximum participation in the payment of fixed fees is related to ICBC Bank. Among the minimum and maximum participation values proposed, Sberbank Bank has offered the minimum values and ICBC Bank has proposed the maximum participation values; Both of them are efficient. To determine the fixed cost for each unit using Equation (7), we first obtain the value of μ from Equation (8). The value of μ is equal to 0.54.

Using Equation (7), the cost of each DMU is calculated by Excel software and is located in the sixth column of the table. The fixed fee value that each DMU has to pay, is in the suggested range. ICBC Bank and the Agricultural Bank of China have proposed the maximum value of participation; Both of them are efficient. Sberbank has the lowest participation rate. The seventh column of the table is related to the implementation of the model (12), which is related to the second model proposed in this manuscript. In this model, ICBC Bank and China Construction Bank have the maximum participation in the payment of fixed costs; Both of them are efficient. RBS Bank has the lowest participation in the payment of fixed costs, which is an inefficient DMU.

5 Conclusions

DEA was initially introduced by Cook and Kress to evaluate the performance of units with positive inputs and outputs. In the real world, data can be categorized as desirable or undesirable. The desirable and undesirable output must be respectively increased and decreased to improve efficiency. Fixed cost allocation is one of the topics considered by DEA researchers. In this manuscript, it was presented two DEA-based approaches to equitable fixed cost allocation among a set of DMUs with undesirable data. In the first approach, a model has been presented that allows each DMU to determine the minimum and maximum values which can contribute to the payment of costs so that the efficiency of itself and other DMUs after the allocation is equal to the efficiency before the allocation. The decision maker then selects the fixed cost value in the proposed range. In this model, DMUs are allowed to set a minimum value of zero as the suggested value for the minimum participation. Here, to determine the fixed cost allocated to each unit, we have assumed the weights are equal. Using the method presented by Li et al. [29] can be beneficial in future research.

In the second approach, fixed cost allocation was performed using the CCR multiplicative model. In total, according to a survey of 27 commercial banks, 33% of banks with 37% participating in the payment of fixed fees are efficient. Finding fixed costs in fuzzy programming mode as well as finding fixed costs with inaccurate data can be considered for future research. Both approaches are illustrated using a practical example in commercial banks. The maximum value of participation in both proposed methods is related to efficient banks. In the first method, the minimum participation is related to an inefficient bank and in the second method, the minimum participation is related to an efficient DMU.

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